

Deep Learning Stochastic Process with QCD Phase Transition

Speaker: **Lingxiao Wang** (FIAS)

With: **Lijia Jiang** (Northwest Uni. and FIAS) and **Kai Zhou** (FIAS)

Phys. Rev. D 103, 116023 (2021)

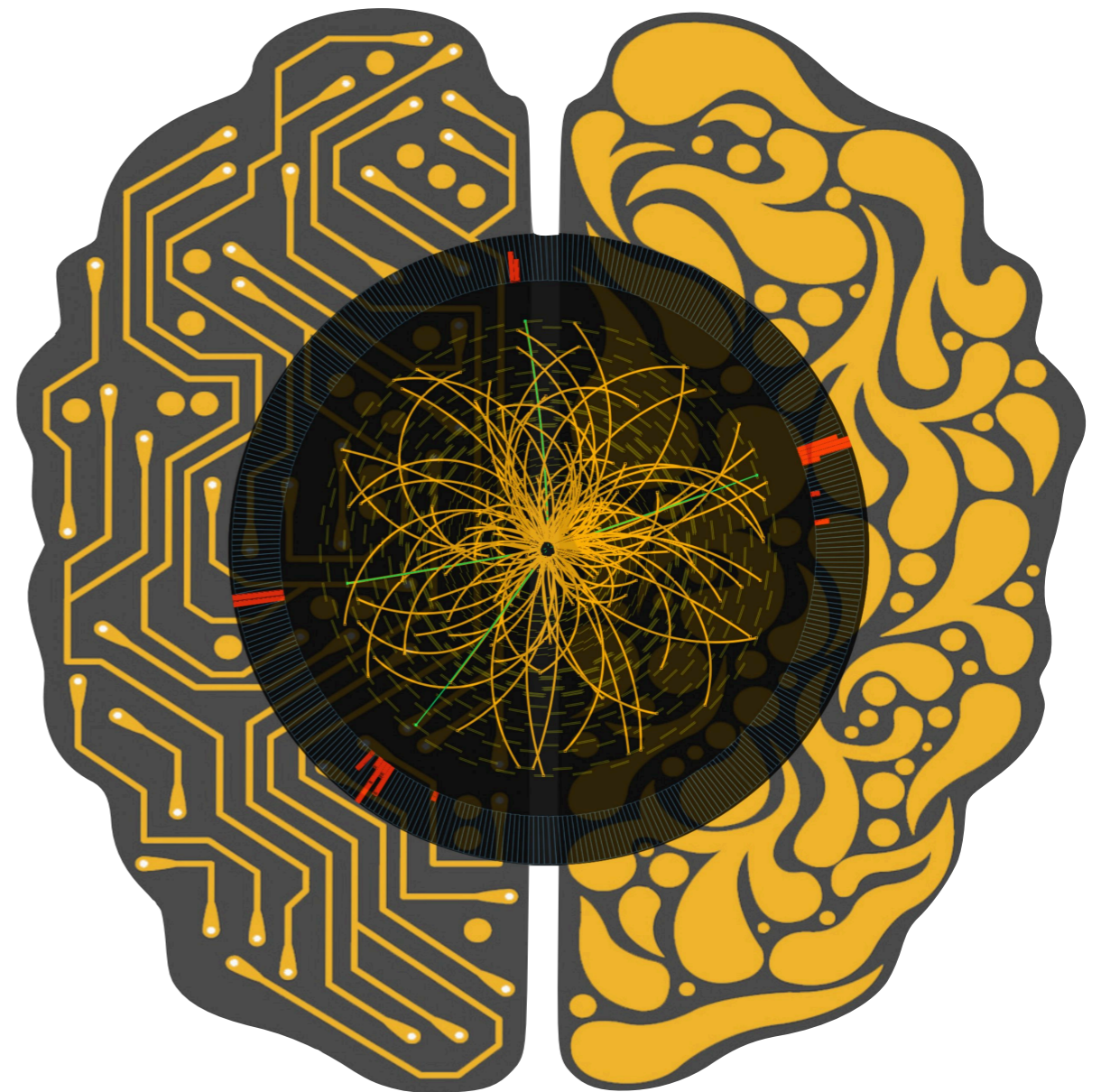
15 July, 2021 Transport Meeting



Main Content

Deep Learning Meets Physics

- Background
 - QCD phase transition
 - Deep learning in physics
- Deep Learning Dynamics
 - Understanding stochastic processes as images
 - Recognizing phase transition orders
 - Learning dynamical parameters
- Summary



@copyright Fermilab

Background

Phase Transition

Critical Point

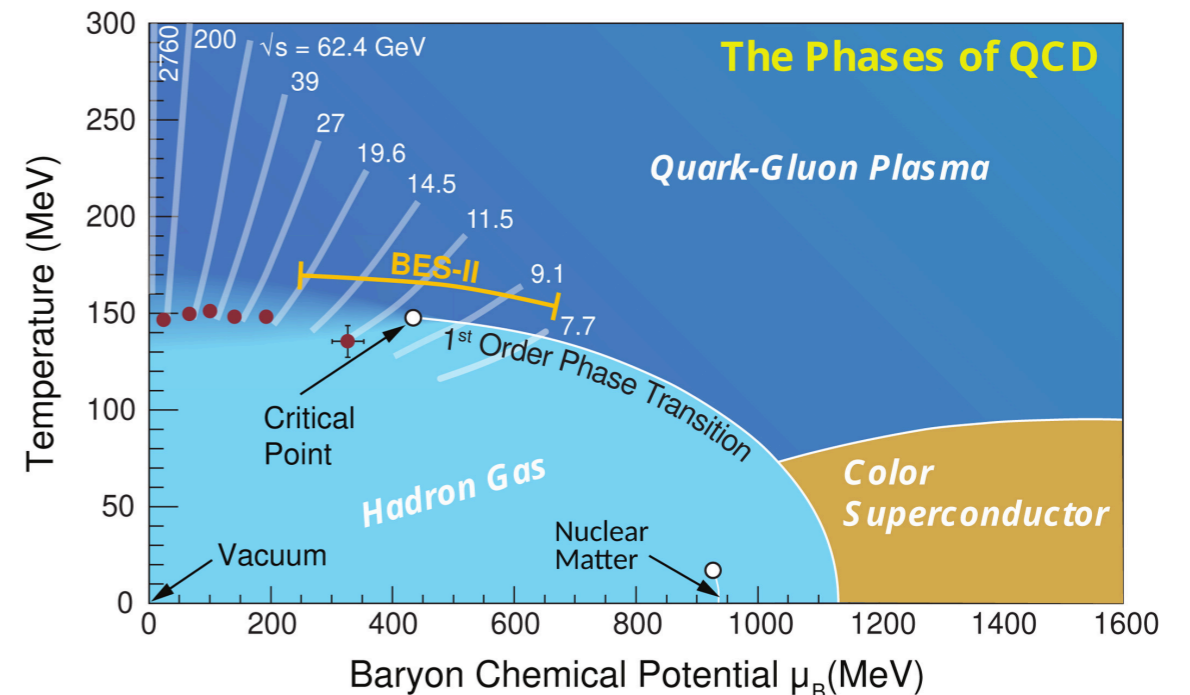
- **QCD Phase Structures**

- Chiral symmetry restoration
- Deconfinement
- Finite chemical potential μ_B, μ_I, B, ω

- **Searching Critical End Point**

- Theoretical calculations: (P)NJL, QM, FRG, DSE, etc
- Lattice calculations
- **Experiments: BES**

A schematic QCD phase diagram



A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, and N. Xu, Physics Reports (2020).

Stochastic Process

Criticality

- Along the freeze-out surface
 - Critical fluctuations
 - Statistical fluctuations
 - **Dynamical** critical fluctuations?

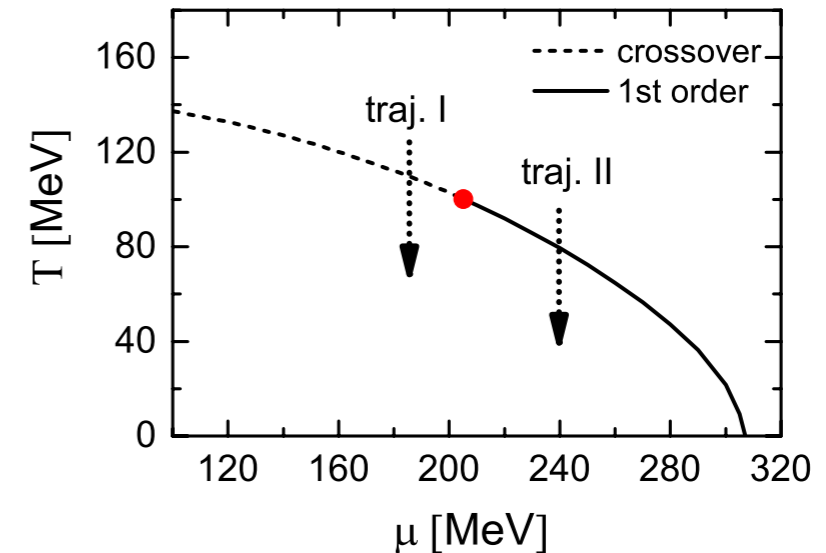
S. Mukherjee, R. Venugopalan, and Y. Yin, Phys. Rev. C **92**, 034912 (2015).
 L. Jiang, S. Wu, and H. Song, EPJ Web Conf. **171**, 16003 (2018).

- Memory effects from dynamical evolution
- The Skewness and Kurtosis of the high order cumulants could be different from the equilibrated

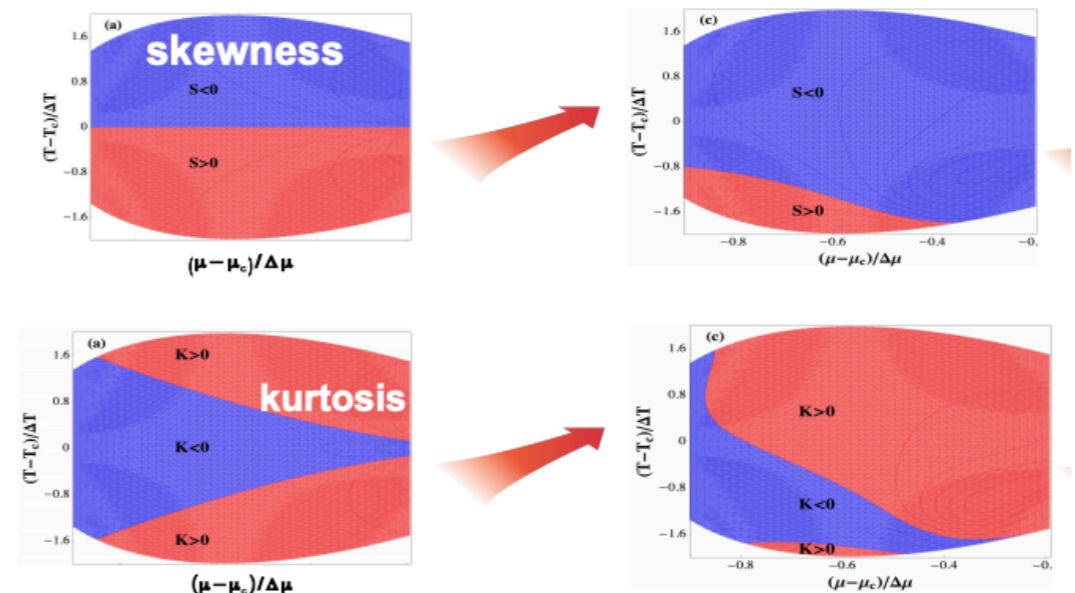
- **With Deep Learning**

- Detecting phase transition
- Extracting dynamical information
- ... in stochastic processes

QCD phase diagram from the linear sigma model with constituent quarks



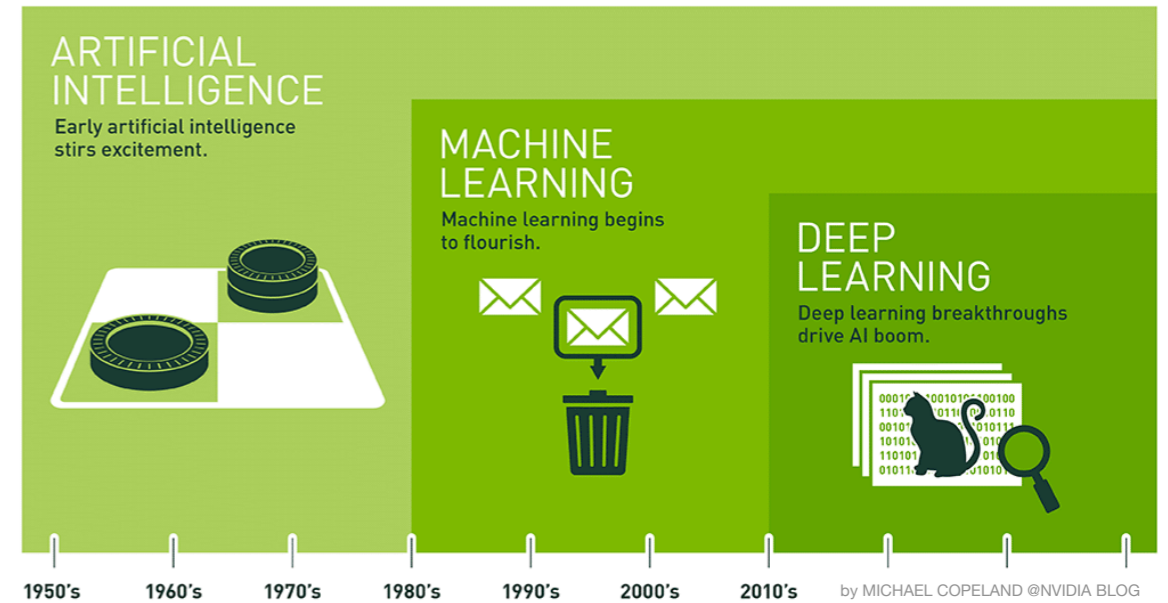
L. Jiang, S. Wu, and H. Song, Nuclear Physics A **967**, 441 (2017).



from Swagato's talk @201509,Kobe

Deep Learning

AI in Physics



Since an early flush of optimism in the 1950s, smaller subsets of artificial intelligence – first machine learning, then deep learning, a subset of machine learning – have created ever larger disruptions.

- **Classify phases**

Phys. Rev. B 94, 195105 (2016), Phys. Rev. X 7, 031038 (2017), Nat. Phys. 13, 431 (2017).

- **Predict properties/signals**

Phys. Rev. Lett. 120, 241602 (2018), Phys. Rev. Materials 3, 104405 (2019).

- **Innovate computation**

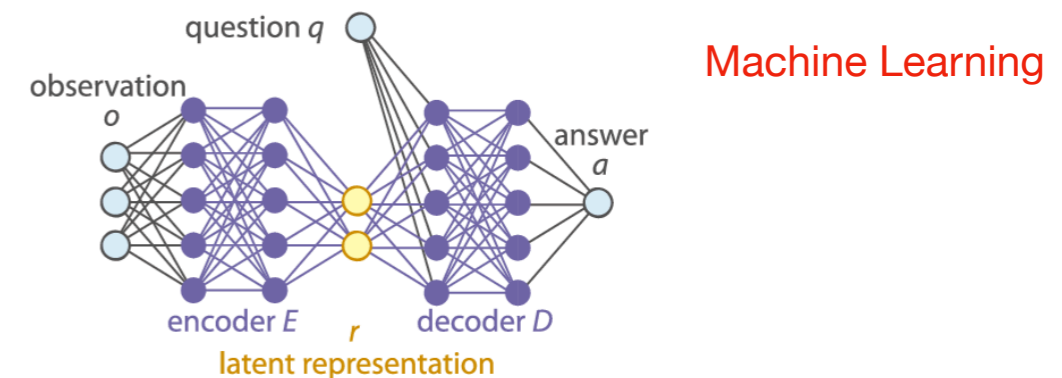
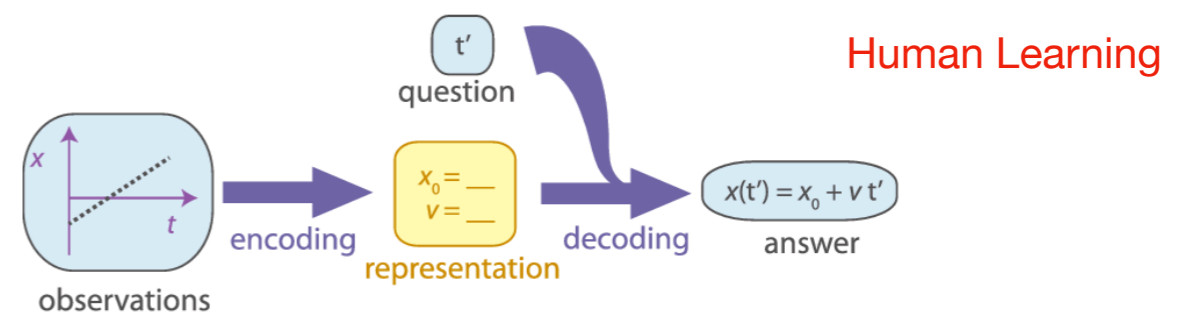
Science 355, 602 (2017), Phys. Rev. B 97, 205140 (2018), Phys. Rev. Lett. 122, 080602 (2019).

- **New developments**

arXiv:1410.3831, Nat. Phys. 14, 578 (2018), Science Bulletin 64, 1228 (2019), Phys. Rev. Lett. 124, 010508 (2020).

Machine Learning and the Physical Sciences, Rev. Mod. Phys. **91**, 045002 (2019).
Machine Learning Meets Quantum Physics, Physics Today **72**, 48 (2019).

Learning physical representations



Phys. Rev. Lett. 124, 010508 (2020).

Deep Learning

AI in High Energy Nuclear Physics

- **White Paper**

P. Bedaque, A. Boehnlein, M. Cromaz, M. Diefenthaler, L. Elouadrhiri, T. Horn, M. Kuchera, D. Lawrence, D. Lee, S. Lidia, R. McKeown, W. Melnitchouk, W. Nazarewicz, K. Orginos, Y. Roblin, M. S. Smith, M. Schram, and X.-N. Wang, *Report from the A.I. For Nuclear Physics Workshop*, (2020).

- **Experiments**

Phys. Rev. Lett. 120, 241602 (2018), SciPost Physics 7, 014 (2019), Nat. Phys. 15,1113-1117 (2019), ...

- **Lattice calculations**

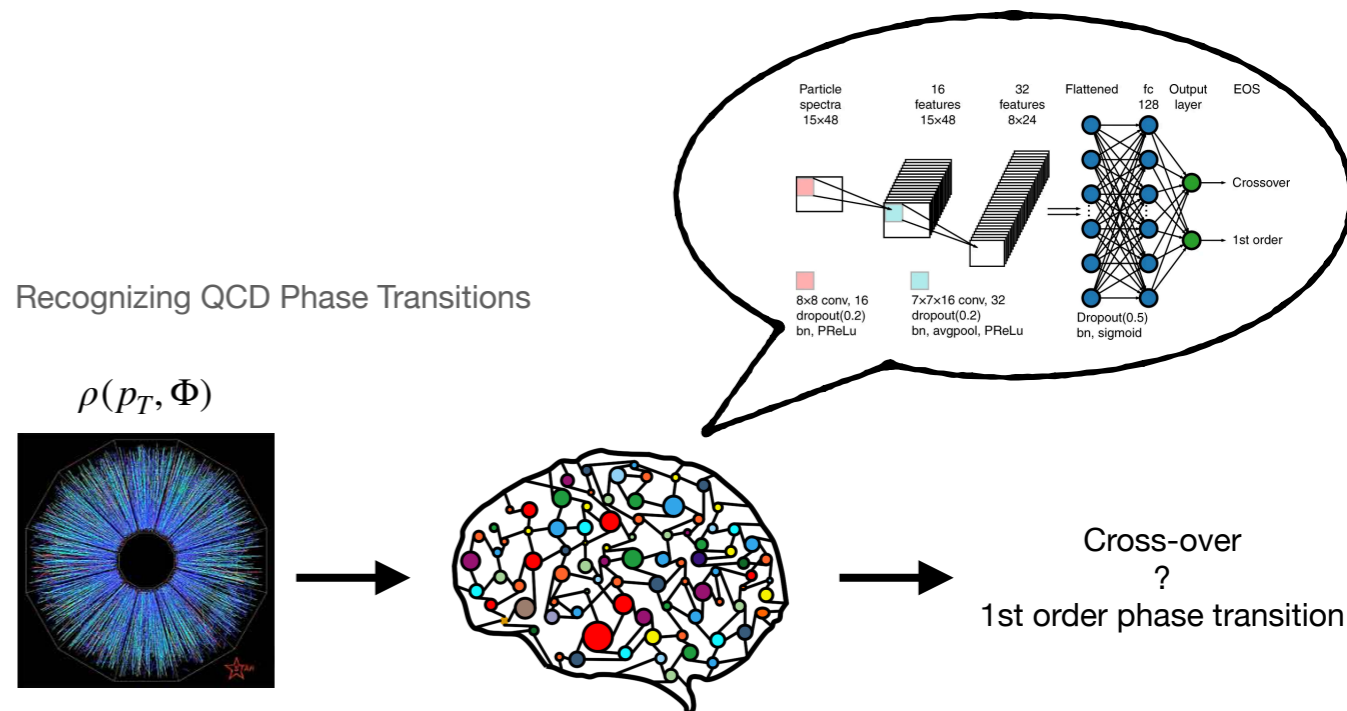
Phys. Rev. D 100, 011501 (2019), Phys. Rev. D 101, 094507 (2020), Phys. Rev. Lett. 125, 121601 (2020), Phys. Rev. D 102, 096001 (2020), ...

- From lattice QCD to in-medium heavy-quark interactions via deep learning
Kai Zhou 24 June, Transport Meeting 2021

- **New Developments**

Phys. Rev. D 98, 023019 (2018), A&A 642, A78 (2020), Eur. Phys. J. C 79, 870 (2019), JHEP, 2019, 122 (2019), ...

- **Compact stars**
- **Phase structures**
- **Dynamical processes**



L.-G. Pang, K. Zhou, N. Su, H. Petersen, H. Stöcker, and X.-N. Wang, *Nature Commun.* 9, 210 (2018).

Deep Learning Critical Dynamics

Problem Set-Up

Stochastics Process

$$\mathcal{L} = \bar{q} \left[i\gamma^\mu \partial_\mu - g (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) + \gamma_0 \mu \right] q + \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} \right) - U(\sigma, \pi)$$

Linear Sigma Model

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{q\bar{q}}(\sigma; T, \mu)$$

Effective potential

$$U(\sigma) = \frac{1}{4} \lambda^2 (\sigma^2 - v^2)^2 - h\sigma - U_0$$

$$\Omega_{q\bar{q}}(\sigma; T, \mu) = -d_q \int \frac{d^3p}{(2\pi)^3} \left\{ E + T \ln [1 + e^{-(E-\mu)/T}] + T \ln (1 + e^{-(E+\mu)/T}) \right\}$$

- **QCD phase transition in Langevin equation**

$$\partial^\mu \partial_\mu \sigma(t, x) + \eta \partial_t \sigma(t, x) + \frac{\delta V_{eff}(\sigma)}{\delta \sigma} = \xi(t, x)$$

η : damping coefficient

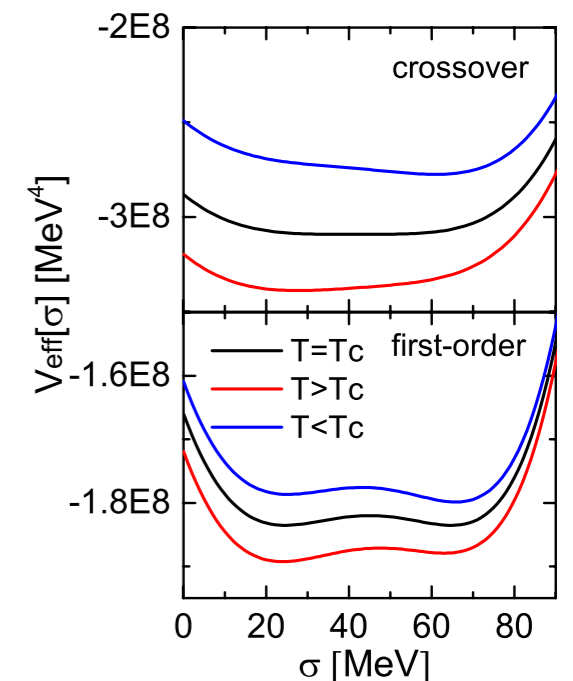
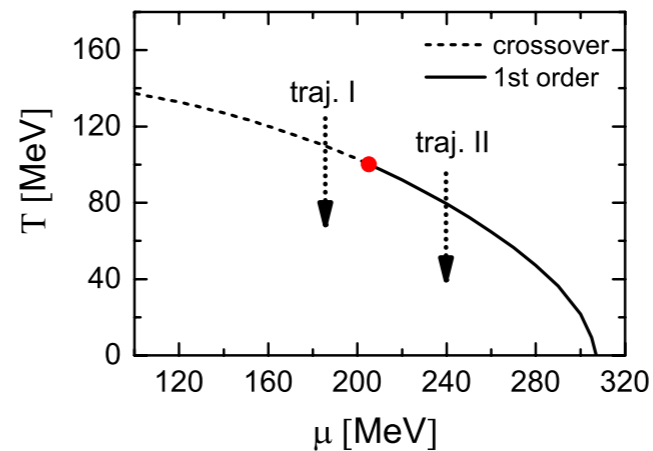
$$\frac{T(t)}{T_0} = \left(\frac{t}{t_0} \right)^{-0.45}$$

Hubble-like

- **Fluctuations**

$$\langle \xi(t) \xi(t') \rangle = \frac{1}{V} m_\sigma \eta \coth \left(\frac{m_\sigma}{2T} \right) \delta(t - t')$$

$$\xi(x) = B \sqrt{\frac{1}{V} m_\sigma \eta \coth \left(\frac{m_\sigma}{2T} \right)} \frac{1}{dt} G(x)$$



Problem Set-Up

Stochastics Process

- **QCD phase transition in Langevin equation**

$$\partial^\mu \partial_\mu \sigma(t, x) + \eta \partial_t \sigma(t, x) + \frac{\delta V_{eff}(\sigma)}{\delta \sigma} = \xi(t, x)$$

η : damping coefficient

- **Fluctuations**

$$\langle \xi(t) \xi(t') \rangle = \frac{1}{V} m_\sigma \eta \coth \left(\frac{m_\sigma}{2T} \right) \delta(t - t')$$

$$\xi(x) = B \sqrt{\frac{1}{V} m_\sigma \eta \coth \left(\frac{m_\sigma}{2T} \right)} \frac{1}{dt} G(x)$$

$$\mathcal{L} = \bar{q} \left[i\gamma^\mu \partial_\mu - g (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) + \gamma_0 \mu \right] q + \frac{1}{2} \left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} \right) - U(\sigma, \pi)$$

Linear Sigma Model

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{q\bar{q}}(\sigma; T, \mu)$$

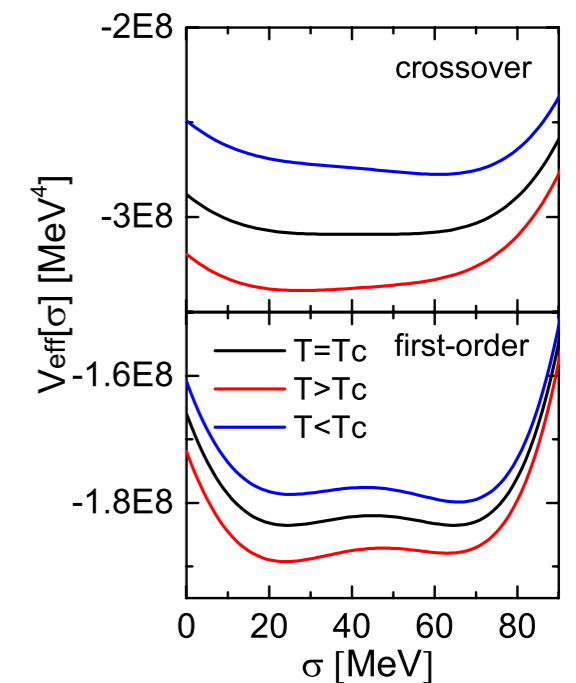
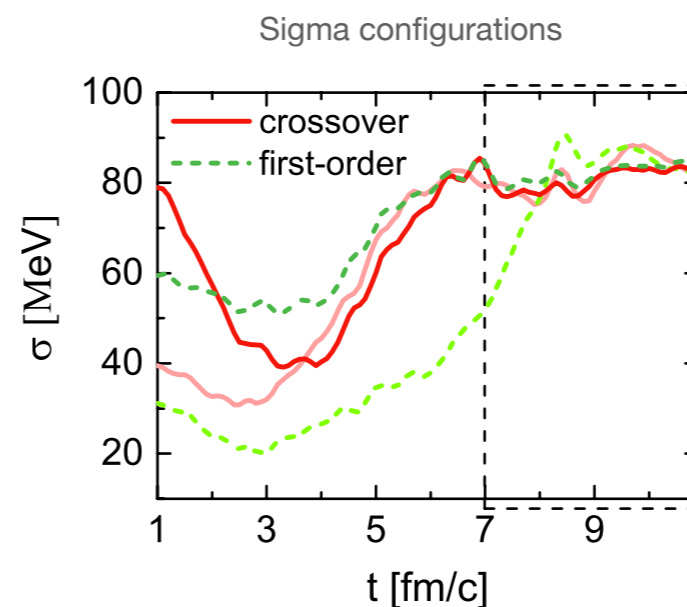
Effective potential

$$U(\sigma) = \frac{1}{4} \lambda^2 (\sigma^2 - v^2)^2 - h\sigma - U_0$$

$$\Omega_{q\bar{q}}(\sigma; T, \mu) = -d_q \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \ln [1 + e^{-(E-\mu)/T}] + T \ln (1 + e^{-(E+\mu)/T}) \right\}$$

$$\frac{T(t)}{T_0} = \left(\frac{t}{t_0} \right)^{-0.45}$$

Hubble-like



Problem Set-Up

Stochastics Processes as Images

- **Preparing configurations**

- Initial profiles sample with

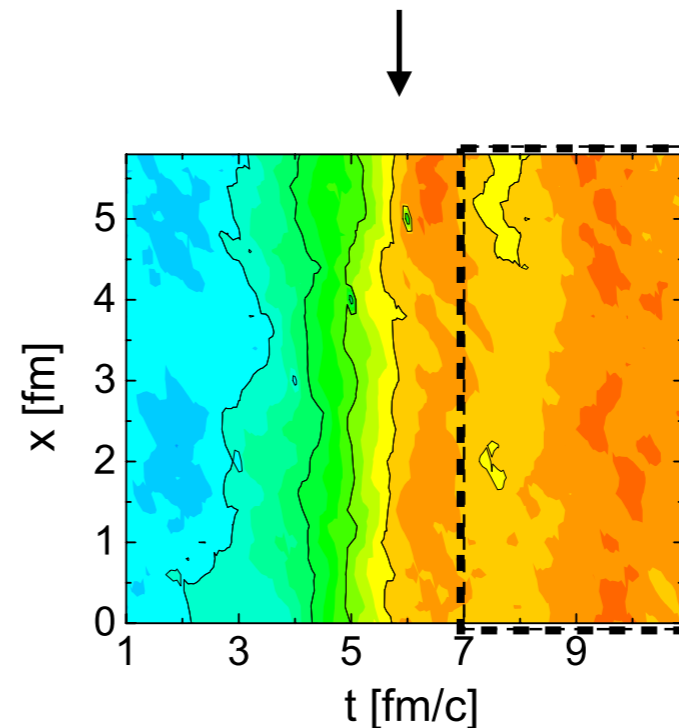
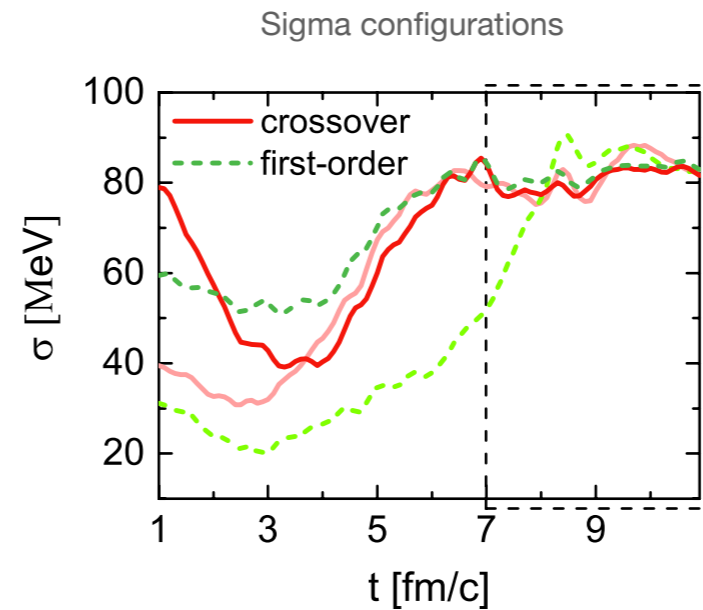
$$P[\sigma(\mathbf{x})] \sim \exp(-\varepsilon(\sigma)/T) \quad \varepsilon(\sigma) = \int dx \left[\frac{1}{2}(\nabla\sigma(x))^2 + V_{eff}(\sigma(x)) \right]$$

- Simulated 10,000 events at each parameter

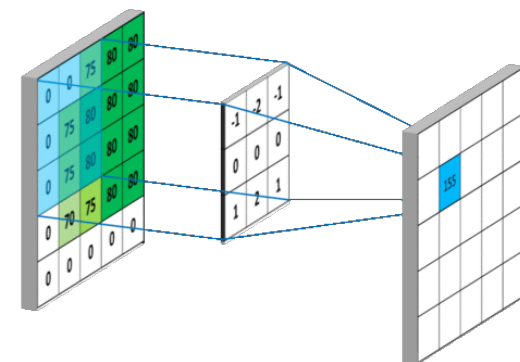
- $\mu = 180$ MeV as “cross-over”
- $\mu = 240$ MeV as “1st order”
- $B = 0.5, 1$

- **Compressing $\sigma(x, t)$ as images**

- $dx = 0.2$ fm, $dt = 0.1$ fm/c
- $L = 6.0$ fm, t in $[7, 11]$ fm/c
- $N_t \times N_x = 40 \times 30 = 1200$ pixels



Convolutional Operation



Spatial-Temporal Correlation!

Neural Networks

Deep CNNs

- **Compressing $\sigma(x, t)$ as images**

- $N_t \times N_x = 40 \times 30 = 1200$ pixels
- Data-sets: 8:2 = training: validation

- **Architecture**

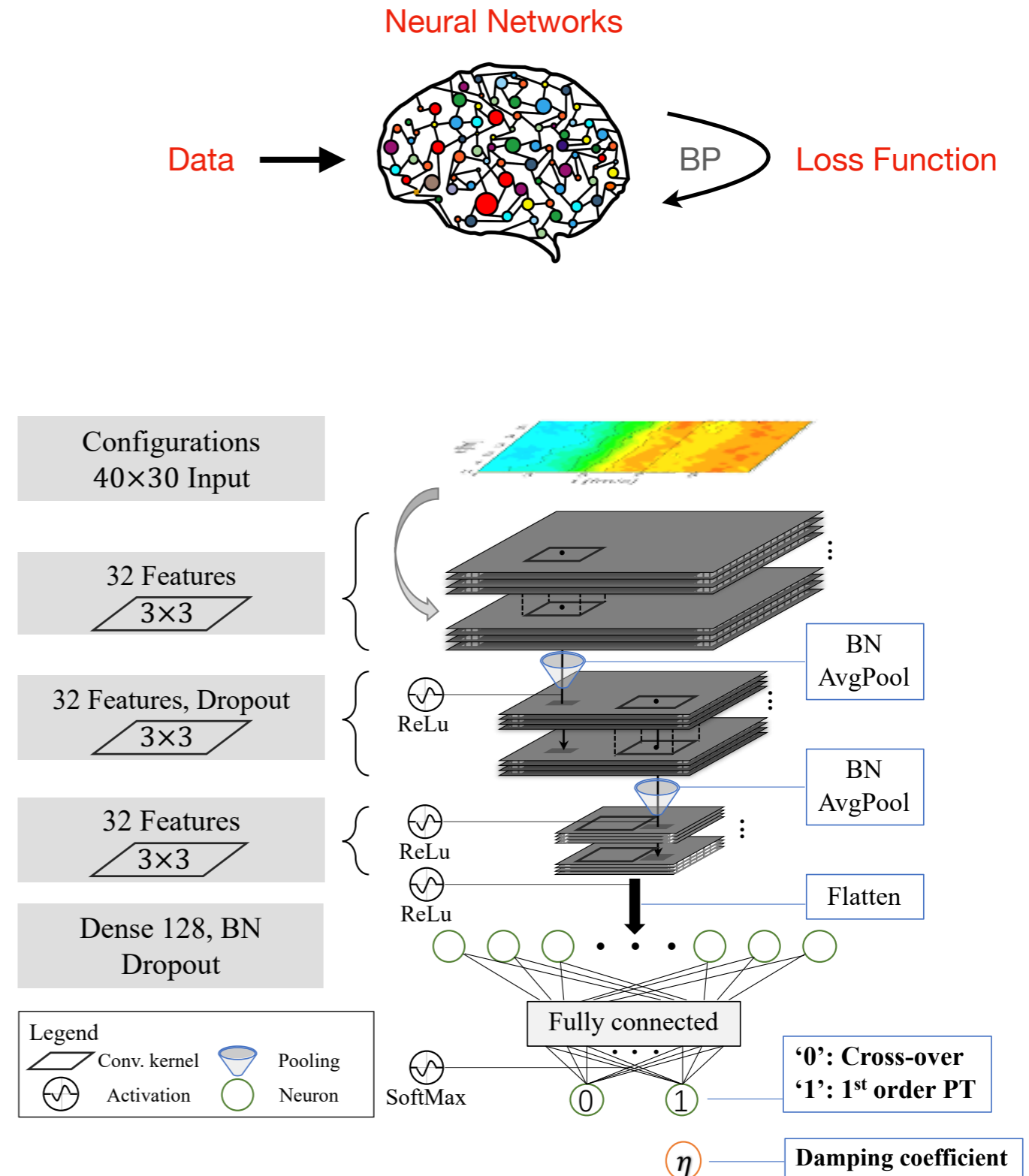
- Inputs : $\sigma(x, t)$ configurations
- 3 CNNs
- 1 Fully-Connected layer
- Outputs: 0,1 / damping coefficient η

- **Loss function**

- Categorical cross entropy

$$\text{loss} = - \sum_{i=1}^c y_i \log f_i(x)$$

- Mean Square Error



Recognition

Phase Transition Orders

- **Fluctuations**

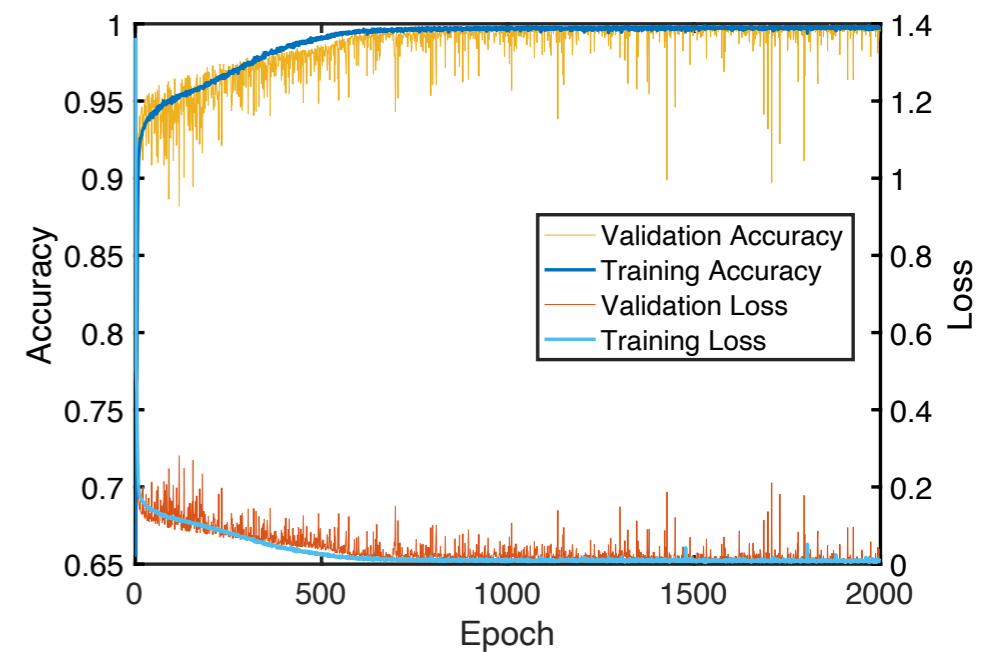
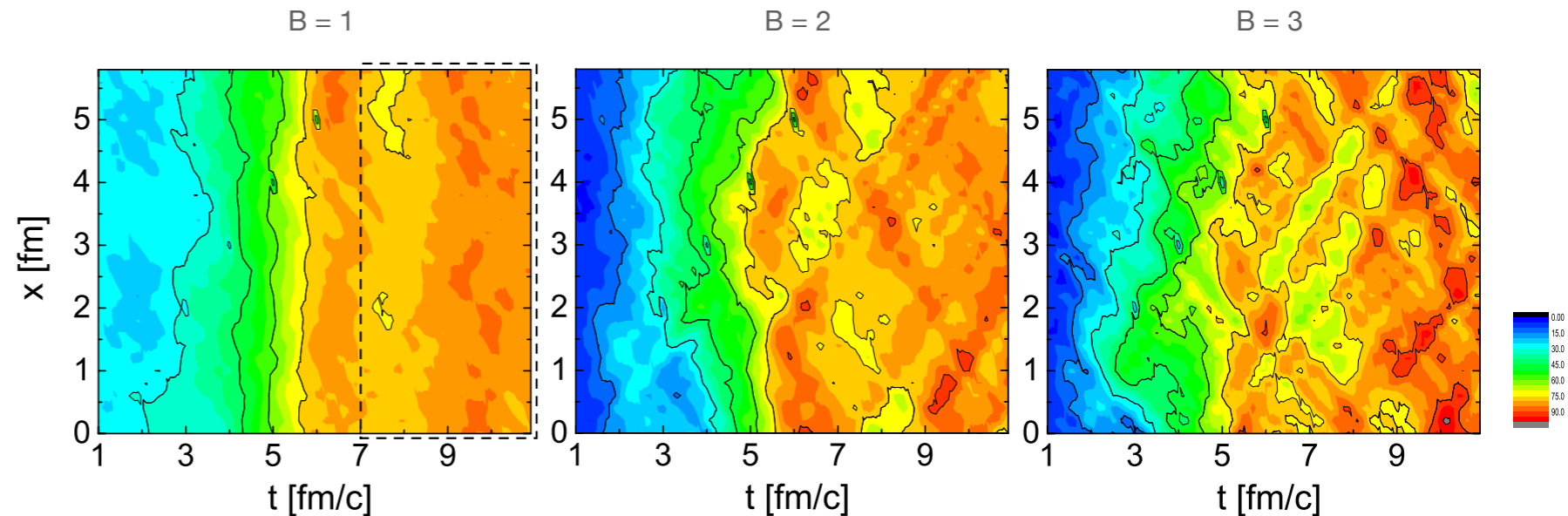
$$\xi(x) = B \sqrt{\frac{1}{V} m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right) \frac{1}{dt} G(x)}$$

- **Training**

- $B = 0.5, 1$
- Reaching saturation without over-fitting

- **Testing**

- $B = 0.5, 1, 1.5, 2, 2.5$
- Accurate on low-noise configurations
- Robust to the noises



Recognition

Phase Transition Orders

- **Training**

- $B = 0.5, 1$

- **Testing**

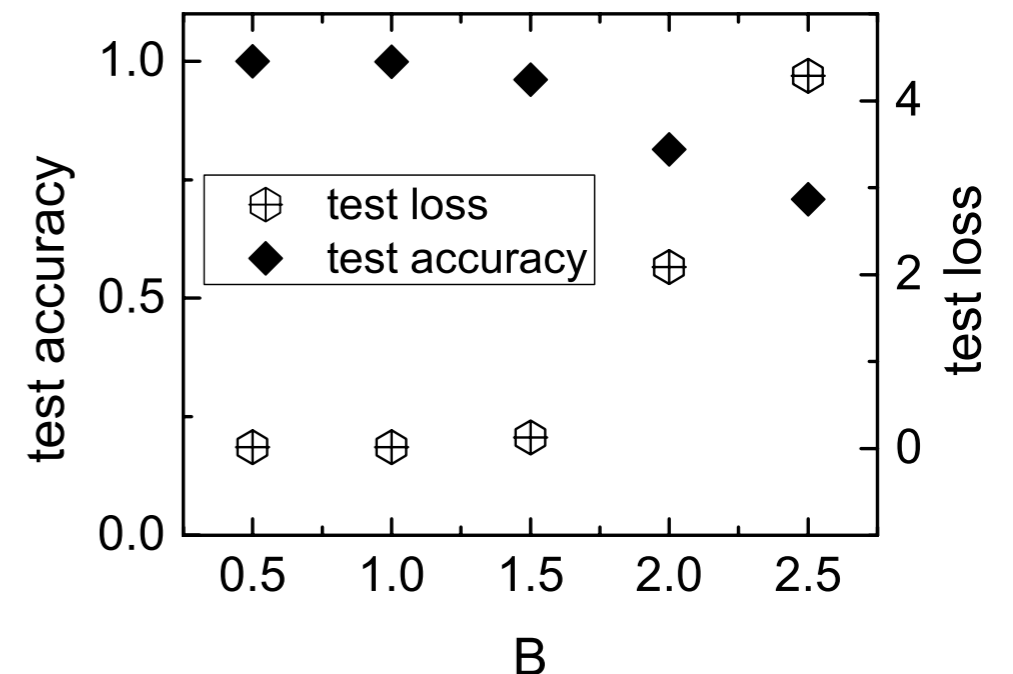
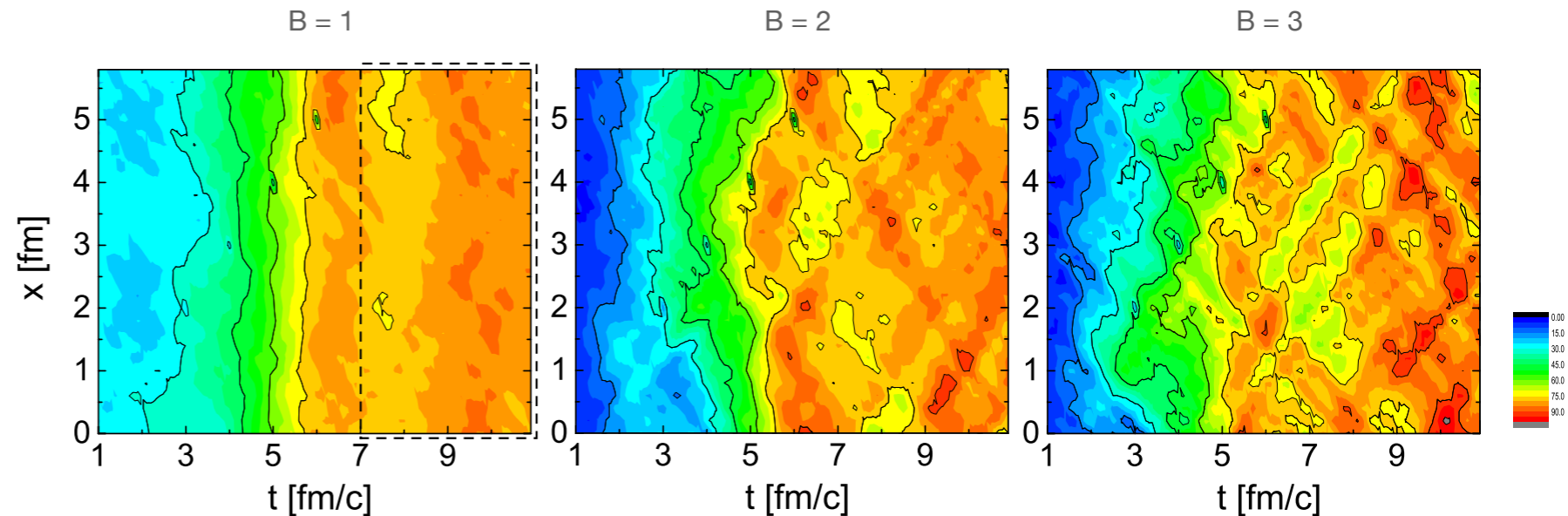
- $B = 0.5, 1, 1.5, 2, 2.5$

- Accurate on low-noise configurations

- Robust to the noises

- **Recognizing the phase transition orders on the spatial-temporal configurations**

- with spatial and initial fluctuations



Learning Dynamics

Damping Coefficients

- **Training**

- $L = 6.0 \text{ fm}$, t in $[12, 16] \text{ fm/c}$
- $N_t \times N_x = 50 \times 30 = 1500 \text{ pixels}$
- $\eta = (1.0 - 2.5), (4.6 - 5.5) \text{ fm}^{-1}$
 - $d\eta = 0.1 \text{ fm}^{-1}$
 - 1000 events in each bin

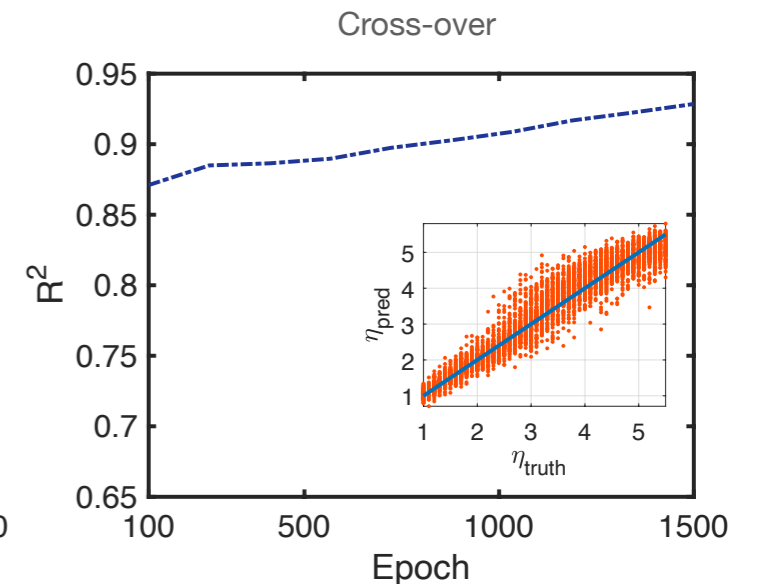
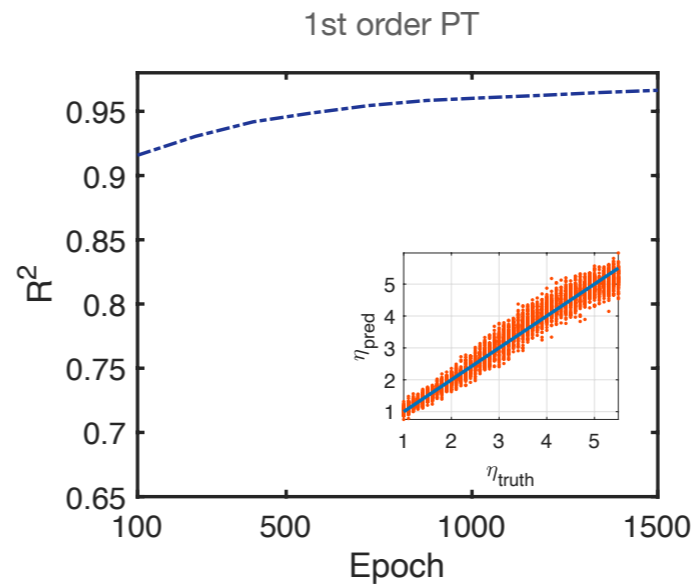
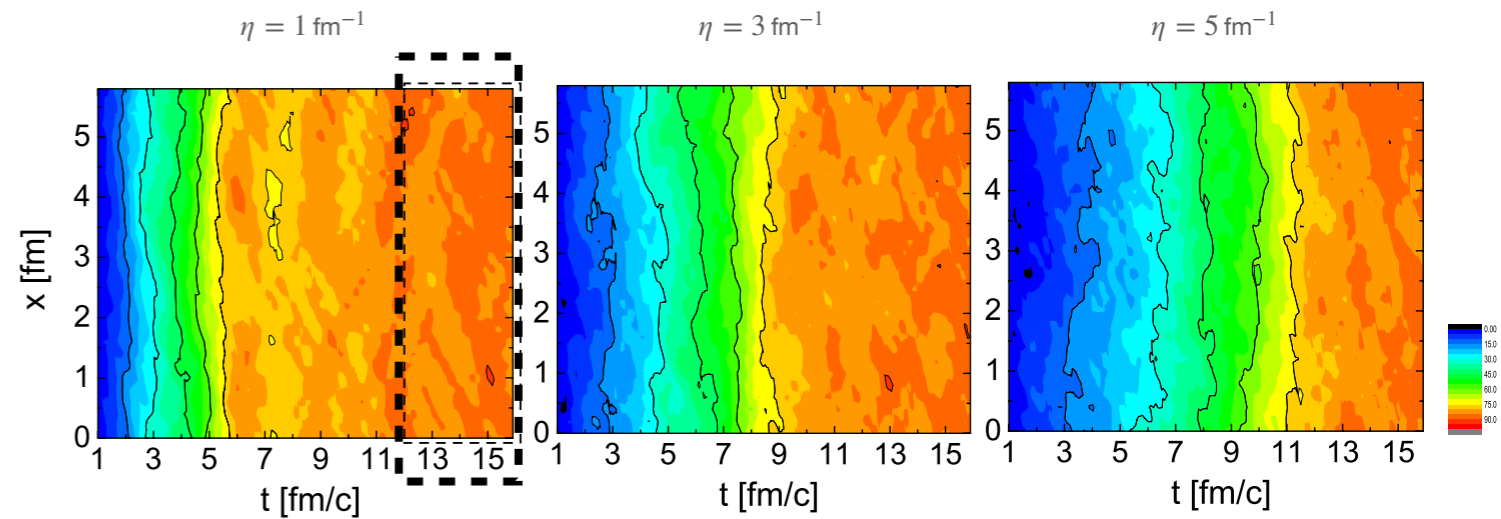
$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

$$SS_{\text{res}} = \sum_i (\eta_{i, \text{truth}} - \bar{\eta}_{\text{truth}})^2$$

$$SS_{\text{tot}} = \sum_i (\eta_{i, \text{truth}} - \eta_{i, \text{pred}})^2$$

- **Testing**

- Ground truth vs Predictions
- $\eta = (2.6 - 4.5) \text{ fm}^{-1}$



Learning Dynamics

Damping Coefficients

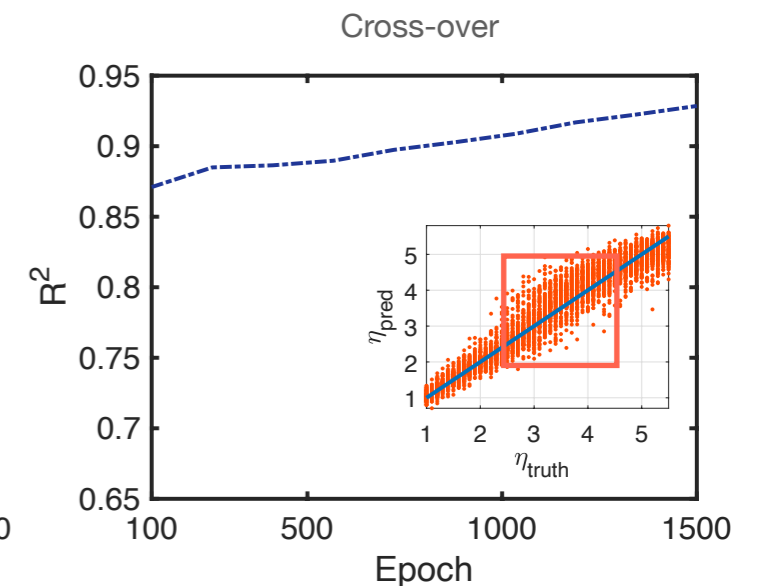
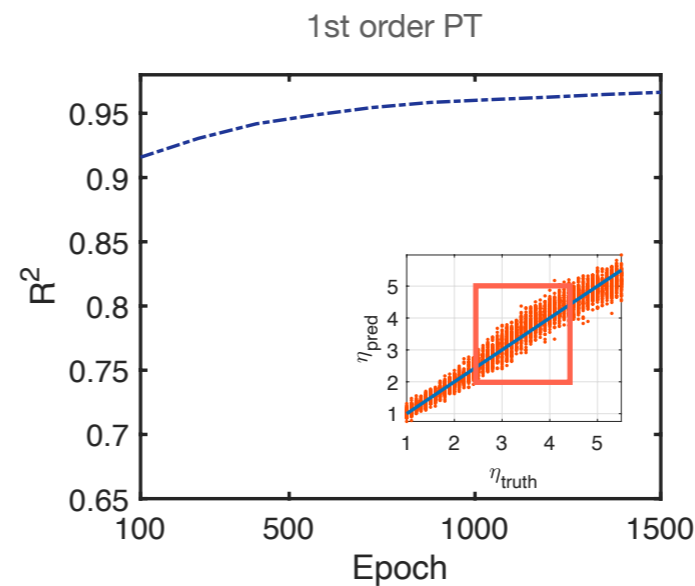
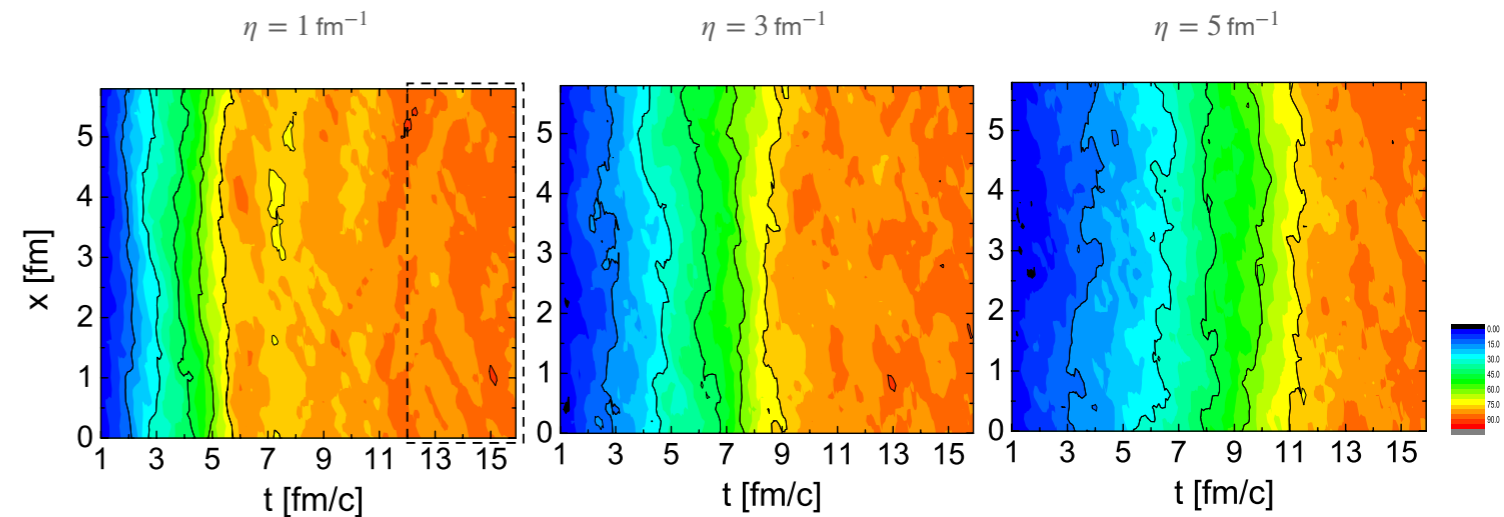
- **Training**

- $\eta = (1.0 - 2.5), (4.6 - 5.5) \text{ fm}^{-1}$
- $d\eta = 0.1 \text{ fm}^{-1}$
- 1000 events in each bin

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}}$$

- **Testing**

- Ground truth vs Predictions
- $\eta = (2.6 - 4.5) \text{ fm}^{-1}$
- Learning dynamics of stochastic processes from configurations driven by the damping coefficient



Summary

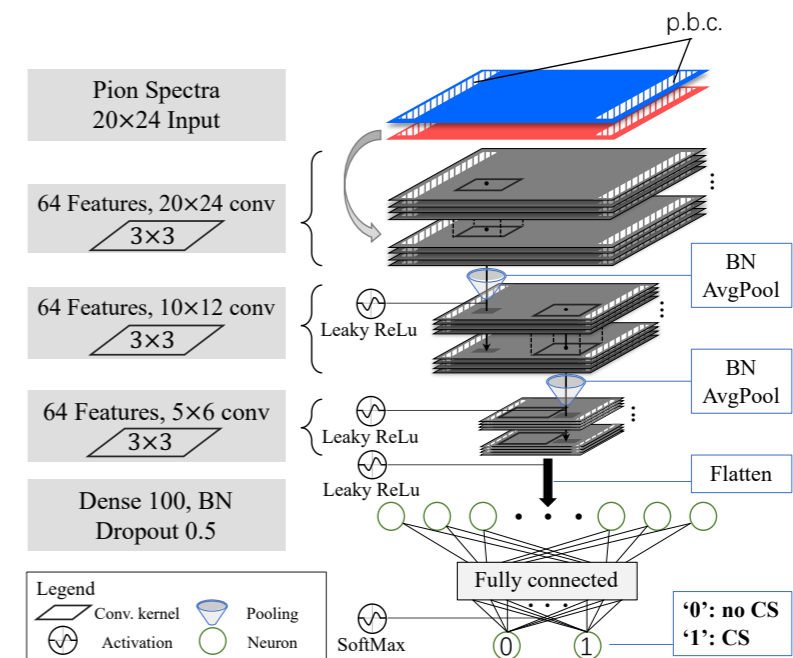
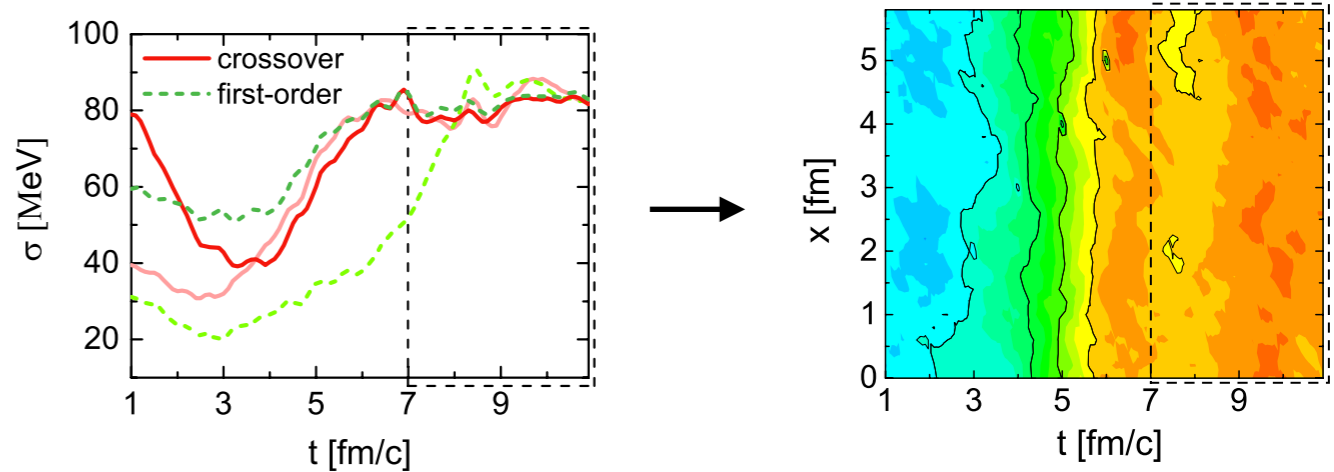
Summary and Outlooks

- **Take-home messages**

- Treat time-series as images
- Learn dynamics from a stochastic process
- The phase transition informations are encoded in the latent stage

- **Related works**

- Detecting CME
 - from Pion Spectra
 - as a CME-meter
 - validated in AuAu and ZrZr, RuRu collision systems



Summary and Outlooks

- **Take-home messages**

- Treat time-series as images
- Learn dynamics from a stochastic process
- The phase transition informations are encoded in the latent stage

- **Future works**

- 2+1D Langevin equation
 - Preparing $\sigma(x, y, t)$
 - Conv3D layers
- Track dynamics with Generative Models



Faces generated by Generative Adversarial Networks(GANs)





Future

AI in Physics, opportunities and challenges