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Towards Constraints on the Equation of State using SMASH

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Transport Meeting

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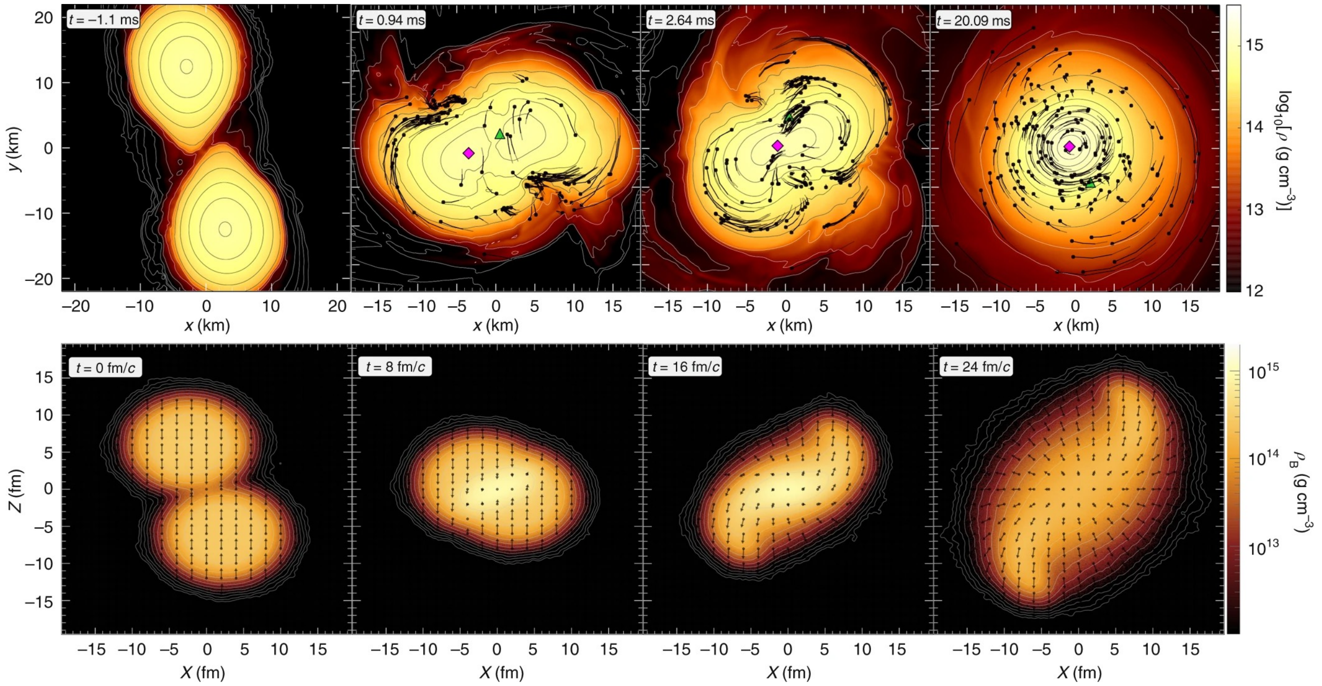
GSI

Motivation

- Gravitational waves from neutron star mergers renewed interest in equation of state of nuclear matter

- Heavy ion collisions produce nuclear matter under similar conditions as mergers

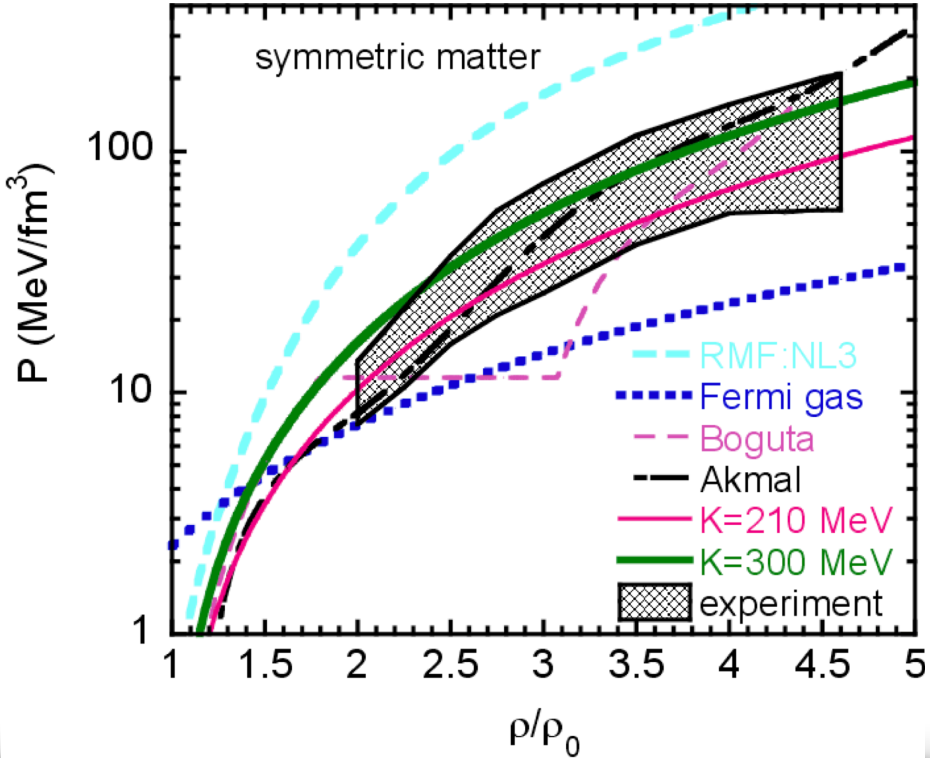
- Constrain the equation of state from high precision data from heavy ions



HADES Nature Phys. 15 (2019)

Equation of State from Transport

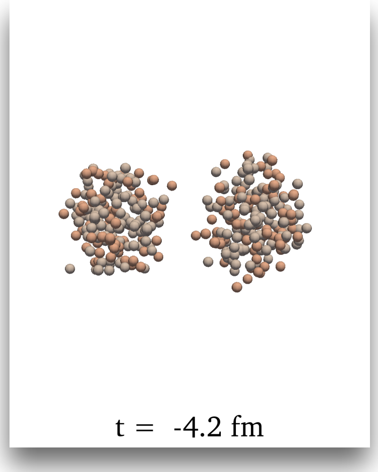
- Equation of state enters transport calculations via nuclear potentials
- Compare to experimental data to constrain parameters of the potential
- Sensitive experimental observables are anisotropic flow of protons and sub-threshold strangeness production



Danielewicz et al.
Science 298 (2002)

Transport Model SMASH

- Based on relativistic Boltzmann equation
- Hadron degrees of freedom including resonances from Particle Data Group
- Collisions between hadrons according to geometric collision criterion $d_{\text{trans}} < \sqrt{\sigma/\pi}$
- Publicly available at smash-transport.github.io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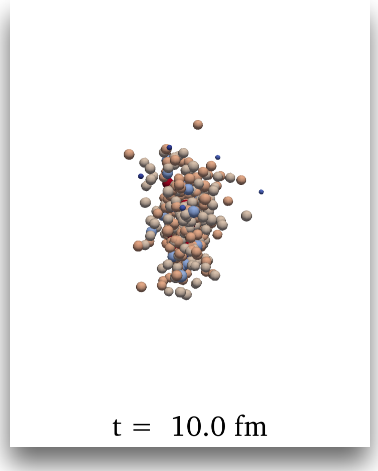
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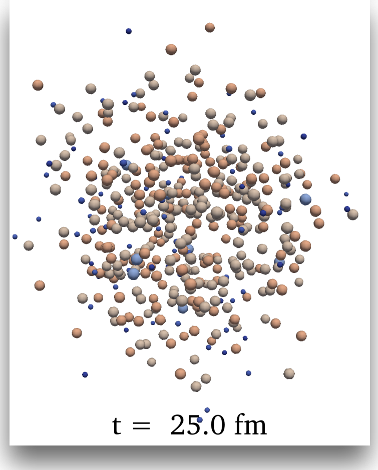

t = 25.0 fm

smash

Potentials in SMASH

- Equation of state enters the calculation with nuclear potentials

$$U_{\mathrm{Sk}} = A\left(\frac{\rho_B}{\rho_0}\right) + B\left(\frac{\rho_B}{\rho_0}\right)^{\tau}$$

- Basic Skyrme potential for the baryon density dependence

$$U_{\mathrm{Sym}} = \pm\, 2S_{\mathrm{pot}}\frac{\rho_{I_3}}{\rho_0}$$

- Symmetry potential counteracts an excess of neutrons or protons
$I_3 = (n_{\mathrm{up}} - n_{\mathrm{down}})/2$

Momentum-Dependent Potentials

- Nuclear potential should include a momentum dependence

- Implement the parametrisation by Welke et al.

  G. M. Welke et al. Phys.Rev.C 38 (1988)
  Used in GiBUU: O. Buss et al. Phys.Rept. 512 (2012)

$$U(\mathbf{r},\mathbf{p}) = \underbrace{\boxed{A\frac{\rho(\mathbf{r})}{\rho_0} + B\left(\frac{\rho(\mathbf{r})}{\rho_0}\right)^{\tau}}}_{\text{Skyrme Potential}} + \underbrace{\boxed{\frac{2C}{\rho_0} g \int \frac{d^3p'}{(2\pi)^3} \frac{f(\mathbf{r},\mathbf{p}')}{1+\left(\frac{\mathbf{p}-\mathbf{p}'}{\Lambda}\right)^2}}}_{\text{Momentum-dependent part}}$$

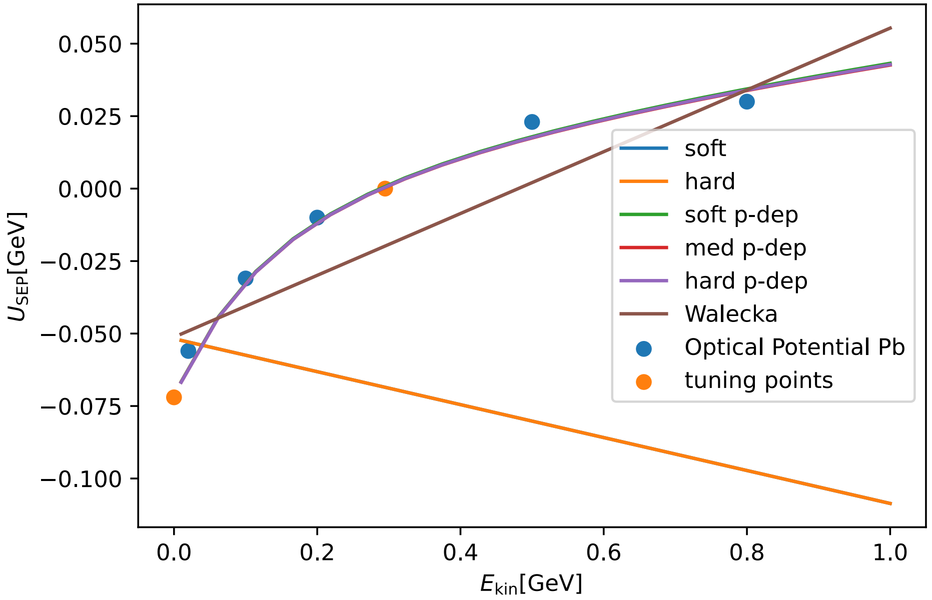
- Integral is simplified assuming cold nuclear matter: $f(\mathbf{r},\mathbf{p}) = \Theta(p_F - p)$

- Single particle energy evaluated in local rest-frame for equation of motion $\dot{\mathbf{p}} = -\nabla E$

Parameter Estimation

- Binding energy at saturation density
- Minimum of the binding energy at saturation density
- Two parameters of the momentum-dependent part constrained by the optical potential
- Require a given incompressibility

$$\left(\frac{E_b}{A}\right)_{\rho=\rho_0} = -16\,\mathrm{MeV} \qquad \frac{\partial}{\partial\rho}\left(\frac{E_b}{A}\right)_{\rho=\rho_0} = 0$$



$$\kappa = 9\rho^2 \frac{\partial^2}{\partial\rho^2}\left(\frac{E_b}{A}\right)_{\rho=\rho_0}$$

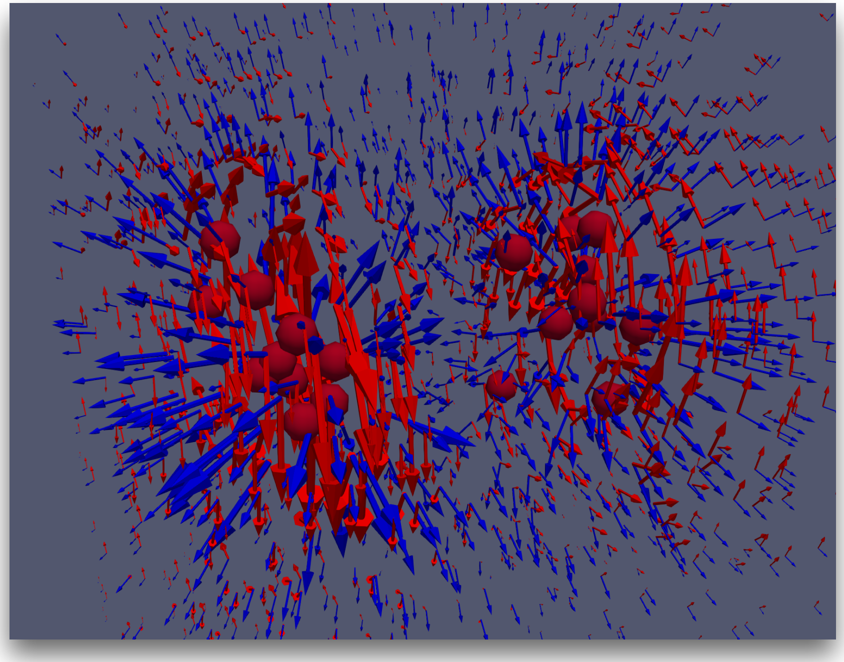
Coulomb Potential

- Assume stationary current for simplicity
- Calculate fields by integrating over a lattice:

$$\mathbf{E}(\mathbf{r}) = \int \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dV' \text{ and}$$

$$\mathbf{B}(\mathbf{r}) = \int \mathbf{j}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} dV'$$

- Update momenta using Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$



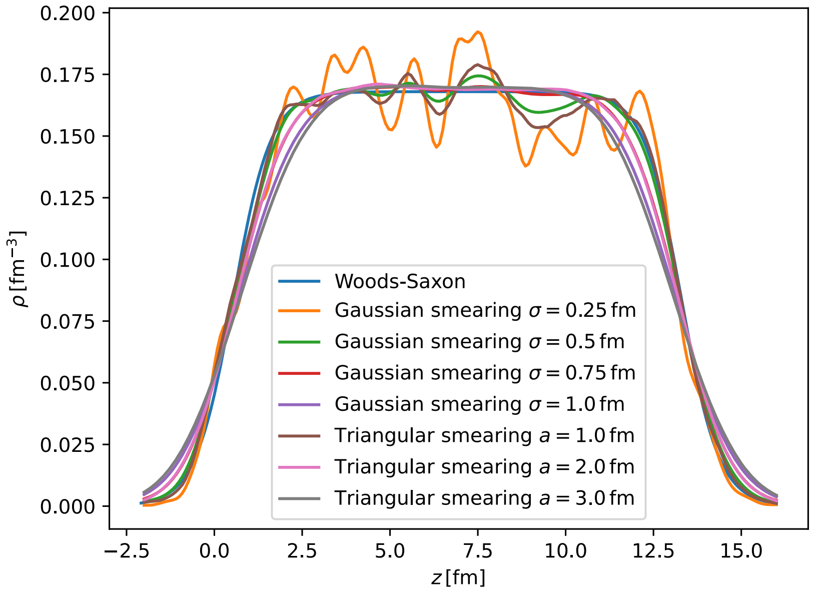
Electric (blue) and magnetic (fields) in a SMASH calculation

Density Calculation

- Apply covariant Gaussian smearing
- Average over parallel ensembles
- Smearing width should be large enough for stable calculation
- Can’t reproduce large gradients if too wide

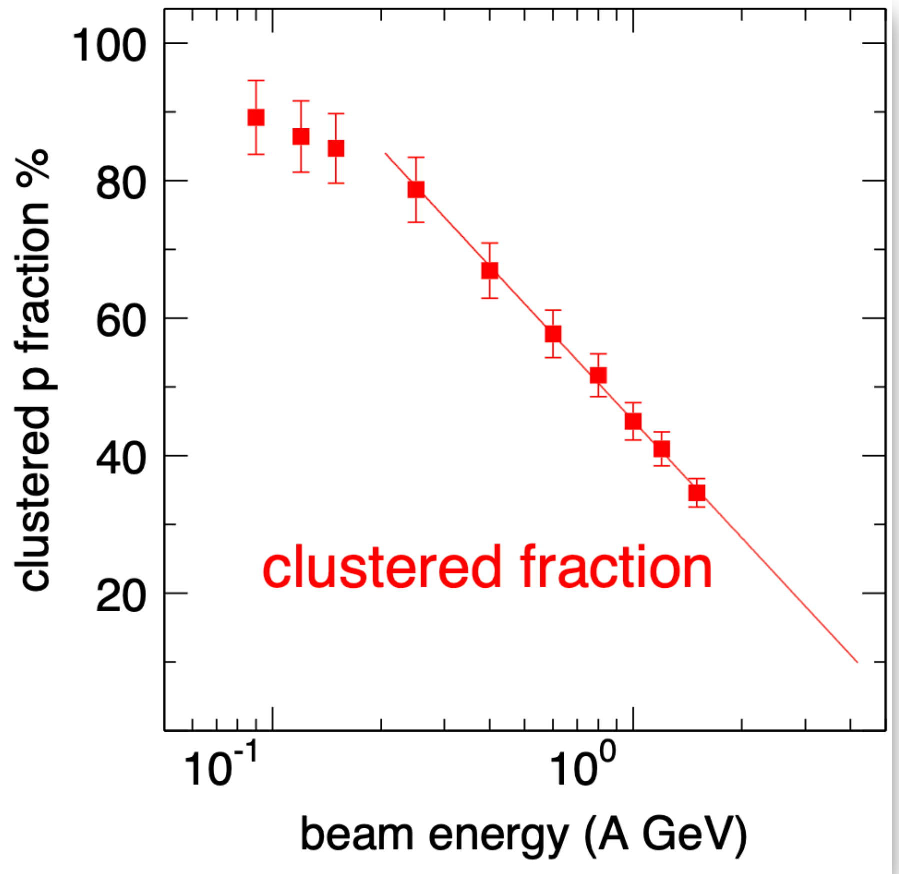
$$f(\mathbf{r},\mathbf{p}) = \frac{1}{N_{\text{test}}} \sum_{i=1}^{N_{\text{test}}} K(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{p}-\mathbf{p_i})$$

$$K(\mathbf{r}) = (2\pi\sigma^2)^{-\frac{3}{2}}\gamma \exp\left(-\frac{r^2+(\mathbf{r}\cdot\mathbf{u})^2}{2\sigma^2}\right)$$



Gold nucleus with 1000 testparticles

Light Nuclei Formation

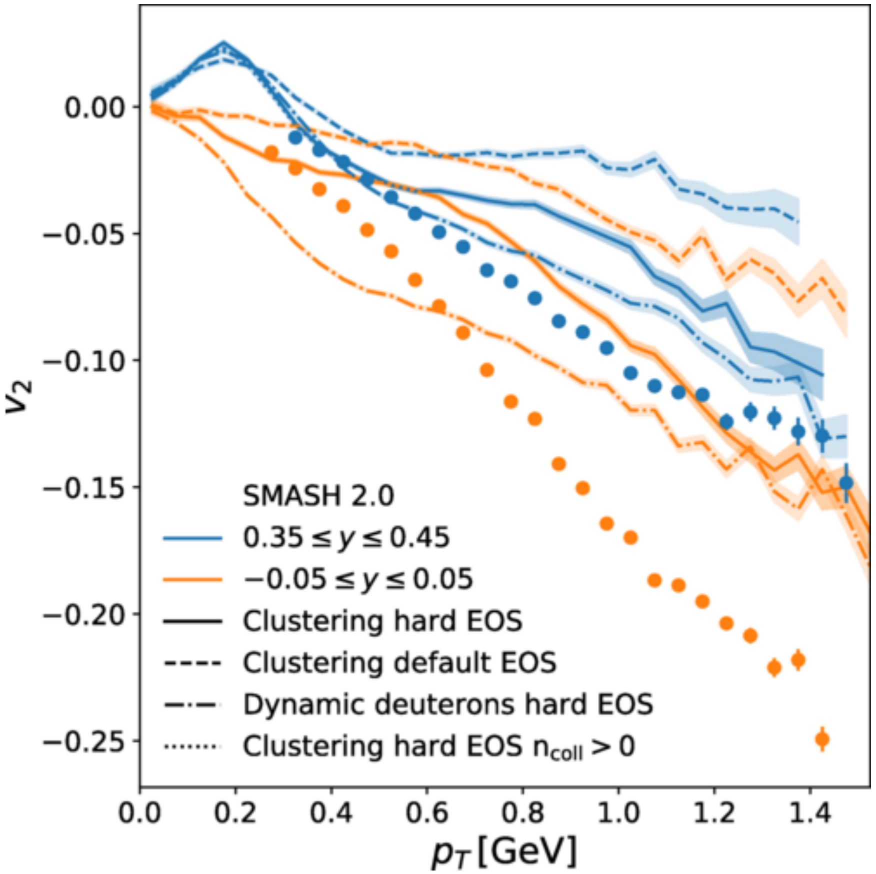


FOPI Nucl.Phys.A 848 (2010)

- Large fraction of protons are bound in light nuclei at low collision energies
- We’ll look at HADES data at 1.23A GeV. A third of protons is bound
- It is important to understand the formation of light nuclei even if one is only interested in protons

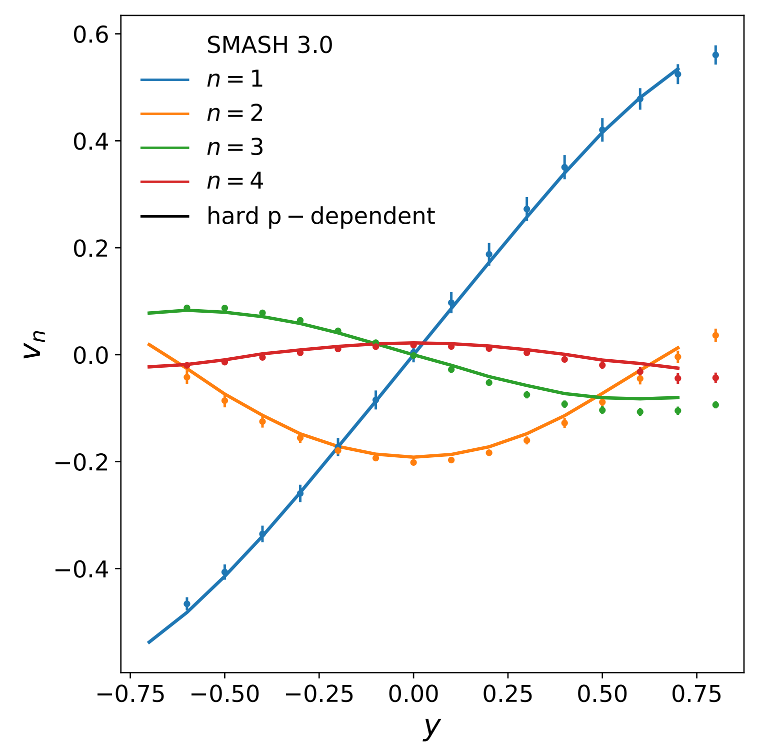
Light Nuclei Formation

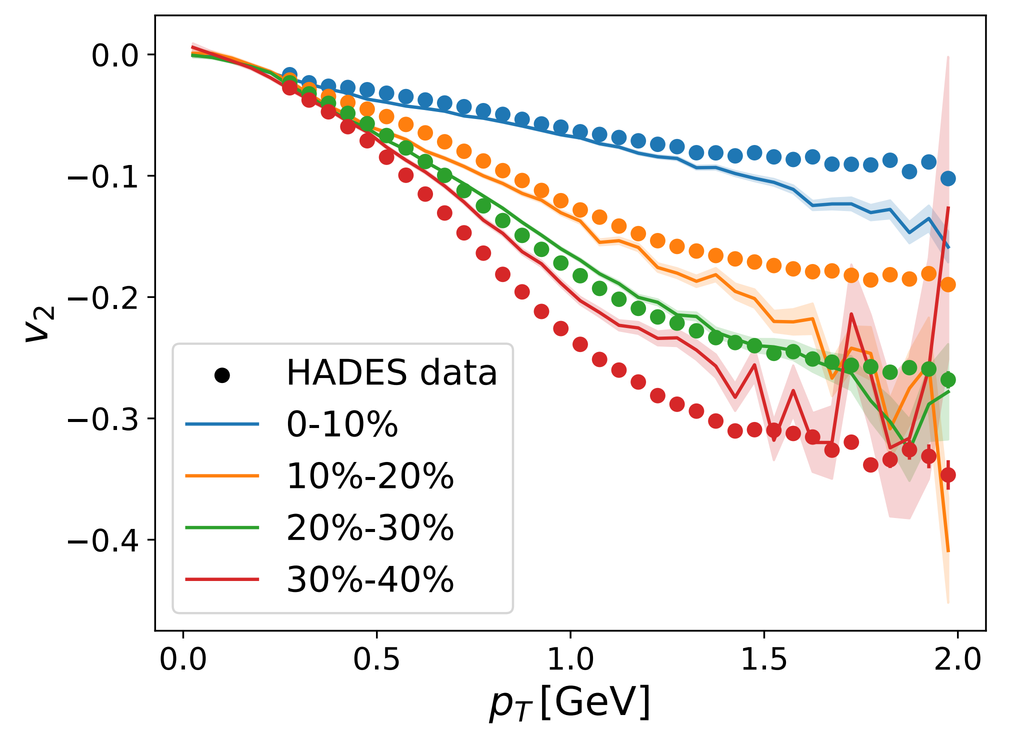
- Previous study without momentum-dependence
- Comparing final states coalescence to “dynamic” light nuclei formation
- Observe strong dependence of elliptic flow on light nuclei treatment at low p_T
- Flow at larger transverse momentum is a cleaner probe of the EoS



Proton elliptic flow at 1.23A GeV Au+Au

Proton Flow Overview





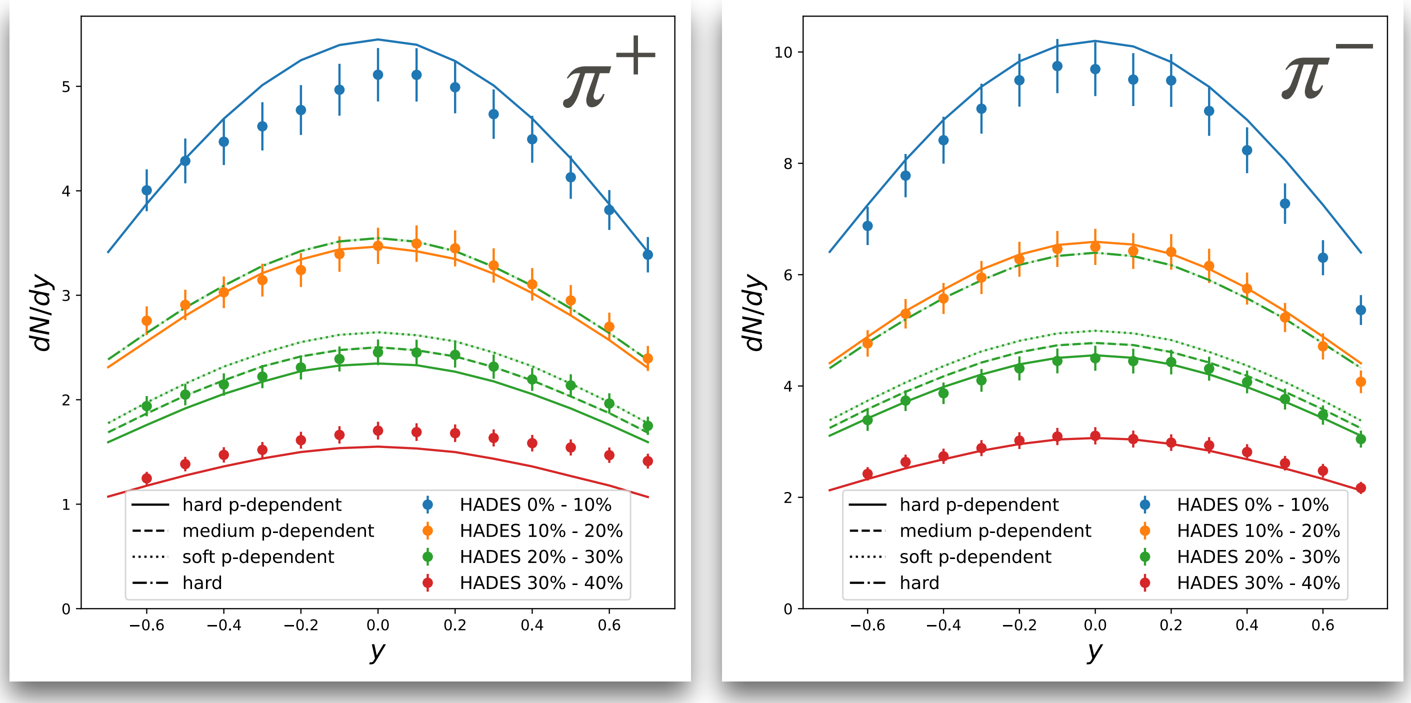
- Decent description with momentum dependence

HADES data
Phys. Rev. Lett. 125 (2020)

Hard momentum-dependent EOS
HADES data
Eur. Phys. J. A 59 (2023)

Pion Production

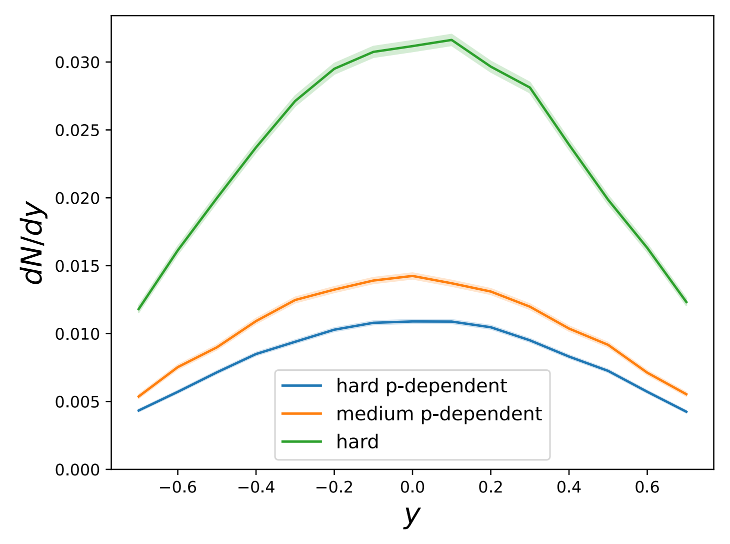
- Compare pion production to HADES data
- Softer equation of state allows for more pion production
- Including momentum dependence strongly reduces the pion yield



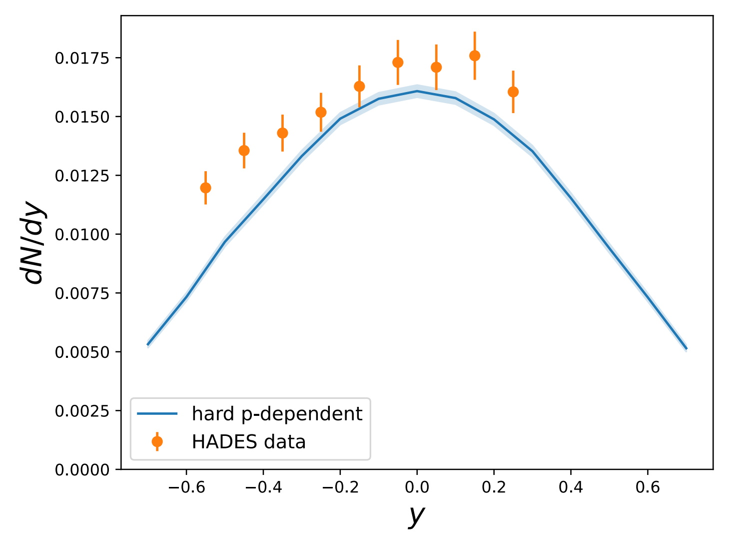
Pion spectra from HADES for different centrality classes

Kaon production

- Kaon yield is larger for softer equation of state
- Momentum-dependence reduces the Kaon yield
- Obtain good description of the Kaon yield with momentum-dependence



20%-30% centrality



0%-40% centrality
HADES data
Phys. Lett. B 778 (2018)

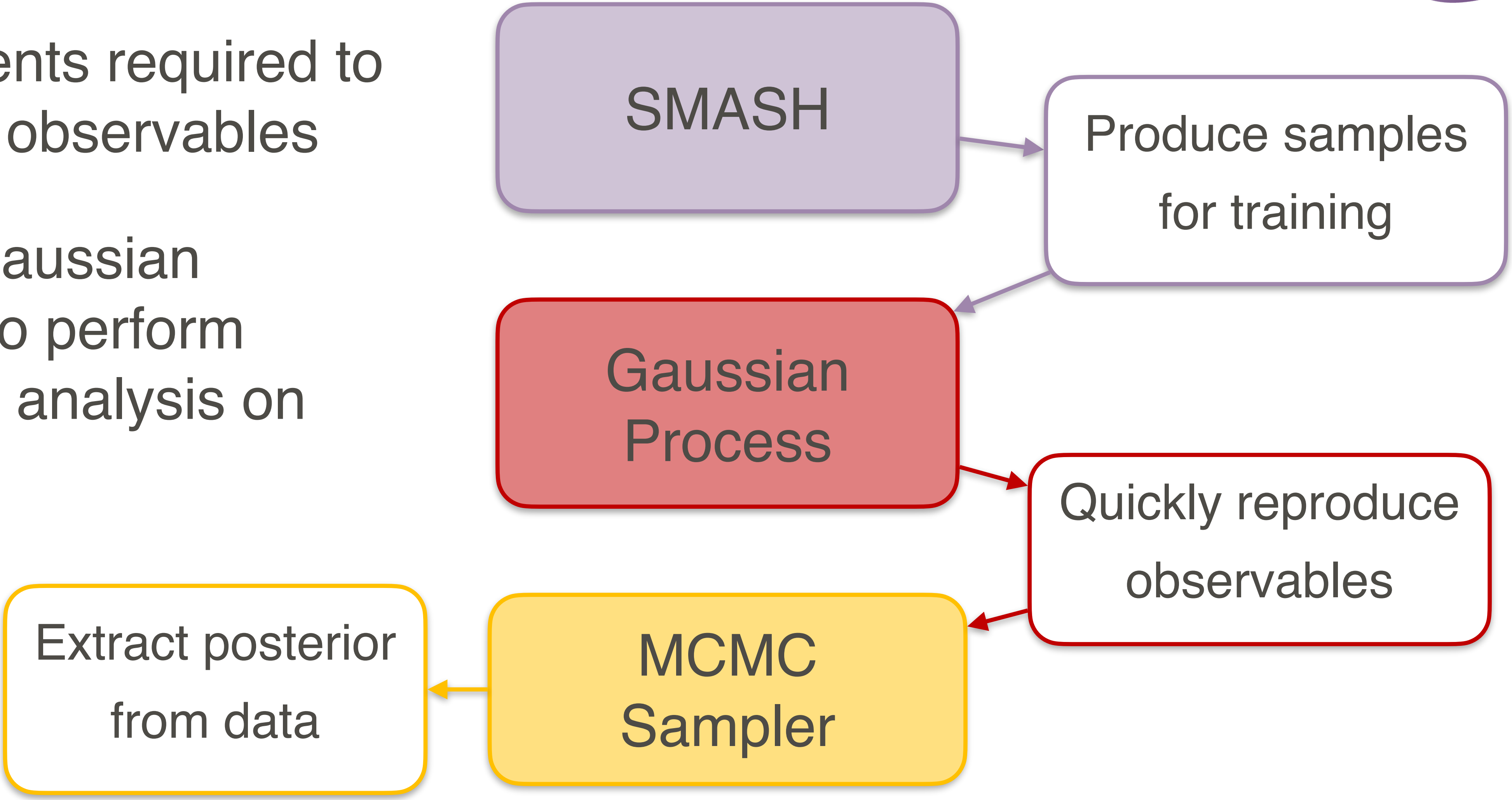
Bayesian Inference

- Goal: put systematic constraints on the equation of state
- Parameters are stiffness κ and symmetry potential S_{Pot}
- Priors chosen to be flat in κ and S_{Pot}
- Extract information from proton and deuteron flow in gold-gold collisions at HADES

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayesian Analysis Structure

- Many events required to calculate observables
- Train a Gaussian process to perform Bayesian analysis on



SMASH

Produce samples for training

Gaussian Process

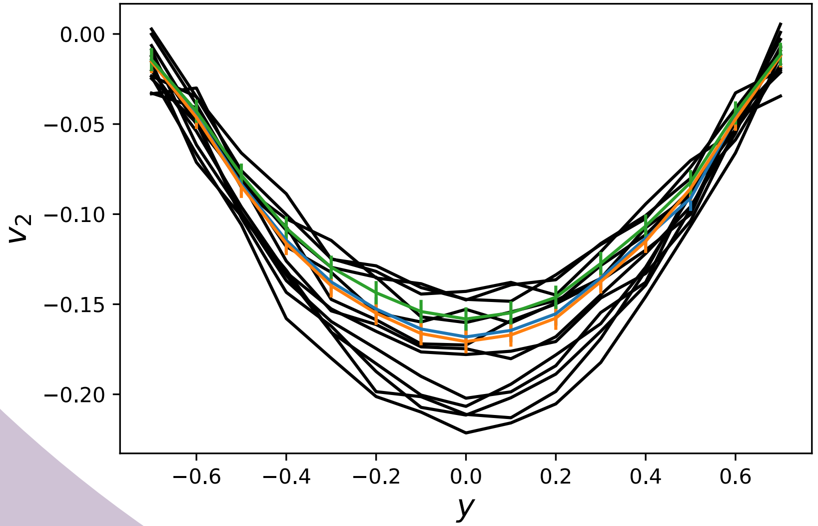
Quickly reproduce observables

MCMC Sampler

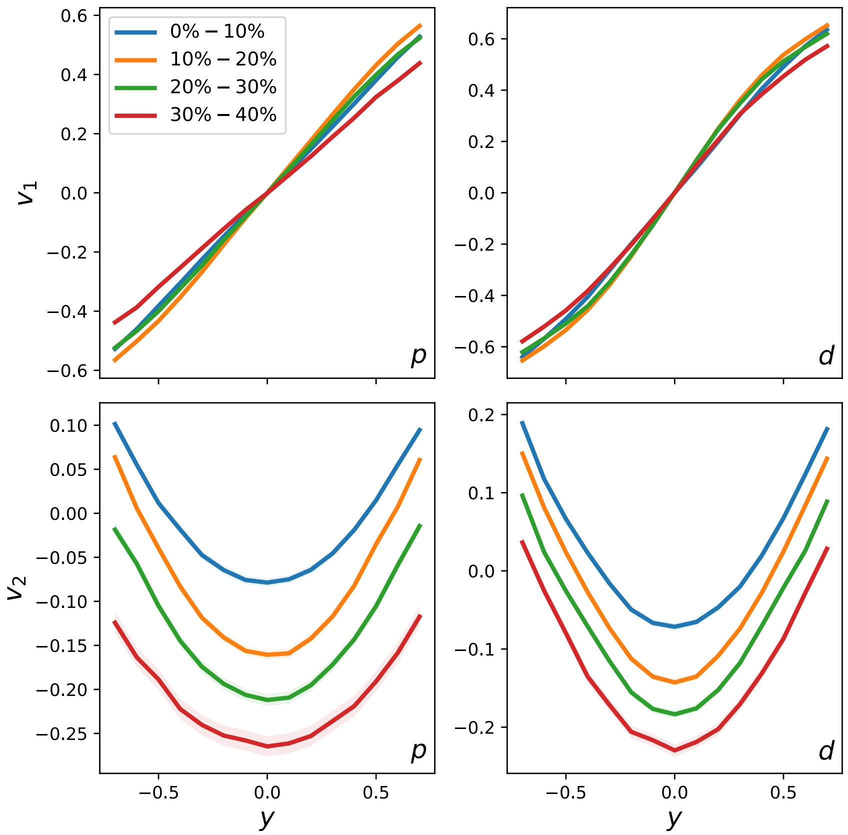
Extract posterior from data

Creating the Gaussian Process

- Run smash events for parameters sampled using latin hypercube
- Train the Gaussian process and compare to training data



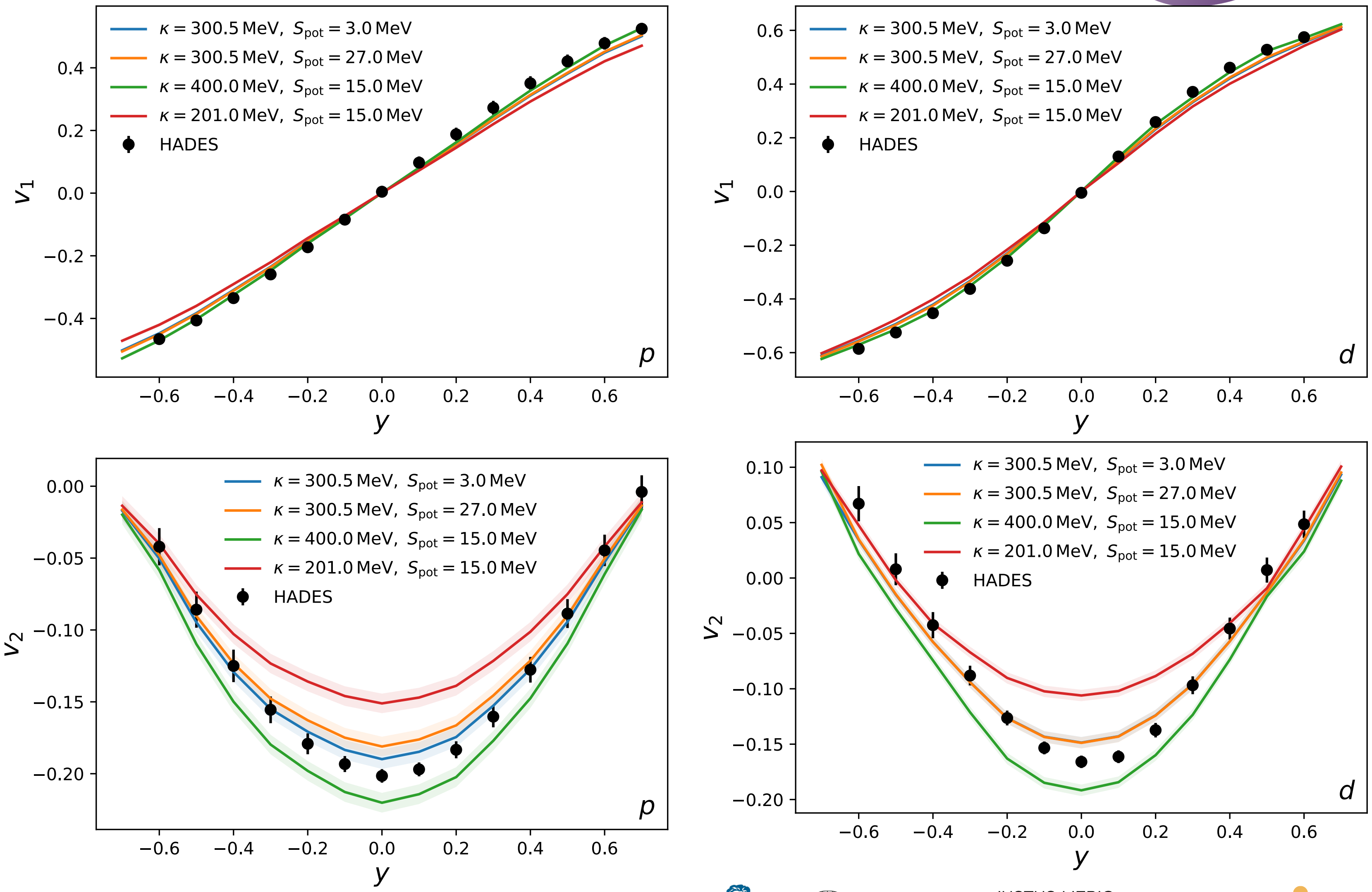
Smash samples compared to GP



Output from GP for one point in parameter space

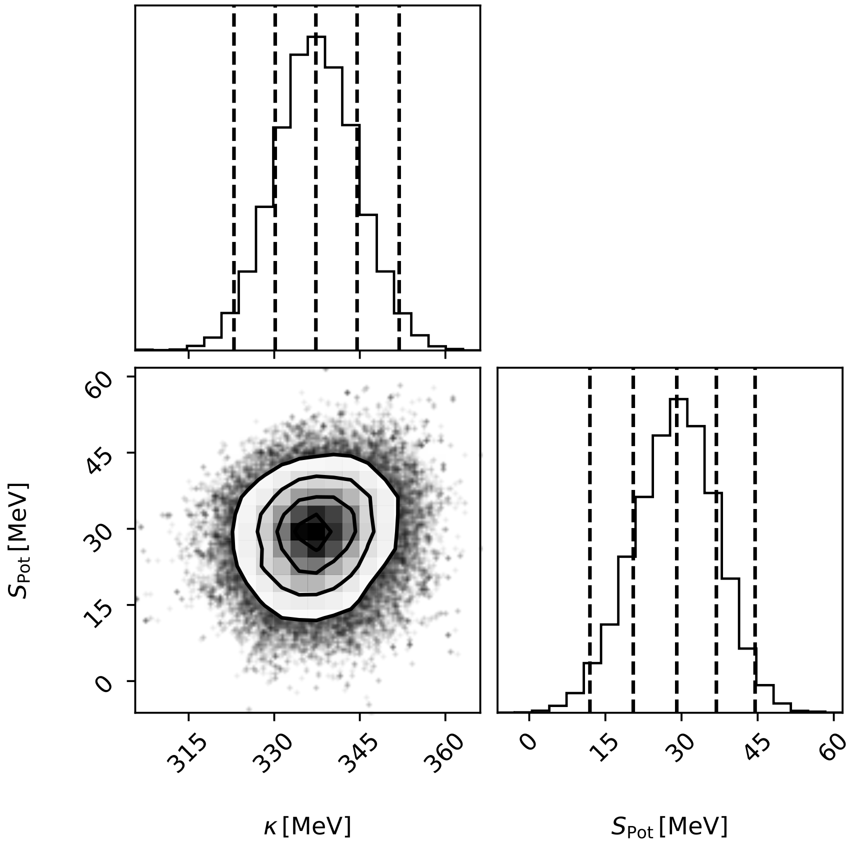
Sensitivity of Gaussian Process

- Evaluate Gaussian process for different parameters
- Observe stronger flow for larger stiffness κ
- Larger proton flow for smaller symmetry potential
- Deuterons unaffected by symmetry potential

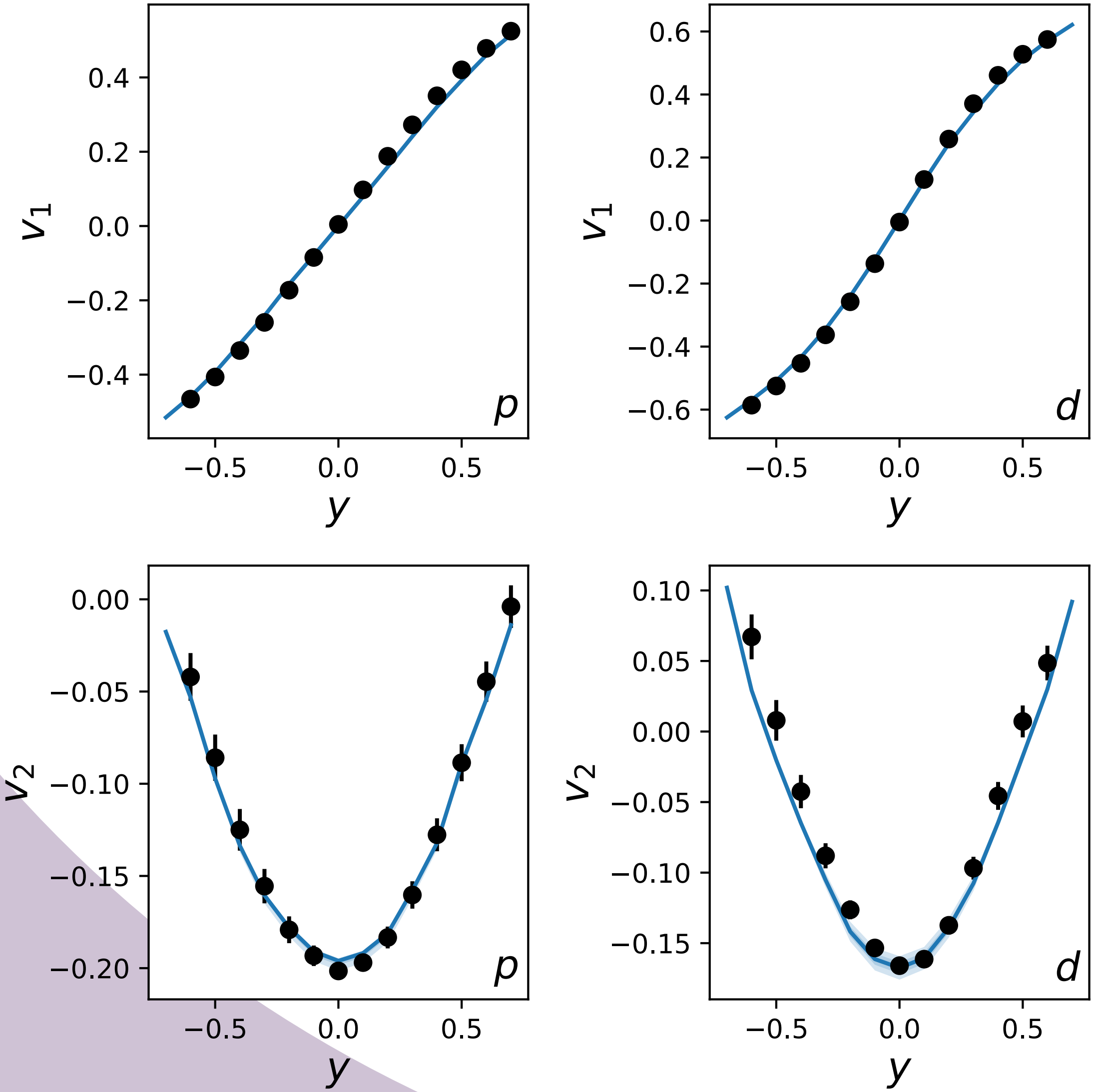


Posterior Distribution

- Obtained the posterior distribution from Markov chain Monte Carlo sampling
- Preliminary results:
  - $\kappa = 337^{+7.2}_{-7.1}\,\mathrm{MeV}$
  - $S_{\mathrm{Pot}} = 29.1^{+7.8}_{-8.5}\,\mathrm{MeV}$
- Relatively stiff equation of state and large uncertainty S_{Pot}



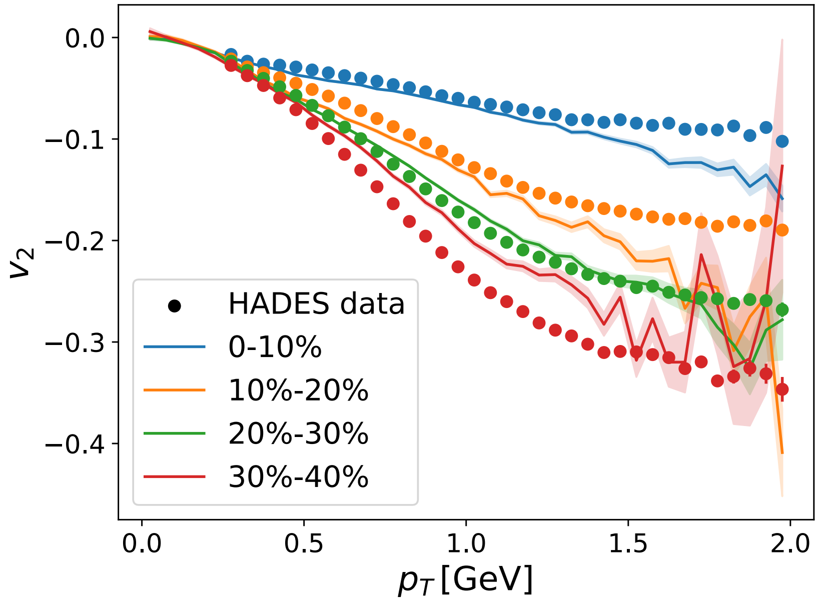
Maximum Posterior



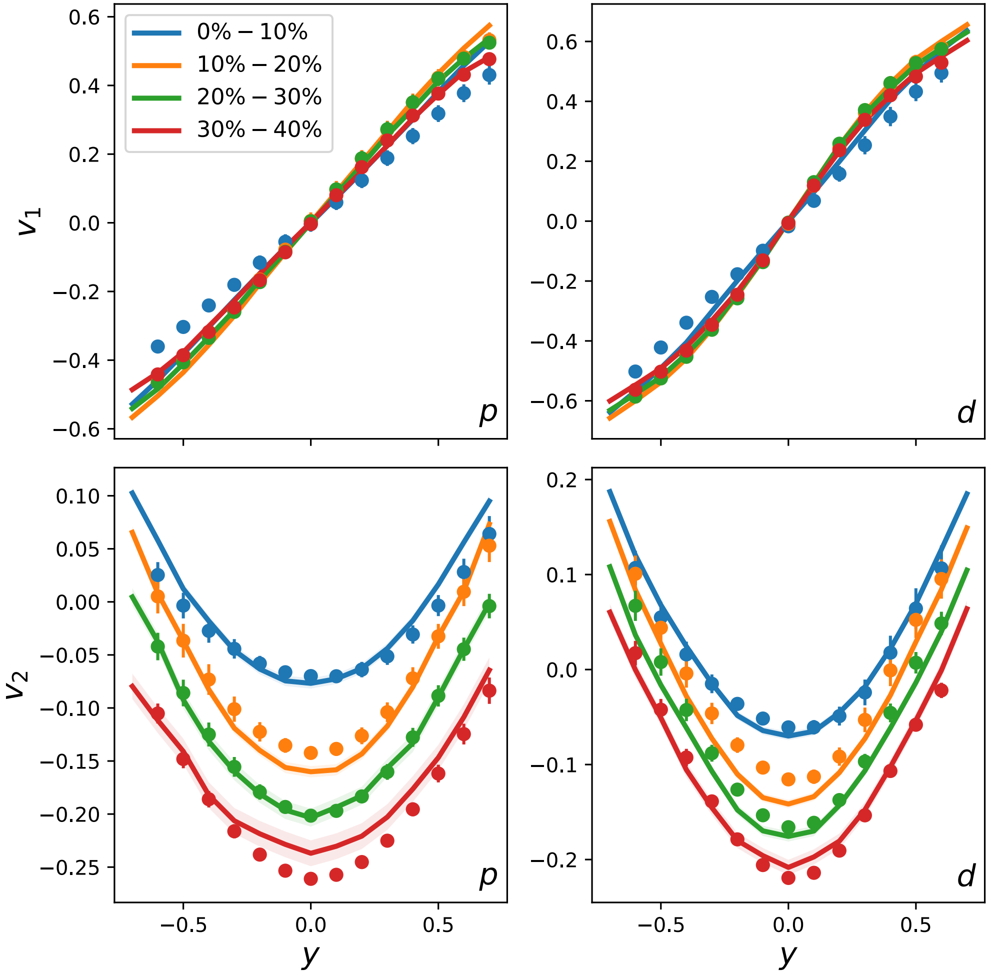
- Comparing the maximum of the posterior distribution to data
- Cross check for parameter extraction
- Obtained good fit to the data
- More data points will be included

Repeat with more Data!

- Add more experimental data for other centrality classes



HADES data
Eur. Phys. J. A 59 (2023)



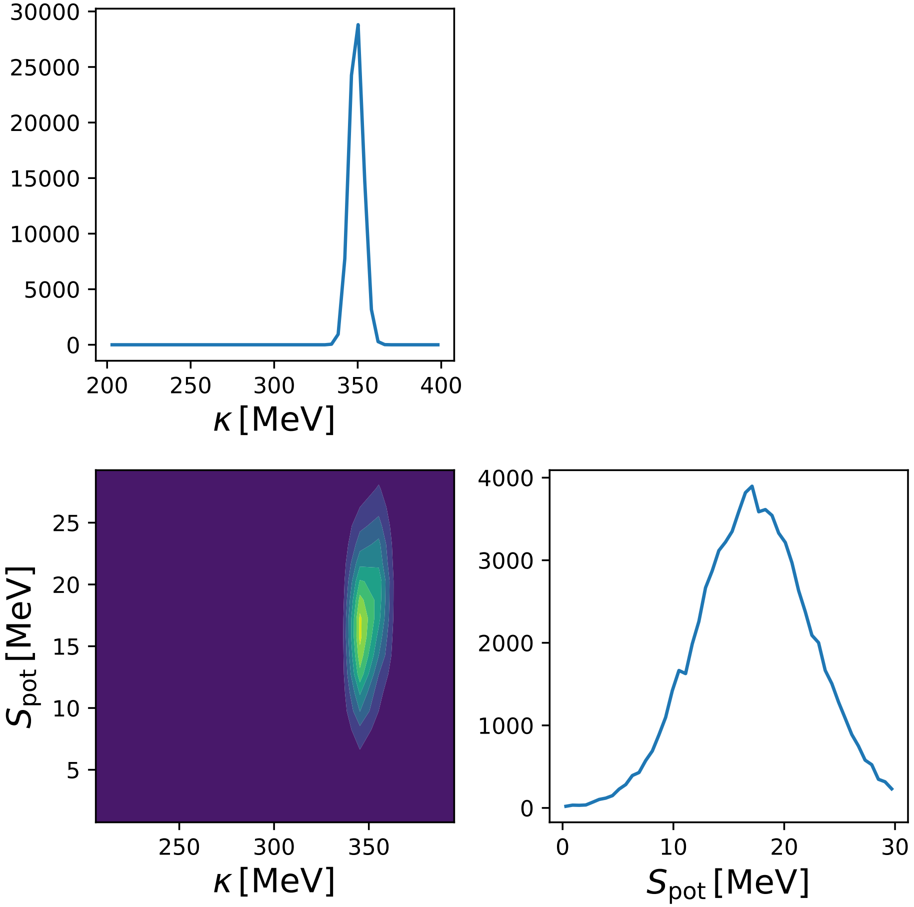
Posterior Distribution

- Redo analysis and obtain an improved Posterior distribution

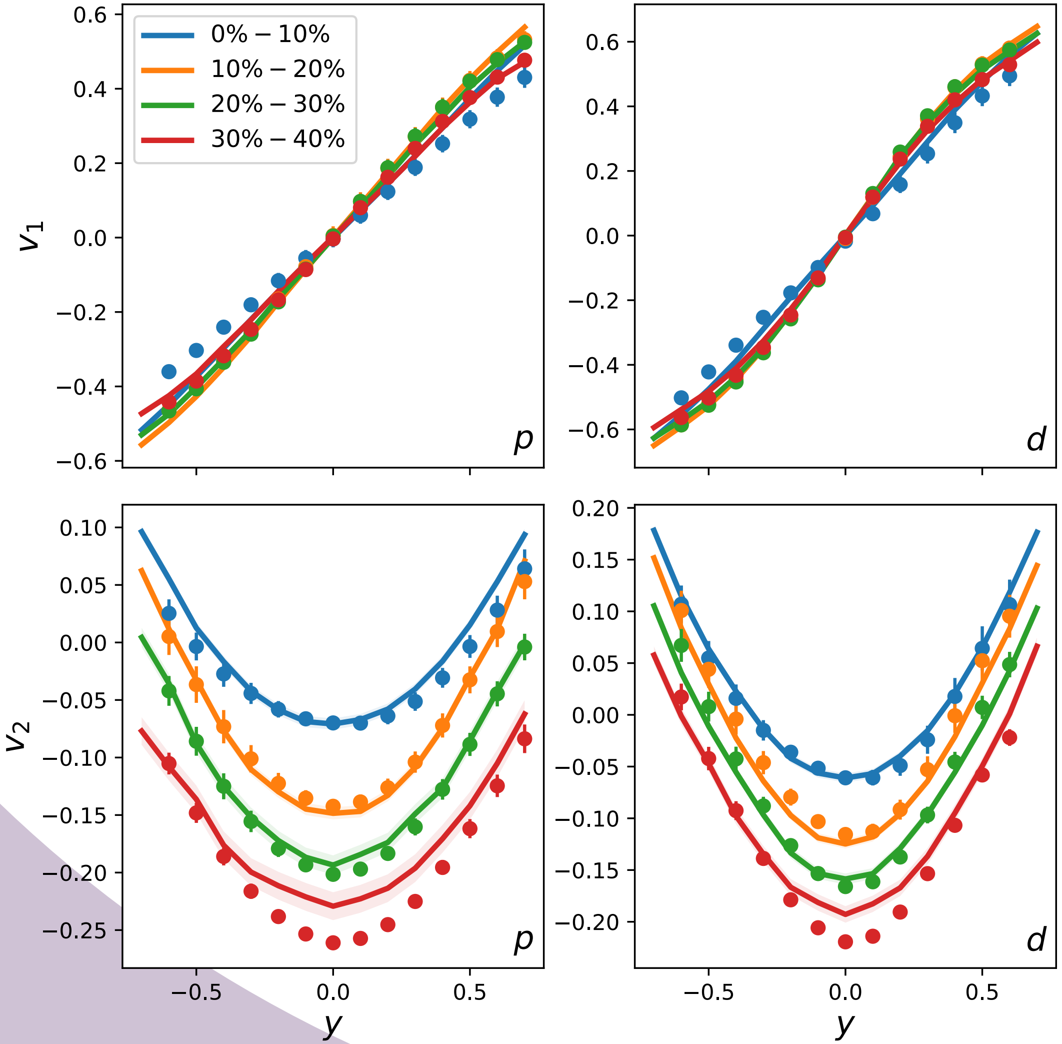
- Preliminary results:
    - $\kappa = 349.3^{+4.0}_{-3.0}\,\mathrm{MeV}$
    - $S_{\mathrm{Pot}} = 17.4^{+5.1}_{-4.7}\,\mathrm{MeV}$

- S_{Pot} corresponds to $25.6\,\mathrm{MeV} < S_0 < 35.4\,\mathrm{MeV}$

- Incompressibility is relatively large



Maximum Posterior



- Comparing the maximum of the posterior distribution to data
- Not all data perfectly reproduced but good agreement overall

Summary

- Add momentum-dependence to the nuclear potentials in the SAMSH transport model
- Performed a Bayesian analysis to determine stiffness of EoS and symmetry potential
- $\kappa = 349.3^{+4.0}_{-3.0}\,\mathrm{MeV}$ and $S_{\mathrm{Pot}} = 17.4^{+5.1}_{-4.7}\,\mathrm{MeV}$

Outlook

- Closure test and detailed flow analysis with maximum posterior
- Vary saturation density
- Vary number of Resonances in the model