Kinetic theory for massive spin-1 particles in electromagnetic fields

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in collaboration with

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CRC–TR 211 Strong-interaction matter under extreme conditions

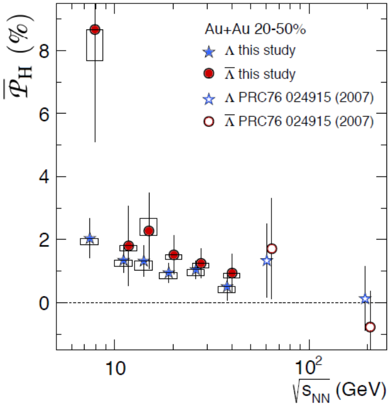
DFG

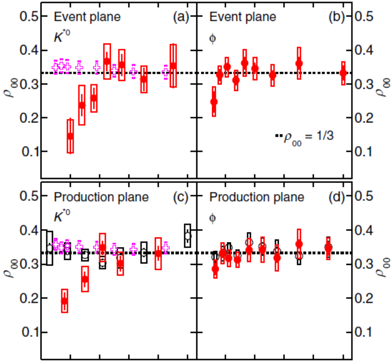
Motivation

- Heavy-ion collisions provide several polarization observables
  - Spin $1/2$: Decay of Λ-Hyperons
  - Spin 1: Decay of ϕ/K^{*0}-Mesons
- Both feature significant global polarization at lower energies

L. Adamczyk et al. (STAR), Nature 548 62-65 (2017)

S. Acharya et al. (ALICE), Physical Review Letters 125, 012301 (2020)





Free charged vector fields

Proca Lagrangian

$$\mathcal{L}_0 = \hbar \left(-\frac{1}{2} V_0^{*\mu\nu} V_{0,\mu\nu} + \frac{m^2}{\hbar^2} V^{*\mu} V_\mu \right) \qquad (1)$$

Field strength tensor $V_0^{\mu\nu} := \partial^\mu V^\nu - \partial^\nu V^\mu$

- Spin-1 particles have three degrees of freedom $(\lambda = -1, 0, 1)$
  - But a four-vector has four components
  - $\rightarrow$ One component fixed by constraint $\partial^\mu V_\mu = 0$
- All components fulfill the Klein-Gordon equation

$$\left(\Box + \frac{m^2}{\hbar^2} \right) V^\mu = 0 \,. \qquad (2)$$

Coupling to electromagnetic fields

Maxwell-Proca Lagrangian

$$\mathcal{L} = \hbar \left(-\frac{1}{2} V^{*\mu\nu} V_{\mu\nu} + \frac{m^2}{\hbar^2} V^{*\mu} V_\mu \right) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - iq\kappa F_{\mu\nu} V^\mu V^{*\nu} \quad (3)$$

Field strength tensors $V^{\mu\nu} := D^\mu V^\nu - D^\nu V^\mu \,, \quad F^{\mu\nu} := \partial^\mu A^\nu - \partial^\nu A^\mu .$

- Adjustments when taking into account electromagnetic fields:
  - Include Maxwell Lagrangian
  - Introduce (gauge-)covariant derivative $D^\mu := \partial^\mu + iq/\hbar A^\mu$
  - Take into account magnetic moment $\mu := (1+\kappa) q\hbar/2m$

    H. C. Corben, J. Schwinger, Physical Review 58, no. 11, 953-968 (1940)
- Which value to choose for κ?

Coupling to electromagnetic fields

- Demanding well-behaved high-energy cross-section suggests $\kappa = 1$

  M. Napsuciale, S. Rodriguez E. G. Delgado-Acosta, M. Kirchbach, Physical Review D 77, no. 1, 430 (2008)

  → Gyromagnetic ratio equals two, i.e. $\mu = q\hbar/m$
- Total electric current contains magnetization current

$$J^\mu := iq\left[V^{*\mu\nu}V_\nu - V^{\mu\nu}V^*_\nu - \partial_\nu\left(V^\nu V^{*\mu} - V^{*\nu}V^\mu\right)\right] . \tag{4}$$

Equations of motion of the coupled system

$$\left(D^\nu D_\nu + \frac{m^2}{\hbar^2}\right)V^\mu = i\frac{q}{\hbar}\left[2V_\nu F^{\nu\mu} - \frac{\hbar^2}{m^2}D^\mu\left(V_\nu J^\nu\right)\right] \tag{5}$$

$$\partial_\nu F^{\nu\mu} = J^\mu \tag{6}$$

Constraint equation

$$D^\mu V_\mu = -i\frac{q\hbar}{m^2}J^\mu V_\mu . \tag{7}$$

Wigner function

- Idea: Introduce a quantum-mechanical analogue of the one-particle distribution function H. -W. Lee, Physics Reports 259, no. 3, 147-211 (1995)
- Definition:
  - QM: Wigner transform of the density matrix
  - QFT: Wigner transform of the normal-ordered two-point function
- Significance of the Wigner function:
  - Determines polarization observables
  - Appears in conserved quantities (energy-momentum tensor, spin tensor, electric current)
  - Follows Boltzmann-like evolution equation

Wigner function

Wigner function for vector fields

$$W^{\mu\nu}(x,k) := \frac{1}{(2\pi\hbar)^4}\int \mathrm{d}^4 v e^{-\frac{i}{\hbar}k^\alpha v_\alpha}\left\langle : V_+^{*\mu} U_{+-} V_-^{\nu} : \right\rangle \tag{8}$$

with

$$V_\pm^\mu := V^\mu\left(x \pm \frac{v}{2}\right) , \tag{9}$$

$$U_{+-} := \hat{T}\exp\left[-i\frac{q}{\hbar}v^\alpha \int_{-1/2}^{1/2} \mathrm{d}t A_\alpha(x+tv)\right] . \tag{10}$$

- U_{+-} is the gauge link such that the Wigner function is gauge invariant

How to proceed?

- Goals:
  - Identify the relevant components of the (4×4) Wigner function
  - Obtain their dynamics
  - Relate to observables
- Plan of action:
  1. Formulate equations of motion for $W^{\mu\nu}$
     - $\longrightarrow$ Constraint equations
     - $\longrightarrow$ Mass-shell equations
     - $\longrightarrow$ Kinetic (Boltzmann) equations
  2. Simplify via an expansion around the classical limit ($\hbar$ expansion)
  3. Identify different components (scalar, vector, tensor)
  4. Formulate global equilibrium

Equations of motion

- Want to use equations of motion for the fields V^μ

  $\rightarrow$ Introduce Bopp operator $\hat{K}^\mu := \hat{\Pi}^\mu + \frac{i\hbar}{2}\hat{\nabla}^\mu$

  F. Bopp, Annales de l'institut Henri Poincaré 15, no.2, 81-112 (1956)
  D. Vasak, M. Gyulassy, H.-T. Elze, Annaly of Physics 173, no.2, 462-492 (1987)

  - $\hat{\Pi}^\mu := k^\mu - \frac{\hbar q}{2} j_1(\Delta) F^{\mu\nu}\partial_{k,\nu}$
  - $\hat{\nabla}^\mu := \partial^\mu - q j_0(\Delta) F^{\mu\nu}\partial_{k,\nu}$
  - $\Delta := \frac{\hbar}{2}\partial^\mu \partial_{k,\mu}$

Action of Bopp operators

$$\hat{K}^\alpha W^{\mu\nu} = \frac{i\hbar}{(2\pi\hbar)^4}\int \mathrm{d}^4 v e^{-\frac{i}{\hbar}k^\alpha v_\alpha} \left\langle : V_+^{*\mu} U_{+-} (D^\alpha V^\nu)_- : \right\rangle \tag{11}$$

- Equations of motion for $W^{\mu\nu}$ straightforward to obtain, but lengthy

$j_0(x) := \sin(x)/x \ , \quad j_1(x) := [\sin(x) - x\cos(x)]/x^2$

Power counting

- Expand the equations of motion around the semiclassical limit
  - $\rightarrow$ Approximate wavepackets as point particles, treat quantum corrections perturbatively
- Example: Bopp operator $\hat{K}^\mu = k^\mu + \frac{i\hbar}{2}\hat{\nabla}^{(0),\mu} + \mathcal{O}(\hbar^2)$
  - $\hat{\nabla}^{(0),\mu} := \partial^\mu - qF^{\mu\nu}\partial_{k,\nu}$
- Split Wigner function into symmetric and antisymmetric parts: $W^{\mu\nu} = W_S^{\mu\nu} + W_A^{\mu\nu}$
  - First step in separating effects (antisymmetric part connected to spin)

Equations of motion to order $\mathcal{O}(\hbar)$

Constraint equations

$$\begin{cases} 0 = k_\alpha W_S^{\mu\alpha} + \frac{i\hbar}{2}\hat{\nabla}_\alpha^{(0)} W_A^{\mu\alpha} \\ 0 = k_\alpha W_A^{\mu\alpha} + \frac{i\hbar}{2}\hat{\nabla}_\alpha^{(0)} W_S^{\mu\alpha} \end{cases} \quad (12)$$

Mass-shell equations

$$\begin{cases} 0 = (k^2 - m^2) W_S^{\mu\nu} + iq\hbar F_\alpha{}^{(\mu} W_A^{\nu)\alpha} \\ 0 = (k^2 - m^2) W_A^{\mu\nu} - iq\hbar F_\alpha{}^{[\mu} W_S^{\nu]\alpha} \end{cases} \quad (13)$$

Kinetic equations

$$\begin{cases} 0 = ik^\alpha \hat{\nabla}_\alpha^{(0)} W_S^{\mu\nu} + iqF_\alpha{}^{(\mu} W_S^{\nu)\alpha} \\ +\frac{q\hbar}{2}\left[(\partial^\gamma F_\alpha{}^{(\mu})\partial_{k,\gamma} W_A^{\nu)\alpha} + \frac{1}{m^2} J_\alpha W_A^{\alpha(\mu} k^{\nu)}\right] \\ 0 = ik^\alpha \hat{\nabla}_\alpha^{(0)} W_A^{\mu\nu} - iqF_\alpha{}^{[\mu} W_A^{\nu]\alpha} \\ -\frac{q\hbar}{2}\left[(\partial^\gamma F_\alpha{}^{[\mu})\partial_{k,\gamma} W_S^{\nu]\alpha} + \frac{1}{m^2} J_\alpha W_S^{\alpha[\mu} k^{\nu]}\right] \end{cases} \quad (14)$$

$A^{(\mu} B^{\nu)} := A^\mu B^\nu + A^\nu B^\mu \,, \quad A^{[\mu} B^{\nu]} := A^\mu B^\nu - A^\nu B^\mu$

Analysis of equations

- Three types of equations have different effects:
  - Kinetic equations determine evolution in phase space
  - Mass-shell equations determine dispersion relation
  - Constraint equations remove seven degrees of freedom

    $\longrightarrow$ Decompose $W^{\mu\nu}_{A/S}$ to recover nine independent components

Decomposition with respect to k^μ

$$W_S^{\mu\nu} = E^{\mu\nu} f_E + K^{\mu\nu} f_K + \frac{k^{(\mu}}{2k} F_S^{\nu)} + F_K^{\mu\nu} \tag{15}$$

$$W_A^{\mu\nu} = \frac{k^{[\mu}}{2k} F_A^{\nu]} + \epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} G_\beta \tag{16}$$

- $F_S^\mu k_\mu = F_A^\mu k_\mu = G^\mu k_\mu = F^\mu_{K,\mu} = 0 \,, \quad k_\mu F_K^{\mu\nu} = 0 \,, \quad F_K^{\mu\nu} = F_K^{\nu\mu}$

$E^{\mu\nu} := k^\mu k^\nu / k^2 \,, \quad K^{\mu\nu} := g^{\mu\nu} - E^{\mu\nu}$

Comparison to spin 1/2

- Spin-1/2 Wigner function: matrix in spinor space

  N. Weickgenannt, X. -L. Sheng, E. Speranza, Q. Wang, D. Rischke, Physical Review D 100, no. 5, 152 (2019)

- Decomposable according to Clifford algebra

  $\longrightarrow$ Four independent components

Decomposition of spin-1/2 Wigner function

$$W = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{i}{4}\left[\gamma_\mu, \gamma_\nu\right]\mathcal{S}^{\mu\nu}\right) \tag{17}$$

- Straightforward to compare independent components:
  - Scalar: $f_K \leftrightarrow \mathcal{F}$

    $\longrightarrow$ distribution function
  - Axial vector: $G^\mu \leftrightarrow \mathcal{A}^\mu$

    $\longrightarrow$ vector polarization
  - Traceless symmetric tensor: $F_K^{\mu\nu} \leftrightarrow \emptyset$

    $\longrightarrow$ tensor polarization?

Perturbative solution

- Goal: Solve equations of motion perturbatively
  - $\rightarrow$ Expansion $W^{\mu\nu} = W^{(0),\mu\nu} + \hbar W^{(1),\mu\nu} + \cdots$
- Zeroth order: Wigner function on shell, $W^{\mu\nu} \propto \delta(k^2 - m^2)$
  - $f_E^{(0)} = F_A^{(0),\mu} = F_S^{(0),\mu} = 0$
  - Independent components follow evolution equations

Equations of motion at order $\mathcal{O}(\hbar^0)$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} f_K^{(0)} \tag{18}$$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} G^{(0),\mu} - qF^{\mu\nu} G_\nu^{(0)} \tag{19}$$

$$0 = k^\alpha \hat{\nabla}_\alpha^{(0)} F_K^{(0),\mu\nu} - qF_{K,\alpha}^{(0),(\mu} F^{\nu)\alpha} \tag{20}$$

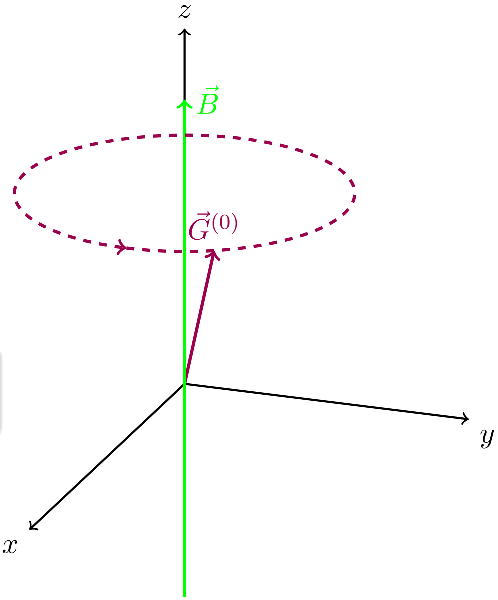
- Eq. (18) has transparent interpretation
- What about the others?

- Particle rest frame: Familiar BMT equation
- Describes rotation of vector polarization around the (rest-frame-) magnetic field

BMT equation

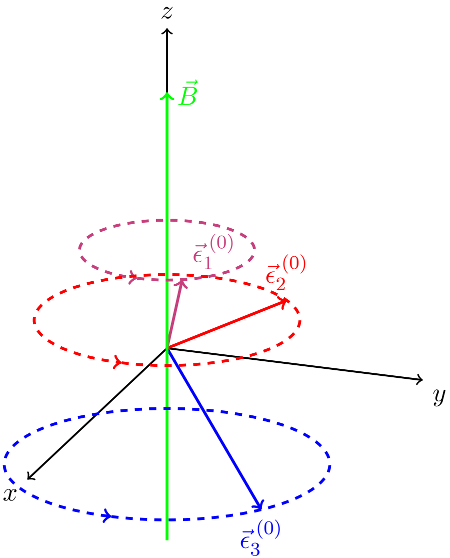
$$\dot{\vec{G}}^{(0)} = q\vec{G}^{(0)} \times \vec{B} \tag{21}$$

$G^{(0),\mu} \equiv (G^{(0),0}, \vec{G}^{(0)})$



Evolution of tensor polarization

- Decompose $F_K^{(0),\mu\nu}$ into three orthogonal main axes $\vec{\epsilon}_i^{(0)}, i = 1, 2, 3$
- All main axes fulfill BMT equation separately



Results: Zeroth order

- Recovered Boltzmann-Vlasov equation for classical distribution function
- Vector and tensor polarization follow BMT equations
- Intuitive classical geometric picture:
  - Spin-1/2 particles: spinning sphere
    - $\rightarrow$ Spin direction given by $\mathcal{A}^\mu$
  - Spin-1 particles: spinning ellipsoid
    - $\rightarrow$ Spin direction given by G^μ
    - $\rightarrow$ Relative orientation specified by $F_K^{\mu\nu}$

Solution: First order in $\hbar$

- Constraint and mass-shell equations nontrivial
  - $\rightarrow$ $f_E, F^\mu_A, F^\mu_S \neq 0$
  - $\rightarrow$ Independent parts of the Wigner function not on-shell anymore

$$W^{(1),\mu\nu} = \delta(k^2 - m^2) W^{(1),\mu\nu}_{\text{on-shell}} + \delta'(k^2 - m^2) W^{(1),\mu\nu}_{\text{off-shell}} \quad (22)$$

- Introduces Zeeman splitting $m \rightarrow m + \lambda \frac{\hbar q}{2} F^{\alpha\beta} \Sigma_{\alpha\beta}$
  - Twice as large as in spin-1/2 case
- Kinetic equations:
  - Contain off-shell-contributions as well
    - Can be eliminated by a suitable transformation (not changing dynamics)

      N. Weickgenannt, X. -L. Sheng, E. Speranza, Q. Wang, D. Rischke, Physical Review D 100, no. 5, 152 (2019)
  - Describe dynamics of modified quantities
    - Consequence of constraints

$\Sigma^{\mu\nu} := -\epsilon^{\mu\nu\alpha\beta} \frac{k_\alpha}{m} \frac{G_\beta}{|G|}$

Boltzmann equation for spin-1 particles

- Introduce scalar distribution function for (anti-)particles of spin λ

$$W^{\mu}_{\text{on-shell},\mu} = \frac{1}{(2\pi\hbar)^3}\frac{1}{3}\sum_{\lambda=-1}^{1}\sum_{e=\pm}\Theta(ek^0)f^e_\lambda \tag{23}$$

Scalar Boltzmann equation to order $\mathcal{O}(\hbar)$

$$\sum_{\lambda,e}\Theta(ek^0)\delta\left(k^2-m^2\right)\left[k\cdot\hat{\nabla}^{(0)}+\frac{q\hbar\lambda}{2}(\partial^\gamma F^{\rho\sigma})\partial_{k,\gamma}\Sigma_{\rho\sigma}\right]f^e_\lambda = 0 \tag{24}$$

- Contains free-streaming and Vlasov terms as well as Mathisson force (twice as large as in spin-1/2 case)
  → Naive picture of spinning balls with twice the spin magnitude holds up to this order
- However, dynamics of polarization still different

$f^e_\lambda := f^{(0),e}_\lambda + \hbar f^{(1),e}_\lambda$

Global equilibrium

- Two kinds of equilibrium:
  - Local: Collision term in Boltzmann equation vanishes
  - Global: Local+streaming term (other side of Boltzmann equation) vanishes
- Collision term not yet included [$\rightarrow$ (near) future]
- Use Ansatz for local equilibrium distribution function, Boltzmann equation then determines conditions for global equilibrium

Ansatz for local equilibrium distribution function

$$f_\lambda^{e,\mathrm{eq}} := \left(e^{g_\lambda^e} - 1\right)^{-1} \tag{25}$$

$$g_\lambda^e := a_\lambda^e + k_\mu \beta^\mu + \lambda \frac{\hbar}{2} \Omega_{\mu\nu} \Sigma^{\mu\nu} \tag{26}$$

- $a_\lambda^e, \beta^\mu, \Omega^{\mu\nu}$ Lagrange multipliers

Global equilibrium: distribution function

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Necessary conditions for global equilibrium

$$\partial^{(\mu}\beta^{\nu)} = 0 \quad (27)$$

$$\partial_\mu a^e_\lambda = F_{\mu\nu}\beta^\nu \quad (28)$$

$$\partial_\mu \Omega_{\alpha\nu} = 0 \quad (29)$$

- In global equilibrium, $\Omega_{\mu\nu}$ equals thermal vorticity $\varpi_{\mu\nu} := \frac{1}{2}\partial_{[\mu}\beta_{\nu]}$
- $\beta^\mu \equiv U^\mu/T$ related to fluid velocity
- Discussion identical to spin-1/2 case due to same structure of Boltzmann equation
- What about polarization in equilibrium?

Global equilibrium: polarization

- Assumption: polarization at least of order $\mathcal{O}(\hbar)$
  $\rightarrow$ No large initial polarization
- Idea: Split first-order polarization into free and induced parts
  - Free parts follow BMT equation
  - Induced parts determined from force terms in kinetic equations
- Induced parts determined by $V = \frac{1}{(2\pi\hbar)^3}\frac{1}{3}\sum_{\lambda,e}\Theta(ek^0)f^e_\lambda$
- Induced vector polarization by thermal vorticity and magnetic fields
- No induced tensor polarization to first order in $\hbar$

Global equilibrium: polarization

Wigner function in global equilibrium

$$W^{\mu\nu}_{S,\text{eq},\text{on-shell}} = K^{\mu\nu}\left(V^{(0)} + \hbar V^{(1)}\right) + \hbar\Phi^{\mu\nu} \quad (30)$$

$$W^{\mu\nu}_{A,\text{eq},\text{on-shell}} = -i\hbar\left[\varpi_{\mu\nu}V^{(0)\prime} + \frac{q}{2k^2}F_{\mu\nu}V^{(0)} + \frac{1}{2}E^{\alpha}{}_{[\mu}\left(\varpi_{\nu]\alpha}V^{(0)\prime} + \frac{2q}{k^2}F_{\nu]\alpha}V^{(0)}\right) + \Xi^{\mu\nu}\right] \quad (31)$$

$$W^{\mu\nu}_{S,\text{eq},\text{off-shell}} = 0 \quad (32)$$

$$W^{\mu\nu}_{A,\text{eq},\text{off-shell}} = -i\hbar q F_{\alpha}{}^{[\mu}K^{\nu]\alpha}V^{(0)} \quad (33)$$

- $\Phi^{\mu\nu}$, $\Xi^{\mu\nu}$ follow BMT equations, unconstrained otherwise
- Terms $\propto E^{\alpha}{}_{\mu}$ do not contribute to polarization density P^{μ}_{eq}

$$P^{\mu}_{\text{eq}}(x,k) \propto \frac{i}{2m}\epsilon^{\mu\nu\alpha\beta}k_\nu W_{A,\text{eq},\alpha\beta} \quad (34)$$

F. Becattini, arXiv:2004.04050v2 (2020)

Polarization in equilibrium: results

- Alignment of polarization with magnetic field and fluid vorticity
$$\Pi^\mu_{\text{eq}}(x) := \int \mathrm{d}^4k P^\mu_{\text{eq}}(x,k) \propto B^\mu \;, \omega^\mu \tag{35}$$

- Alignment with vorticity (magnetic field) in same (opposite) directions for antiparticles
- In global equilibrium to order $\mathcal{O}(\hbar)$, spin-1 particles behave like spin-1/2 particles with double spin magnitude
  - Not necessarily true in local equilibrium/ out of equilibrium! ($\rightarrow$ future work)

$\omega^\mu := 1/2\epsilon^{\mu\nu\alpha\beta}U_\nu\partial_\alpha(U_\beta/T)\;, \quad B^\mu := 1/2\epsilon^{\mu\nu\alpha\beta}U_\nu F_{\alpha\beta}$

Conserved quantities

- Action invariant under $\mathrm{SO}^+(1,3) \times \mathbf{R}^{1,3} \times \mathrm{U}(1)$
- Non-conservation of spin tensor $S^{\lambda\mu\nu}$
- Conservation of
  - Energy-momentum tensor $T^{\mu\nu}$
  - Electric current J^{μ}

(Non-)conservation equations

$$\partial_\mu J^\mu = 0 \tag{36}$$

$$\partial_\mu T^{\mu\nu} = 0 \tag{37}$$

$$\hbar \partial_\lambda S^{\lambda\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \tag{38}$$

- Can be expressed via Wigner functions

Pseudo-gauge transformations

- Conserved currents not unique, determined up to pseudo-gauge transformations

  E. Speranza, N. Weickgenannt, The European Physical Journal A 57, 155 (2021)

- Idea: Find a spin tensor conserved in the absence of interactions
  - Reasoning: Spin and angular momentum should only be exchanged in interactions
  - We have so far included only (self-consistent) mean fields

Electric current

$$J^\mu = \frac{2q}{\hbar}\int \mathrm{d}K \left(k^\mu V + \frac{q\hbar}{2}F^{\alpha\beta}\partial_k^\mu \bar{\Sigma}_{\alpha\beta} + \hbar\partial_\nu \bar{\Sigma}^{\mu\nu}\right) + \mathcal{O}(\hbar^2) \qquad (39)$$

- Recover magnetization current (separately conserved)

$\bar{\Sigma}^{\mu\nu} := -\epsilon^{\mu\nu\alpha\beta}\frac{k_\alpha}{m}G_\beta\ ,\quad \mathrm{d}K := \mathrm{d}^4k\,\delta(k^2-m^2)$

Conserved currents: spin tensor

- Comparison to spin-1/2:

E. Speranza, N. Weickgenannt, The European Physical Journal A 57, 155 (2021)

Spin tensors

$$\text{Spin}-1: \qquad S^{\lambda\mu\nu}_{s=1} = i g^{[\mu}_{\alpha} g^{\nu]}_{\beta} \int \mathrm{d}^4k\, k^\lambda W^{\alpha\beta}_{(s=1)} \tag{40}$$

$$\text{Spin}-\frac{1}{2}: \qquad S^{\lambda\mu\nu}_{s=1/2} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]_{ab} \int \mathrm{d}^4k\, k^\lambda W^{ab}_{(s=1/2)} \tag{41}$$

- Connection between spin-1 and spin-1/2: generators of $\mathrm{SO}^+(1,3)$ in respective representations (as expected)
- Spin tensor not conserved only due to electromagnetic interactions

Conserved currents: Energy-momentum tensor

Energy-momentum tensor

$$T^{\mu\nu} = -\frac{2}{\hbar}\sum_{\lambda=-1}^{1}\sum_{e=\pm}\int \mathrm{d}K^{(\lambda,e)} k^\mu k^\nu f_\lambda^e + H^{\mu\alpha}F_\alpha{}^\nu - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \quad (42)$$

- Spin- and energy-dependent mass-shell
  $\mathrm{d}K^{(\lambda,e)} := \mathrm{d}^4k\delta\left(k^2 - m^2 - \hbar\lambda q F^{\alpha\beta}\Sigma_{\alpha\beta}\right)\Theta(ek^0)$
- Familiar objects from macroscopic electrodynamics:
  - Displacement tensor $H^{\mu\nu} := F^{\mu\nu} - M^{\mu\nu}$
  - Dipole tensor $M^{\mu\nu} := -2q\int \mathrm{d}KW^{\mu\nu}_{A,\mathrm{on-shell}}$
- Energy-momentum tensor of fluid and electromagnetic field split in a familiar way

W. Israel, General Relativity and Gravitation 9, no. 5, 451-468 (1978)

Conclusion

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- Formulated kinetic theory for massive spin-1 particles in electromagnetic fields
- Started from the full quantum equations of motion
  $\longrightarrow$ Considered semiclassical limit
- Computed global equilibrium
- Clarified connection to conserved currents

Future perspectives

- Formulate extended phase space $(x, k) \to (x, k, \mathfrak{s})$
  - Already done ($\to$ future talk)
- Include collisions
  - In progress
  - Next logical step towards dissipative hydrodynamics

    N. Weickgenannt, D. W., E. Speranza, D. Rischke, in preparation
  - Compare to spin-$1/2$

    N. Weickgenannt, E. Speranza, X. -L. Sheng, Q. Wang, D. Rischke, arXiv: 2103.04896 (2021)
- Consider gauge fields
  - Clarify issues with gauge invariance

    $\to$ Connection to massive case via Lorenz gauge?

    X. -G. Huang, P. Mitkin, A. V. Sadofyev, E. Speranza, J. High Energ. Phys. 2020, 117 (2020)

Appendix

Kinetic equations to order $\mathcal{O}(\hbar)$

- Definitions:
  - $3V := K_{\mu\nu} W^{\mu\nu}_{\text{on-shell}}$
  - $\bar{\Sigma}^{\mu\nu} := -iW^{\mu\nu}_{A,\text{on-shell}} - \frac{2q\hbar}{k^2} E^{[\mu}_{\alpha} F^{\nu]\alpha} V + \frac{q\hbar}{k^2} F^{\mu\nu} V$
  - $\mathcal{F}^{\mu\nu}_{K} := K^{\mu\nu}_{\alpha\beta} W^{\alpha\beta}_{\text{on-shell}}$
- Modifications arise due to constraint equations

Kinetic equations to order $\mathcal{O}(\hbar)$

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Combined kinetic equations

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)} 3\left(V + \frac{1}{3}\frac{q\hbar}{4k^2} F^{\mu\nu}\bar{\Sigma}_{\mu\nu}\right) + \frac{q\hbar}{2}(\partial^\gamma F^{\alpha\beta})\partial_{k,\gamma}\bar{\Sigma}_{\alpha\beta}\right] \tag{43}$$

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)} \bar{\Sigma}^{\mu\nu} - qF_\rho{}^{[\mu}\bar{\Sigma}^{\nu]\rho} - \frac{q\hbar}{2}(\partial^\gamma F_\rho{}^{[\mu})\partial_{k,\gamma}\left(\mathcal{F}_K^{\nu]\rho} + g^{\nu]\rho}\mathcal{F}_K\right) - \frac{q}{k^2} J_\alpha \mathcal{F}_K^{\alpha[\mu} k^{\nu]}\right] \tag{44}$$

$$0 = \delta(k^2 - m^2)\left[k \cdot \hat{\nabla}^{(0)} \mathcal{F}_K^{\rho\sigma} + qF_\alpha{}^{(\rho}\mathcal{F}_K^{\sigma)\alpha} + \frac{q\hbar}{2} K^{\rho\sigma}_{\mu\nu}(\partial^\gamma F^\mu{}_\alpha)\partial_{k,\gamma}\bar{\Sigma}^{\nu\alpha}\right] \tag{45}$$