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Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA
UNIVERSITÀ DEGLI STUDI FIRENZE DIPARTIMENTO DI FISICA E STRONOMIA

Recent progress on spin hydrodynamics

David Wagner

based mainly on

Sapna, S. K. Singh, DW, 2503.22552 (2025)
DW, Phys.Rev.D 111 (2025) 1, 016008
DW, M. Shokri, D. H. Rischke, Phys.Rev.Res. 6 (2024) 4, 4
DW, N. Weickgenannt, D. H. Rischke, Phys.Rev.D 106 (2022) 11, 116021

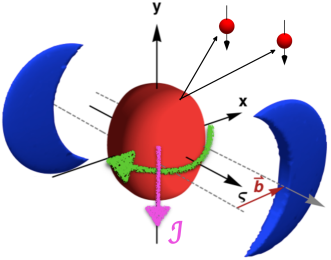
02.04.2025

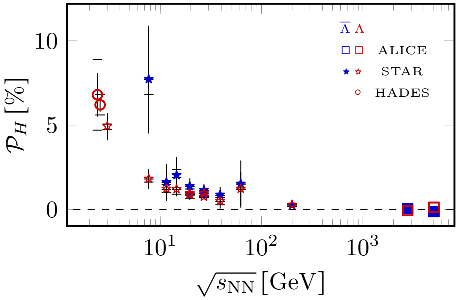
INFN FIRENZE
CRC-TR 211 Strong-interaction matter under extreme conditions
DFG
EL EM EN TS

Overview

Motivation & open questions

Global Λ-Polarization

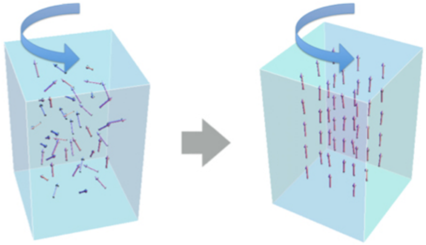




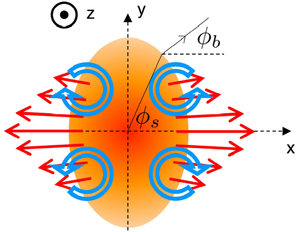
- "Global": Integrated polarization along the direction of orbital angular momentum
- Can be explained by assuming spins in equilibrium

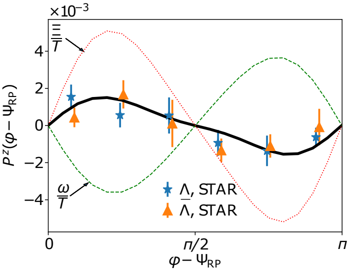
→ "Polarization through rotation"

- Analogous to Barnett effect



Local Λ-Polarization



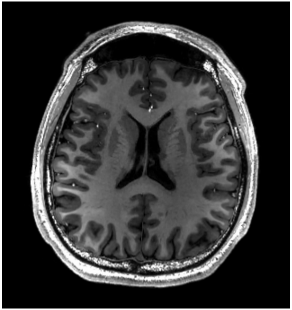


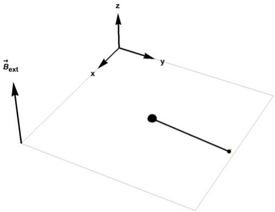
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, PRL 127, 272302 (2021)

- "Local": Angle-dependent polarization along beam-direction
- Can only be explained by incorporating shear effects
  - $\longrightarrow$ Simple picture of equilibrated spins not complete
- Possible answer: develop a theory of **spin hydrodynamics** to describe polarization dynamics

Analogy: Magnetic resonance imaging (MRI)

- MRI: Large constant B-field in z-direction and short-lived alternating field in x, y-plane
- Identify materials by relaxation times T_1, T_2





https://en.wikipedia.org/wiki/Bloch_equations

Bloch equations

$$T_2 \dot{M}_{x,y} + M_{x,y} = \mu_2 \left(\mathbf{M} \times \mathbf{B}\right)_{x,y} \;,$$
$$T_1 \dot{M}_z + M_z = \mu_1 \left(\mathbf{M} \times \mathbf{B}\right)_z + M_0 \;.$$

$\mu_1 := T_1 \frac{gq}{2m}$, $\mu_2 := T_2 \frac{gq}{2m}$

Spin hydrodynamics

Spin hydrodynamics: Basics

- Theory is based on conservation laws
- Uncharged fluid: consider energy-momentum tensor and total angular momentum tensor

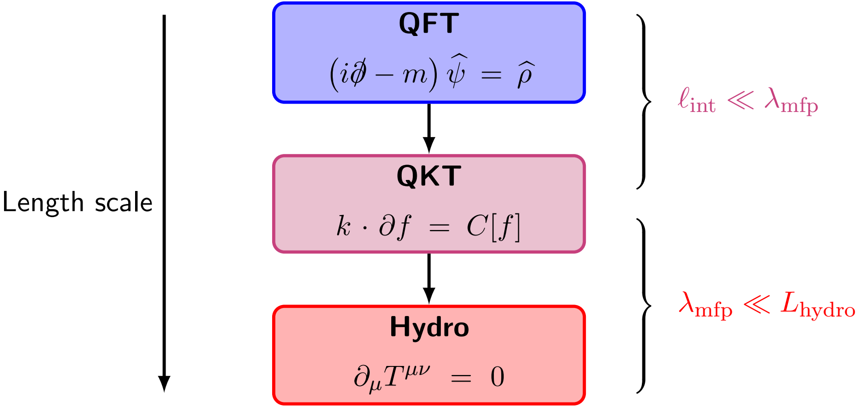
Conservation equations

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\lambda J^{\lambda\mu\nu} =: \hbar \partial_\lambda S^{\lambda\mu\nu} + T^{[\mu\nu]} = 0$$

- **10** equations for **16**+**24** quantities
  - Underdetermined system
  - Additional information about dissipative quantities has to be provided
- Here: Use *quantum kinetic theory* as the microscopic basis

$A^{[\mu}B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$


Length scale
QFT
$(i\partial\!\!\!/ - m)\,\widehat{\psi} = \widehat{\rho}$
QKT
$k \cdot \partial f = C[f]$
Hydro
$\partial_\mu T^{\mu\nu} = 0$
$\ell_{\rm int} \ll \lambda_{\rm mfp}$
$\lambda_{\rm mfp} \ll L_{\rm hydro}$

QKT: Boltzmann equation

DW, NW, DHR, Phys.Rev.D 106 (2022) 11, 116021

Boltzmann equation with collisions

$$k \cdot \partial f(x, k, \mathfrak{s}) = \frac{1}{2} \int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma_2 \mathrm{d}\Gamma' \delta^{(4)}(k_1 + k_2 - k - k') \mathcal{W}$$
$$\times \Big[f(x + \Delta_1 - \Delta, k_1, \mathfrak{s}_1) f(x + \Delta_2 - \Delta, k_2, \mathfrak{s}_2)$$
$$\times \widetilde{f}(x, k, \mathfrak{s}) \widetilde{f}(x + \Delta' - \Delta, k', \mathfrak{s}')$$
$$- \widetilde{f}(x + \Delta_1 - \Delta, k_1, \mathfrak{s}_1) \widetilde{f}(x + \Delta_2 - \Delta, k_2, \mathfrak{s}_2)$$
$$\times f(x, k, \mathfrak{s}) f(x + \Delta' - \Delta, k', \mathfrak{s}') \Big] \ .$$

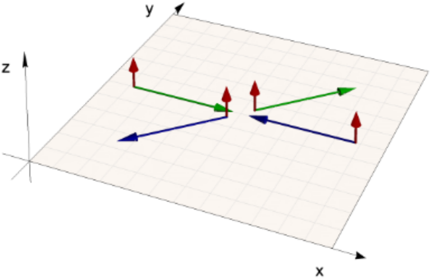
- Equivalence up to first order in quantum corrections:

$$f(x, k, \mathfrak{s}) + \Delta^\mu \partial_\mu f(x, k, \mathfrak{s}) \approx f(x + \Delta, k, \mathfrak{s})$$

- A (momentum- and spin-dependent) **spacetime shift** Δ^μ enters
  $\rightarrow$ Particles do not scatter at the same spacetime point!

$\mathrm{d}\Gamma := 2\mathrm{d}^4 k \delta(k^2 - m^2) \mathrm{d}S(k)$, $\widetilde{f} := 1 - f$

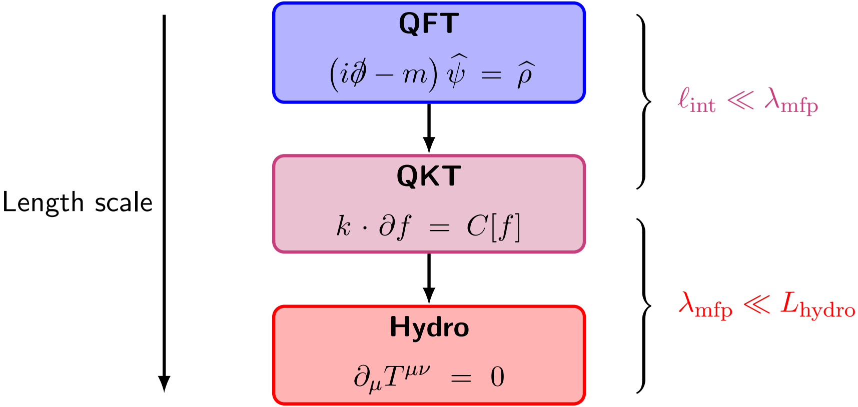
Nonlocal collisions



W. Florkowski, A. Kumar, R. Ryblewski, Prog. Part. Nucl. Phys. 108, 103709 (2019)

- Assume that collisions take place in a point
  - $\rightarrow$ Total orbital angular momentum vanishes
  - $\rightarrow$ Spin is conserved on its own
  - $\rightarrow$ No exchange of spin and orbital angular momenta
- Collisions must be **nonlocal** for spin equilibration!
- Becomes manifest through a spacetime shift Δ^μ that is fixed by the microscopic interaction

Spin hydrodynamics: Procedure



“Semiclassical” spin hydrodynamics & equilibrium

Conservation equations

$$\partial_\mu T^{(\mu\nu)} = 0 + \mathcal{O}(\hbar^2) ,$$
$$\partial_\lambda S^{\lambda\mu\nu} = \frac{1}{\hbar} T^{[\nu\mu]} .$$

- No backreaction of spin on fluid evolution, fluid profile serves as input for spin potential
- “Ideal” fluid: Determined through local equilibrium, i.e., by
  $$f_{\text{eq}} = \left[\exp\left(-\alpha_0 + \beta_0 u \cdot k + \tfrac{\hbar}{4m}\epsilon^{\mu\nu\alpha\beta}\Omega_{0,\mu\nu} k_\alpha \mathfrak{s}_\beta\right) + 1\right]^{-1}$$
- “Ideal” spin evolution determined by **spin potential**
  $\Omega_0^{\mu\nu} = u^{[\mu}\kappa_0^{\nu]} + \epsilon^{\mu\nu\alpha\beta} u_\alpha \omega_{0,\beta}$

Beyond equilibrium: Moment method

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f(x,k,\mathfrak{s})$$

$$\tau_r^{\mu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f(x,k,\mathfrak{s})$$

- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

$k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1}\cdots k^{\nu_\ell}$

Beyond equilibrium: Moment method

- Split distribution function $f = f_{\text{eq}} + \delta f$
- Perform moment expansion including spin degrees of freedom

Irreducible moments

Standard dissipation

$$\rho_r^{\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s})$$

$$\tau_r^{\mu,\mu_1\cdots\mu_\ell}(x) := \int \mathrm{d}\Gamma \mathfrak{s}^\mu E_{\mathbf{k}}^r k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} \delta f(x,k,\mathfrak{s})$$

Spin dissipation

- Equations of motion can be derived from Boltzmann equation
- Knowing the evolution of all moments is equivalent to solving the Boltzmann equation

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$k^{\langle\mu_1} \cdots k^{\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} k^{\nu_1} \cdots k^{\nu_\ell}$

Resumming (spin) hydrodynamics: IReD

DW, A. Palermo, V. E. Ambruș, Phys. Rev. D **106**, 016013 (2022)
DW, Phys.Rev.D 111 (2025) 1, 016008)

- Basic idea: Power-counting scheme to second order in
  - Knudsen number $\mathrm{Kn} := \lambda_{\mathrm{mfp}}/L_{\mathrm{hydro}}$
  - inverse Reynolds numbers $\mathrm{Re}^{-1} \sim \delta f/f_{\mathrm{eq}}$
- Derive asymptotic (Navier-Stokes) relations to close the system

Asymptotic matching (example)

$$\rho_r^{\mu\nu} = \eta_r \sigma^{\mu\nu} + \mathcal{O}(\mathrm{Kn}\mathrm{Re}^{-1}) = \frac{\eta_r}{\eta_0}\pi^{\mu\nu} + \mathcal{O}(\mathrm{Kn}\mathrm{Re}^{-1})$$

- The same procedure can be done for the moments $\tau_r^{\mu,\mu_1\cdots\mu_\ell}$
- Many moments can be related to ω_0^μ and κ_0^μ
  - No need to introduce more dynamical quantities
- Exception: tensor-valued moments $t_r^{\mu\nu} := \tau_{r,\alpha,\beta}{}^{\langle\mu}\epsilon^{\nu\rangle\alpha\beta\rho}u_\rho$
  - Additional dynamical quantity $\mathfrak{t}^{\mu\nu}$ is needed, $S^{\lambda\mu\nu} \sim \mathfrak{t}^{\lambda[\mu}u^{\nu]}$

Dissipative spin hydrodynamics

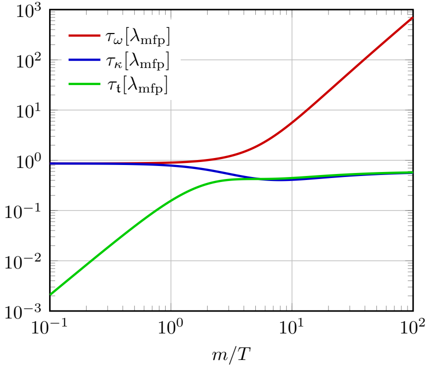
DW, Phys.Rev.D 111 (2025) 1, 016008

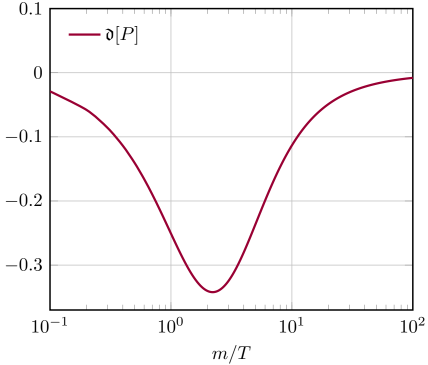
$$\begin{aligned}
\tau_\omega \dot{\omega}_0^{\langle\mu\rangle} + \omega_0^\mu &= -\beta_0 \omega^\mu + \delta_{\omega\omega}\omega_0^\mu \theta + \lambda_{\omega\omega}\sigma^{\mu\nu}\omega_{0,\nu} + \lambda_{\omega\mathfrak{t}}\mathfrak{t}^{\mu\nu}\omega_\nu \\
&\quad + \epsilon^{\mu\nu\alpha\beta}u_\nu \left(\ell_{\omega\kappa}\nabla_\alpha \kappa_{0,\beta} - \tau_\omega \dot{u}_\alpha \kappa_{0,\beta} + \lambda_{\omega\kappa} I_\alpha \kappa_{0,\beta}\right) \\
\tau_\kappa \dot{\kappa}_0^{\langle\mu\rangle} + \kappa_0^\mu &= -\beta_0 \dot{u}^\mu + \mathfrak{b} I^\mu + \delta_{\kappa\kappa}\kappa_0^\mu \theta + \left(\lambda_{\kappa\kappa}\sigma^{\mu\nu} + \frac{\tau_\kappa}{2}\omega^{\mu\nu}\right)\kappa_{0,\nu} \\
&\quad + \epsilon^{\mu\nu\alpha\beta}u_\nu \left(\frac{\tau_\kappa}{2}\nabla_\alpha \omega_{0,\beta} + \tau_\kappa \dot{u}_\alpha \omega_{0,\beta} + \lambda_{\kappa\omega} I_\alpha \omega_{0,\beta}\right) \\
&\quad + \mathfrak{t}^{\mu\nu}\left(\tau_{\kappa\mathfrak{t}}\dot{u}_\nu + \lambda_{\kappa\mathfrak{t}} I_\nu\right) + \ell_{\kappa\mathfrak{t}}\Delta^\mu_\lambda \nabla_\nu \mathfrak{t}^{\nu\lambda} \\
\tau_\mathfrak{t} \dot{\mathfrak{t}}^{\langle\mu\nu\rangle} + \mathfrak{t}^{\mu\nu} &= \mathfrak{d}\beta_0 \sigma^{\mu\nu} + \delta_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}^{\mu\nu}\theta + \lambda_{\mathfrak{t}\mathfrak{t}}\mathfrak{t}_\lambda{}^{\langle\mu}\sigma^{\nu\rangle\lambda} + \frac{5}{3}\tau_\mathfrak{t}\mathfrak{t}_\lambda{}^{\langle\mu}\omega^{\nu\rangle\lambda} + \ell_{\mathfrak{t}\kappa}\nabla^{\langle\mu}\kappa_0^{\nu\rangle} \\
&\quad + \lambda_{\mathfrak{t}\kappa} I^{\langle\mu}\kappa_0^{\nu\rangle} + \tau_{\mathfrak{t}\omega}\omega^{\langle\mu}\omega_0^{\nu\rangle} + \lambda_{\mathfrak{t}\omega}\sigma_\lambda{}^{\langle\mu}\epsilon^{\nu\rangle\lambda\alpha\beta}u_\alpha \omega_{0,\beta}
\end{aligned}$$

$I^\mu := \nabla^\mu \alpha_0$

Relaxation times and first-order coefficient

DW, Phys.Rev.D 111 (2025) 1, 016008

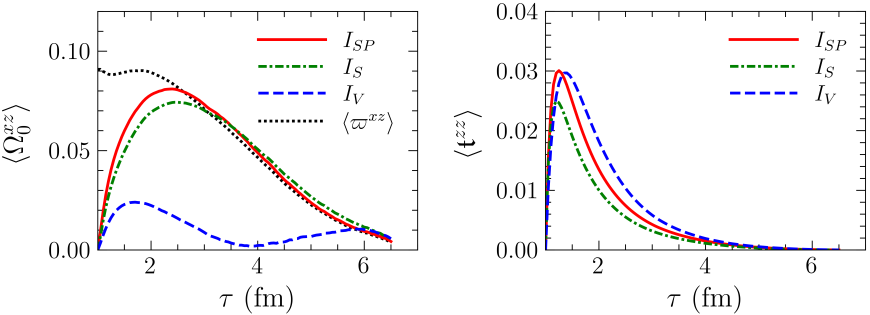




- τ_ω grows with z^2 compared to τ_κ and $\tau_{\rm t}$
- $\tau_{\rm t}$ vanishes for $z \to 0$

Numerical results

Sapna, S.K. Singh, DW, 2503.22552 (2025)



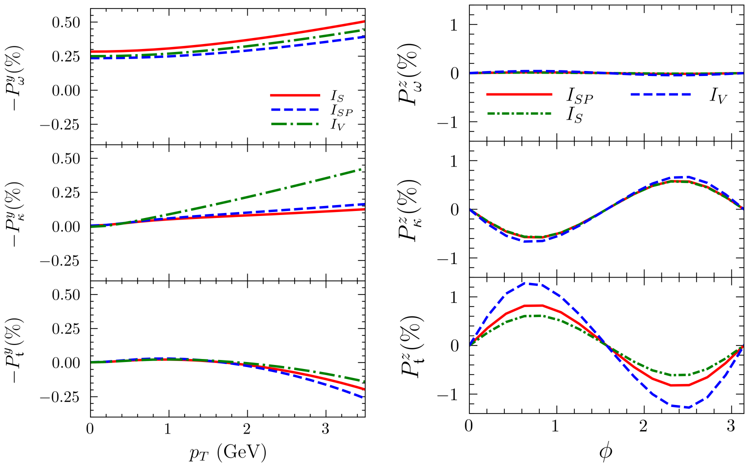
$$SP: \mathcal{L}_{\text{int}} \sim G\left[\left(\overline{\psi}\psi\right)^2 - \left(\overline{\psi}\gamma_5\psi\right)^2\right] ,$$

$$S: \mathcal{L}_{\text{int}} \sim G\left(\overline{\psi}\psi\right)^2 ,$$

$$V: \mathcal{L}_{\text{int}} \sim G\left(\overline{\psi}\gamma^\mu\psi\right)\left(\overline{\psi}\gamma_\mu\psi\right) .$$

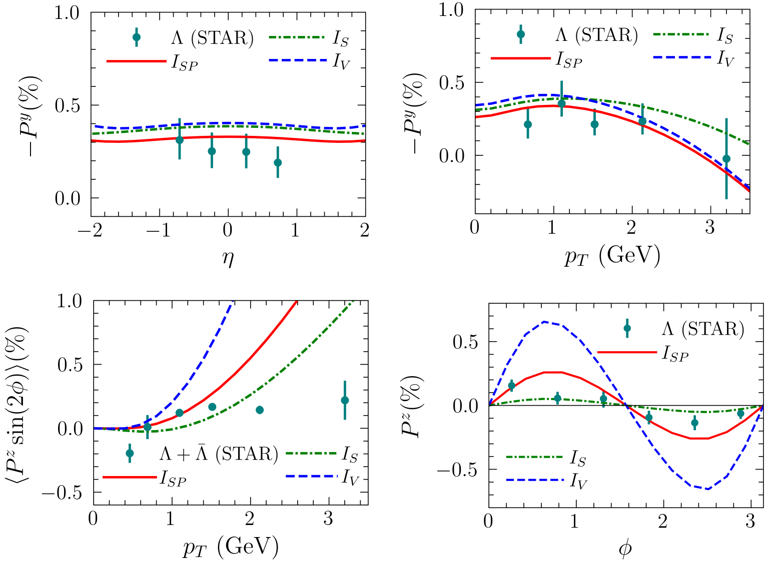
Polarization contributions

Sapna, S.K. Singh, DW, 2503.22552 (2025)



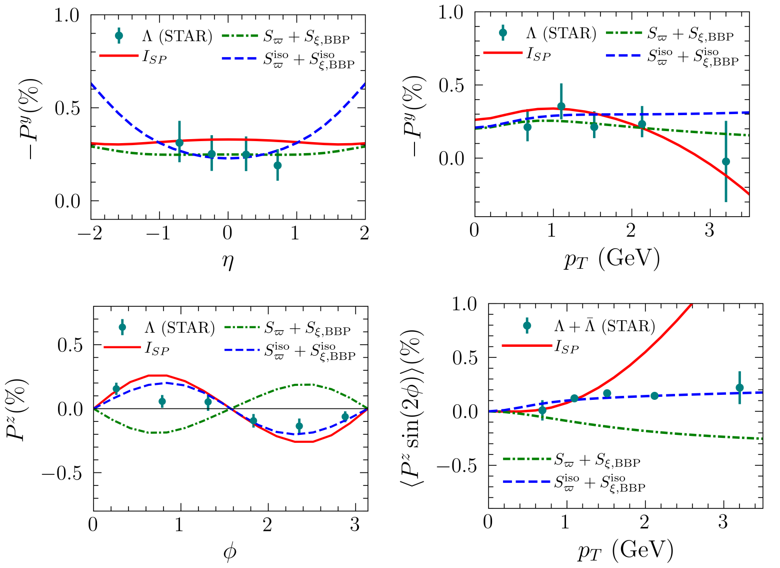
Polarization results

Sapna, S.K. Singh, DW, 2503.22552 (2025)



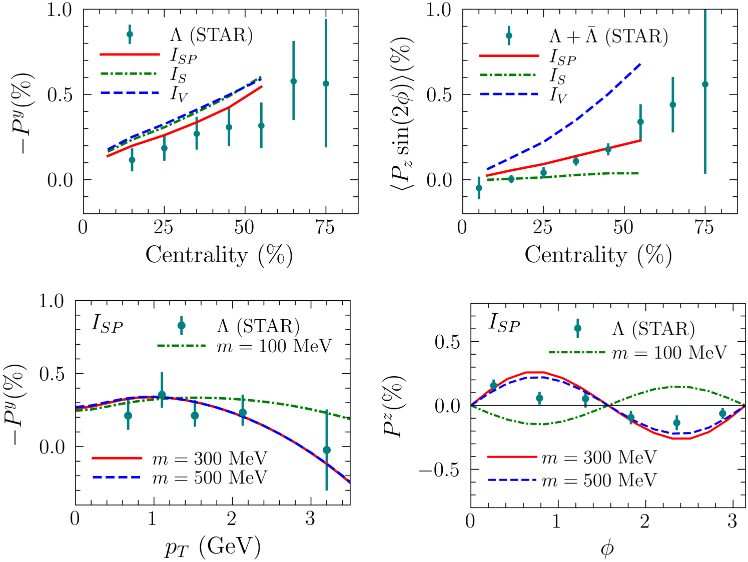
Comparison to BBP

Sapna, S.K. Singh, DW, 2503.22552 (2025)



Centrality and mass dependence

Sapna, S.K. Singh, DW, 2503.22552 (2025)

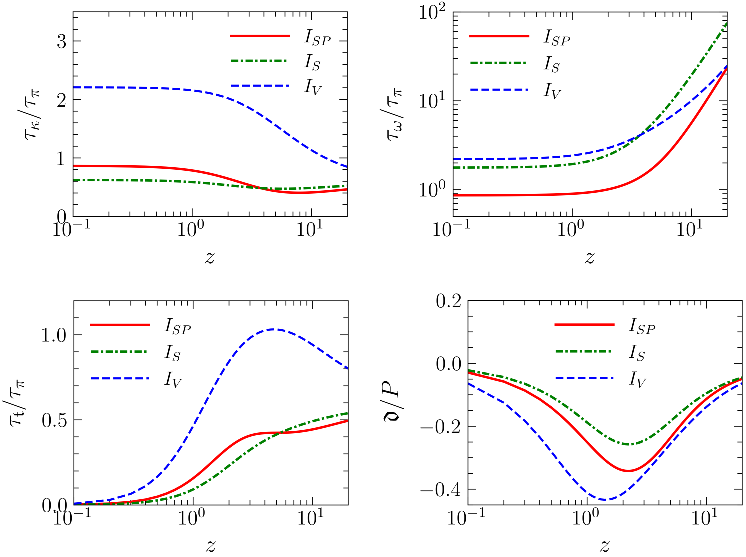


Summary and outlook

- Resummed spin hydrodynamics was derived from QKT
  - Main result: Only the quantities ω_0^μ, κ_0^μ, $\mathfrak{t}^{\mu\nu}$ need dynamical treatment
  - All transport coefficients have been computed
- Numerical implementation shows promising results
  - Global polarization rather robust
  - Local polarization depends on interaction, correct sign can be reproduced
- Future avenues:
  - Study of the spin dynamics in various setups
  - Expected to become especially relevant at lower collision energies

Appendix

Sapna, S.K. Singh, DW, 2503.22552 (2025)



Conserved currents in QKT

Conserved currents

$$\frac{1}{2}T^{(\mu\nu)} = \int \mathrm{d}\Gamma k^\mu k^\nu f \ ,$$

$$S^{\lambda\mu\nu} = \frac{1}{2m}\int \mathrm{d}\Gamma k^\lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta f \ .$$

$$T^{[\mu\nu]} = \frac{1}{2}\int [\mathrm{d}\Gamma] \widetilde{\mathcal{W}} \Delta^{[\mu} k^{\nu]} \left(f_1 f_2 - f f'\right)$$

Conservation laws

$$\int \mathrm{d}\Gamma k^\mu C[f] = 0$$

$$\frac{\hbar}{2m}\int \mathrm{d}\Gamma \epsilon^{\mu\nu\alpha\beta} k_\alpha \mathfrak{s}_\beta C[f] = \frac{\hbar}{m}\int \frac{\mathrm{d}^4 k}{(2\pi\hbar)^4} k^{[\mu} \mathcal{D}_{\mathcal{V}}^{\nu]}$$

$[\mathrm{d}\Gamma] \coloneqq \mathrm{d}\Gamma_1\, \mathrm{d}\Gamma_2\, \mathrm{d}\Gamma\, \mathrm{d}\Gamma'$

Polarization observables in kinetic theory

Vector Polarization (Pauli-Lubanski Pseudovector)

$$S^\mu(k) \coloneqq \mathrm{Tr}\left[\hat{S}^\mu\,\hat{\rho}(k)\right] = \frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s})$$

Tensor Polarization

$$\begin{aligned}
\rho_{00}(k) &= \frac{1}{3} - \sqrt{\frac{2}{3}}\epsilon^{(0)}_\mu(k)\epsilon^{(0)}_\nu(k)\Theta^{\mu\nu}(k)\\
\Theta^{\mu\nu}(k) &\coloneqq \frac{1}{2}\sqrt{\frac{3}{2}}\mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3}K^{\mu\nu}\right)\hat{\rho}(k)\right]\\
&= \frac{1}{2}\sqrt{\frac{3}{2}}\frac{1}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda \int \mathrm{d}S(k) K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta f(x,k,\mathfrak{s})
\end{aligned}$$

$N(k) \coloneqq \int \mathrm{d}\Sigma_\gamma k^\gamma \int \mathrm{d}S(k) f(x,k,\mathfrak{s})$, $\quad \hat{S}^\mu \coloneqq -(1/2m)\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta$

Polarization in spin hydrodynamics

Local Polarization

$$S_0^\mu = \frac{2\sigma^2\hbar}{N(k)m}\int \mathrm{d}\Sigma_\lambda k^\lambda \left(u^\mu \omega_0^\nu k_\nu - E_{\mathbf{k}}\omega_0^\mu + \epsilon^{\mu\nu\alpha\beta}u_\nu k_\alpha \kappa_{0,\beta}\right) f_0 \widetilde{f}_0$$

$$\delta S^\mu = -\frac{2\sigma}{N(k)}\int \mathrm{d}\Sigma_\lambda k^\lambda K^{\mu\gamma}\Xi_{\gamma\alpha} f_0 \widetilde{f}_0 \times \left(\mathfrak{x}_n \epsilon^{\alpha\beta\rho\sigma}u_\beta k_\rho n_\sigma + \mathfrak{x}_\mathfrak{t} \mathfrak{t}_\rho{}^{\langle\beta}\epsilon^{\gamma\rangle\alpha\sigma\rho}u_\sigma k_{\langle\beta}k_{\gamma\rangle}\right)$$

Global Polarization

$$\overline{S}_0^\mu = -\frac{2\sigma^2\hbar}{\overline{N}m}\int \mathrm{d}\Sigma_\lambda \left(J_{21}u^\mu \omega_0^\lambda + J_{20}\omega_0^\mu u^\lambda + J_{21}\epsilon^{\mu\nu\lambda\beta}u_\nu \kappa_{0,\beta}\right)$$

$$\delta\overline{S}^\mu = \frac{\sigma}{\overline{N}}\frac{1}{2}\int \mathrm{d}\Sigma_\lambda\, B_0 \epsilon^{\mu\lambda\alpha\beta}u_\alpha n_\beta$$

$\mathfrak{x}_n := \frac{1}{2}\sum_n \mathcal{H}_{\mathbf{k}n}^{(1,1)}\frac{\mathfrak{b}_n^{(1)}}{\varkappa}$, $\mathfrak{x}_\mathfrak{t} := \frac{2}{3}\sum_n \mathcal{H}_{\mathbf{k}n}^{(1,2)}\frac{\mathfrak{d}_n}{\mathfrak{d}_0}$

Nonlocal collisions

DW, NW, ES, 2306.05936 (2023)

Spacetime shifts

$$\Delta^\mu \quad := \quad -\frac{i\hbar}{8m}\frac{m^4}{\mathcal{W}} M^{\gamma_1\gamma_2\delta_1\delta_2} M^{\zeta_1\zeta_2\eta_1\eta_2} h_{1,\gamma_1\eta_1} h_{2,\gamma_2\eta_2} h'_{\zeta_2\delta_2} [h, \gamma^\mu]_{\zeta_1\delta_1}$$

- Depend on the transfer-matrix elements

$$\langle 11' | \, \widehat{t} \, | 22' \rangle = \bar{u}_{1,\alpha} \bar{u}_{1',\beta} u_{2,\gamma} u_{2',\delta} M^{\alpha\beta\gamma\delta}$$

- Manifestly covariant
  - $\rightarrow$ no "no-jump" frame

$h := \frac{1}{4}(\mathbb{1} + \gamma_5 \not{s})(\not{k} + m)$

Appendix: Some interactions

Scalar interaction

$$M_{\alpha\alpha'\alpha_1\alpha_2} = \frac{2G}{\hbar}\left(\delta_{\alpha\alpha_1}\delta_{\alpha'\alpha_2} - \delta_{\alpha\alpha_2}\delta_{\alpha'\alpha_1}\right)$$

Thermal gluon exchange

$$M_{\alpha\alpha'\alpha_1\alpha_2} = \frac{2g}{\hbar}\left[\gamma^{\mu}_{\alpha\alpha_1}\frac{g_{\mu\nu}}{(k-k_1)^2 - m_{\text{th}}^2}\gamma^{\nu}_{\alpha'\alpha_2} - \gamma^{\mu}_{\alpha\alpha_2}\frac{g_{\mu\nu}}{(k-k_2)^2 - m_{\text{th}}^2}\gamma^{\nu}_{\alpha'\alpha_1}\right]$$

- $m_{\text{th}} = \sqrt{2N_C + N_f}gT/(3\sqrt{2})$

Moment equations: Spin-rank 1

- Same procedure as for the moments of spin-rank 0

Moment equation for $\ell = 2$

$$\dot{\tau}_r^{\langle\mu\rangle,\nu\lambda} - \mathfrak{C}_{r-1}^{\langle\mu\rangle,\nu\lambda} = \cdots$$

- Navier-Stokes limit: $\mathfrak{C}_{r-1}^{\langle\mu\rangle,\nu\lambda} = 0$
- Contains local and nonlocal contributions
  - $\mathfrak{C}_{r,\text{local}}^{\langle\mu\rangle,\nu\lambda} \sim \tau_r^{\mu,\nu\lambda}$
  - $\mathfrak{C}_{r,\text{nonlocal}}^{\langle\mu\rangle,\nu\lambda} \sim \sigma_\rho{}^{\langle\nu}\epsilon^{\lambda\rangle\mu\alpha\rho}u_\alpha$
- Leads to shear-induced polarization, coefficient independent of total cross-section
- Magnitude not yet clear

N. Weickgenannt, DW, E. Speranza, D. H. Rischke, Phys. Rev. D 106, L091901 (2022)
N. Weickgenannt, DW, E. Speranza, D. H. Rischke, Phys. Rev. D 106, 096014 (2022)