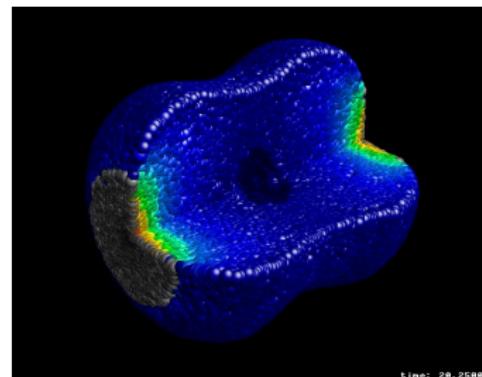
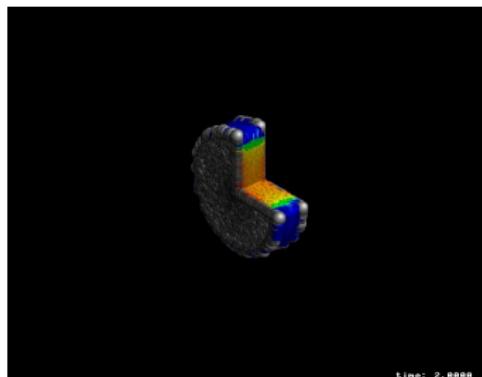
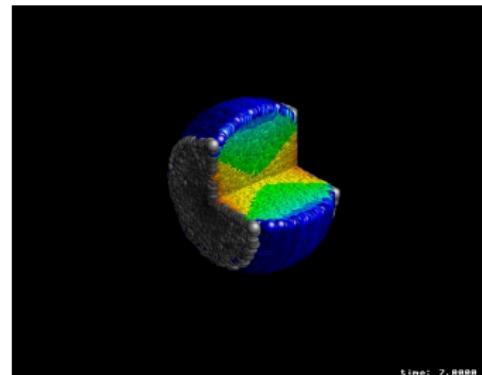


# From fluctuating minijet initial state to global observables in AA collisions at LHC and RHIC

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Frankfurt – 11.2.2015

with  
**R. Paatelainen and K. J. Eskola**



# Initial energy density from pQCD

- NLO pQCD calculation of transverse energy  $E_T$
- EPS09 nuclear parton distributions (Eskola et. al. JHEP **0904**, 065 (2009)) with impact parameter dependence (Helenius et. al. JHEP **1207** 073 (2012))

$$d\sigma^{AB \rightarrow kl\dots} \sim f_{i/A}(x_1, Q^2) \otimes f_{j/B}(x_2, Q^2) \otimes \hat{\sigma}$$

Essential quantity  $\sigma \langle E_T \rangle$  with  $p_T$  cut-off  $p_0$

$$\sigma \langle E_T \rangle (p_0, \Delta y, \beta) = \int_0^{\sqrt{s}} dE_T E_T \frac{d\sigma}{dE_T} \theta(y_i \in \Delta y, p_T > p_0, E_T > \beta p_0)$$

- 2 → 2 processes  $p_{T1} + p_{T2} > 2p_0$
- 2 → 3 processes  $p_{T1} + p_{T2} + p_{T3} > 2p_0$
- In 2 → 3 processes can still require for the total  $E_T$  in the rapidity window  $\Delta y$ :  $E_T > \beta p_0$ , with  $\beta \in [0, 1]$

$$\frac{dE_T}{d^2\mathbf{s}} = T_A(\mathbf{s} - \frac{\mathbf{b}}{2}) T_A(\mathbf{s} + \frac{\mathbf{b}}{2}) \sigma \langle E_T \rangle_{p_0, \Delta y}$$

$$e = \frac{dE_T}{\tau_0 \Delta y d^2\mathbf{s}} = T_A(\mathbf{s} - \frac{\mathbf{b}}{2}) T_A(\mathbf{s} + \frac{\mathbf{b}}{2}) \frac{\sigma \langle E_T \rangle_{p_0, \Delta y}}{\tau_0 \Delta y}$$



# Saturation condition (average) for central AA collisions

Original EKRT saturation condition

(K. J. Eskola, K. Kajantie, P. V. Ruuskanen and K. Tuominen, Nucl. Phys. B **570**, 379 (2000).)

$$N_{AA}(p_0, \sqrt{s_{NN}}, \Delta Y = 1, \mathbf{b} = \mathbf{0}) \times \frac{\pi}{p_0^2} = K_{\text{sat}} \pi R_A^2,$$

Average saturation condition in terms of transverse energy (R. Paatelainen, K. J. Eskola, H. Holopainen and K. Tuominen, Phys. Rev. C 87, 044904 (2013))

$$E_T^{AA}(p_0, \sqrt{s_{NN}}, \Delta Y, \mathbf{0}) = K_{\text{sat}} R_A^2 p_0^3 \Delta Y,$$

# Local saturation condition

- Lower cut-off  $p_0$  determined from a local saturation condition

$$\frac{dE_T}{d^2\mathbf{s}}(p_0, \sqrt{s}, \mathbf{s}, \mathbf{b}, \Delta y) = \frac{K_{\text{sat}}}{\pi} p_0^3 \Delta y$$

or equivalently

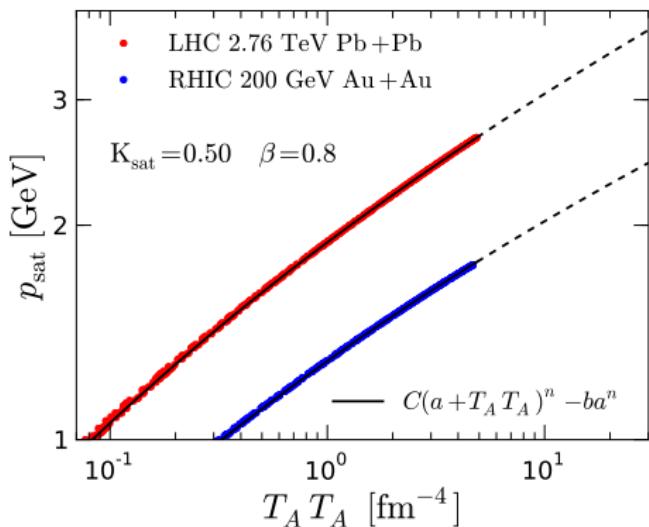
$$T_A(\mathbf{s} - \frac{\mathbf{b}}{2}) T_A(\mathbf{s} + \frac{\mathbf{b}}{2}) \sigma \langle E_T \rangle_{p_0, \Delta y} = \frac{K_{\text{sat}}}{\pi} p_0^3 \Delta y$$

- In principle  $\sigma \langle E_T \rangle_{p_0, \Delta y}$  depends also on the transverse coordinate  $\mathbf{s}$  through the  $\mathbf{s}$ -dependent nuclear parton distributions, but it turns out that in this particular application the dependence is weak.
- Parametrize the solution of the saturation condition  $p_0 = p_{\text{sat}}$  to be a function of  $T_A T_A$  alone.

Once we know the solution of the saturation equation we can write energy density at time  $\tau_0 = 1/p_{\text{sat}}$

$$e(\mathbf{s}, \tau_0 = 1/p_{\text{sat}}) = K_{\text{sat}} p_{\text{sat}}(\mathbf{s})^4 / \pi$$

- Two parameters:  $K_{\text{sat}}$  in the saturation condition, and  $\beta$  in the definition of transverse energy in the measurement function.

$p_{sat}$ 

- The full calculation can be summarized by a simple parametrization
- This also shows that the assumption  $p_{sat} = f(T_A T_A)$  works

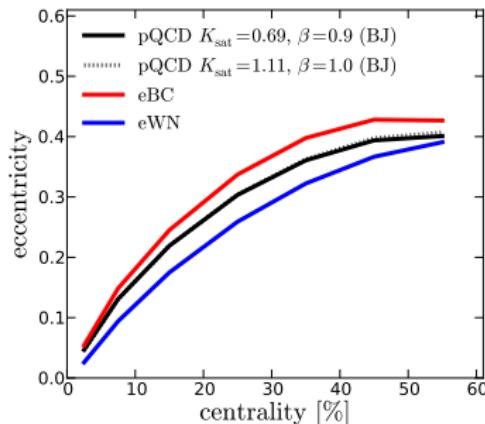
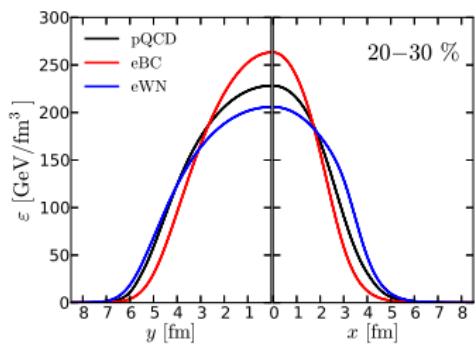
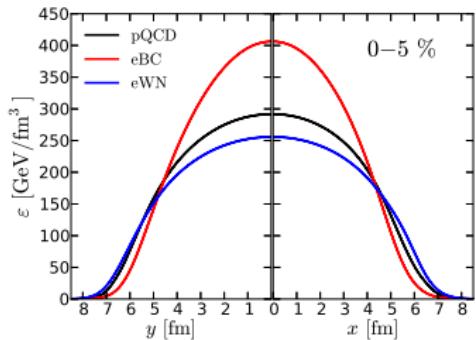
evolution to same  $\tau$ 

Naively  $e = K_{\text{sat}} p_{\text{sat}}^4 / \pi$ , **but all at different times**  $\tau_0 = 1/p_{\text{sat}}$ !

- For fluid dynamics we need  $e(x, y)$  at fixed proper time  $\tau_0$ .
  - Need to evolve all the energy densities to a same time.
  - Latest time given by  $\tau_{\text{max}} = 1/p_{\text{min}}$ , where  $p_{\text{min}} \sim 1 \text{ GeV}$ , the smallest scale we think we can still trust the pQCD calculation
- 
- Here: Bjorken scaling ( $\sim$  conserves entropy)

$$e(\tau_{\text{max}}) = e(\tau = 1/p_{\text{sat}}) \left( \frac{\tau}{\tau_{\text{max}}} \right)^{4/3}$$

## Energy density profiles



- Initial energy density and eccentricities compared to eBC and eWN profiles with the same initial entropy  $dS/d\eta$ .

$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\}/\{r^m\},$$

$$\{\dots\} = \int dx dy \epsilon(x, y, \tau_0)(\dots)$$

# Ebye fluctuations

How to generalize the nuclear thickness function to ebye case:

- Nucleon (gluonic) profile from HERA  $\gamma^* p \rightarrow J/\Psi + p$  data

$$T_n(r) = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}},$$

with  $\sigma = 0.43$  fm.

- Sample nucleon positions from the Wood-Saxon profile.

$$T_A(\mathbf{r}) = \sum_{i=1}^A T_n(|\mathbf{r} - \mathbf{r}_i|),$$

$$\longrightarrow T_A T_A \longrightarrow p_{\text{sat}} \longrightarrow e$$

# Israel-Stewart hydrodynamics

Model the space-time evolution of A+A collisions by relativistic fluid dynamics:

Neglect net-baryon number, bulk viscosity & heat flow

$$\partial_\mu T^{\mu\nu} = 0$$

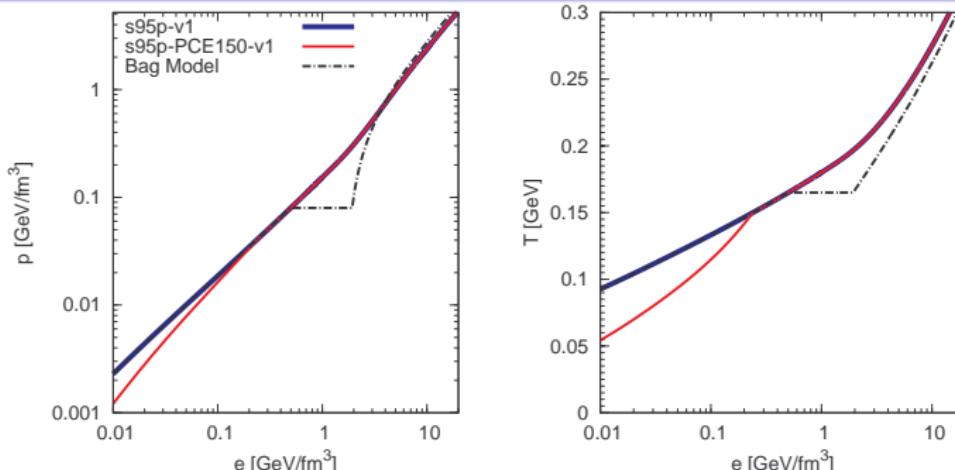
$$D\pi^{\langle\mu\nu\rangle} = -\frac{1}{\tau_\pi} \left( \pi^{\mu\nu} - 2\eta \nabla^{\langle\mu} u^{\nu\rangle} \right) - \frac{4}{3} \pi^{\mu\nu} \left( \nabla_\lambda u^\lambda \right) - \frac{10}{7} \pi_\lambda^{\langle\mu} \sigma^{\nu\rangle\lambda}$$

Longitudinal expansion is treated using boost invariance:  $\frac{\partial p}{\partial \eta_s} = 0$ ,  $v_z = \frac{z}{t}$

To solve this set of equations we need at  $\tau = \tau_0$

- Equation of state  $p = p(e)$  and  $T = T(e)$
- Initial condition  $T^{\mu\nu}(\tau_0, x, y)$
- Shear viscous coefficient  $\eta(T)$  and relaxation time  $\tau_\pi(T)$ .

# Equation of State



- Lattice parametrization by Petreczky/Huovinen:  
Nucl. Phys. **A837**, 26-53 (2010), [[arXiv:0912.2541 \[hep-ph\]](https://arxiv.org/abs/0912.2541)].
- Chemical equilibrium (s95p-v1)
- **(partial) chemical freeze-out at  $T_{\text{chem}} = 175 \text{ MeV}$  (s95p-PCE150-v1)**
- for comparison bag-model EoS
- Hadron Resonance Gas (HRG) includes all hadronic states up to  $m \sim 2 \text{ GeV}$



## Freeze-out

**Converting fluid to particles**

$$e, u^\mu, \pi^{\mu\nu} \longrightarrow E \frac{dN}{d^3\mathbf{p}}$$

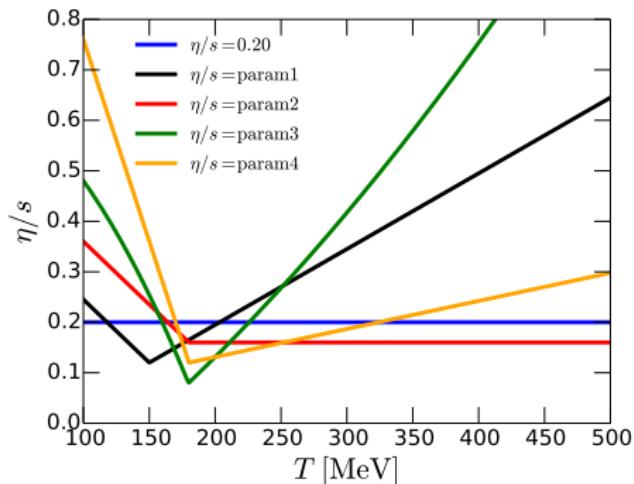
- Standard Cooper-Frye freeze-out for particle  $i$

$$E \frac{dN}{d^3\mathbf{p}} = \frac{g_i}{(2\pi)^3} \int d\sigma^\mu p_\mu f_i(\mathbf{p}, x),$$

where

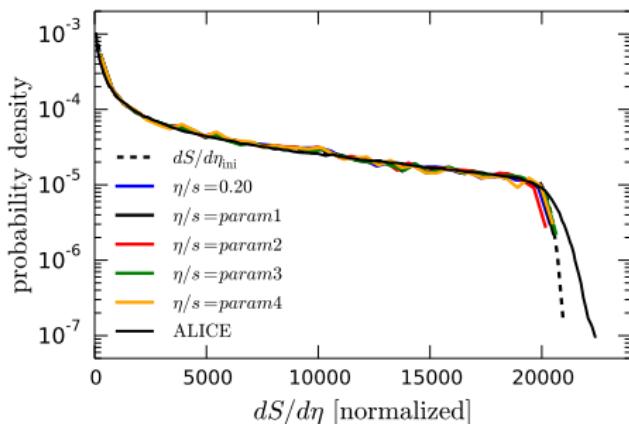
$$f_i(\mathbf{p}, x) = f_{i,\text{eq}}(\mathbf{p}, u^\mu, T, \{\mu_i\}) \left[ 1 + \frac{\pi^{\mu\nu} p_\mu p_\nu}{2T^2(e + p)} \right]$$

- Integral over constant temperature hypersurface
- 2- and 3-body decays of unstable hadrons included
- Here  $T_{\text{dec}} = 100$  MeV

Temperature dependent  $\eta/s$ 

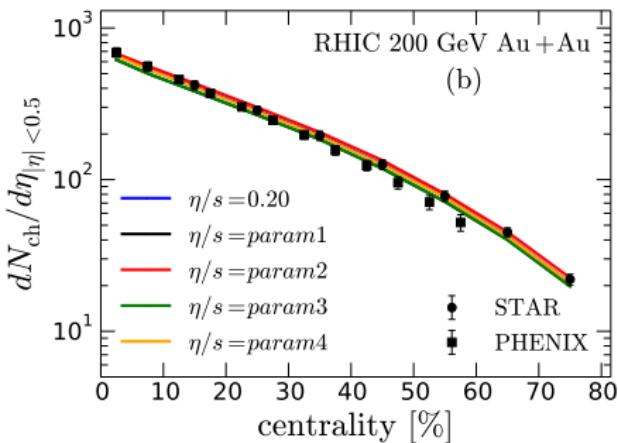
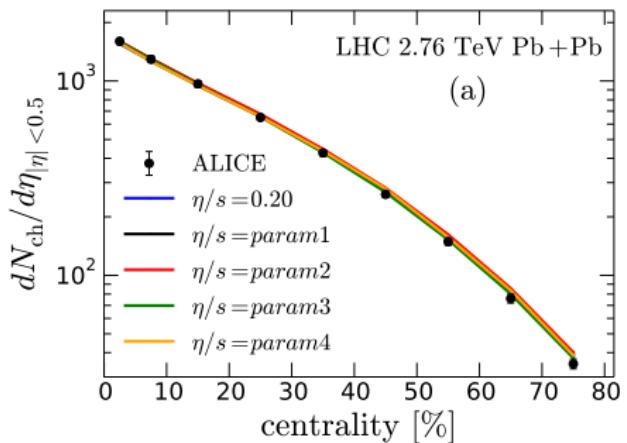
- relaxation time  $\tau_\pi(T) = \frac{5\eta}{\varepsilon + p}$ .

## Centrality selection



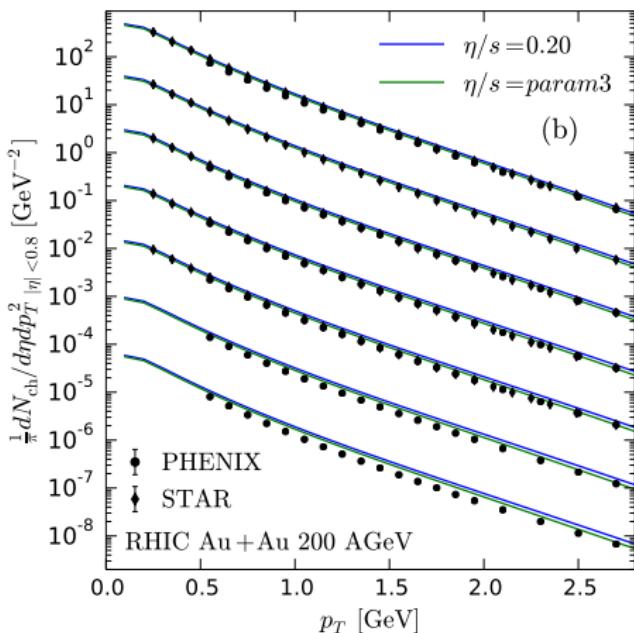
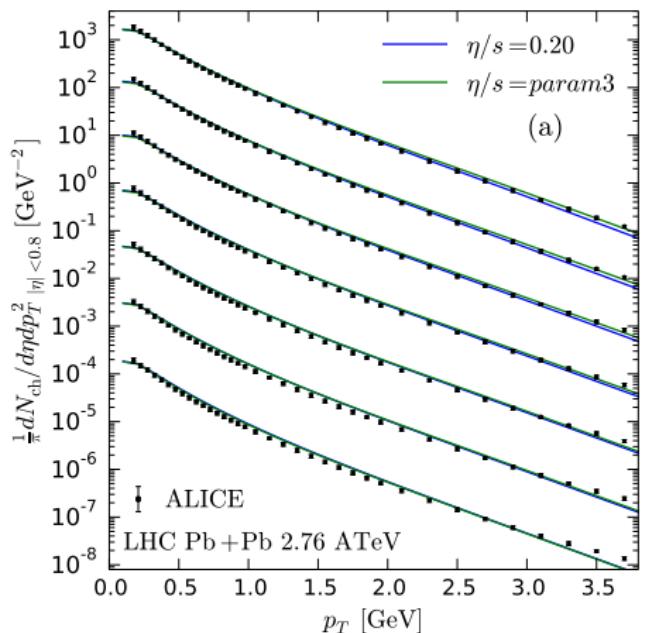
- Run random events
- Divide events into centrality classes according to final multiplicity.
- Still missing: ebye multiplicity fluctuations (with fixed  $T_A T_A$ )

## multiplicity

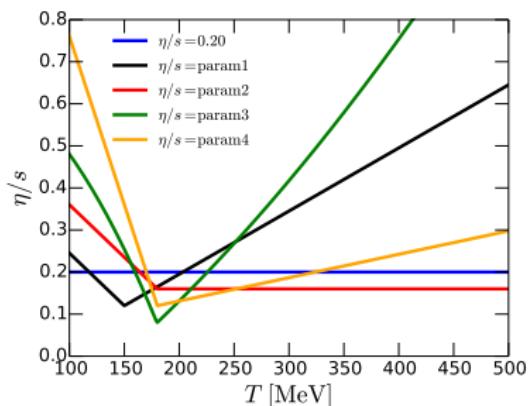
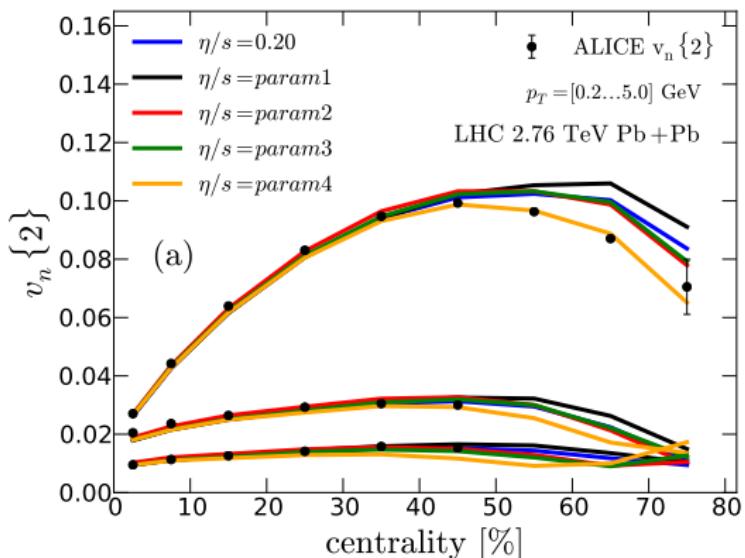


- $\beta = 0.8$
- $K_{\text{sat}} \sim 1$  fixed to reproduce the charged hadron multiplicity in 0-5 % Pb+Pb collisions at the LHC.
- Centrality and  $\sqrt{s}$  dependence prediction of the model

## Transverse momentum spectra

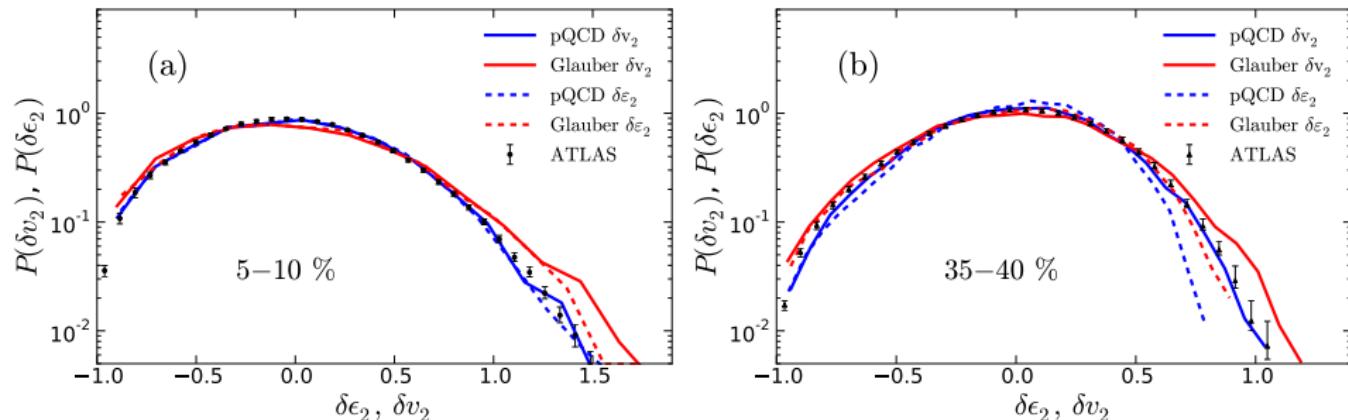


- kinetic ( $T_{\text{dec}}$ ) and chemical ( $T_{\text{chem}}$ ) decoupling temperatures are the most important parameters that determine the shape of  $p_T$ -spectra.
- $T_{\text{dec}} = 100$  MeV
- $T_{\text{chem}} = 175$  MeV

$\eta/s(T)$  from  $v_n$  data

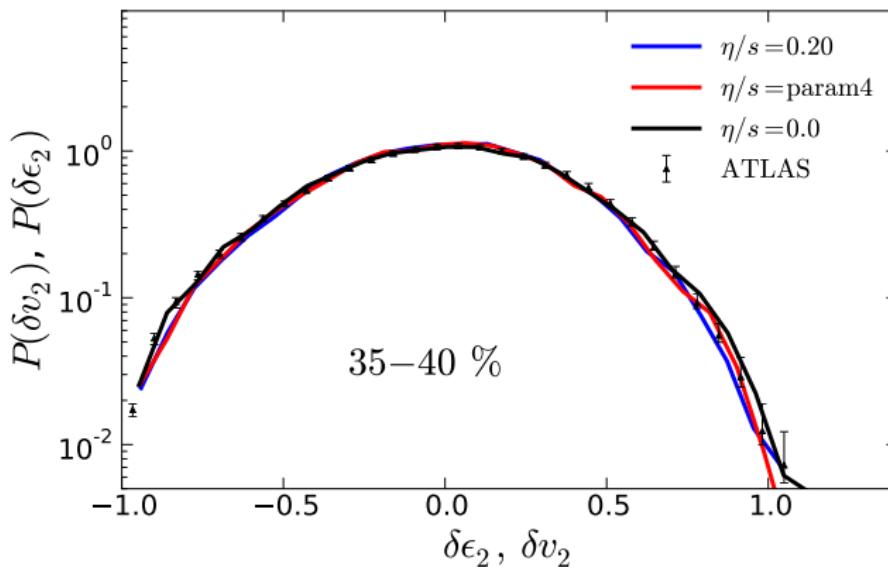
- $\eta/s(T)$  parametrizations tuned to reproduce the  $v_n$  data at the LHC.
- No strong constraints to the temperature dependence (all give equally good agreement)
- Deviations mainly in peripheral collisions, where the applicability of the framework most uncertain.

## Flow fluctuations



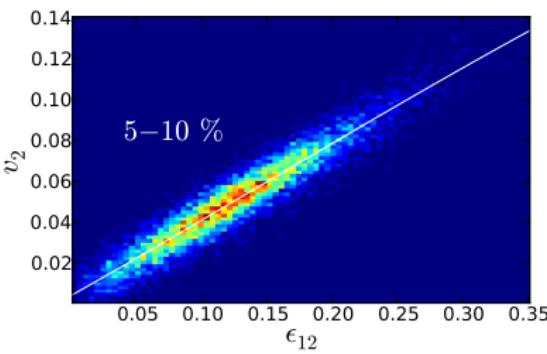
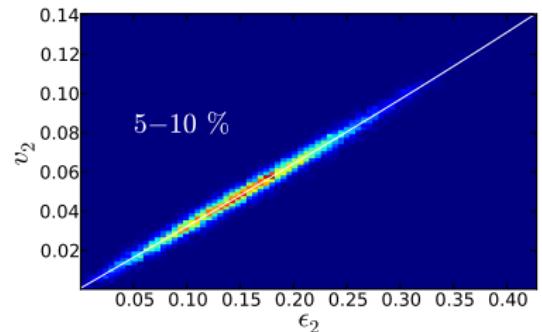
$$\delta v_n = \frac{v_n - \langle v_n \rangle_{ev}}{\langle v_n \rangle_{ev}}$$

- Direct constrain to initial conditions.
- $\delta v_2$  spectra well described in all centralities (Glauber eWN+eBC mixture as comparison)
- Non-linear hydro response in peripheral collisions?

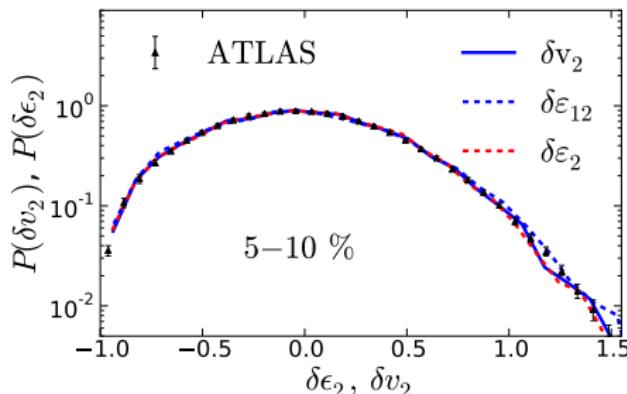
Sensitivity of  $P(\delta v_2)$  to viscosity

- Hydro response shows no sensitivity to  $\eta/s$ . (Note: we scale out the average  $v_2$ )

## (non)linear-response?



5-10 %

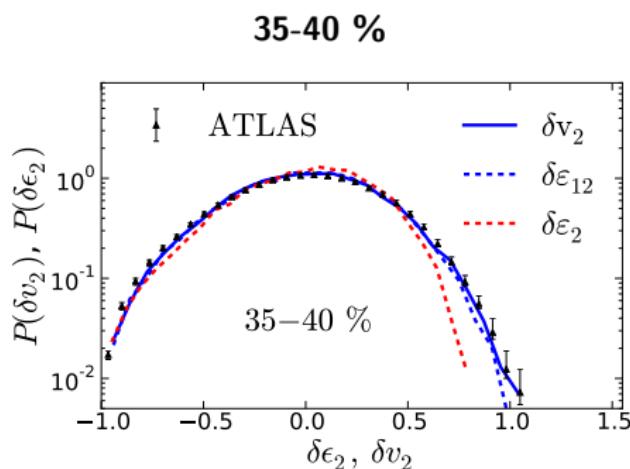
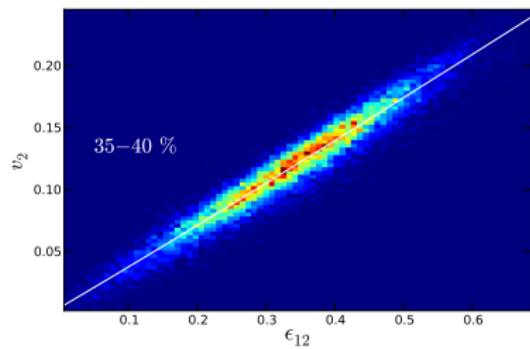
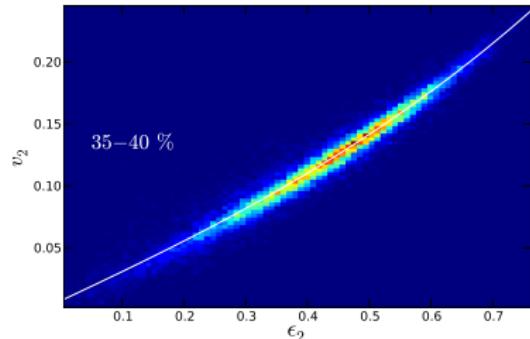


$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\}/\{r^m\},$$

$$\varepsilon_2 \equiv \varepsilon_{2,2} \text{ vs } \varepsilon_{1,2}$$

Full azimuthal structure:  $m = 0, \dots, \infty$

## (non)linear-response?

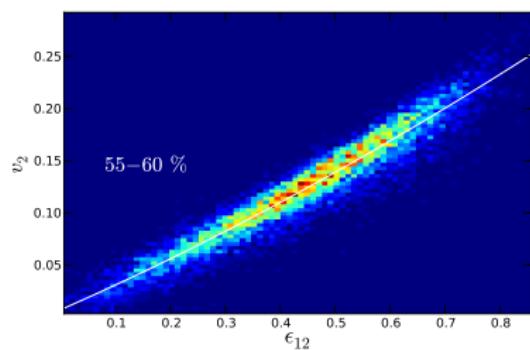
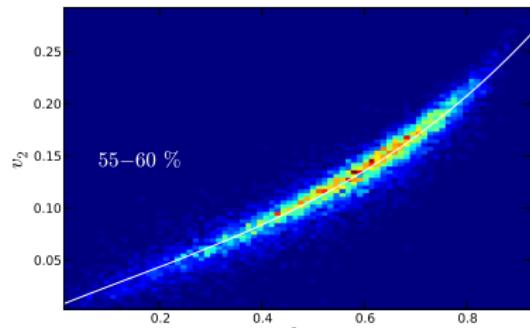


$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\}/\{r^m\},$$

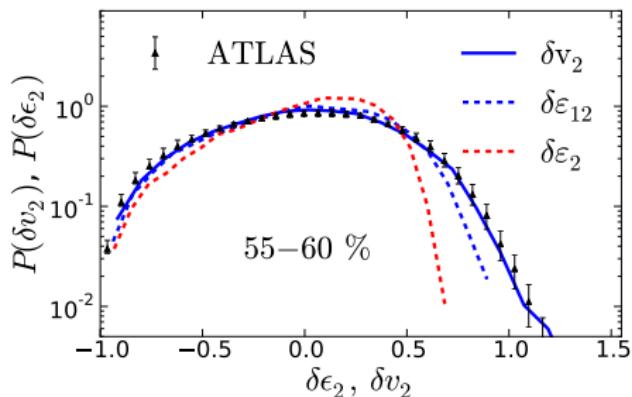
$$\varepsilon_2 \equiv \varepsilon_{2,2} \text{ vs } \varepsilon_{1,2}$$

Full azimuthal structure:  $m = 0, \dots, \infty$

## (non)linear-response?



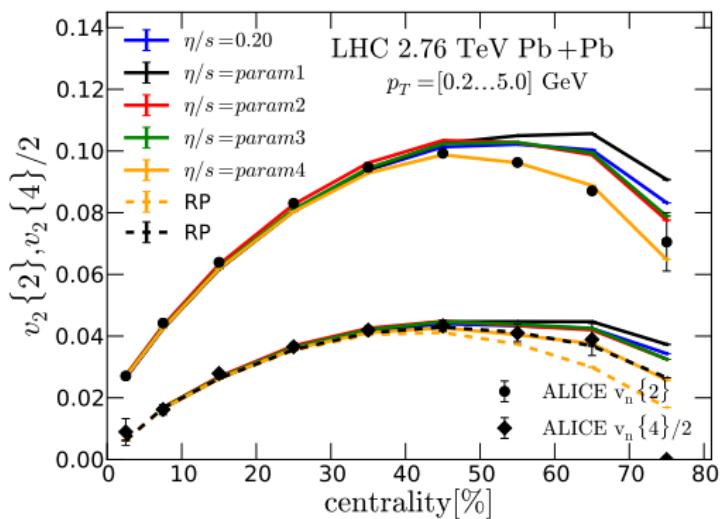
55–60 %



$$\varepsilon_{m,n} e^{in\Psi_{m,n}} = -\{r^m e^{in\phi}\}/\{r^m\},$$

$$\varepsilon_2 \equiv \varepsilon_{2,2} \text{ vs } \varepsilon_{1,2}$$

Full azimuthal structure:  $m = 0, \dots, \infty$

Flow fluctuations from  $v_n\{2\}$  and  $v_n\{4\}$ 

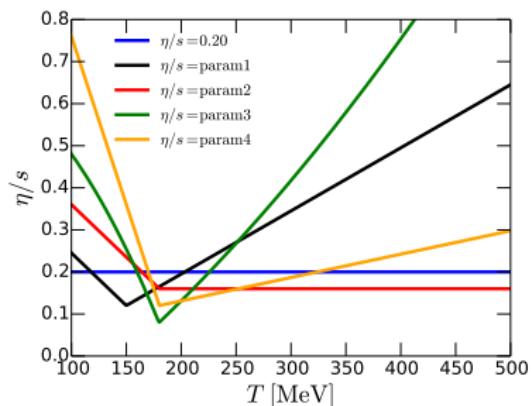
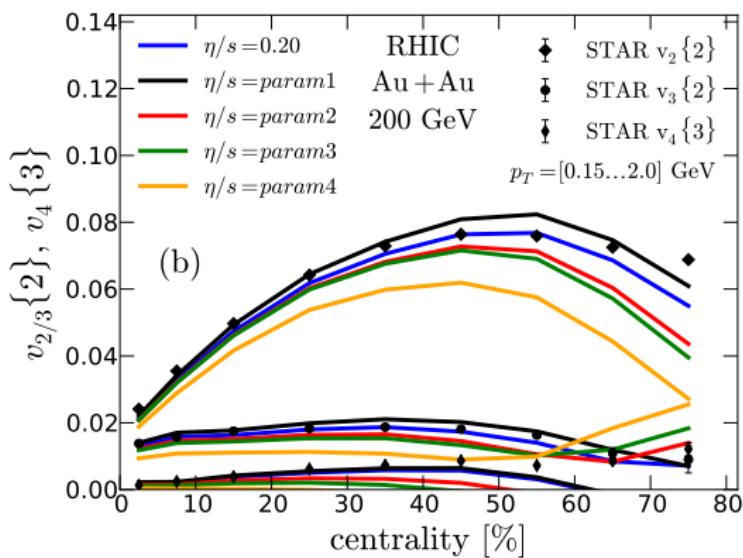
$v_n\{2\}$  and  $v_n\{4\}$  measure different moments of the  $v_n$ -fluctuation spectrum:

$$v_n\{2\}^{\text{flow}} \equiv \langle v_n^2 \rangle_{ev}^{1/2}$$

$$v_n\{4\}^{\text{flow}} \equiv \left( 2\langle v_n^2 \rangle_{ev}^2 - \langle v_n^4 \rangle_{ev} \right)^{1/4}$$

- Same information as in the fluctuation spectrum (assuming non-flow is not significant)

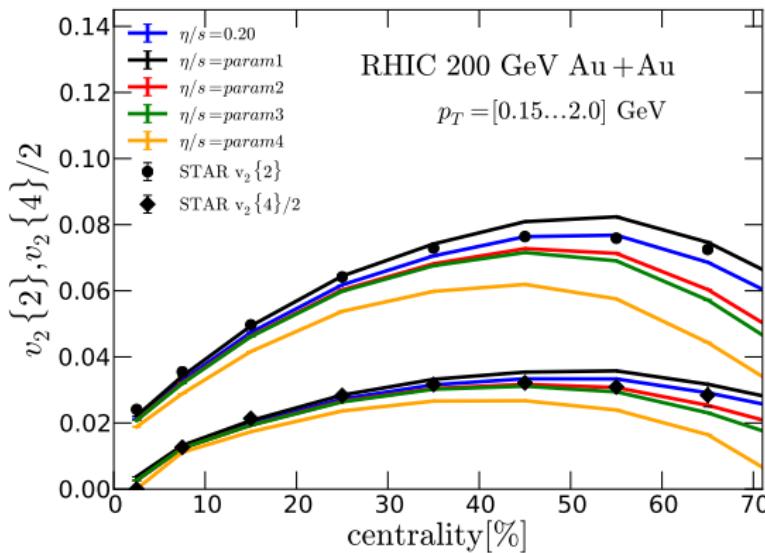
# RHIC 200 AGeV Au+Au: more constraints to $\eta/s(T)$

Constraints for  $\eta/s(T)$  from RHIC  $v_n$  data

$$v_4\{3\} \equiv \frac{\langle v_2^2 v_4 \cos(4[\Psi_2 - \Psi_4]) \rangle_{ev}}{\langle v_2^2 \rangle_{ev}}.$$

- $\eta/s = \text{param4}$  clearly below data

## Flow fluctuations

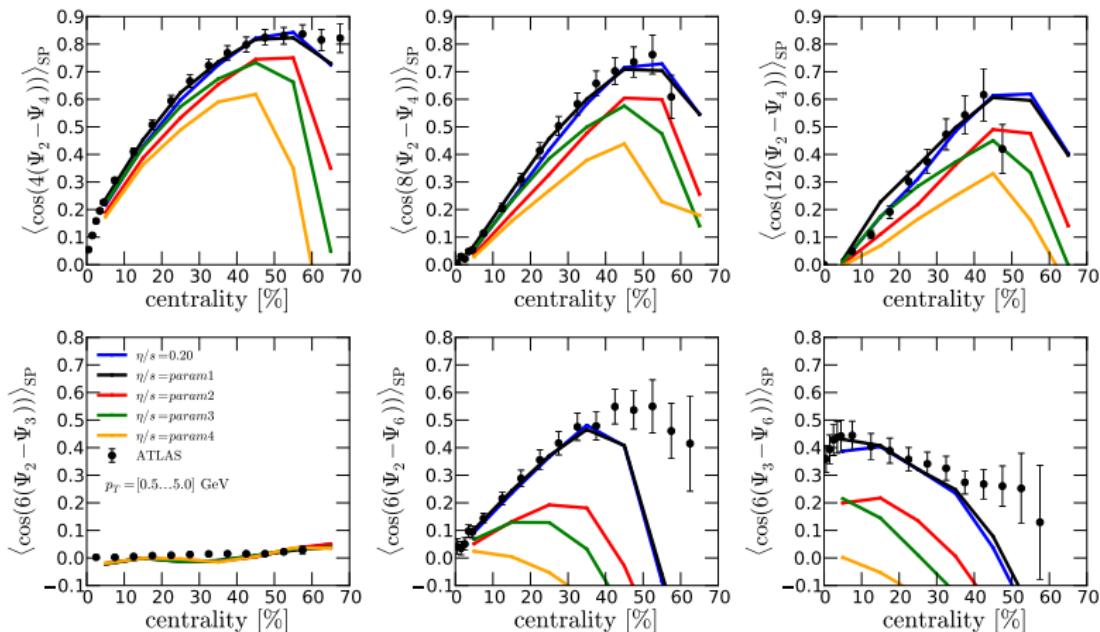


- No distributions directly, but  $v_2\{2\}$  and  $v_2\{4\}$  simultaneously described

# Event-plane correlations

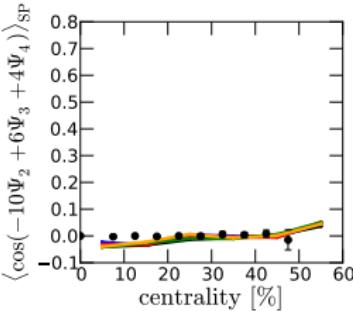
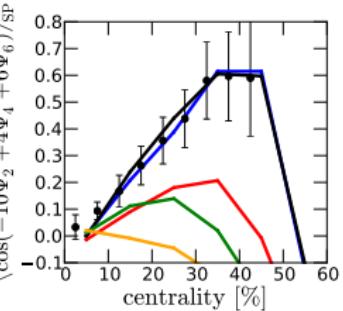
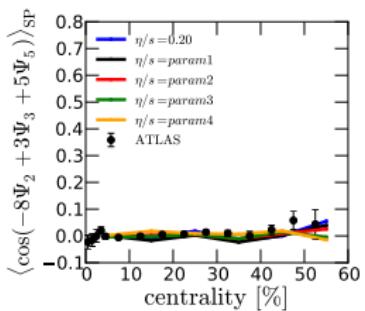
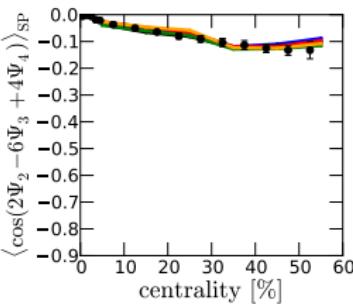
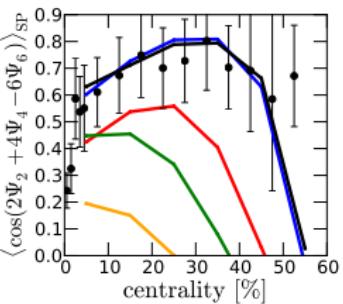
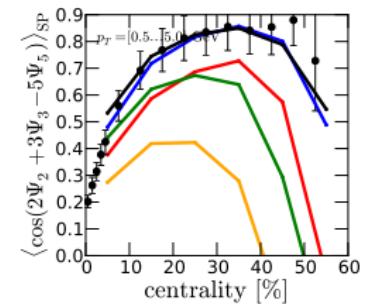
$$\langle \cos(k_1 \Psi_1 + \cdots + nk_n \Psi_n) \rangle_{\text{SP}} \equiv \frac{\langle v_1^{|k_1|} \cdots v_n^{|k_n|} \cos(k_1 \Psi_1 + \cdots + nk_n \Psi_n) \rangle_{ev}}{\sqrt{\langle v_1^{2|k_1|} \rangle_{ev} \cdots \langle v_n^{2|k_n|} \rangle_{ev}}},$$

## Event-plane correlations: 2 angles

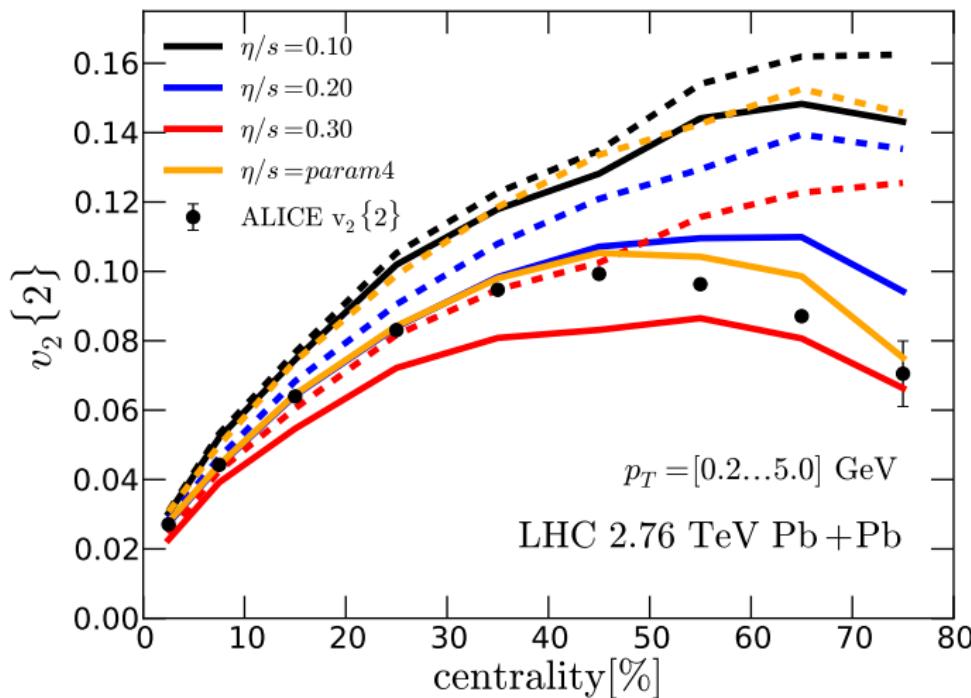


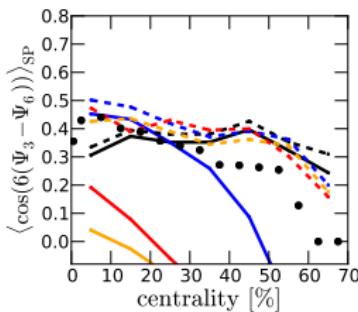
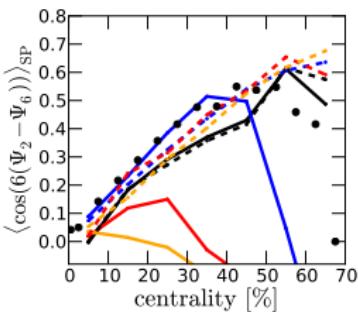
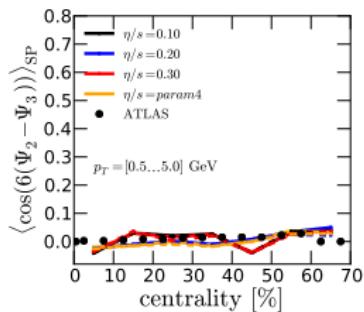
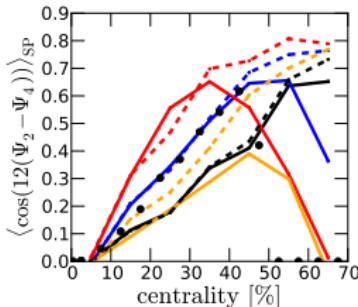
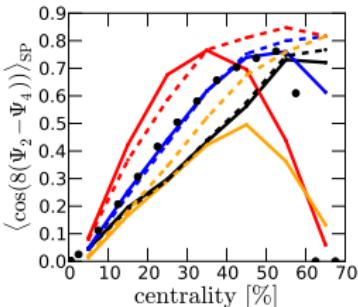
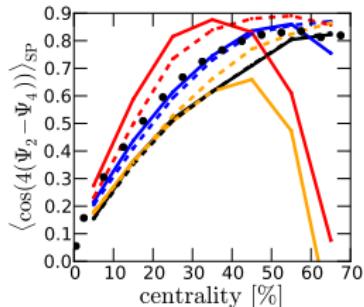
- Already from the LHC data more constraints to  $\eta/s(T)$ .
- Small hadronic viscosity needed to reproduce the data.

## Event-plane correlations: 3 angles



- Equally well described by the same parametrizations that describe 2-angle correlations.

$\delta f$  in  $v_2$  $\bullet \delta f$

$\delta f$  in event-plane correlations

- $\delta f$

## Summary

- Presented a new EbyE framework for NLO pQCD + saturation & viscous hydro
- The computed  $\sqrt{s}$  and centrality dependence of  $dN_{\text{ch}}/d\eta$  agree very well with LHC and RHIC data: predictive power!
- Most direct constraints for the IS come from the  $v_2$  fluctuations and the ratio  $v_2/v_3$  both are now very well reproduced!
- LHC  $v_n s$  alone do not stringently constrain the  $T$ -dependence of  $1\eta/s$
- Further constraints for  $\eta/s(T)$  from the  $v_n s$  at RHIC and the EP correlations at the LHC
- $\eta/s = 0.2$  (blue) and param1 with minimum at  $T = 150$  MeV (black) and small hadronic  $\eta/s$  work best in our framework
- Very promising results but we should keep in mind the uncertainties when ruling out a large hadronic viscosity: peripheral collisions and large hadronic  $\eta/s \rightarrow$  large  $\delta f$  at decoupling  $\rightarrow$  applicability of fluid dynamics ?

