Vorticity and polarization in SMASH via a coarse-grained approach

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Motivation
STAR Collaboration (at RHIC) findings in 2017

- Angular momentum of order $1000\hbar$ (in non-central collisions)
- Alignment between angular momentum and spin of emitted particles
- $\Lambda$ (and $\bar{\Lambda}$) hyperons chosen for analysis
  - Proton of weak decay ($\Lambda \rightarrow p + \pi^-$) tends to be emitted along spin direction of $\Lambda$ (self-analyzing)

L. Adamczyk et al. (STAR), Nature 548, 62 (2017)
SMASH transport approach
Simulating Many Accelerated Strongly-interacting Hadrons

- Relativistic hadronic transport approach based on
  
  Relativistic Boltzmann equation
  \[ p^\mu \partial_\mu f_i(x, p) + m_i F^\alpha_{\alpha} \partial^\alpha f_i(x, p) = C^t_{\text{coll}} \]
  (no potentials used in this work, hence \( F^\alpha = 0 \))

  Geometrical collision criterion
  \[ d_{\text{trans}} < d_{\text{int}} = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}} \]

- Includes all hadrons from the PDG (2018) up to \( m \sim 2.35 \text{ GeV} \)

- Publicly available at [www.smash-transport.github.io](http://www.smash-transport.github.io)


Global polarization of $\Lambda$ hyperons

Global polarization

\[ P = \frac{\langle S^* \rangle \cdot J}{|\langle S^* \rangle| \cdot |J|} \quad P = 2 \cdot \langle S^* \rangle_y \]


$\Lambda$ spin 4-vector

\[ S^\mu(x, p) = -\frac{1}{8m} e^{\mu\nu\rho\sigma} p_\nu \sigma_{\rho\sigma}(x) \]


Thermal vorticity

\[ \sigma_{\mu\nu} = \frac{1}{2} (\partial_\nu \beta_\mu - \partial_\mu \beta_\nu) \quad \text{with} \quad \beta_\mu = \frac{u_\mu}{T} \]

L. Adamczyk et al. (STAR), Nature 548, 62 (2017)
Coarse-grained approach

• Space divided into many small cells
  ▷ Done by SMASH which provides thermodynamic output
    \( T^{\mu\nu}, \text{Landau velocity, } j^\mu_Q, j^\mu_B, j^\mu_S \)

• Averaging over many events calculating vorticity and spins of emitted \( \Lambda \)
  ▷ Done by codes written by Gabriele Inghirami
    Publicly available at www.github.com/gabriele-inghirami/
Global angular momentum in SMASH

- SMASH-3.0 shows high global angular momentum in line with STAR statement of order $1000\hbar$ (in the first few Fermi)

Picture
R. Snellings,
New J. Phys. 13,
055008 (2011)
Angular momentum per octant
Coordinate space

- Au+Au collisions at \( b = 6 \text{ fm} \) and \( \sqrt{s_{NN}} = 7.7 \text{ GeV} \)

Collision sketch (modified)
Z. Liang et al.,
arXiv:nucl-th/0410079v5

Cube sketch (modified)
https://de-academic.com/dic.nsf/dewiki/1045183

Octant 4 \((x \geq 0, y < 0, z \geq 0)\)
Octant 1 \((x \geq 0, y \geq 0, z \geq 0)\)
Octant 3 \((x < 0, y < 0, z \geq 0)\)
Octant 2 \((x < 0, y \geq 0, z \geq 0)\)

- \(-L_y\) vs. \(t\) in [fm]
- \(L_{\text{tot}}\), \(L_r\), \(L_{\text{ap}}\)

SMASH-3.0
Angular momentum per octant

Momentum space

- Au+Au collisions at $b = 6$ fm and $\sqrt{s_{NN}} = 7.7$ GeV

https://de-academic.com/dic.nsf/dewiki/1045183
Hyperon decay in SMASH

- Weak & electromagnetic decays in SMASH disabled by default
- Following decays do not occur and mother particles are stable in SMASH
  - \( \Lambda \rightarrow p + \pi^- \)
  - \( \Sigma^0 \rightarrow \Lambda + \gamma \)
  - \( \Xi^0 \rightarrow \Lambda + \pi^0 \)
  - \( \Xi^- \rightarrow \Lambda + \pi^- \)
  - \( \Omega \) decays into \( \Lambda, \Xi^0 \) or \( \Xi^- \)
\( \Lambda \) and \( \bar{\Lambda} \) emission time evolution

- **Chemical freeze-out**: Point in time where last inelastic collision of a particle takes place

- **Kinetic freeze-out**: Interaction point of last elastic collision

L. Kumar, Nucl. Phys. A 931 (2014)
$\Lambda$ and $\Lambda + \Sigma^0$ emission time evolution

Chemical and kinetic freeze-out compared to UrQMD data

$\Lambda + \Sigma^0 + \Xi^0 + \Xi^- + \Omega$ emission time evolution

Chemical and kinetic freeze-out compared to UrQMD data

\( \Lambda + \Sigma^0 \) production yields

SMASH, UrQMD, and NA49 experimental data

- Deviation by factor of 2 from UrQMD and experimental data
  - Strangeness production in SMASH too low…
    V. Steinberg et al., Phys. Rev. C 99.6 (2019), 064908
  - … leading to lower hyperon yields

UrQMD data: H. Petersen et al., arXiv:0805.0567v1 [hep-ph]
NA49 data: C. Blume et al., Journal of Physics G: Nuclear and Particle Physics 31.6 (2005), s685
Vorticity in the reaction plane

Au+Au, $\sqrt{s_{NN}} = 7.7$ GeV, $b = 6$ fm

UrQMD EoS used to calculate vorticity
Vorticity in the reaction plane

\[ \text{Au+Au}, \sqrt{s_{NN}} = 7.7 \text{ GeV}, b = 6 \text{ fm} \]

UrQMD EoS used to calculate vorticity
Vorticity in the reaction plane

\[ \text{Au} + \text{Au}, \sqrt{s_{NN}} = 7.7 \text{ GeV}, b = 6 \text{ fm}, t = 15 \text{ fm} \]

- “Central area” of SMASH and UrQMD vorticity similar

- “Upper and lower area” different

- Vorticity dependent on used equation of state

UrQMD EoS used to calculate SMASH vorticity

Global Hyperon Polarization

- Computation of polarization with time and spatial emission points of the $\Lambda$ hyperons and vorticity of fluid cells

Global polarization

$$P = \frac{\langle S^* \rangle \cdot J}{|\langle S^* \rangle| \cdot |J|} \quad \frac{|\langle S^* \rangle| = 1/2}{|J| \approx J_y} \quad P = 2 \cdot \langle S^* \rangle_y$$


$\Lambda$ spin 4-vector

$$S^\mu(x, p) = -\frac{1}{8m} \varepsilon^{\mu\nu\rho\sigma} p_\nu \sigma_{\rho\sigma}(x)$$


Thermal vorticity

$$\varpi_{\mu\nu} = \frac{1}{2} (\partial_\nu \beta_\mu - \partial_\mu \beta_\nu) \quad \text{with} \quad \beta_\mu = \frac{u_\mu}{T}$$
Global Hyperon Polarization

Feed-down contributions

Chemical freeze-out

K arterialm, Au+Au, $b = 6$ fm, $\sqrt{s_{NN}} = 7.7$ GeV

$P_H$ vs $t$ (fm)

- $\Lambda$
- $\Lambda + \Sigma^0$
- $\Lambda + \Sigma^0 + \Xi^0 + \Xi^- + \Omega^-$

Kinetic freeze-out

$P_H$ vs $t$ (fm)

- $\Lambda$
- $\Lambda + \Sigma^0$
- $\Lambda + \Sigma^0 + \Xi^0 + \Xi^- + \Omega^-$
Global $\Lambda$ Polarization

Chemical vs. kinetic freeze-out

- Feed-down contributions negligible
  - Only $\Lambda$ polarization taken into account
- Chemical freeze-out polarization higher than kinetic one
  - Possible explanation: Vorticity drop per cell due to system expansion $\Rightarrow$ lower “kinetic freeze-out” polarization
Global $\Lambda$ Polarization
SMASH-3.0 vs. UrQMD

- Possible reasons for differences:
  - EoS has impact on vorticity  
    $\Rightarrow$ impact on polarization
  - Last elastic and inelastic interaction dependent on cross sections leading to a shift in emission times of the hyperons  
    $\Rightarrow$ impact on polarization
Summary and outlook

• Summary
  ▶ Global angular momentum and per octant meet expectations
  ▶ Polarization dependent on freeze-out definition and EoS
  ▶ Feed-down contributions from $\Sigma^0 + \Xi^0 + \Xi^- + \Omega$ negligible for polarization
  ▶ SMASH’s global $\Lambda$ polarization differs significantly from UrQMD’s

• Outlook
  ▶ Study the vorticity and polarization dependence of different EoS
  ▶ Investigate the impact of the difference in cross sections between SMASH and UrQMD
Backup Slides
SMASH transport approach
Simulating Many Accelerated Strongly-interacting Hadrons

- Effective solution of the relativistic Boltzmann equation
  \[ p^\mu \partial_\mu f_i(x,p) + m_i F^\alpha \partial_\alpha f_i(x,p) = C_{coll}^i \]

- Geometrical collision criterion
  \[ d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}} \quad \text{with} \quad d_{trans}^2 = (\vec{r}_a - \vec{r}_b)^2 - \frac{((\vec{r}_a - \vec{r}_b) \cdot (\vec{p}_a - \vec{p}_b))^2}{(\vec{p}_a - \vec{p}_b)^2} \]

- Includes all hadrons from the PDG(2018) up to \( m \sim 2.35 \) GeV

- Publicly available at www.smash-transport.github.io

DOI 10.5281/zenodo.3484711

Global polarization of $\Lambda$ hyperons

- $\Lambda$ spin 4-vector at space-time point $x$
  $$S^\mu(x, p) = -\frac{1}{8m}e^{\mu\nu\rho\sigma}p_\nu\sigma(x)$$

- Thermal vorticity
  $$\omega_{\mu\nu} = \frac{1}{2}(\partial_\nu \beta_\mu - \partial_\mu \beta_\nu)$$
  with $\beta_\mu = \frac{u_\mu}{T}$

- $\Lambda$ spin 4-vector in local rest frame
  $$S^*(x, p) = S - \frac{p \cdot S}{E_p(m + E_p)} \cdot p$$
  with $E_p = \sqrt{p^2 + m^2}$

- Average of over all $\Lambda$'s emitted
  $$\langle S^* \rangle = \frac{1}{N} \sum_{i=1}^{N} S^*(x_i, p_i)$$

- Global polarization
  $$p = \frac{\langle S^* \rangle \cdot J}{|\langle S^* \rangle| \cdot |J|}$$

Global angular momentum in SMASH

- Angular momentum conservation is violated in SMASH (as in all transport approaches)
  - Geometrical collision criterion enables instantaneous interactions over finite distances

Geometrical collision criterion

\[ d_{\text{trans}} < d_{\text{int}} = \sqrt{\frac{\sigma_{\text{tot}}}{\pi}} \]

Angular momentum per octant

Coordinate space, Au+Au collisions at $b = 6$ fm and $\sqrt{s_{NN}} = 7.7$ GeV

SMASH-3.0
Angular momentum per octant

Momentum space, Au+Au collisions at $b = 6$ fm and $\sqrt{s_{NN}} = 7.7$ GeV

SMASH-3.0
\( \Lambda \) and \( \Lambda + \Sigma^0 \) emission time evolution

Chemical freeze-out

**Chemical freeze-out**

\[
dN/dt [1/fm] \\
0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \quad 300
\]

**Chemical freeze-out**

\[
dN/dt [1/fm] \\
0 \quad 50 \quad 100 \quad 150 \quad 200 \quad 250 \quad 300
\]

SMASH-3.0, Au+Au, \( b = 6 \text{ fm} \)

- \( \Lambda \) @ 7.7 GeV
- \( \Lambda \) @ 11.5 GeV
- \( \Lambda \) @ 14.5 GeV
- \( \Lambda \) @ 19.6 GeV
- \( \Lambda + \Sigma^0 \) @ 7.7 GeV
- \( \Lambda + \Sigma^0 \) @ 11.5 GeV
- \( \Lambda + \Sigma^0 \) @ 14.5 GeV
- \( \Lambda + \Sigma^0 \) @ 19.6 GeV

And emission time evolution
Λ and ¯Λ emission time evolution

Chemical vs. kinetic freeze-out
\( \Lambda + \Sigma^0 \) and \( \bar{\Lambda} + \bar{\Sigma}^0 \) emission time evolution

Chemical vs. kinetic freeze-out

**Chemical freeze-out**

**Kinetic freeze-out**
\( \Lambda \) and \( \bar{\Lambda} \) production yields

SMASH vs. UrQMD vs. NA49 experimental data

UrQMD data: H. Petersen et al., arXiv:0805.0567v1 [hep-ph]
NA49 data: C. Blume et al., Journal of Physics G: Nuclear and Particle Physics 31.6 (2005), s685
Vorticity in the reaction plane

- SMASH hadron resonance gas EoS
  - Au+Au, $\sqrt{s_{NN}} = 7.7$ GeV, $b = 6$ fm
  - Reaction plane: $\overrightarrow{p}_{\text{beam}} \times \overrightarrow{b}$
SMASH settings

• Angular momentum $N = 2000$:
  ▶ Global Fermi motion frozen, per octant Fermi motion off

• $\Lambda$ emission time evolution $N = 10^6$

• Vorticity and polarization plots:
  ▶ $\sqrt{s_{NN}} = 7.7$ GeV $N = 50000$, $\sqrt{s_{NN}} = 14.5$ GeV $N = 30000$

• $\Lambda$ rapidity spectra $N = 2000$

• Fermi motion “frozen” was used if not stated otherwise