Dynamical equilibration and transport coefficients of strongly-interacting ‘infinite’ parton matter

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In order to study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma – we need a consistent non-equilibrium (transport) model with

- explicit parton-parton interactions (i.e. between quarks and gluons) beyond strings!
- explicit phase transition from hadronic to partonic degrees of freedom
- IQCD EoS for partonic phase

Transport theory: off-shell Kadanoff-Baym equations for the Green-functions $S^{<}_{h}(x,p)$ in phase-space representation for the partonic and hadronic phase

QGP phase described by

Dynamical QuasiParticle Model (DQPM)
**DQPM spectral function**

**Basic idea:** effective strongly-interacting quasiparticles
- massive quarks, antiquarks and gluons \((q, q_{\text{bar}}, g)\) with broad spectral functions

Breit-Wigner spectral function:

\[
\rho(\omega, p) = \frac{4\omega \Gamma}{(\omega^2 - p^2 - M^2)^2 + 4\Gamma^2 \omega^2} \equiv \frac{\Gamma}{E} \left[ \frac{1}{(\omega - E)^2 + \Gamma^2} - \frac{1}{(\omega + E)^2 + \Gamma^2} \right]
\]

**notation:** \(E^2 = p^2 + M^2 - \Gamma^2\)

- mass and width:
  \(\Rightarrow\) quasiparticle properties

- finite width:
  \(\Rightarrow\) two-particle correlations

running coupling $\Rightarrow$ fit to the lattice QCD results

$$\alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f) \ln[\lambda^2(T/T_c - T_s/T_c)^2]}$$

- **number of colors** $\Rightarrow N_c = 3$
- **number of flavors** $\Rightarrow N_f = 3$
- **3 fitting parameters**
  $\Rightarrow \lambda = 2.42$  $T_s/T_c = 0.56$
  $c = 14.4$
## DQPM mass and width

- **Spectral function:**
  \[
  \rho(\omega, \mathbf{p}) = \frac{4\omega \Gamma}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\Gamma^2 \omega^2}
  \]

<table>
<thead>
<tr>
<th>Quark (antiquark):</th>
<th>Gluon:</th>
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| \[
  M_{q(\bar{q})}^2(T) = \frac{N_c^2 - 1}{8N_c} g^2 \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)
  \] |
| \[
  \Gamma_{q(\bar{q})}(T) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} g^2 T \frac{8}{2c} \ln \left( \frac{2c}{g^2} + 1 \right)
  \] |
| \[
  M_g^2(T) = \frac{g^2}{6} \left( \frac{N_c + \frac{N_f}{2}}{T} \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2}
  \] |
| \[
  \Gamma_g(T) = \frac{1}{3} N_c \frac{g^2 T}{8\pi} \ln \left( \frac{2c}{g^2} + 1 \right)
  \] |

Peshier, PRD 70 (2004) 034016

- **High temperature regime**
  - \( \Rightarrow \) one-loop perturbative QCD results

- **Mass and width** define quasiparticle properties

![Graph](image)
DQPM thermodynamics ($N_f=3$)

- **entropy:** $s = \frac{\partial P}{\partial T} \Rightarrow$ pressure

- **energy density:** $\varepsilon = Ts - P$

- **interaction measure:**

$$W = \varepsilon - 3P = Ts - 4P$$

**DQPM gives a good description of IQCD results!**

**IQCD: Wuppertal-Budapest group**

Y. Aoki et al., JHEP 0906 (2009) 088
DQPM overview

- fit to lattice QCD results:
  - thermodynamics quantities (pressure, entropy density, energy density) in equilibrium
  - running coupling:
    \[ \alpha_s(T) = \frac{g^2(T)}{4\pi} = \frac{12\pi}{(11N_c - 2N_f) \ln[\lambda^2(T/T_c - T_s/T_c)^2]} \]

- DQPM provides:
  - spectral function (mass and width) \(\Rightarrow\) off-shell quasiparticle properties
    \[ \rho(\omega, p) = \frac{4\omega\Gamma}{(\omega^2 - p^2 - M^2)^2 + 4\Gamma^2\omega^2} \]
  - mean fields (1 PI) for quarks (antiquarks) and gluons as well as effective 2-body interactions (2 PI)

Peshier, Cassing, PRL 94 (2005) 172301;
Our goals

- study of the partonic system out of equilibrium (beyond the DQPM)
  - dynamical equilibration of QGP within the non-equilibrium off-shell PHSD transport approach
  - influence of the partonic elastic and inelastic cross sections;

- study of the thermal properties of equilibrated partonic system in PHSD
  - transport coefficients (shear and bulk viscosities) of strongly-interacting partonic matter;
  - particle number fluctuations (scaled variance, skewness, kurtosis).

Ozvenchuk, Linnyk, Gorenstein, Bratkovskaya, Cassing, arXiv: 1203.4734
Parton interactions in PHSD

- **degrees of freedom in PHSD:**
  - colored quarks (u, d, s), antiquarks (ubar, dbar, sbar) and gluons

- **interaction processes:**

  **(quasi)-elastic**
  
  \[
  q(m_1) + q(m_2) \rightarrow q(m_3) + q(m_4) \\
  q + \bar{q} \rightarrow q + \bar{q} \\
  \bar{q} + q \rightarrow \bar{q} + q \\
  g + q \rightarrow g + q \\
  g + \bar{q} \rightarrow g + \bar{q} \\
  g + g \rightarrow g + g 
  \]

  **inelastic**
  
  \[
  q + \bar{q} \rightarrow g \\
  g \rightarrow q + \bar{q} \\
  \textbf{basic processes for the chemical equilibration} \leftrightarrow \textbf{flavor exchange} \\
  \text{e.g. } \Rightarrow u + \bar{u} \leftrightarrow g \ldots g \leftrightarrow s + \bar{s} \\
  \textbf{suppressed} (<1\%) \text{ due to the large mass of gluon}
  \]

Initialization

- **cubic box:**
  - periodic boundary conditions;
  - size is fixed to $9^3 \text{ fm}^3$;
  - light and strange quarks, antiquarks and gluons;
  - various values for the energy density and quark chemical potential.

- **Initialization is:**
  - close to the thermal equilibrium with thermal distribution for the momenta;
  - far out of the chemical equilibrium due to the strangeness suppression:

$$N_u \div N_d \div N_s = 3 \div 3 \div 1$$
Partial widths

- **DQPM** provides the total width $\Gamma$ of the dynamical quasiparticles

  $$\Gamma_{total} = \Gamma_{elastic} + \Gamma_{inelastic}$$

- **Partial widths** - (quasi)-elastic and inelastic - **cannot** be defined from the DQPM

  **For gluons:**
  $$\Gamma_{g}^{DQPM}(\varepsilon) = \Gamma_{g \rightarrow q + \bar{q}}^{inelastic}(\varepsilon) + \Gamma_{gg}^{elastic}(\varepsilon) + \Gamma_{gq}^{elastic}(\varepsilon) + \Gamma_{g\bar{q}}^{elastic}(\varepsilon)$$

  **For quarks:**
  $$\Gamma_{j}^{DQPM}(\varepsilon) = \Gamma_{\bar{q}q \rightarrow g}^{inelastic}(\varepsilon) + \Gamma_{jg}^{elastic}(\varepsilon) + \Gamma_{jq}^{elastic}(\varepsilon) + \Gamma_{j\bar{q}}^{elastic}(\varepsilon)$$

- Obtain the partial widths $\leftrightarrow$ cross sections for different channels from the PHSD simulations in the box

- **Final check** $\Rightarrow$ reproduce the lQCD EoS within PHSD in the box
Parton cross sections in PHSD

(Quasi)-elastic cross sections

\begin{align*}
q + q &\rightarrow q + q \\
q + \bar{q} &\rightarrow q + \bar{q} \\
\bar{q} + \bar{q} &\rightarrow \bar{q} + \bar{q} \\
g + q &\rightarrow g + q \\
g + \bar{q} &\rightarrow g + \bar{q} \\
g + g &\rightarrow g + g
\end{align*}

\[ \Rightarrow \]

Inelastic channels

Breit-Wigner cross section

\[ \sigma_{q\bar{q} \rightarrow g}(\varepsilon) = \frac{2}{4 \pi} \frac{s \Gamma_g^2}{(s - M_g^2(\varepsilon))^2 + s \Gamma_g^2} / P_{rel}^2 \]
Detailed balance

- Reactions rates are practically constant and obey detailed balance for:
  - Gluon splitting
  - Quark + antiquark fusion

- (quasi)-elastic collisions lead to the thermalization of all particle species

\[
\begin{align*}
q + q &\rightarrow q + q \\
q + \bar{q} &\rightarrow q + \bar{q} \\
\bar{q} + \bar{q} &\rightarrow \bar{q} + \bar{q} \\
g + q &\rightarrow g + q \\
g + \bar{q} &\rightarrow g + \bar{q} \\
g + g &\rightarrow g + g
\end{align*}
\]

- The numbers of partons dynamically reach their equilibrium values through the inelastic collisions

\[
\begin{align*}
q + \bar{q} &\rightarrow g \\
g &\rightarrow q + \bar{q}
\end{align*}
\]
Chemical equilibrium

- A sign of chemical equilibrium is the stabilization of the numbers of partons of the different species in time.

- Final abundancies vary with energy density.
Chemical equilibration of strange partons

- **Slow increase** of the total number of strange quarks and antiquarks
- **Long equilibration times** through inelastic processes involving strange partons

\[ N_u : N_d : N_s = 3 : 3 : 1 \]

- **Initial rate** for \( s + \bar{s} \rightarrow g \) is suppressed by a factor of 9
Equation of state

- equation of state implemented in PHSD
  - is well in agreement with the DQPM and the IQCD results;
  - includes the potential energy density from the DQPM.

**IQCD data:** Borsanyi et al., JHEP 1009, 073 (2010); JHEP 1011, 077 (2010)
Shear viscosity (Kubo formalism)

- **Kubo formula for the shear viscosity:**

\[
\eta = \frac{1}{T} \int d^3r \int_0^\infty dt \langle \pi^{xy}(0, 0) \pi^{xy}(r, t) \rangle
\]


- **shear component (traceless part):**

\[
\pi^{xy}(r, t) = \int \frac{d^3p}{(2\pi)^3} \frac{p_x p_y}{E} f(r, p, t)
\]

- **test-particles ansatz** \(\Rightarrow \pi^{xy} = \frac{1}{V} \sum_{j=1}^{N} \frac{p_{xj} p_{yj}}{E_j}\)

- **correlation functions** are empirically found to decay exponentially in time:

\[
\langle \pi^{xy}(0) \pi^{xy}(t) \rangle = \langle \pi^{xy}(0) \pi^{xy}(0) \rangle \exp\left(-\frac{t}{\tau}\right) \Rightarrow \eta = \frac{V}{T} \langle \pi^{xy}(0)^2 \rangle \tau
\]
relaxation time depends on the number of test-particles
⇒ reaches the constant value for large number of TP

shear viscosity does not depend on the volume of the system
starting hypothesis:  \[ C[f] = -\frac{f - f^{eq}}{\tau} \]

\(\tau = \Gamma^{-1}\)

shear and bulk viscosities assume the following expressions:

\[ \eta = \frac{1}{15T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{|p|^4}{E_a^2} \tau_a(E_a)f^{eq}(E_a/T) \]

\[ \zeta = \frac{1}{9T} \sum_a \int \frac{d^3p}{(2\pi)^3} \frac{\tau_a(E_a)}{E_a^2} \left[ (1 - 3v_s^2)E_a^2 - m_a^2 \right] f^{eq}(E_a/T) \]


in numerical simulations  \[\Rightarrow\] test-particle ansatz:

\[ \eta = \frac{1}{15TV} \sum_{i=1}^{N} \frac{|p_i|^4}{E_i^2} \Gamma_i^{-1} \]

\[ \zeta = \frac{1}{9TV} \sum_{i=1}^{N} \frac{\Gamma_i^{-1}}{E_i^2} \left[ (1 - 3v_s^2)E_i^2 - m_i^2 \right]^2 \]
Specific shear viscosity

Kubo $\approx$ RTA

- minimum close to the critical temperature
- pQCD limit at higher temperatures
- fast increase of the ratio $\eta/s$ for $T < T_c$
  $\Rightarrow$ lower interaction rate of the hadronic system;
  $\Rightarrow$ smaller number of degrees of freedom (or entropy density).

QGP in PHSD $\Rightarrow$ strongly-interacting liquid
Bulk viscosity (mean-field effects)

- **bulk viscosity with mean-field effects:**
  
  \[ \zeta = \frac{1}{TV} \sum_{i=1}^{N} \frac{\Gamma_i - 1}{E_i^2} \left[ \left( \frac{1}{3} - v_s^2 \right) |p|^2 - v_s^2 \left( m_i^2 - T^2 \frac{\partial m_i^2}{\partial T^2} \right) \right]^2 \]

- **DQPM expressions for masses:**

  \[ m_q^2 = \frac{1}{3} g^2 T^2, \quad m_g^2 = \frac{3}{4} g^2 T^2 \]

- **significant rise in the vicinity of critical temperature**

- **in line with the ratio from the IQCD calculations**

Bulk to shear viscosity ratio

- **without** mean-field effects: almost temperature independent behavior
- **with** mean-field effects: strong increase close to the critical temperature
Scaled variance

- Scaled variance: \[ \omega = \frac{\sigma^2}{\mu} \]
  \[ \sigma^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu)^2 \]

- Scaled variances reach a plateau in time for all observables.
- Equilibrium values are less than 1 for all observables \( \Rightarrow \) MCE.
- Particle number fluctuations are flavor blind.
Impact of total energy conservation in the sub-volume $V_n$ is less than in the total volume $V$

$\Rightarrow \omega \approx 1$ for all scaled variances for large number of cells $\Rightarrow$ GCE

- for larger box sizes by up to about a factor of 8 ($n \approx 0.15$)
- $\Rightarrow$ scaled variances reach the continuum limit
Skewness

- **skewness**: 
  \[ g_1 = \frac{m_3}{m_2^{3/2}} = \frac{m_3}{\sigma^3}, \quad m_3 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^3 \]

- **skewness** characterizes the **asymmetry** of the distribution function with respect to its average value.
Kurtosis

- **Kurtosis:** \( \beta_2 = \frac{m_4}{m_2^2} = \frac{m_4}{\sigma^4} \), \( m_4 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^4 \)

- **Kurtosis is equal to 3 for normal distribution**

**Excess Kurtosis:** \( g_2 = \beta_2 - 3 \)

Summary

- Partonic systems in PHSD achieve kinetic and chemical equilibrium in time.
- Kubo formalism and the relaxation time approximation show the same results for the shear viscosity to entropy density ratio.
- QGP in PHSD behaves as a strongly-interacting liquid.
- Significant rise of the bulk viscosity to entropy density ratio in the vicinity of the critical temperature when including the scalar mean-field from PHSD.
- Scaled variances for the different particle number fluctuations in the box reach equilibrium values in time and behave as in a micro-canonical ensemble.
- Scaled variances for all observables approach the Poissonian limit (GCE) when the cell volume is much smaller than that of the total box.
- Skewness for all observables are compatible with zero.
- Excess kurtosis is compatible with IQCD results for gluons and charged particles.
Back up
Initial momentum distributions and abundancies

- initial number of partons is given by:
  \[ N_{g(q,\bar{q})} = \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^3p}{(2\pi)^3} f_{g(q,\bar{q})}(\omega, \mathbf{p}) \]

- with a ‘thermal’ distribution:
  \[ f(\omega, \mathbf{p}) = C_i p^2 \omega \rho_i(\omega, \mathbf{p}) n_{F(B)}(\omega/T_{in}) \]

- spectral function:
  \[ \rho_i(\omega, \mathbf{p}) = \frac{\gamma_i}{E_i} \left( \frac{1}{(\omega - E_i)^2 + \gamma_i^2} - \frac{1}{(\omega + E_i)^2 + \gamma_i^2} \right) \]
  \[ = \frac{4\omega \gamma_i}{(\omega^2 - \mathbf{p}^2 - M_i^2)^2 + 4\gamma_i^2 \omega^2} \]

- Fermi and Bose distributions:
  \[ n_{F(B)} = \frac{1}{e^{(\omega - \mu)/T_{in}} \pm 1} \]

- initial parameters:
  \[ T_{in}, \mu, C_i \]

- four-momenta are distributed according to the \( f(\omega, \mathbf{p}) \) by Monte Carlo
Determination of mean-field parton potentials

Partonic potential energy density:
\[ V := T_{00,g}^- + T_{00,q}^- + T_{00,q}^- = \tilde{V}_{gg} + \tilde{V}_{qq} + \tilde{V}_{qg} \]

+ Constrain:
\[ P = \langle P_{xx} \rangle - V_s + V_0 \]
\[ \varepsilon = \langle p_0 \rangle + V_s + V_0 \]

Mean-field potential:
\[ U_s = \frac{dV_s}{d\rho_s} \quad U_0 = \frac{dV_0}{d\rho_0} \]

Parton density:
\[ \rho_p = N_g^+ + N_q^+ + N_{\bar{q}}^+, \quad N_x^+ = \tilde{T}_{r_x}^+ \]

Scalar parton density:
\[ \rho_x^s(T) = \tilde{T}_{r_x}^s \left( \sqrt{\frac{P^2}{\omega}} \right), \quad x : g, q, \bar{q} \]

\[ \rightarrow \text{PHSD} \]
Effective 2-body interactions of time-like partons

2\textsuperscript{nd} derivatives of interaction densities

\begin{align*}
\bar{v}_{gg}(\rho_p) := \frac{\partial^2 \tilde{V}_{gg}}{\partial N_g^+ \partial N_g^+} &\approx \frac{1}{2} \frac{\partial^2 (1 - \beta - \kappa) V}{\partial \rho_p^2} \left( \frac{\partial \rho_p}{\partial N_g^+} \right)^2 \\
\bar{v}_{qq}(\rho_p) := \frac{\partial^2 \tilde{V}_{qq}}{\partial (N_q^+ + N_q^-)^2} &\approx \frac{1}{2} \frac{\partial^2 (1 - \beta + \kappa) V}{\partial \rho_p^2} \left( \frac{\partial \rho_p}{\partial (N_q^+ + N_q^-)} \right)^2 \\
\bar{v}_{qg}(\rho_p) := \frac{\partial^2 \tilde{V}_{qg}}{\partial (N_q^+ + N_q^-) \partial N_g^+} &\approx \frac{\partial^2 (\beta V)}{\partial \rho_p^2} \left( \frac{\partial \rho_p}{\partial (N_q^+ + N_q^-)} \right) \left( \frac{\partial \rho_p}{\partial N_g^+} \right)
\end{align*}

effective interactions turn strongly attractive below 2.2 fm\textsuperscript{3}!
Dynamical phase transition & different initializations

- The transition from partonic to hadronic degrees-of-freedom is complete after about 9 fm/c.

- A small non-vanishing fraction of partons – local fluctuations of energy density from cell to cell.

- The equilibrium values of the parton numbers do not depend on the initial flavor ratios.

- Our calculations are stable with respect to the different initializations.
the phase transition happens at the same critical energy $\varepsilon_c$ for all $\mu_q$

in the present version the DQPM and PHSD treat the quark-hadron transition as a smooth crossover at all $\mu_q$
The dynamical spectral function is well described by the DQPM form in the fermionic sector for time-like partons.

\[ \rho_j(\omega, p) = \frac{\Gamma_j}{E_j} \left( \frac{1}{(\omega - E_j)^2 + \Gamma_j^2} - \frac{1}{(\omega + E_j)^2 + \Gamma_j^2} \right) \]
Deviation in the gluonic sector

- The inelastic collisions are more important at higher parton energies.

- The elastic scattering rate of gluons is lower than that of quarks.

- The inelastic interaction of partons generates a mass-dependent width for the gluon spectral function in contrast to the DQPM assumption of the constant width.
PHSD: Hadronization details

Local covariant off-shell transition rate for q+qbar fusion

\[ \frac{dN_m(x, p)}{d^4x d^4p} = Tr_q Tr_q \delta^4(p - p_q - p_{\bar{q}}) \delta^4 \left( \frac{x_q + x_{\bar{q}}}{2} - x \right) \]

\[ \times \omega_q \rho_q(p_q) \omega_{\bar{q}} \rho_{\bar{q}}(p_{\bar{q}}) |\nu_{q\bar{q}}|^2 W_m(x_q - x_{\bar{q}}, p_q - p_{\bar{q}}) \]

\[ \times N_q(x_q, p_q) N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}}) \delta(\text{flavor, color}). \]

using \( Tr_j = \sum_j \int d^4x_j d^4p_j / (2\pi)^4 \)

- \( N_j(x,p) \) is the phase-space density of parton \( j \) at space-time position \( x \) and 4-momentum \( p \)
- \( W_m \) is the phase-space distribution of the formed 'pre-hadrons': (Gaussian in phase space) \( \sqrt{<r^2>} = 0.66 \text{ fm} \)
- \( \nu_{q\bar{q}} \) is the effective quark-antiquark interaction from the DQPM
Why do we need broad quasiparticles?

Shear viscosity ratio to entropy density:

$$\eta^{\text{DQP}} = -\frac{d_g}{60} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n}{\partial \omega} \rho^2(\omega) \left[7\omega^4 - 10\omega^2p^2 + 7p^4\right]$$

\[\eta/s\] will be too high!