Equation of state and sound velocity of hadronic gas with hard-core interaction

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Recent results -> arXiv:1411.0959

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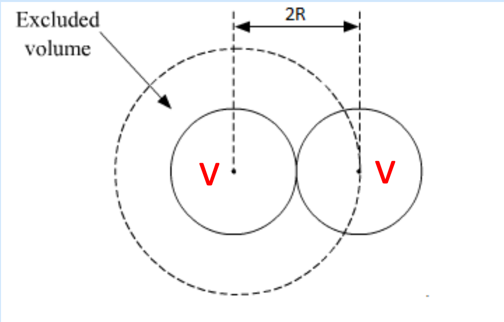
Equation of state in excluded volume approach

Van-der-Waals (1910): $V \to V - Nb$ in the ideal gas EoS



hard-sphere particles with radius R:

excluded volume per particle $b = \frac{1}{2} \times \frac{4\pi}{3}(2R)^3 = 4v$

($v = 4\pi R^3/3$ - single particle proper volume)

In the Boltzmann approximation

partition function $$\mathcal{Z}(T,V,N) \simeq \mathcal{Z}_{id}(T, V - bN, N) = \frac{\phi^N(T)}{N!}(V - Nb)^N$$

ideal gas density at zero chem. pot. $$\phi(T) = \frac{g}{(2\pi)^3}\int d^3p\, e^{-\sqrt{m^2+p^2}/T} = \frac{g m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right)$$

pressure $$P = T\frac{\partial}{\partial V}\ln \mathcal{Z}(T,V,N) = \frac{NT}{V - Nb} = \frac{nT}{1 - 4\eta}$$ ($\eta<0.25$)

$n = N/V$ - number density $\eta = nv$ - packing fraction

this EoS is not applicable at $\eta \gtrsim 0.2$

Excluded volume model in grand canonical variables (μ,T)

Van-der-Waals (canonical ensemble): shift $n \rightarrow n(1-bn)^{-1}$ in the ideal gas EoS

Cleymans et al. (1983), Rischke et al. (1991) (equivalent grand canonical formulation):

shift of chemical potentials $\mu_i \rightarrow \mu_i - bP$ due to hard sphere interactions (HSI)

$P = \sum_i P_i^{id}(\mu_i - bP, T)$ where $P_i^{id}(\mu_i, T)$ - ideal gas pressure of i-th particles

In the Boltzmann approximation: $P_i^{id}(\mu_i, T) = T\phi_i(T)\, e^{\mu_i/T}$

$$\Rightarrow \quad n_i = \frac{\partial P}{\partial \mu_i} = \frac{\phi_i(T)\, e^{(\mu_i - bP)/T}}{1 + bP/T} \quad \Rightarrow \quad n = \sum_i n_i = \frac{P}{T + bP} \quad \Rightarrow \quad Z \equiv \frac{P}{nT} = \frac{1}{1 - bn}$$

For chemically equilibrated system of hadrons: $\mu_i = B_i\mu_B + S_i\mu_S$ where

$B_i = 0, \pm 1$ - baryon charge, $S_i = 0, \pm 1, \pm 2, \pm 3$ - strangeness of i-th hadron

At given μ_B (baryon chem. potential) one can determine μ_S (strange chem. potential)

from the condition of strangeness neutrality $\sum S_i n_i = 0 \rightarrow P = P(\mu_B, T)$

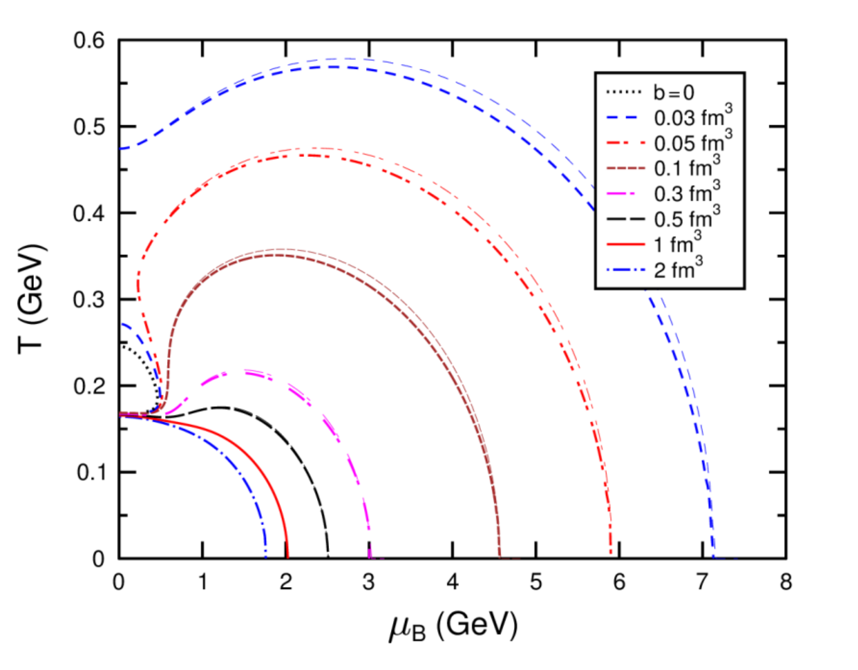
- this approach is satisfactory only at $bn \ll 1$ (strong overestimation of repulsion effects, superluminal sound velocities at $n \sim b^{-1}$)

Phase diagram of strongly interacting matter

Sensitivity to hadron sizes: Satarov, Dmitriev, Mishustin, Phys. Atom. Nucl. 72 (2009)

Gibbs condition for mixed phase (MP) $P_H(\mu_B, \mu_S, T) = P_Q(\mu_B, \mu_S, T)$

P_H is calculated in EVM, P_Q in MIT bag model, μ_S from (global) strangeness neutrality



all known hadrons with masses below 2 GeV are taken into account

the same radius R is assumed for all hadrons ($b = 1\ \mathrm{fm}^3$ corresponds to $R \simeq 0.39\ \mathrm{fm}$)

MIT bag parameters:

$$B^{1/4} = 227\ \mathrm{MeV}, m_s = 150\ \mathrm{MeV}$$

MP: thin strip instead of critical line in μ_B-T plane due to presence of μ_S

unrealistic phase diagram at $b \to 0$ (in this case P_H increases with T faster than P_Q)

Virial expansion for one-component Boltzmann gas

change of free energy F due to interaction (β=1/T): $e^{\beta(F_{id}-F)} = V^{-N}\int d^3r_1\dots d^3r_N \exp\left(-\beta\sum_{i<j}u_{ij}\right)$

$\exp\left(-\beta\sum_{i<j}u_{ij}\right) = \prod_{i<j}(1-f_{ij})$ Mayer function: $f_{ij} = 1 - e^{-\beta u(r_{ij})} \underset{\text{HSI}}{\longrightarrow} \Theta(2R - r_{ij})$

(no energy scale in HSI)

➡ virial expansion of EoS (H.K. Onnes, 1901)

compressibility function: $Z \equiv \frac{P}{P_{id}} = 1 + \sum_{i=1}^{\infty} B_{i+1}(T)n^i$ $(P_{id} = nT)$

virial coefficients:

HSI: $B_i(T) = \text{const} \propto v^{i-1} \to Z = Z(\eta),\ \eta = nv$

$$B_2 = \frac{1}{2V}\int d^3r_1 d^3r_2 f_{12} \underset{\text{HSI}}{\longrightarrow} b = 4v$$

- contribution of binary interactions $(v = 4\pi R^3/3)$

$$B_3 = \frac{1}{3V}\int d^3r_1 d^3r_2 d^3r_3 f_{12}f_{23}f_{31} \underset{\text{HSI}}{\longrightarrow} 10v^2$$

- contribution of three particle interactions

Monte Carlo calculation for HSI (van Rensburg, 1993):

$$Z = 1 + 4\eta + 10\eta^2 + 18.36\eta^3 + 28.23\eta^4 + 39.74\eta^5 + 53.5\eta^6 + 70.8\eta^7 + \dots$$

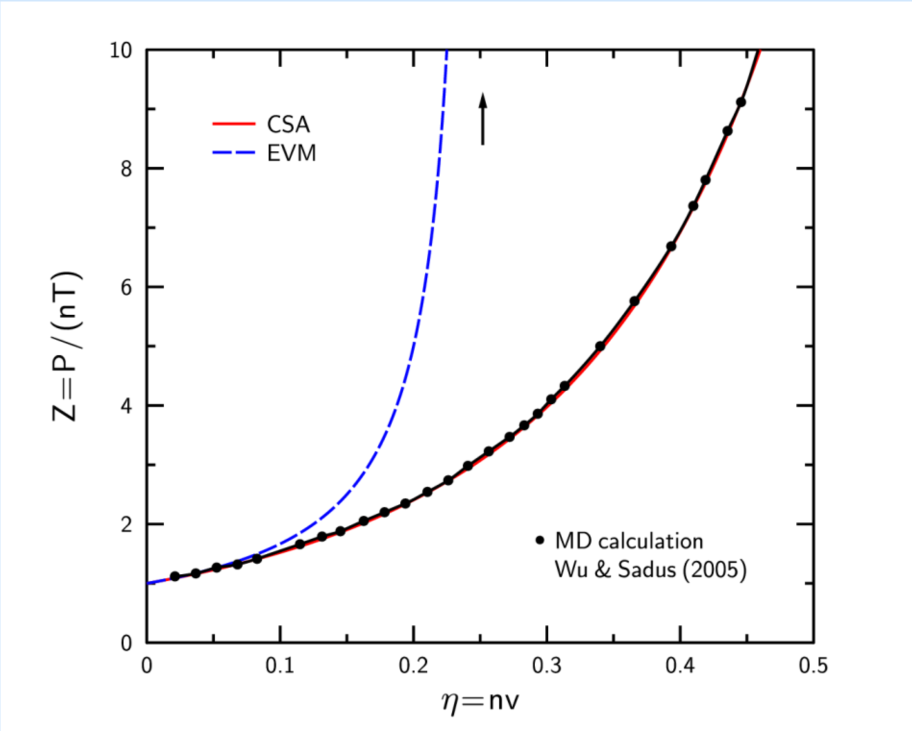
4 10 18 28 40 54 70

only first two terms are correctly reproduced in the EVM: Z=1/(1-4η)

red numbers - coefficients in the Carnahan-Starling approximation (CSA): J. Chem. Phys. 51 (1969) 635, decomposed in powers of η

this expansion works well at $\eta \lesssim 0.5$

Analytic approximations for pressure of Boltzmann gas with HSI



Excluded volume model (EVM)

$$Z = (1-4\eta)^{-1}$$

Carnahan-Starling approximation (CSA)

$$Z = \frac{1+\eta+\eta^2-\eta^3}{(1-\eta)^3}$$

$$\text{at } \eta \ll 1 \rightarrow Z_{CSA} \simeq Z_{EVM} \simeq 1 + 4\eta$$

liquid phase:
$0 < \eta < 0.49$
mixed phase:
$0.49 < \eta < 0.55$
solid phase:
$0.55 < \eta < 0.74$

- CSA predicts softer EoS as compared to EVM at $\eta \gtrsim 0.1$
- CSA agrees well with numerical calculation at $\eta \lesssim 0.5$

One-component ideal gas in the Boltzmann approximation

$$e^{-F_{id}/T} = \mathcal{Z}_{id}(T,V,N) = \frac{\phi^N(T)V^N}{N!}, \quad \phi(T) = \frac{g m^2 T}{2\pi^2} K_2(x), \quad x \equiv \frac{m}{T}$$

free energy density $f_{id} = \frac{F_{id}}{V} = -\frac{T}{V} \ln \mathcal{Z}_{id} = nT \left[\ln \frac{n}{\phi(T)} - 1 \right]$ $(N \gg 1)$

chemical potential $\mu_{id} = \left(\frac{\partial f_{id}}{\partial n} \right)_T = T \ln \frac{n}{\phi(T)} \quad \rightarrow \quad n = \phi(T) e^{\mu_{id}/T}$

pressure $P_{id} = \mu_{id} n - f_{id} = nT$

entropy density $s_{id} = -\left(\frac{\partial f_{id}}{\partial T} \right)_n = n \left(\ln \frac{\phi(T)}{n} + \xi(T) \right), \quad \xi(T) \equiv T \frac{\phi'(T)}{\phi(T)} + 1 = x \frac{K_3(x)}{K_2(x)}$

energy density $\varepsilon_{id} = f_{id} + T s_{id} = nT \left[\xi(T) - 1 \right]$

isochoric heat capacity $C_V = \left(\frac{\partial \varepsilon_{id}}{\partial T} \right)_n = n \left[x^2 + 3\xi - (\xi - 1)^2 \right] \equiv n \widetilde{C}(T)$

➡ ideal Boltzmann gas: energy (ε/n) and heat capacity (C_V/n) per particle depend only on T

Sound velocity of ideal classic gas

Adiabatic sound velocity squared $c_s^2 = \left(\frac{\partial P}{\partial \varepsilon}\right)_\sigma$

the derivative is taken along the Poisson adiabat $\sigma = s/n = \mathrm{const}$

in general case $c_s^2 = \frac{(\partial P/\partial n)_T + (\partial P/\partial T)_n (\partial T/\partial n)_\sigma}{(\partial \varepsilon/\partial n)_T + (\partial \varepsilon/\partial T)_n (\partial T/\partial n)_\sigma}$

slope of Poisson adiabat for an ideal gas $\left(\frac{\partial T}{\partial n}\right)_\sigma = \frac{T}{n\widetilde{C}(T)}$ ⟹ $c_s^{id} = \sqrt{\xi^{-1}\left(1+\widetilde{C}^{-1}\right)}$

nonrelativistic limit (x=m/T>>1):

$$\xi \simeq x + \frac{5}{2} + \frac{15}{8x} + \dots, \qquad \widetilde{C} \simeq \frac{3}{2} + \frac{15}{4x} - \frac{15}{2x^2} + \dots \to \quad c_s^{id} \simeq \sqrt{\frac{5T}{3m}}$$

ultrarelativistic case (x<<1):

(more appropriate for pions)

$$\xi \simeq 4 + \frac{x^2}{2} + \dots, \qquad \widetilde{C} \simeq 3 - \frac{x^2}{2} + \dots \quad \to \quad c_s^{id} \simeq 1/\sqrt{3} \simeq 0.577$$

c_s^{id} - monotonously increasing function of T with asymptotic value $1/\sqrt{3}$

Chemical potential of one-component matter with HSI

$$d\mu = \frac{1}{n}(dP - s\,dT) \quad \to \quad \Delta\mu(T,n) \equiv \mu - \mu_{id} = \int_0^n \frac{dn_1}{n_1}\frac{\partial \Delta P(T,n_1)}{\partial n_1} = T\psi(n)$$

thermodynamic consistency

$$\Delta P = nT(Z-1) \quad \to \quad \psi(n) = Z(n) - 1 + \int_0^n \frac{dn_1}{n_1}\left[Z(n_1) - 1\right]$$

A. Mulero et al. (1999)

$$\mu = T\left[\ln\frac{n}{\phi(T)} + \psi(n)\right]$$

- equation for n(T,μ): transition to grand canonical ensemble

Equivalent form of this equation: $n = e^{-\psi(n)}\phi(T)\,e^{\mu/T} \quad (n < n_{id})$

$\phi(T)\,e^{\mu/T} \to n_{id}(T,\mu)$

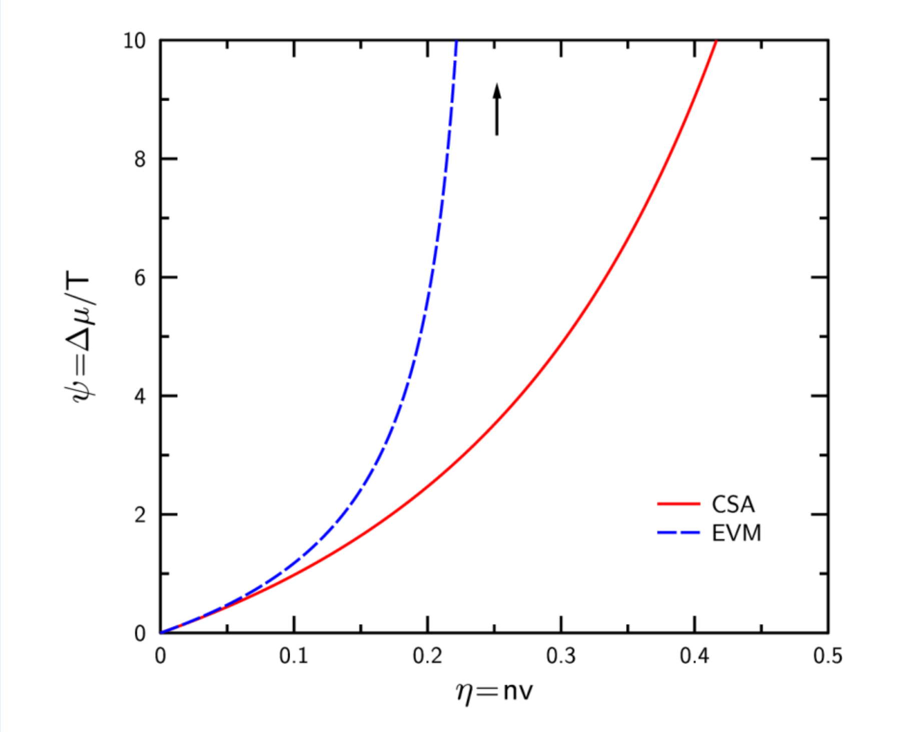
Excluded volume model $\left(Z = \dfrac{1}{1-bn}\right)$

$$\to \quad \mu = T\left[\ln\frac{P}{T\phi(T)} + bP\right]$$

is equivalent to $P = P_{id}(T, \mu - bP)$

these relations are not valid in CSA

Shift of chemical potential for non-ideal Boltzmann gas



$$\mu = \mu_{id}(n,T) + \Delta\mu$$

$$\Delta\mu = T\psi(n)$$

similar to mean field with $U = T\psi(n) \propto T$

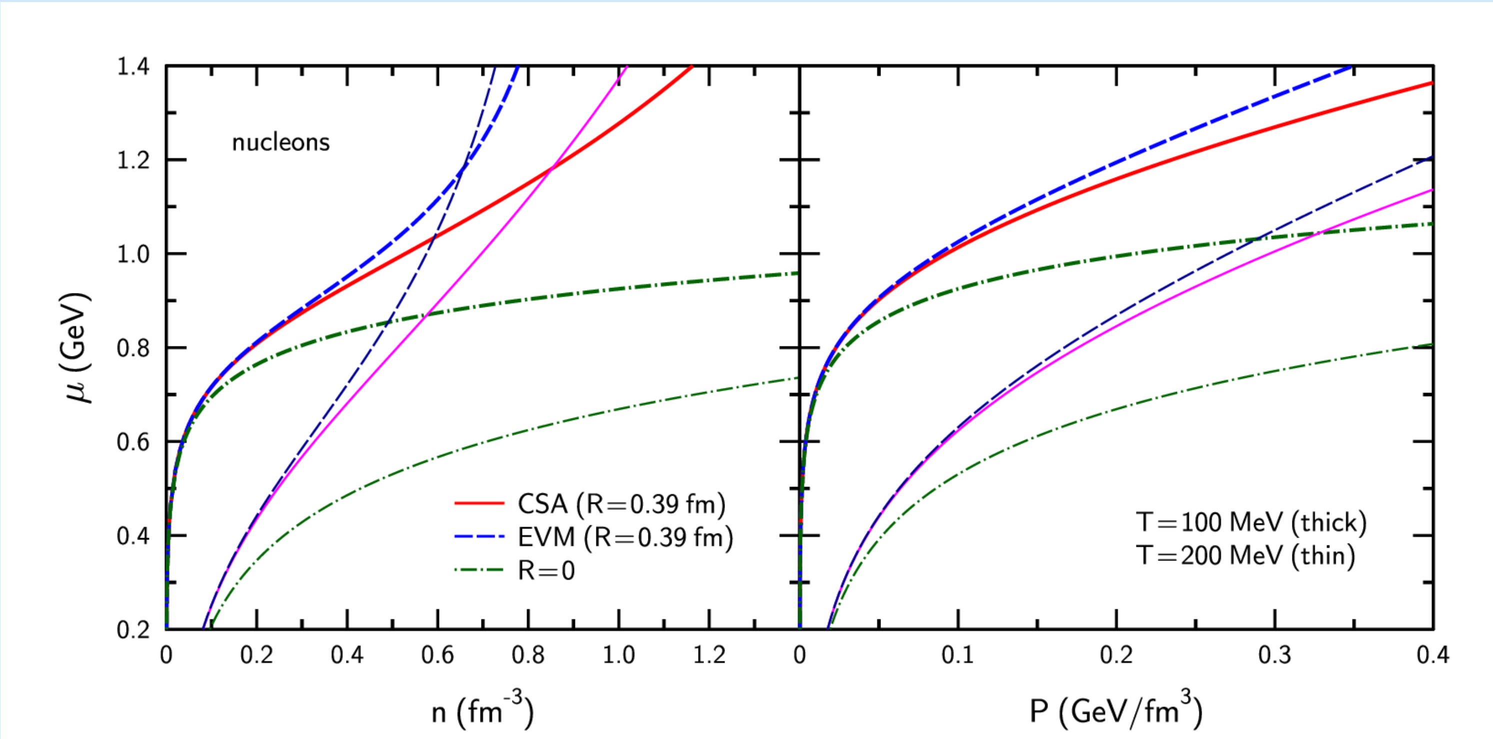
$$\mu_{id} = T \ln \frac{n}{\phi(T)}$$

$$\psi_{EVM} = \frac{4\eta}{1-4\eta} + \ln \frac{1}{1-4\eta}$$

$$\psi_{CSA} = \frac{3-\eta}{(1-\eta)^3} - 3$$

- for both approximations $\Delta\mu \simeq 8\eta T$ at $\eta \lesssim 0.1$
- at given T,n deviation of chem. potential from the ideal gas value is larger in EVM

Chemical potential of nucleonic matter



m=939 MeV, g=4

ideal gas: dash-dotted curves

R=0.39 fm
↓
v=0.25 fm^3
(b=1 fm^3)

at fixed T,μ : $n_{EVM} < n_{CSA} < n_{id}, \quad P_{EVM} < P_{CSA} < P_{id}$

this follows from $\psi_{EVM}(n) > \psi_{CSA}(n)$

Thermodynamic functions of nucleonic matter

$$\Delta f = n\Delta\mu - \Delta P \quad \rightarrow \quad f = nT\left\{ \ln\frac{n}{\phi(T)} - 1 + \int_0^n \frac{dn_1}{n_1}\left[Z(n_1) - 1\right] \right\}$$

free energy density

$$s = -\left(\frac{\partial f}{\partial T}\right)_n = n\left\{ \ln\frac{\phi(T)}{n} + \xi(T) - \int_0^n \frac{dn_1}{n_1}\left[Z(n_1) - 1\right] \right\}, \quad \xi(T) = x\frac{K_3(x)}{K_2(x)} \quad (x = m/T)$$

energy density $\varepsilon = f + Ts = nT\left[\xi(T) - 1\right] = \varepsilon_{id}(T,n) \rightarrow$ energy per particle $\varepsilon/n = F(T)$ is not changed by HSI !

heat capacity $C_V = (\partial\varepsilon/\partial T)_n = C_V^{id}(n,T) = n\widetilde{C}(T)$

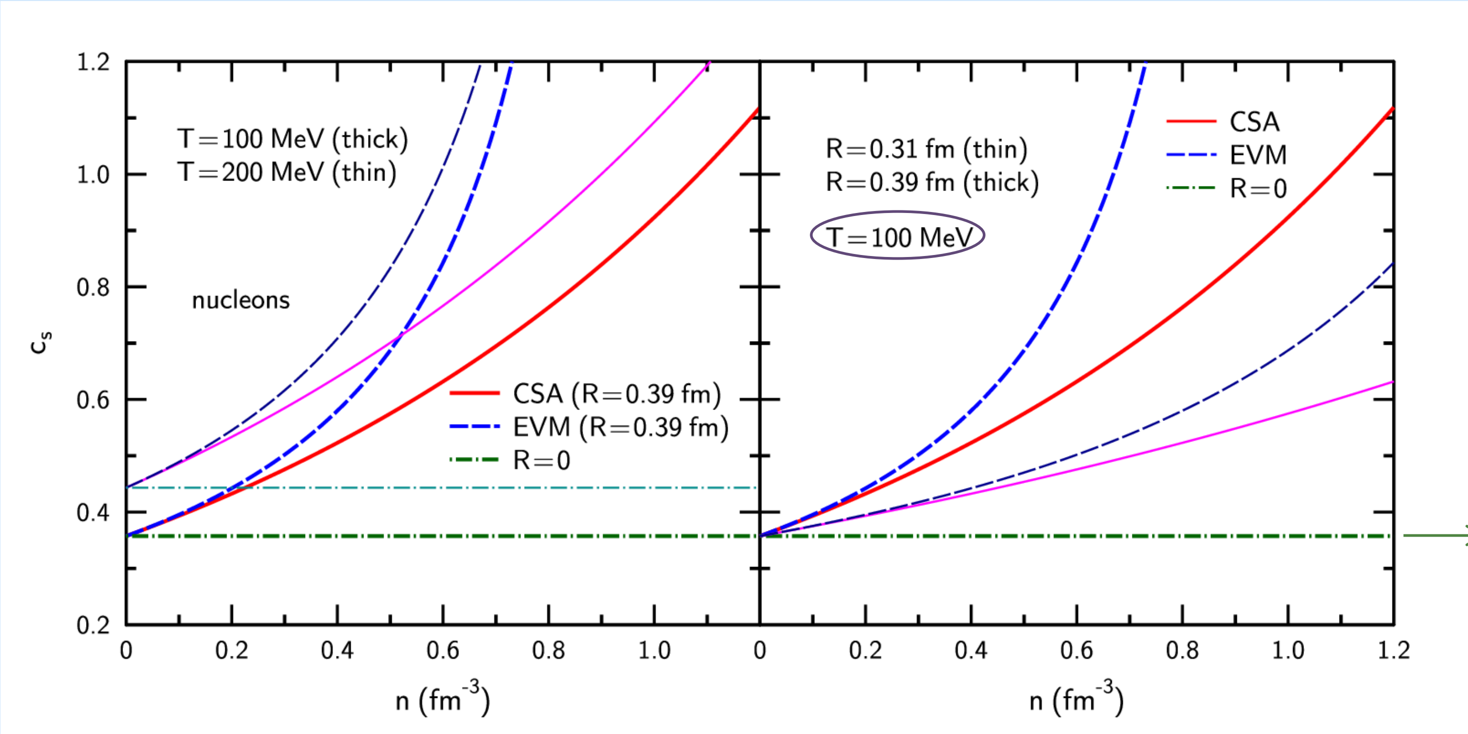
slope of Poisson adiabat $n(\partial T/\partial n)_\sigma = Z(n)T\widetilde{C}^{-1}(T)$ increases with density $(\sigma = s/n)$

sound velocity $c_s = \left(\frac{\partial P}{\partial\varepsilon}\right)_\sigma^{1/2} = \sqrt{\frac{(nZ)' + Z^2\widetilde{C}^{-1}}{\xi + Z - 1}} \underset{Z\rightarrow 1}{\rightarrow} c_s^{id} = \sqrt{\xi^{-1}(1 + \widetilde{C}^{-1})}$

at large enough densities (Z>>1): $c_s>1$

$$c_s(EVM, x \gg 1) \simeq \sqrt{\frac{5Z}{3(1 + x/Z)}}$$

Sound velocity of nucleonic matter



deviations from ideal gas significant at $n \gtrsim 0.1\ \mathrm{fm}^{-3}$

$c_s^{id} \simeq \sqrt{5T/3m}$

- strong sensitivity to nucleon size $(R = 0.31\ \mathrm{fm} \rightarrow b = 4v = 0.5\ \mathrm{fm}^3,\ R = 0.39\ \mathrm{fm} \rightarrow b = 1\ \mathrm{fm}^3)$
- superluminal sound velocity at $n \gtrsim b^{-1}$ in CSA (and even earlier in EVM)

EoS of pion matter with HSI

inelastic scattering and resonance decays: $N_\pi = N_\pi(T) \neq const$

$$\phi_\pi = \frac{g_\pi m_\pi^2 T}{2\pi^2} K_2\left(\frac{m_\pi}{T}\right)$$

condition of chemical equilibrium $\mu_\pi = T\left[\ln\frac{n_\pi}{\phi_\pi(T)} + \psi(n_\pi)\right] = 0$

$$m_\pi = 140 \text{ MeV}, g_\pi = 3$$

$\Rightarrow$ $n_\pi = \phi_\pi(T)\, e^{-\psi(n_\pi)}$ - equation for equilibrium pion density $n_\pi = n_\pi(T) < \phi_\pi(T)$

Using further $P_\pi = n_\pi T Z(n_\pi)$ we get $s_\pi = dP_\pi/dT = n_\pi(Z + \xi_\pi - 1) \rightarrow$

Pion energy density $\varepsilon_\pi = Ts_\pi - P_\pi = n_\pi T\left[\xi_\pi(T) - 1\right] = \varepsilon_\pi^{id}(T, n_\pi)$

Sound velocity $c_s = \sqrt{dP_\pi/d\varepsilon_\pi} = \sqrt{(Z + \xi_\pi - 1)/\widetilde{C}_\pi} \underset{Z\to 1}{\longrightarrow} c_s^{id} = \left(3 + x_\pi^2/\xi_\pi\right)^{-1/2}$

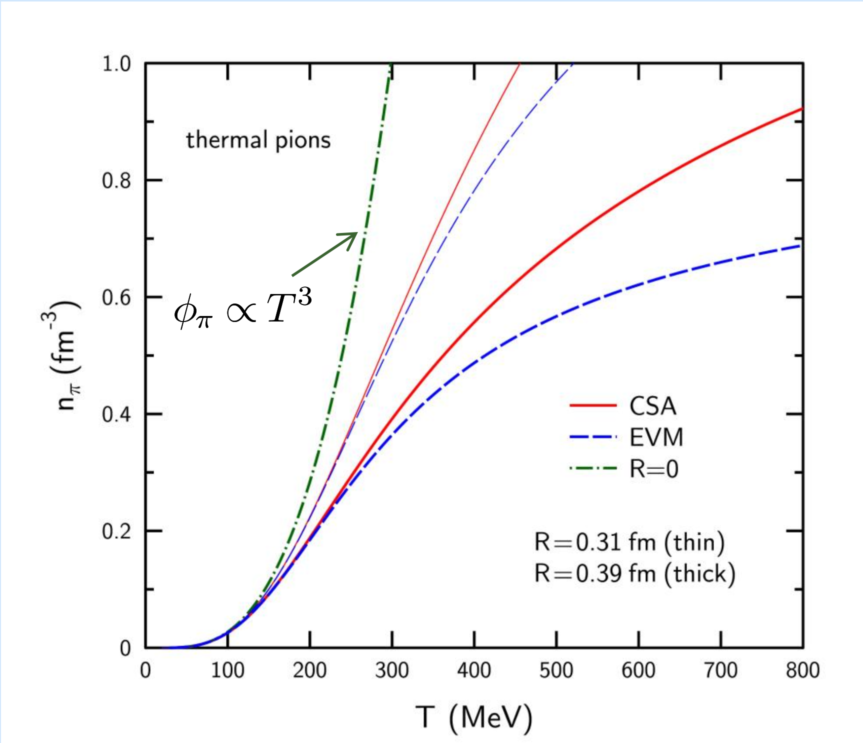
$$x_\pi \equiv m_\pi/T$$

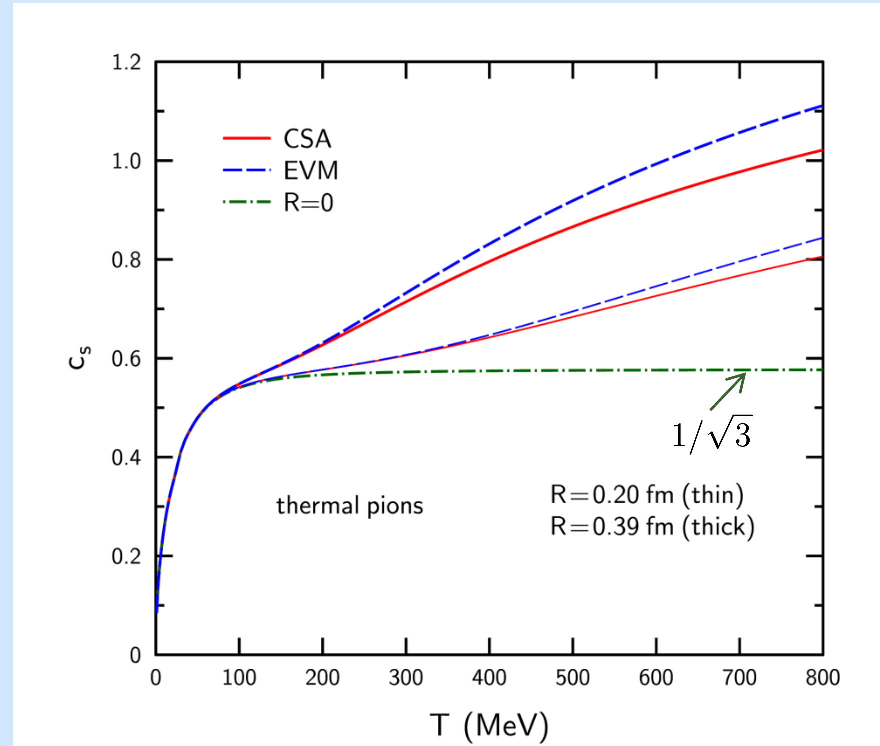
where $\widetilde{C}_\pi = n_\pi^{-1} d\varepsilon_\pi/dT = x_\pi^2 + 3\xi_\pi + (\xi_\pi - 1)^2\left[1/(n_\pi Z)' - 1\right]$

Problems to be considered:

1) significance of quantum effects? $\lambda_\pi \sim \hbar/\sqrt{m_\pi T} \gtrsim 2R_\pi$ for realistic T and R_π
2) strong isospin dependence of ππ interaction: attractive (I=0) and repulsive (I=2) channels nearly cancel each other (Gorenstein et al., 2000)
3) Lorentz contraction of relativistic pions in lab frame (Bugaev et al., 2008) ← effective reduction of v_π

Density and sound velocity of pion gas





- suppression of pion density as compared to ideal gas
- strong sensitivity of n_π and c_s to pion size
- significant differences between the EVM and CSA results only at $T \gtrsim 400$ MeV

Multi-component Boltzmann gas with HSI

Virial expansion $$P = P(T, n_1, n_2 \ldots) = P_{id} \cdot \left(1 + \sum_{i,j} b_{ij} x_i x_j + \ldots\right)$$

higher order terms $\sum b_{ijk} x_i x_j x_k + \ldots$ are not known

$$P_{id} = nT, \quad n = \sum_i n_i, \quad x_i = \frac{n_i}{n}, \quad b_{ij} = \frac{2\pi n}{3}(R_i + R_j)^3$$

➡ old results if radii are the same (R_i=R for all i) 2[nd] term → 4η=4nv

Shift of free energy density $$\Delta f = f - f_{id} = \int_0^1 \frac{d\alpha}{\alpha^2} \Delta P(T, \alpha n_1, \alpha n_2 \ldots)$$

➡ $$\Delta \mu_i = \frac{\partial \Delta f}{\partial n_i}, \quad \Delta s = -\frac{\partial \Delta f}{\partial T}, \quad \Delta \varepsilon = \Delta f + T \Delta s$$

Two-component mixture with $R_2 << R_1$

Important limiting case: large difference in particle sizes

particles of second component (i=2) can be considered as point-like, but in the reduced volume $V_{red}=V-N_1v_1$

$$P(T,n_1,n_2) = Tn_1Z(n_1) + \frac{n_2T}{1-\eta_1} \qquad (\eta_1 \equiv n_1v_1)$$

Partial pressure of 2nd component is determined by 'local' density $N_2/V_{red} > n_2 = N_2/V$

$$\Delta f = T\left\{n_1\int_0^{n_1}\frac{dn}{n}[Z(n)-1] + n_2\ln\frac{1}{1-\eta_1}\right\}$$

$$\Delta\mu_1 = \frac{\partial\Delta f}{\partial n_1} = T\left[\psi(n_1) + \frac{n_2v_1}{1-\eta_1}\right] = T\psi(n_1) + P_2v_1$$

Additional energy for creating a cavity with volume v_1 inside a gas of particles i=2

This term exists even at small n_1!

N+Δ matter

$m_\Delta = 1232$ MeV, $g_\Delta = 16$
$\Gamma_\Delta \to 0$

assuming $R_N = R_\Delta \equiv R \to$ $P = n_B T Z(n_B)$ (conserved) baryon density $n_B = n_N + n_\Delta$

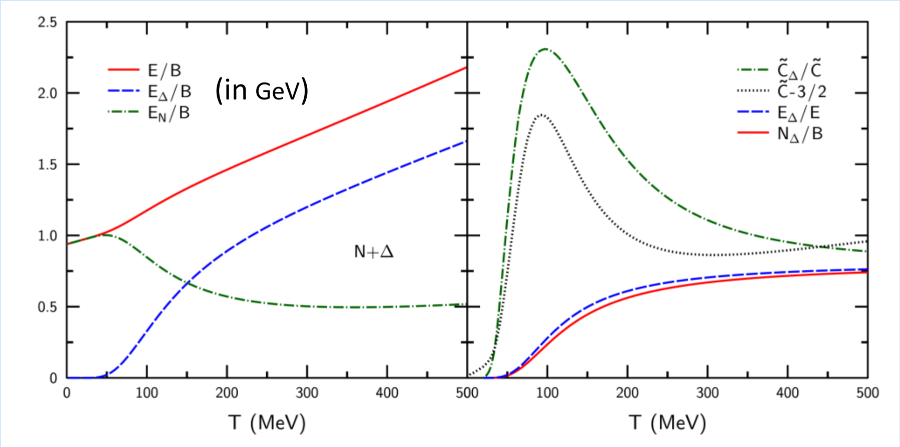
similarly to one-component matter: $\mu_i = T\left[\ln\frac{n_i}{\phi_i(T)} + \psi(n_B)\right]$ $(i = N, \Delta)$

From the condition of chemical equilibrium $\mu_N = \mu_\Delta = \mu_B$

➡ baryon chemical potential $\mu_B = T\left[\ln\frac{n_B}{\phi_N + \phi_\Delta} + \psi(n_B)\right]$

same as in ideal N+Δ gas (Galitsky & Mishustin, 1979)

relative concentration of Δ-isobars: $n_\Delta / n_B = w_\Delta(T) \equiv \phi_\Delta(\phi_N + \phi_\Delta)^{-1} = 1 - w_N(T)$ increases with T



$$\varepsilon = n_B T \langle \xi - 1 \rangle$$

$$C_V / n_B = \langle x^2 + 3\xi \rangle - \langle \xi - 1 \rangle^2 \equiv \widetilde{C}(T)$$

where $< A > \equiv A_N w_N + A_\Delta w_\Delta$

➡ excitation of resonances is important at $T \gtrsim 50$ MeV

General formula for sound velocity

For arbitrary thermodynamically equilibrium matter at fixed n_B and T

$$c_s^2 = \frac{1}{\varepsilon + P}\left[n_B \left(\frac{\partial P}{\partial n_B}\right)_T + \frac{T}{C_V}\left(\frac{\partial P}{\partial T}\right)_{n_B}^2 \right]$$

one should know ε, P as functions of n_B,T

$\sigma = s/n_B$

can be derived by using thermodynamic identity $(\partial T/\partial n_B)_\sigma = T(\partial P/\partial T)_{n_B}(n_B C_V)^{-1}$

Rosenfeld (1998): analogous relation for one-component matter with nonrelativistic particles $(n_B \to n,\ \varepsilon + P \to mn)$

The above expression:

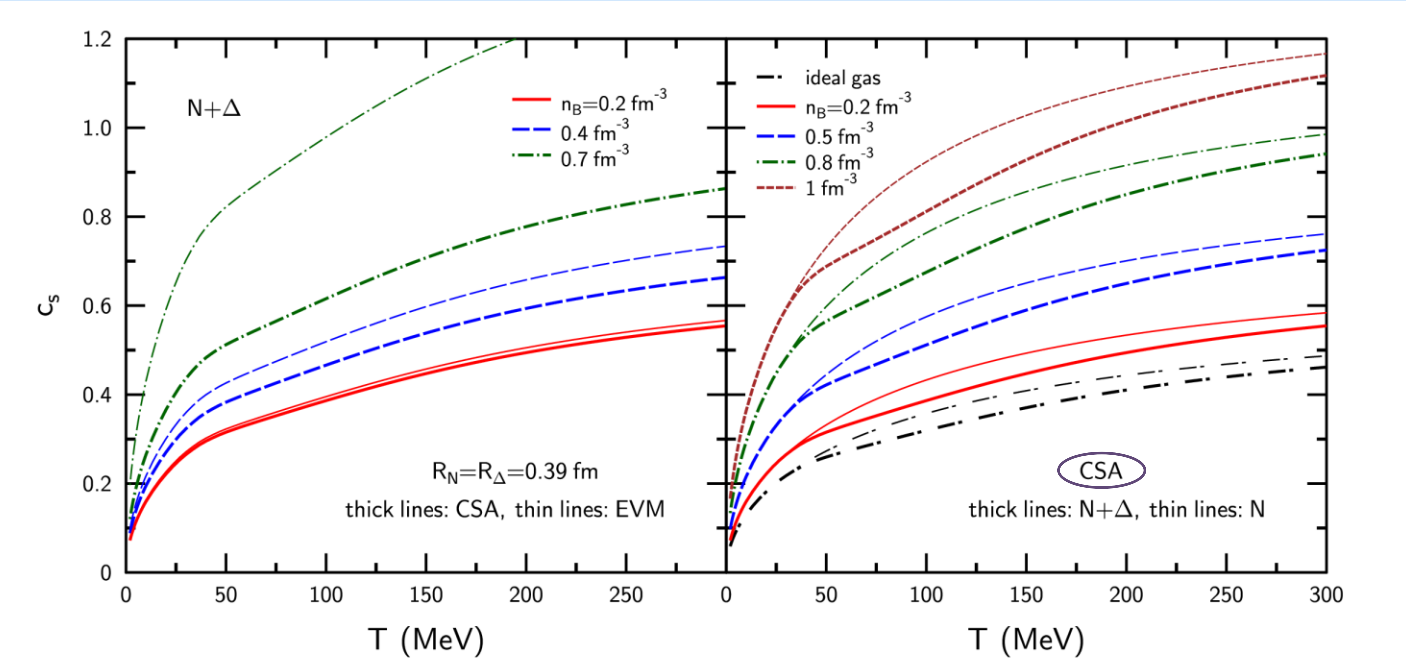
- is applicable for any form of short-range interaction
- is valid for arbitrary number of particle species $(B, \overline{B}, M \ldots)$
- works even in the case of quantum statistics
- can be used to check constraints from the causality condition $c_s \leqslant 1$

Example: polytropic EoS $P = a n_B^\gamma$ at $T = 0$ $d\mu_B = \frac{dP}{n_B} \to \mu_B = \frac{\varepsilon + P}{n_B} = \frac{a\gamma}{\gamma - 1} n_B^{\gamma-1}$

⇒ $c_s^2 = \gamma - 1 \to 1 \leqslant \gamma \leqslant 2$ ⇒ the hardest polytropic EoS: $P \propto n_B^2$ (Zeldovich, 1962)

Sound velocity of N+Δ matter

general formula → $c_s^2 = \frac{(n_B Z)' + Z^2 \widetilde{C}^{-1}}{< \xi > + Z - 1}$ $Z = Z(n_B)$



- $c_s^{(id)} < c_s^{(CSA)} < c_s^{(EVM)}$, superluminal c_s in CSA only for $n_B \gtrsim 1\ \mathrm{fm}^{-3}$
- noticeable reduction of sound velocity due to excitation of Δ's (right panel)

π+N+Δ matter

more realistic system

1) first calculation: same sizes of hadrons $(R_N = R_\Delta = R_\pi)$

$$\to \; P = nTZ(n), \; n \equiv n_B + n_\pi, \; \mu_i = T\left\{\ln\left[n_i/\phi_i(T)\right] + \psi(n)\right\} \quad (i = \pi, N, \Delta)$$

at given n_B and T, we get from $\mu_\pi = 0, \; \mu_N = \mu_\Delta \equiv \mu_B$

$$n_\pi = \phi_\pi(T)\, e^{-\psi(n_\pi + n_B)}, \quad \mu_B = T\left\{\ln\left[n_B/(\phi_N + \phi_\Delta)\right] + \psi(n_\pi + n_B)\right\}$$

$\to n_i, P, \varepsilon, c_s, \mu_B$ as functions of n_B and T

2) second calculation: point-like pions $(R_\pi = 0)$ + finite-size baryons $(R_N = R_\Delta)$

$$\to \; P = T\left[\phi_\pi(T) + n_B Z(n_B)\right], \quad n_\Delta/\phi_\Delta = n_N/\phi_N = n_B/(\phi_N + \phi_\Delta), \quad n_\pi = \phi_\pi(1 - v n_B)$$

baryon volume

$$\mu_B = T\left\{\ln\left[n_B/(\phi_N + \phi_\Delta)\right] + \psi(n_B) + v\phi_\pi\right\}$$

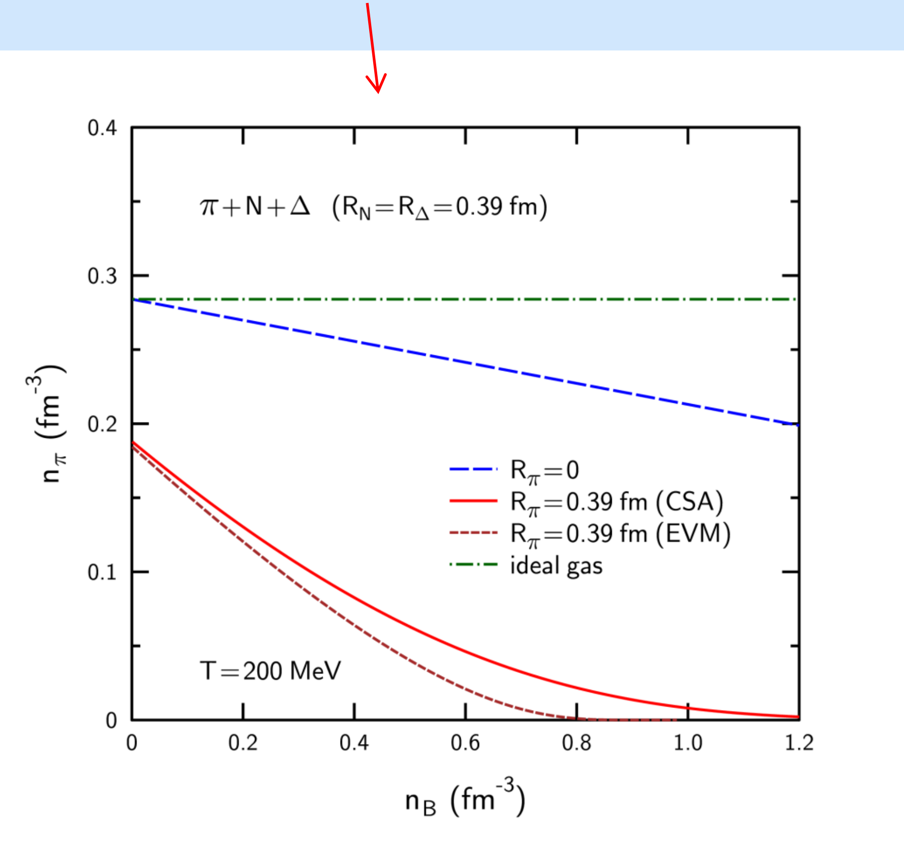
$$\varepsilon = T\left[n_\pi(\xi_\pi - 1) + n_B \langle \xi - 1 \rangle\right]$$

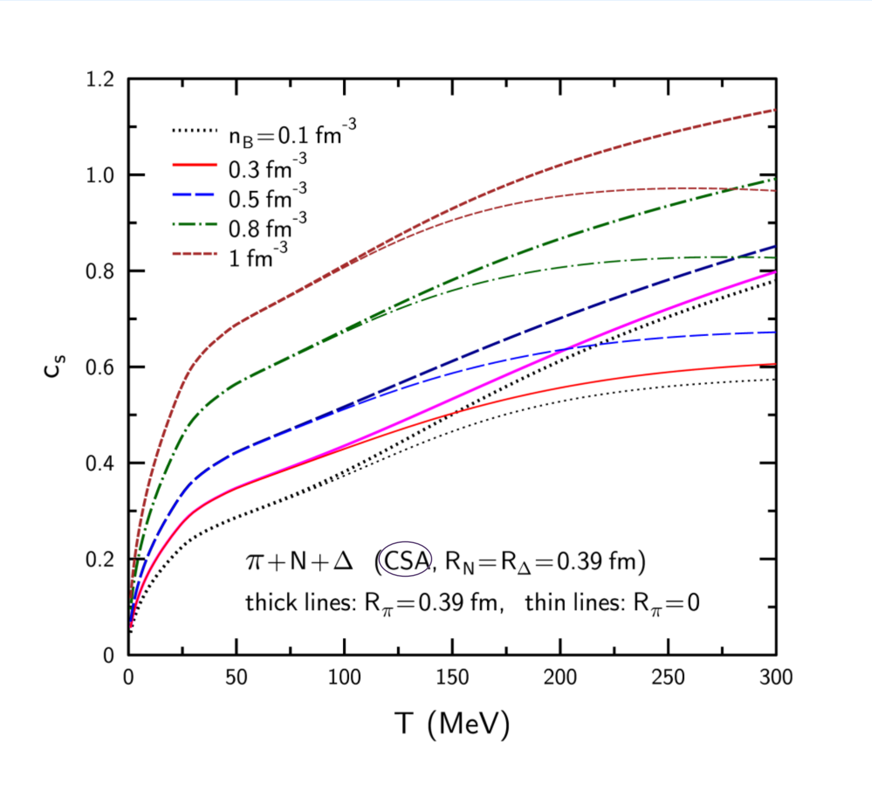
additional shift due to πB interactions

$$\to c_s = c_s(n_B, T)$$

⇒ in both cases $\varepsilon(n_B, T)$ has the same form as for ideal π+N+Δ gas, but with reduced pion density $n_\pi < \phi_\pi(T)$

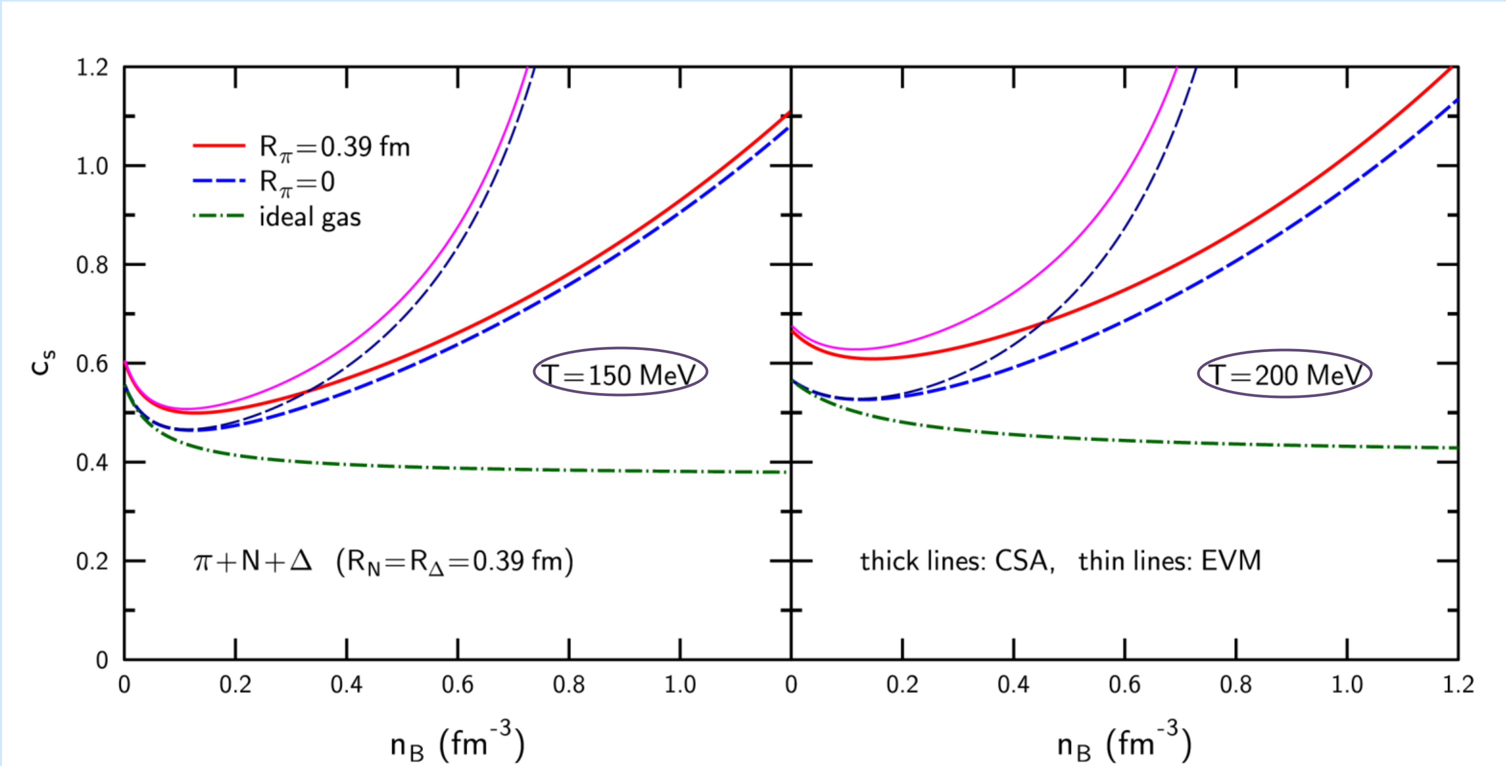
Pion density and sound velocity in π+N+Δ matter





- strong sensitivity to pion size
- $c_s>1$ in CSA calculation only at very large n_B and T

Sound velocity of π+N+Δ matter



- $c_s^{(id)} < c_s^{(CSA)} < c_s^{(EVM)}, \quad c_s(R_\pi = 0) < c_s(R_\pi = R_B)$
- acausal states in CSA are shifted to larger n_B as compared to EVM

Conclusions

- EVM calculations become unrealistic at packing fractions $\eta \gtrsim 0.2$
- The Carnahan-Starling EoS:
  - agrees with EVM at $\eta \lesssim 0.1$
  - is much softer than in the EVM at larger η
  - becomes acausal only at very high n_B, T (presumably outside the region of hadronic phase)
- The strong sensitivity of sound velocities to finite sizes of hadrons
- First results for pion-baryon mixtures with $R_\pi \ll R_B$

Outlook

In the future we are going to:

- include heavier hadrons in CSA calculations
- investigate sensitivity of phase diagram to finite sizes of hadrons
- include quantum-statistical effects for systems with HSI