# Production of $K^+K^-$ pairs through decay of $\phi$ meson

#### Xin-Nan Zhu



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Collaborator: Xin-Li Sheng and Defu Hou

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Special topics in heavy-ion collision dynamics (Transport and Magnetohydrodynamics Meeting)

# Outline

**01** Introdution on  $\phi$  meson's spin alignment



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Relating  $K^+K^-$  pair production to  $\phi$  meson's properties

**03** Calculations in SU(3) NJL model

# Summary

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# Introdution on φ meson's spin alignment

### **Relativistic heavy-ion collision**



### Global spin polarization of $\Lambda$ hyperon



> Serves as an important probe of the system's angular momentum and vorticity dynamics in heavy-ion collisions.

STAR, Nature 548, 62 (2017)

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### Global spin alignment of $\phi$ meson



> The spin alignment of  $\phi$  meson is above 1/3, while the spin alignment of  $K^{*0}$  is consistent with 1/3.

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Spin density matrix of vector meson  $(J^P = 1^-)$ :

$$\bar{\rho}_{\lambda\lambda'} = \begin{pmatrix} \bar{\rho}_{11} & 0 & 0\\ 0 & \bar{\rho}_{00} & 0\\ 0 & 0 & \bar{\rho}_{-1,-1} \end{pmatrix}$$

 $\overline{\rho}_{\lambda\lambda}$ : the number density of particles in spin state  $\lambda$ 

Spin alignment

$$\rho_{00} = \frac{\overline{\rho}_{00}}{\sum_{\lambda=0,\pm 1} \overline{\rho}_{\lambda\lambda}}$$

- > The 00-element  $\rho_{00}$  of its normalized spin density matrix
- The probability of mesons in the spin-0 states

X.-L. Sheng, L. Oliva, Q. Wang, PRD 101, 096005 (2020); PRD 105, 099903 (2022) STAR, Nature, 614(7947):244–248, 2023.

### Meson's motion leads to energy splitting

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the rotation axis is parallel and perpendicular to

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Different resistence:  $f_a \neq f_b$ 

- Power:  $P = \vec{F} \cdot \vec{v}$
- Different applied power:  $P_a \neq P_b$

Disc in water flow	Mesons pass through QGP	
Reference system	Rest frame of meson	
Moving water flow	Moving QGP background relative to meson	
Direction of rotation axis	Spin state of mesons	
Different applied power	Different energy	

the flow direction, respectively. power

>Motion leads to mesons in different spin states having different energy.

### Energy splitting may lead to spin alignment

Meson's motion break the rotation symmetry

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The vector meson's mass will depend on its spin.

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Diagonal spin density matrix of meson in bound state:

$$f_{\lambda} \sim \frac{1}{\exp\left(M_{\phi,\lambda}/T\right) - 1}$$

Mean value of of mesons under different spin states:

$$\overline{M}_{\phi} = \frac{1}{3} \sum_{\lambda=0,\pm 1} M_{\phi,\lambda}$$

$$M_{\phi,0} = \overline{M}_{\phi} + \Delta, \quad M_{\phi,\pm 1} = \overline{M}_{\phi} - \frac{\Delta}{2},$$
Spin alignment of vector meson:  

$$\rho_{00} \equiv \frac{f_0}{f_1 + f_0 + f_{-1}} \simeq \frac{1}{3} - \frac{\Delta}{3T} \left[ 1 + \frac{1}{\exp(\overline{M}_{\Phi}/T) - 1} \right] + O\left[ \left(\frac{\Delta}{T}\right)^2 \right]$$

Production of K<sup>+</sup>K<sup>-</sup> pairs through  $\phi$  meson decay

# Relating $K^+K^$ pair production 2. to $\phi$ meson's properties

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### How to measure $\phi$ meson?



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Breit-Wigner distirbution:

$$egin{aligned} f(E) &= rac{k}{(E^2-M^2)^2+M^2\Gamma^2} \ k &= rac{2\sqrt{2}\,M\Gamma\gamma}{\pi\sqrt{M^2+\gamma}}, \quad \gamma &= \sqrt{M^2(M^2+\Gamma^2)} \end{aligned}$$

STAR, Nature 614(7947): 244–248, 2023.

#### $\phi$ (1020) DECAY MODES

	Mode	Fraction $(\Gamma_i/\Gamma)$	Scale factor/ Confidence level
Γ <sub>1</sub>	$K^+K^-$	$(49.9 \pm 0.5)$	% S=1.5
Γ <sub>2</sub>	$K_{L}^{0}K_{S}^{0}$	(33.6 ±0.4 )	% S=1.3

# How to extract $\rho_{00}$ ?



Angular distribution of daughter particles's momenta in the mother meson's rest frame:

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$$\frac{dN}{d\cos\theta^*} = \frac{3}{4} \left[ 1 - \rho_{00} + (3\rho_{00} - 1)\cos^2\theta^* \right]$$



STAR, Nature 614(7947): 244–248, 2023. Chen J, Liang Z T, Ma Y G, Wang Q, Sci.Bull. 68 (2023), 874-877.

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## $K^+K^-$ pairs production through $\phi$ meson decay

S-matrix element for scattering from an initial state i to a final state f with a  $K^+K^-$  pair:

Kaon's current:

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$$J_{\nu}^{K}(x) \equiv \int d^{4}x_{1} \int d^{4}x_{2} \varphi^{*}(x_{1}) \Gamma_{\nu}^{\text{vac}}(x;x_{1},x_{2}) \varphi(x_{2})$$

Applying  $J_{\nu}^{K}(x)$  to the state  $\langle f, K^{+}K^{-}|$   $\langle f, K^{+}K^{-}|J_{\nu}^{K}(x) = \frac{1}{2V\sqrt{E_{+}E_{-}}}\int \frac{d^{4}q}{(2\pi)^{4}}\Gamma_{\nu}^{\text{vac}}(q;p_{+},p_{-})e^{-iq\cdot x}\langle f|$ C. Gale , J. I. Kapusta , Nucl.Phys.B 357 (1991), 65-89, Nucl.Phys.B 357 (1991), 65-89. arXiv:2503.23919v1 [hep-ph].  $E_{\pm} = \sqrt{p_{\pm}^{2}} + M_{K}^{2}$ 

## $K^+K^-$ pairs production through $\phi$ meson decay

The transition probability per unit space time volume:

$$\begin{split} R_{fi} &\equiv \lim_{\tau V \to \infty} \frac{|S_{fi}|^2}{\tau V} = \int \frac{d^3 \mathbf{p}_+}{(2\pi)^3 2E_+} \frac{d^3 \mathbf{p}_-}{(2\pi)^3 2E_-} \int d^4 y \, e^{-ip \cdot y} \left\langle i \right| J_{\mu_2}^*(y/2) \left| f \right\rangle \left\langle f \right| J_{\mu_1}(-y/2) \left| i \right\rangle \\ &\times D_{\mathrm{R,vac}}^{\mu_1 \nu_1}(p) \tilde{\Gamma}_{\nu_1}^{\mathrm{vac}}(p_+, p_-) \left[ D_{\mathrm{R,vac}}^{\mu_2 \nu_2}(p) \tilde{\Gamma}_{\nu_2}^{\mathrm{vac}}(p_+, p_-) \right]^* \,, \end{split}$$

Production rate of  $K^+K^-$ :

$$n_{K^{+}K^{-}} = \sum_{f} \sum_{i} \frac{1}{Z} e^{-E_{i}/T} R_{fi} \qquad \begin{bmatrix} \text{Total energy:} \\ \omega = E_{+} + E_{-} \end{bmatrix} \qquad \begin{bmatrix} \text{Total momentum:} \\ p^{\mu} = p_{+}^{\mu} + p_{-}^{\mu} \end{bmatrix} \\ = -2 \int \frac{d^{3}\mathbf{p}_{+}}{(2\pi)^{3}2E_{+}} \frac{d^{3}\mathbf{p}_{-}}{(2\pi)^{3}2E_{-}} n_{B}(\omega) \tilde{\Gamma}_{\mu}^{\text{vac}*}(p_{+}, p_{-}) \rho^{\mu\nu}(p) \tilde{\Gamma}_{\nu}^{\text{vac}}(p_{+}, p_{-}) \end{bmatrix}$$

Spectral function of  $\phi$  meson:

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$$\rho^{\mu\nu}(p) = -\left[D_{\mathrm{R,vac}}^{\alpha\mu}(p)\right]^* \left[\mathrm{Im}\Pi_{\alpha\beta}^{\mathrm{R}}(p)\right] D_{\mathrm{R,vac}}^{\beta\nu}(p)$$

Retarded current-current correlator:

$$\Pi^{\rm R}_{\mu\nu}(p) \equiv -i \int d^4 y \,\theta(y^0) e^{-ip \cdot y} \left\langle \left[ J_{\mu}(y), J_{\nu}(0) \right] \right\rangle$$

### **Relating the production rate to spin alignmnet**

The momentum integral in the rest frame of  $K^+K^-$ :  $M_{\phi} = \sqrt{\omega^2 - p^2}$ 

$$d^{3}\mathbf{p}_{+}d^{3}\mathbf{p}_{-} = \frac{1}{2}E_{+}E_{-}\sqrt{1 - \frac{4M_{K}^{2}}{M_{\phi}^{2}}}d^{4}p\,\sin\theta^{*}d\theta^{*}d\phi^{*}$$

Differential production rate:

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$$\frac{dn_{K^+K^-}}{d^4p\,d\cos\theta^*d\phi^*} = -\frac{1}{4(2\pi)^6}\sqrt{1-\frac{4M_K^2}{M_\phi^2}}n_B(\omega)\tilde{\Gamma}_{\mu}^{\rm vac\,*}(p_+,p_-)\rho^{\mu\nu}(p)\tilde{\Gamma}_{\nu}^{\rm vac}(p_+,p_-)$$

Project the spectral function into spin space

$$\rho^{\mu\nu}(p) = -\sum_{\lambda=0,\pm 1} \epsilon^{\mu}(\lambda,p) \epsilon^{*\nu}(\lambda',p) \xi_{\lambda\lambda'}(p)$$
  
Spin polarization vectors:  
$$\epsilon^{\mu}(\lambda,p) = \left(\frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_{\lambda}}{M_{\phi}}, \boldsymbol{\epsilon}_{\lambda} + \frac{\mathbf{p} \cdot \boldsymbol{\epsilon}_{\lambda}}{M_{\phi}(\omega + M_{\phi})} \mathbf{p}\right)$$

The spectral function can be decomposed into longitudinal and transverse components as  $\epsilon_{\rm H}^{\mu} = \frac{1}{M_{\phi}} \left( |\mathbf{p}|, \omega \frac{\mathbf{p}}{|\mathbf{p}|} \right)$ 

$$\rho^{\mu\nu}(p) = -\epsilon^{\mu}_{\mathrm{H}}(p)\epsilon^{\nu}_{\mathrm{H}}(p)\rho_{L}(p) + \left[g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^{2}} + \epsilon^{\mu}_{\mathrm{H}}(p)\epsilon^{\nu}_{\mathrm{H}}(p)\right]\rho_{T}(p)$$

Production of K<sup>+</sup>K<sup>-</sup> pairs through  $\phi$  meson decay

### **Relating the production rate to spin alignmnet**

The differential production rate experssed with  $\rho_L$  and  $\rho_T$ 

$$\frac{dn_{K^+K^-}}{d^4p\,d\cos\theta^*d\phi^*} = \frac{1}{4(2\pi)^6} \sqrt{1 - \frac{4M_K^2}{M_\phi^2}} n_B(\omega) \left\{ \left| \epsilon_{\rm H}^{\mu}(p) \tilde{\Gamma}_{\mu}^{\rm vac}(p_+, p_-) \right|^2 \left[ \rho_L(p) - \rho_T(p) \right] \right. \\ \left. - \tilde{\Gamma}_{\mu}^{\rm vac}(p_+, p_-) \tilde{\Gamma}_{\nu}^{\rm vac}(p_+, p_-) \left( g^{\mu\nu} - \frac{p^{\mu}p^{\nu}}{p^2} \right) \rho_T(p) \right\} ,$$

The effective three meson vertex in vacuum:

$$\tilde{\Gamma}^{\rm vac}_{\mu}(p_+, p_-) = q_{\mu}\Gamma^{\rm vac}_1(p_+, p_-) + p_{\mu}\Gamma^{\rm vac}_2(p_+, p_-)$$
$$p_{\mu}\epsilon^{\mu}_H(p) = 0, \quad p_{\mu}\left(g^{\mu\nu} - p^{\mu}p^{\nu}/p^2\right) = 0$$

Final differential production rate:

$$\frac{dn_{K^+K^-}}{d^4p\,d\cos\theta^*d\phi^*} \propto \left[q_\mu\epsilon_{\rm H}^\mu(p)\right]^2 \left[\rho_L(p) - \rho_T(p)\right] - q^2\rho_T(p)$$

Spin alignment:

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$$\rho_{00}(p) = \frac{\rho_L(p)}{\rho_L(p) + 2\,\rho_T(p)}$$

# Calculation in SU(3) Nambu-Jonas-Lasinio model

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### Properties of SU(3) NJL model

- **¤** Chiral symmetry
- $-SU(3)_L \times SU(3)_R \to SU(3)_V$
- $\langle q \overline{q} \rangle \neq 0 \rightarrow \text{pion/kaon}$
- Free quark states exist

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- Requires a truncation scheme3D hardcut, Pauli-Villars,...
- Coupling constant G fitted to light meson masses

- Tree-level QCD NJL via integrating out gluons
- Approximates gluon exchange with local 4-fermion interaction





contract interaction

### Lagrangian density of SU(3) NJL model

Lagrangian density :

scalar interaction

$$\mathcal{L} = \bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m)\psi + \left[G_{S}\sum_{a=0}^{8}\left[(\bar{\psi}\lambda_{a}\psi)^{2} + (\bar{\psi}i\gamma_{5}\lambda_{a}\psi)^{2}\right]\right] + \left[G_{V}\sum_{a=0}^{8}\left[(\bar{\psi}\gamma_{\mu}\lambda_{a}\psi)^{2} + (\bar{\psi}i\gamma_{\mu}\gamma_{5}\lambda_{a}\psi)^{2}\right]\right] \text{ vector interaction} \\ - \left[K\left[\det\bar{\psi}(1+\gamma_{5})\psi + \det\bar{\psi}(1-\gamma_{5})\psi\right]\right], \text{ six point KMT interaction}$$

Lagrangian density under the Mean Field Approximation:

$$\mathcal{L}_{\rm MF} = \sum_{f=u,d,s} \bar{\psi}_f (i\gamma_\mu \partial^\mu - M_f) \psi_f - 2G_S \sum_{f=u,d,s} \sigma_f^2 + 4K\sigma_u \sigma_d \sigma_s$$

Chiral condensate:  $\sigma_f = \langle \psi_f \bar{\psi}_f \rangle$ 

Dynamic quark mass:  $M_f \equiv m_f - 4G_S\sigma_f + 2K \prod_{f' \neq f} \sigma_{f'}$ 

U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).

T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994).

V. Bernard, R. L. Jaffe, and U. G. Meissner, Nucl. Phys. B 308, 753 (1988).

S. Klimt, M. F. M. Lutz, U. Vogl, and W. Weise, Nucl. Phys. A 516, 429 (1990).

### Kaon and quark-meson coupling $g_{Kq\bar{s}}$

Random phase approximation:



Kaon's self-energy:

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P. Rehberg, S.P. Klevansky, J. Hufner, Rev.C 53 (1996), 410-429.S.P. Klevansky, Rev.Mod.Phys. 64 (1992), 649-708.

### Quark mass, kaon mass and $g_{Kq\bar{s}}$



### **Triangle graph**

Effective three-meson vertex:

 $\left( + \frac{1}{u \cdot p} \epsilon^{\mu\nu\alpha\beta} q_{\nu} p_{\alpha} u_{\beta} \Gamma_4(p_+, p_-) \right)$ 

$$q^{\mu} = p^{\mu}_{+} - p^{\mu}_{-}$$
 $p^{\mu}_{-} = p^{\mu}_{+} + p^{\mu}_{-}$ 

Simplified three-meson vertex :

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$$\tilde{\Gamma}^{\mu}(p_{+},p_{-}) \approx (p_{+}^{\mu} - p_{-}^{\mu})\Gamma_{1}(p_{+},p_{-}) \equiv (p_{+}^{\mu} - p_{-}^{\mu})\Gamma_{\mathrm{on}}(M_{\phi},|\mathbf{p}|)$$

Y.B. He, J. Hufner, S.P. Klevansky, P. Rehberg, Nucl. Phys. A 630 (1998), 719-742.

### **Triangle graph**



If the temperature is high enough, we will have  $\sqrt{M_{\phi}^2 + p^2} < 2M_K$ , the kaon loop does not contribute to the imaginary part of  $\phi$  meson's self energy.

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### $\phi$ meson 's self-energy

Dyson-Schwinger equation:

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Y.B. He, J. Hufner, S.P. Klevansky, P. Rehberg, Nucl.Phys.A 630 (1998), 719-742.M. Oertel, M. Buballa, and J. Wambach, Nucl. Phys. A 676, 247 (2000).M. Oertel, M. Buballa, and J. Wambach, Phys. Atom. Nucl. 64, 698 (2001).M. Oertel, M. Buballa, and J. Wambach, Phys. Lett. B 477, 77 (2000).

### $\phi$ meson 's self-energy

Quark loop contribution(LO):

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Y.B. He, J. Hufner, S.P. Klevansky, P. Rehberg, Nucl. Phys. A 630 (1998), 719-742.

### $\phi$ meson 's self-energy

Kaon tadpole contribition(NLO):

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$$\left[p^{\mu}\left[\Pi_{\mu\nu}^{\mathrm{K-tau}}(p)+\Pi_{\mu\nu}^{\mathrm{K-tau}}(p)\right]\right]$$

$$\Pi_{\mu\nu}^{\text{K-tad}}(p) = -g_{\mu\nu}\frac{p^{\alpha}p^{\beta}}{p^{2}}\Pi_{\alpha\beta}^{\text{K-loop}}(p)$$

Decompose the self-energy into longitidinal mode and transverse mode:

$$\Pi_{\rm tot}^{\mu\nu}(p) = -\epsilon_{H}^{\mu}\epsilon_{H}^{\nu}\Pi_{L}(p) + (g^{\mu\nu} - p^{\mu}p^{\nu}/p^{2} + \epsilon_{H}^{\mu}\epsilon_{H}^{\nu})\Pi_{T}(p)$$



Production of K<sup>+</sup>K<sup>-</sup> pairs through  $\phi$  meson decay

### **Invariant mass spectrum**

Longitidinal / Transverse polarized propagator of  $\phi$  meson:

$$D_{L/T}(p) = \frac{4G_V}{1 + 4G_V \Pi_{L/T}^{\text{tot}}(p)}$$

Spectral functions for longitudinally and transversely polarized modes

$$\rho_{L/T}(p) = -\left|\frac{4G_V}{1 + 4G_V \Pi_{\text{vac}}^{\text{tot}}(p)}\right|^2 \operatorname{Im} \Pi_{L/T}^{\text{tot}}(p)$$

Invariant mass spectrum of  $K^+K^-$  pair :





#### Numerical result for $\phi$ meson's spin alignment Spin alignment

$$\overline{\rho}_{00}(\mathbf{p}) - \frac{1}{3} = \frac{2 \int_{M_{\min}}^{M_{\max}} dM_{\phi} \delta f(p)}{3 \int_{M_{\min}}^{M_{\max}} dM_{\phi} \left[3 f_T(p) + \delta f(p)\right]}$$

Auxiliary function:

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$$\begin{pmatrix} f_T(p)\\\delta f(p) \end{pmatrix} \equiv \frac{1}{\omega} n_B(\omega) (M_{\phi}^2 - M_{K,\text{vac}}^2)^{3/2} \left| \Gamma_{\text{on}}^{\text{vac}}(M_{\phi}) \right|^2 \begin{pmatrix} \rho_T(p)\\\rho_L(p) - \rho_T(p) \end{pmatrix}$$





### Summary

### **Key Findings**

 $\blacksquare$  Derived analytical expression for differential K<sup>+</sup>K<sup>-</sup> production rate.

 $\square$  At zero temperature, the invariant mass spectrum agrees with the experimental data observed in experiments.

□ Mass spectrum of  $K^+K^-$  pair is nearly temperature-independent for T ≤ 0.1 GeV, but quark coalescence broadens it significantly at higher T (disagreeing with data).

 $\square$  Kaon loop corrections in self-energy open physical decay channels, giving  $\phi$  meson a finite width.

# Implication

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 $\blacksquare$  Provides quantitative description of  $\phi$  meson behavior in QCD matter.



# Thank you for listening