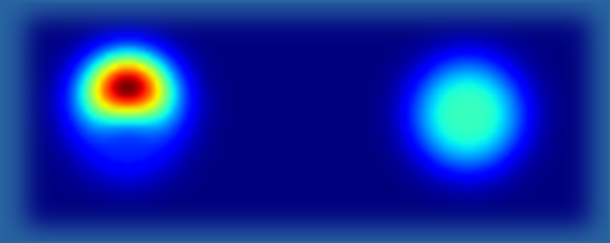
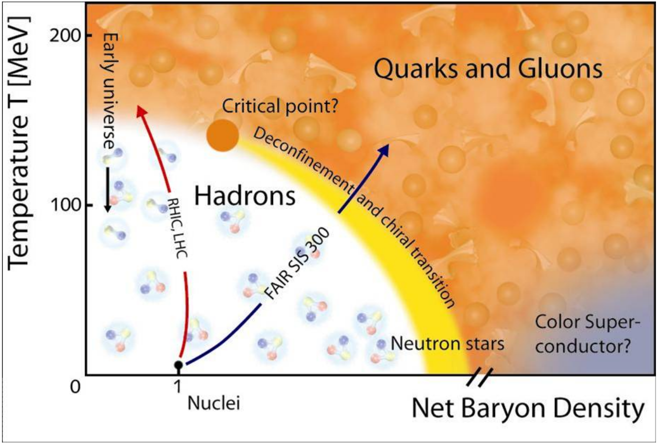


Particle-Field Interactions in a Chiral Transport Model

22.01.2014

C. Wesp with A. Meistrenko, H. van Hees, C. Greiner

Phase Diagram of Nuclear Matter



The Lagrangian

The model: linear σ-model with constituent quarks

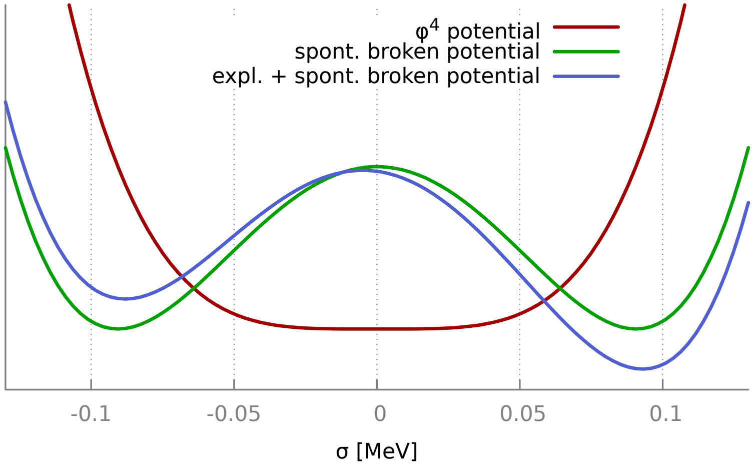
$$\mathcal{L} = \bar{\psi}\left[\imath\partial\!\!\!/ - g\left(\sigma + \imath\vec{\pi}\cdot\vec{\tau}\gamma_5\right)\right]\psi - \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + U_0(f_\pi, m_\pi^2, \sigma)$$

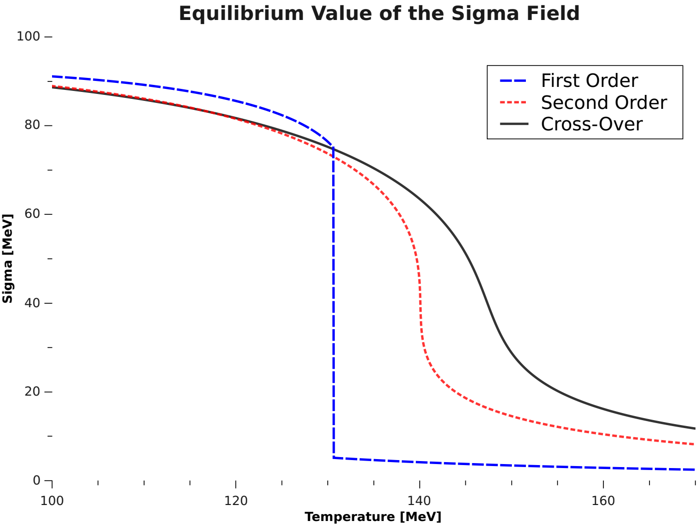
Model Parameter

λ^2	$= 20$	self coupling parameter
g	$\approx 3\ldots6$	Quark-sigma coupling
U_0	$= m_\pi^4/\left(4\lambda^2\right) - f_\pi^2 m_\pi^2$	Ground state
f_π	$= 93\mathrm{MeV}$	Pion Decay Constant
m_π	$= 138\mathrm{MeV}$	Pion mass
ν^2	$= f_\pi^2 - m_\pi^2/\lambda^2$	Field shift term

The Lagrangian



The Lagrangian



Model Implementation

- Quarks: test particles in a vlasov equation
- Fields: 3D+1 Klein-Gordon equations

$$\left[\partial_t + \frac{p}{E(t,\mathbf{r},\mathbf{p})} \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}} E(t,\mathbf{r},\mathbf{p}) \nabla_{\mathbf{p}}\right] \; f(t,\mathbf{r},\mathbf{p}) + I(t) = 0$$

$$\langle \bar{\psi}\psi(\mathbf{r})\rangle = g\sigma(\mathbf{r}) \int d^3\mathbf{p} \; \frac{f(\mathbf{r},\mathbf{p}) + \tilde{f}(\mathbf{r},\mathbf{p})}{E(\mathbf{r},\mathbf{p})}$$

$$\partial_\mu \partial^\mu \; \sigma + \lambda^2 \left(\sigma^2 + \vec{\pi}^2 - \nu^2\right) \sigma + g\langle\bar{\psi}\psi\rangle - f_\pi m_\pi^2 + I(t) = 0$$

Model Implementation

Field interactions:

- via potential: $\sigma \leftrightarrow \sigma$, $\vec{\pi} \leftrightarrow \vec{\pi}$, $\sigma \leftrightarrow \vec{\pi}$
- mean-field coupling: $\langle \bar{\psi}\psi \rangle$

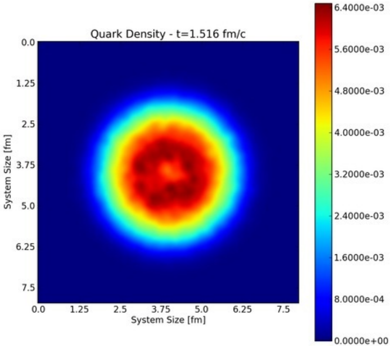
Quark interactions:

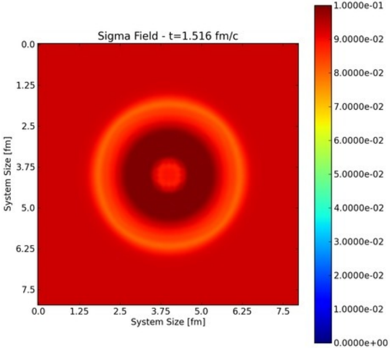
- elastic and binary collisions $\psi\psi \rightarrow \psi\psi$
- elastic interaction with heat-bath
- mean-field coupling: $\nabla_{\mathbf{r}} \sqrt{\sigma^2(\mathbf{r}) + \mathbf{p}^2}$

Under implementation: Quark-Field interactions

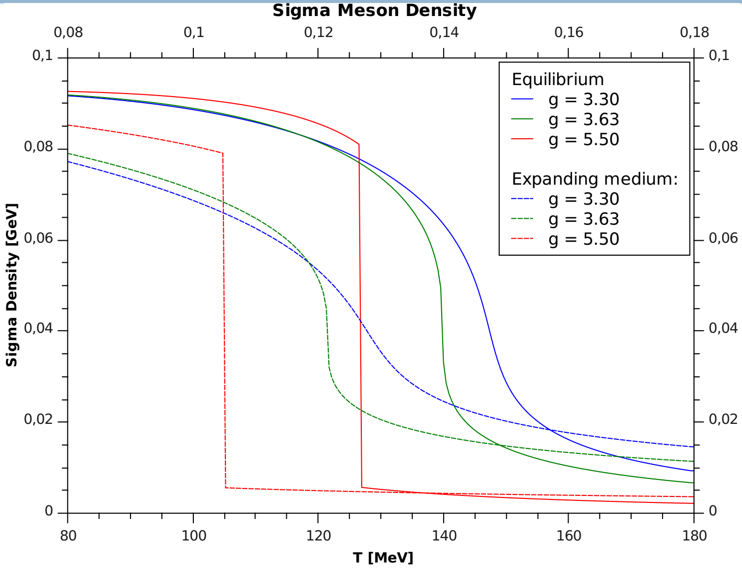
- elastic $\sigma\psi \rightarrow \sigma\psi$
- production $\sigma \rightarrow \bar{\psi}\psi$
- anhilation $\bar{\psi}\psi \rightarrow \sigma$

Dynamic Evolution

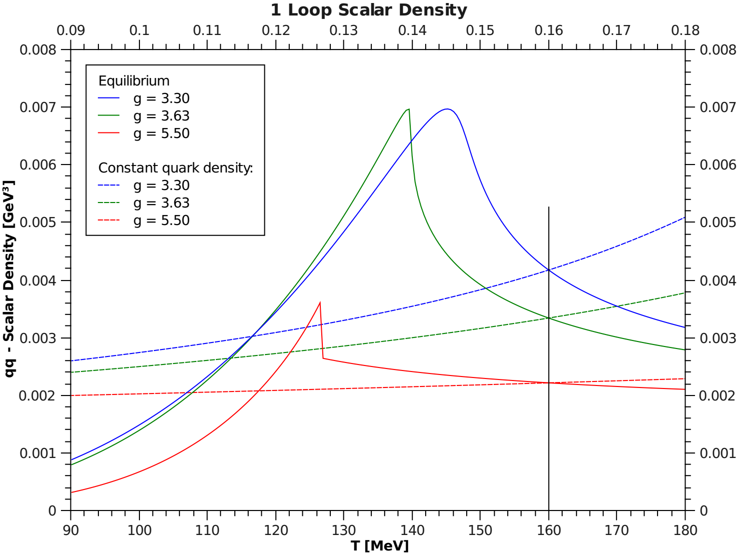




Dynamic Evolution

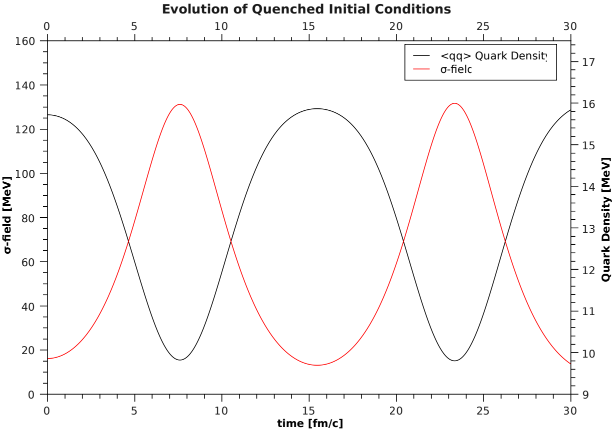


Dynamic Evolution



Equilibration of a field

The Problem: how to equilibrate a energy conserving system?



Equilibration of a field

Possibility: Couple field to a Langevin heat-bath!

$$\partial_\mu \partial^\mu \, \sigma + \frac{\partial U(\sigma, \vec{\pi})}{\partial \sigma} + g \langle \bar{\psi} \psi \rangle = 0$$

$$\partial_\mu \partial^\mu \, \sigma + \frac{\partial U(\sigma, \vec{\pi})}{\partial \sigma} + g \langle \bar{\psi} \psi \rangle = \xi(t) - \gamma \dot{\sigma}$$

- Gain: $\langle \xi(t)^2 \rangle$
- Dissipation: $\gamma \dot{\sigma}$

Equilibration of a field

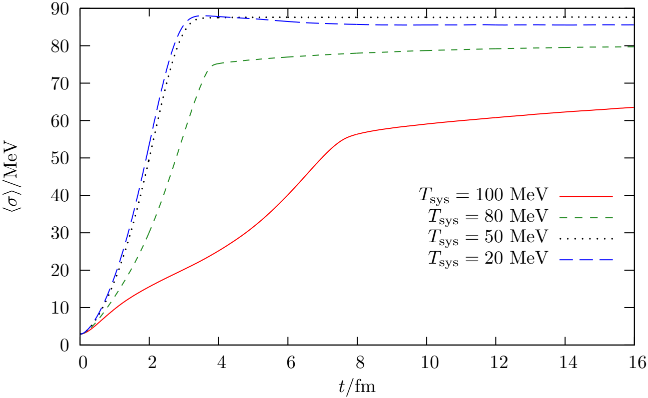
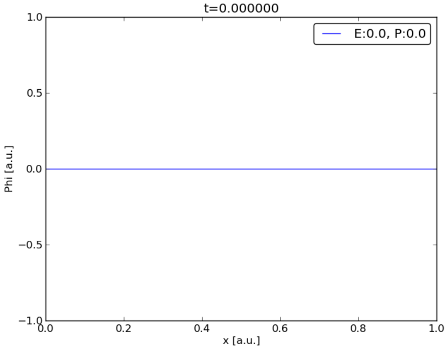


Figure: M. Nahrgang, Phys.Lett. B711 (2012) 109-116

Particle-Field Scattering



damping via $\sim \dot{\sigma}$

Problems:

- continuous energy loss
- how distribute energy to particles?

Equilibration of a field

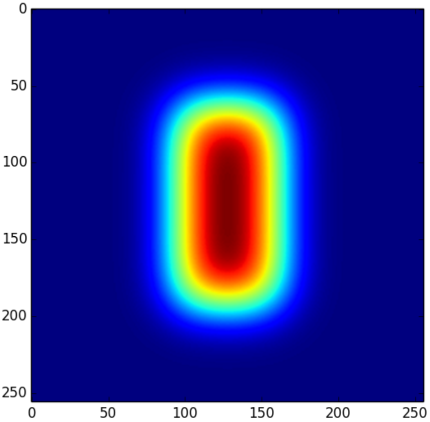
Next Possibility: 'scatter' at Fourier modes

$$\mathcal{F}\left(\sigma(\mathbf{r}, t)\right) \rightarrow \sigma(\mathbf{k}, t)$$

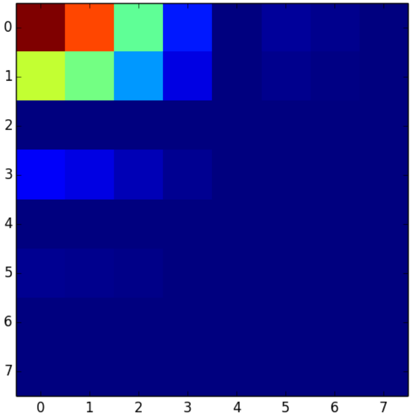
- sample $P_{int}\left(\sigma(\mathbf{k}, t) \rightarrow k_0\tilde{\sigma}(\mathbf{k}, t)\right)$
- calculate energy transfer
- retransform...

$$\mathcal{F}^{-1}\left(\tilde{\sigma}(\mathbf{k}, t)\right) \rightarrow \tilde{\sigma}(\mathbf{r}, t)$$

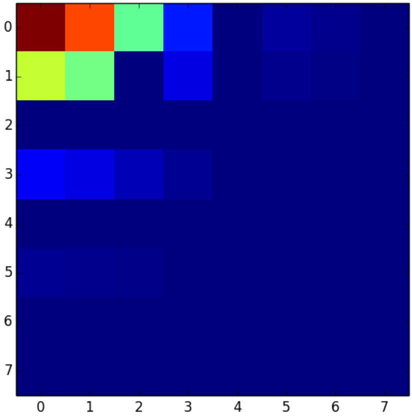
Equilibration of a field



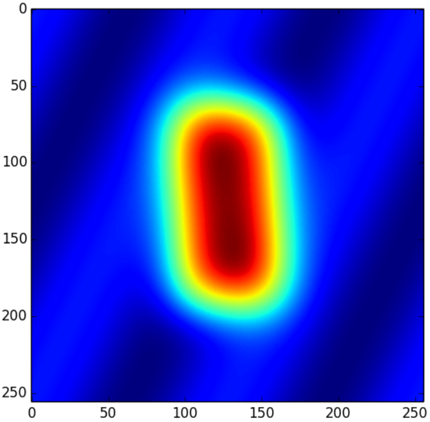
Equilibration of a field



Equilibration of a field



Equilibration of a field



Particle-Field Scattering

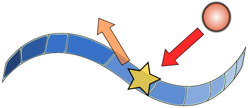
How to scatter a particle on a field?

Classical particles:

- point like position: $\delta(\vec{x}_0 - \vec{x})$
- on shell momentum: $\vec{p}$
- particle-particle scattering well understood

Classical field:

- spacial distribution $\phi(\vec{x})$
- interaction-point?
- momentum?



Particle-Field Scattering

Conserved quantities for a non-linear Klein-Gordon equation:

$$E = \int_V E(\vec{x}) d\vec{x} = \int_V \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi) \right] d\vec{x}$$

$$\vec{P} = \int_V \vec{P}(\vec{x}) d\vec{x} = \int_V \dot{\phi}\vec{\nabla}\phi d\vec{x}$$

$$\vec{A} = \int_V \left[\vec{x} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + V(\phi)\dot{\phi} \right) + t\phi\vec{\nabla}\phi \right] d\vec{x}$$

if $V(\phi)$ is positive defined: $E \geq P$

Particle-Field Scattering

Idea: Break problem into two parts:

- stochastic model for interaction between fields and particles:
  - $P\left(\Delta E, \Delta \mathbf{p}, dt\right)$
- mathematical framework for moment/energy transfer:
  - non-continuous in time
  - spacial constrained
  - defined momentum and energy change

Particle-Field Scattering

Energy / Momentum Remapping

find a new $\phi_f(\mathbf{r})$ which satisfies ΔE and $\Delta\vec{P}$:

$$E(\phi_f(\mathbf{r})) - E(\phi_i(\mathbf{r})) = \Delta E$$
$$\vec{P}(\phi_f(\mathbf{r})) - \vec{P}(\phi_i(\mathbf{r})) = \Delta\vec{P}$$

$$\phi_f(\mathbf{r}) = \phi_i(\mathbf{r}) + \delta(\mathbf{r})$$
$$\delta_x(x) \equiv G(x, A, v) \sim \frac{A}{\sqrt{\sigma\pi}} \; e^{(x-x_0-ctv)^2/\sigma}\Big|_{t\to 0}$$

Test Scenarios

Different test apporaches to mimic known systems:

1. scalar oscillator with discrete damping
2. scalar Oscialltor with Langevin
3. 1D harmonic field with Langevin
4. 3D Energy / Momentum change

Harmonic Oscillator

Test Case 1: Damped Classical Harmonic Oscillator

$$\frac{\partial^2}{\partial t^2}\phi(t) + \gamma\frac{\partial}{\partial t}\phi(t) + \phi(t) = 0$$

Simulate dissipation by discrete, stochastic kicks $\delta\phi(t)$:

$$\frac{\partial^2}{\partial t^2}\phi(t) + \phi(t) = \delta\phi(t)$$

Energy loss probability:

$$P\left(\Delta E\right) = \gamma \cdot \mathrm{d}t \cdot N, \text{ with: } \Delta E = \frac{E_0}{N}$$

Harmonic Oscillator

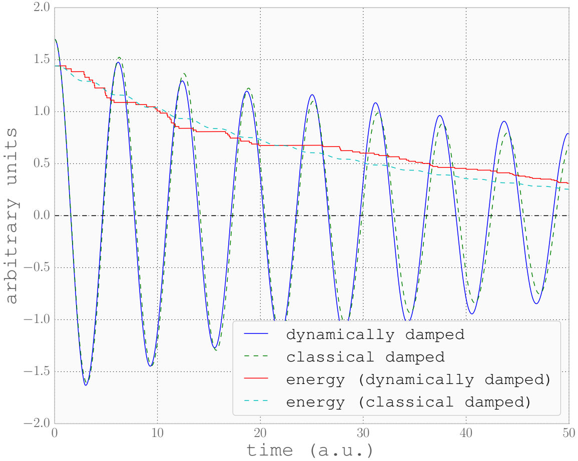


Figure: $\gamma = 0.35$, $N = 50$

Langevin Simulation

Test Case 2: Harmonic oscillator with Langevin force

$$\ddot{\phi}(t) + \phi(t) = -\gamma\dot{\phi}(t) + \kappa\xi(t)$$

$$\kappa = \sqrt{\frac{2k_B\gamma T}{mdt}}$$

Expected properties:

- thermal spectrum
- $\langle x \rangle$ is gaussian distributed with $\sigma = T$
- $1D$ case: field shows Brownian correlations

Scalar Langevin Simulation

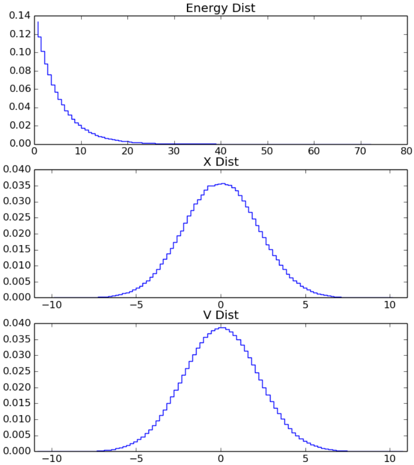


Figure: Scalar oscillator with Langevin force

1D Langevin Simulation

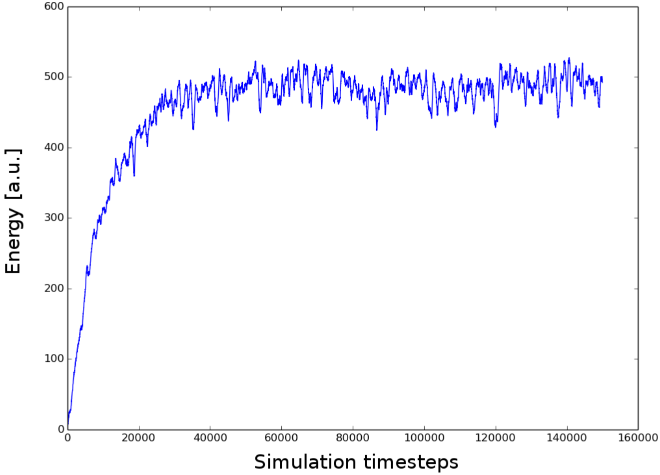


Figure: 1D harmonic field with Langevin force

1D Langevin Simulation

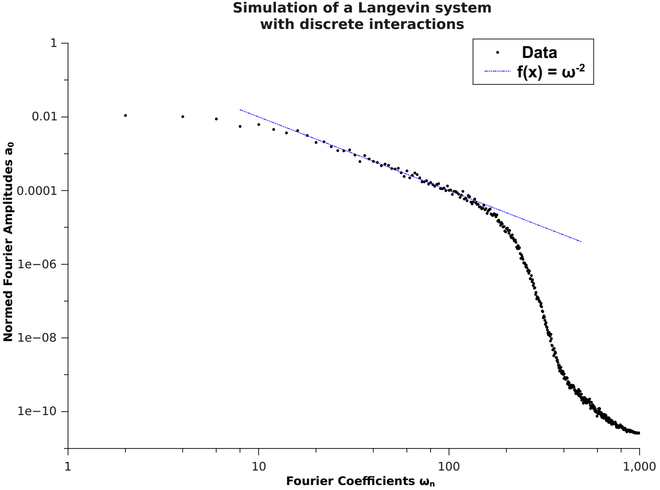
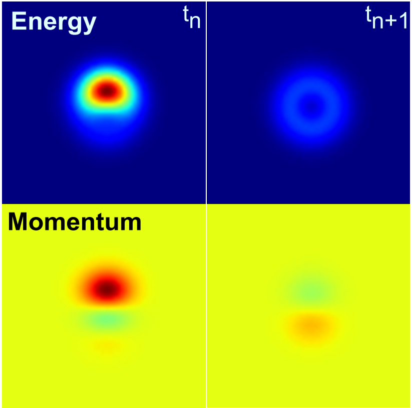


Figure: 1D harmonic field with Langevin force

3D Field Dynamics



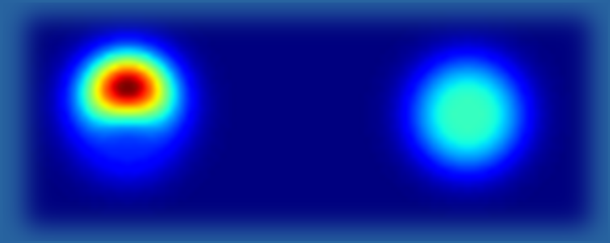
3D dynamics of σ-field with discrete E / $\vec{P}$ gain and loss.
t_n: $|P| = 3\mathrm{MeV}$, $E = 10\mathrm{MeV}$
t_{n+1}: $|P| \approx 0\mathrm{MeV}$, $E = 5\mathrm{MeV}$

Next Steps

The next steps:

- derive physical models for
  - $P(\Delta E, \Delta \mathbf{p}, \psi\sigma \to \psi\sigma)$
  - $P(\Delta E, \Delta \mathbf{p}, \bar{\psi}\psi \to \sigma)$
  - $P(\Delta E, \Delta \mathbf{p}, \psi\sigma \to \bar{\psi}\psi)$

- investigate change of fluctuations at the phase transition



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