Deep Learning Stochastic Process with QCD Phase Transition

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Main Content
Deep Learning Meets Physics

• Background
  • QCD phase transition
  • Deep learning in physics

• Deep Learning Dynamics
  • Understanding stochastic processes as images
  • Recognizing phase transition orders
  • Learning dynamical parameters

• Summary
Background
Phase Transition

Critical Point

- **QCD Phase Structures**
  - Chiral symmetry restoration
  - Deconfinement
  - Finite chemical potential $\mu_B, \mu_I, B, \omega$

- **Searching Critical End Point**
  - Theoretical calculations: (P)NJL, QM, FRG, DSE, etc
  - Lattice calculations
  - Experiments: BES

A schematic QCD phase diagram

A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, and N. Xu, Physics Reports (2020).
Stochastic Process

Criticality

• Along the freeze-out surface
  • Critical fluctuations
  • Statistical fluctuations
  • Dynamical critical fluctuations?


  • Memory effects from dynamical evolution
  • The Skewness and Kurtosis of the high order cumulants could be different from the equilibrated

• With Deep Learning
  • Detecting phase transition
  • Extracting dynamical information

  … in stochastic processes

QCD phase diagram from the linear sigma model with constituent quarks


from Swagato’s talk @201509, Kobe
Deep Learning
AI in Physics

• Classify phases

• Predict properties/signals

• Innovate computation

• New developments

Deep Learning

AI in High Energy Nuclear Physics

- **White Paper**

- **Experiments**

- **Lattice calculations**

  - From lattice QCD to in-medium heavy-quark interactions via deep learning
  *Kai Zhou* 24 June, *Transport Meeting 2021*

- **New Developments**

  - Compact stars
  - Phase structures
  - Dynamical processes

- **Recognizing QCD Phase Transitions**

  \[ \rho(p_T, \Phi) \]

  Cross-over ?

  1st order phase transition

Deep Learning
Critical Dynamics
Problem Set-Up
Stochastics Process

- **QCD phase transition in Langevin equation**

\[
\mathcal{L} = \bar{q} \left[ i\gamma^\mu \partial_\mu - g \left( \sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi} \right) + \gamma_0 \mu \right] q \\
+ \frac{1}{2} \left( \partial_\mu \sigma \partial_\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \right) - U(\sigma, \pi)
\]

\[V_{\text{eff}}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(\sigma; T, \mu)\]

\[U(\sigma) = \frac{1}{4} \lambda^2 \left( \sigma^2 - v^2 \right)^2 - \frac{h}{2} \sigma - U_0\]

\[\Omega_{\bar{q}q}(\sigma; T, \mu) = -d_q \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \ln \left[ 1 + e^{-(E-\mu)/T} \right] + T \ln \left( 1 + e^{-(E+\mu)/T} \right) \right\}\]

\[\frac{T(t)}{T_0} = \left( \frac{t}{t_0} \right)^{-0.45}\]

\[\langle \xi(t)\xi(t') \rangle = \frac{1}{V} m_\sigma \eta \coth \left( \frac{m_\sigma}{2T} \right) \delta(t-t')\]

\[\xi(x) = B \sqrt{\frac{1}{V} m_\sigma \eta \coth \left( \frac{m_\sigma}{2T} \right)} \frac{1}{dt} G(x)\]

- **Flucuations**

Linear Sigma Model
Effective potential
Hubble-like

\[-2E8\]

\[-3E8\]

\[-1.6E8\]

\[-1.8E8\]

\[
\begin{align*}
T & \quad [\text{MeV}] \\
\mu & \quad [\text{MeV}] \\
\sigma & \quad [\text{MeV}] \\
\end{align*}
\]

\[
\begin{align*}
T & = T_c & \text{first-order} \\
T > T_c & \text{crossover} \\
T < T_c & \text{1st order}
\end{align*}
\]
Problem Set-Up

Stochastics Process

• QCD phase transition in Langevin equation

\[ \mathcal{L} = \bar{q} \left[ i \gamma^\mu \partial_\mu - g \left( \sigma + i \gamma_5 \vec{\tau} \cdot \vec{\pi} \right) + \gamma_0 \mu \right] q + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} \right) - U(\sigma, \pi) \]

\[ V_{\text{eff}}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(\sigma; T, \mu) \]

\[ U(\sigma) = \frac{1}{4} \sigma^2 \left( \sigma^2 - v^2 \right)^2 - h \sigma - U_0 \]

\[ \Omega_{\bar{q}q}(\sigma; T, \mu) = - d_\mu \int \frac{d^3 p}{(2\pi)^3} \left\{ E + T \ln \left[ 1 + e^{-(E-\mu)/T} \right] + T \ln \left[ 1 + e^{-(E+\mu)/T} \right] \right\} \]

\[ \frac{T(t)}{T_0} = \left( \frac{t}{t_0} \right)^{-0.45} \]

\[ \langle \xi(t) \xi(t') \rangle = \frac{1}{V m_\sigma \eta} \coth \left( \frac{m_\sigma}{2T} \right) \delta(t - t') \]

\[ \xi(x) = B \sqrt{\frac{1}{V m_\sigma \eta} \coth \left( \frac{m_\sigma}{2T} \right) \frac{1}{dt} G(x)} \]

\[ \text{Sigma configurations} \]

\[ \text{Effective potential} \]

\[ \text{Hubble-like} \]

\[ \text{Linear Sigma Model} \]
Problem Set-Up

Stochastics Processes as Images

• Preparing configurations
  • Initial profiles sample with
    \[ P[\sigma(x)] \sim \exp(-\varepsilon(\sigma)/T) \]
    \[ \varepsilon(\sigma) = \int dx \left[ \frac{1}{2}(\nabla \sigma(x))^2 + V_{\text{eff}}(\sigma(x)) \right] \]
  • Simulated 10,000 events at each parameter
    • \( \mu = 180 \text{ MeV} \) as “cross-over”
    • \( \mu = 240 \text{ MeV} \) as “1st order”
    • \( B = 0.5, 1 \)
  • Compressing \( \sigma(x, t) \) as images
    • \( dx = 0.2 \text{ fm}, dt = 0.1 \text{ fm/c} \)
    • \( L = 6.0 \text{ fm}, t \text{ in } [7, 11] \text{ fm/c} \)
    • \( N_t \times N_x = 40 \times 30 = 1200 \text{ pixels} \)
Neural Networks

Deep CNNs

- Compressing $\sigma(x, t)$ as images
  - $N_t \times N_x = 40 \times 30 = 1200$ pixels
  - Data-sets: 8:2 = training: validation

- Architecture
  - Inputs: $\sigma(x, t)$ configurations
  - 3 CNNs
  - 1 Fully-Connected layer
  - Outputs: 0,1 / damping coefficient $\eta$

- Loss function
  - Categorical cross entropy
    \[
    \text{loss} = - \sum_{i=1}^{C} y_i \log f_i(x)
    \]
  - Mean Square Error
Recognition
Phase Transition Orders

- **Fluctuations**

\[ \xi(x) = B \sqrt{\frac{1}{V} m_{\eta} \eta \coth \left( \frac{m_x}{2T} \right) \frac{1}{dt} G(x)} \]

- **Training**
  - \( B = 0.5, 1 \)
  - Reaching saturation without over-fitting

- **Testing**
  - \( B = 0.5, 1, 1.5, 2, 2.5 \)
  - Accurate on low-noise configurations
  - Robust to the noises
Recognition

Phase Transition Orders

- **Training**
  - $B = 0.5, 1$

- **Testing**
  - $B = 0.5, 1, 1.5, 2, 2.5$
  - Accurate on low-noise configurations
  - Robust to the noises
  - Recognizing the phase transition orders on the spatial-temporal configurations
    - with spatial and initial fluctuations
Learning Dynamics

Damping Coefficients

• Training
  - $L = 6.0 \text{ fm}$, $t$ in $[12, 16] \text{ fm/c}$
  - $N_t \times N_x = 50 \times 30 = 1500$ pixels
  - $\eta = (1.0 - 2.5), (4.6 - 5.5) \text{ fm}^{-1}$
    - $d \eta = 0.1 \text{ fm}^{-1}$
    - 1000 events in each bin

\[ R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \]
\[ SS_{res} = \sum_i \left( \eta_i, \text{truth} - \bar{\eta}_{\text{truth}} \right) \]
\[ SS_{tot} = \sum_i \left( \eta_i, \text{truth} - \eta_i, \text{pred} \right) \]

• Testing
  - Ground truth vs Predictions
  - $\eta = (2.6 - 4.5) \text{ fm}^{-1}$
Learning Dynamics

Damping Coefficients

• Training
  • $\eta = (1.0 - 2.5), (4.6 - 5.5) \text{ fm}^{-1}$
  • $d\eta = 0.1 \text{ fm}^{-1}$
  • 1000 events in each bin

\[ R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \]

• Testing
  • Ground truth vs Predictions
  • $\eta = (2.6 - 4.5) \text{ fm}^{-1}$
  • Learning dynamics of stochastic processes from configurations driven by the damping coefficient
Summary
Summary and Outlooks

• Take-home messages
  • Treat time-series as images
  • Learn dynamics from a stochastic process
  • The phase transition informations are encoded in the latent stage

• Related works
  • Detecting CME
    • from Pion Spectra
    • as a CME-meter
    • validated in AuAu and ZrZr, RuRu collision systems

Yuan-Sheng Zhao, Lingxiao Wang, Kai Zhou, Xu-Guang Huang, ArXiv:2105.13761
• **Take-home messages**
  - Treat time-series as images
  - Learn dynamics from a stochastic process
  - The phase transition informations are encoded in the latent stage

• **Future works**
  - 2+1D Langevin equation
    - Preparing $\sigma(x, y, t)$
    - Conv3D layers
  - Track dynamics with Generative Models
Future

AI in Physics, opportunities and challenges