D mesons in a thermal environment: interactions, generated states and transport properties

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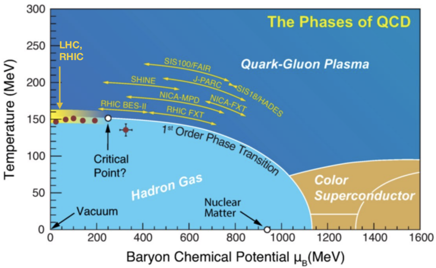
Outline

- Introduction
- **PART I:** Effective Theory for D mesons
  - Unitarization and dynamically-generated states
  - Modifications at finite temperature
  - Thermal evolution of states
- **PART II:** Kinetic Theory for D mesons
  - Transport equation from real-time dynamics
  - Off-shell Fokker-Planck equation
  - Heavy-flavor transport coefficients
- Conclusions

Introduction

A. Bazavov *et al.*, 1904.09951



- Infer QCD properties through final state of RHICs
- Interactions, phase transition(s), decay/regeneration... obscure the connection to QGP properties
- Find clean and solid observables of early stages

Hard Probes: Jets, hard electromagnetic emission, heavy flavor (quarkonia, open-heavy flavor hadrons...)

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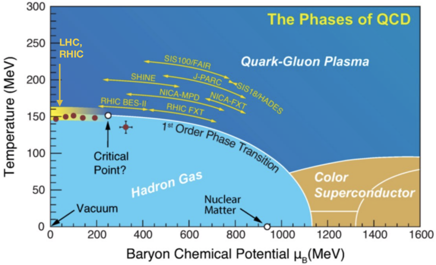

- Infer QCD properties through final state of RHICs
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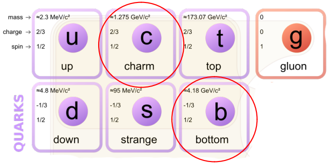
Hard Probes: Jets, hard electromagnetic emission, heavy flavor (quarkonia, open-heavy flavor hadrons...)

Heavy quarks: formed at the initial stage of the collision (short formation time) and difficult to equilibrate along their evolution (large relaxation time)



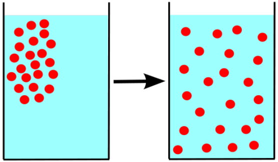
- In this talk I focus on D/D^* mesons, and study their interactions with the abundant light mesons ($\Phi = \{\pi, K, \bar{K}, \eta\}$)
- **Heavy-hadron mass is the dominant scale**

$$M_D \gg m_\Phi, T, \Lambda_{QCD}$$

- Picture: Brownian particle in a thermal bath
  B. Svetitsky, Phys. Rev. D37, 484 (1988)
- Transport properties: (Heavy-flavor) diffusion coefficient, D_s.

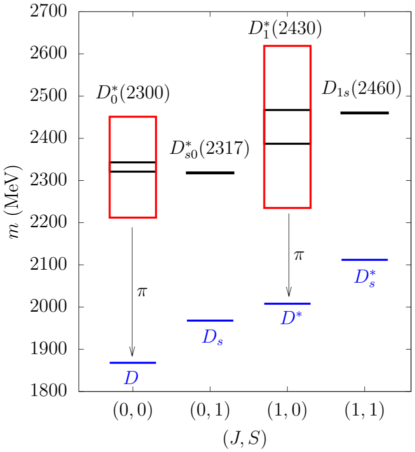
$$\vec{J} = -D_s \vec{\nabla} n$$

D_s depends on interactions (cross sections), and medium properties (T, μ_i)



D-meson spectrum

Vacuum *D*-meson spectroscopy is interesting by itself



- *D* meson: ground state
- D^* meson: effectively stable (HQ partner of *D* meson)
- $D_0^*(2300)$ and $D_1^*(2430)$: resonances decaying into *D* and D^* in *s*-wave
- Strangeness sector: D_s and D_s^* ground states, and $D_{s0}^*(2317)$ and $D_{1s}(2460)$ bound states

P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

- **PART I:** Effective Theory for D mesons
    - Unitarization and dynamically-generated states
    - Modifications at finite temperature
    - Thermal evolution of states

Effective Lagrangian based on **chiral** and **heavy-quark spin-flavor** symmetries ▸ Effective Lagrangian

- **Chiral expansion** is performed up to NLO
  : also explicitly broken due to light-meson masses $(\pi, K, \bar{K}, \eta)$.
- **Heavy-quark mass expansion** is kept to LO
  : broken by heavy meson masses (D, D_s, D^*, D_s^*).

Based on previous works:

E.E. Kolomeitsev and M.F.M. Lutz *Phys.Lett. B582 (2004) 39*

J. Hofmann and M.F.M. Lutz *Nucl.Phys. A733 (2004) 142*

F.K.Guo *et al Phys.Lett. B641 (2006) 278*

M.F.M. Lutz and M. Soyeur *Nucl.Phys. A813 (2008) 14*

F.K.Guo, C.Hanhart. S. Krewald, U.G. Meissner *Phys.Lett. B666 (2008) 251*

F-K.Guo, C. Hanhart, U.G. Meissner *Eur.Phys.J. A40 (2009) 171*

L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

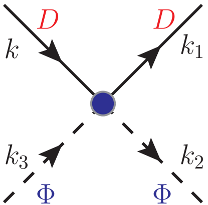
L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada and JMT-R. *Annals Phys. 326 (2011) 2737*

Amplitudes at tree level and lowest-order in m_D^{-1} expansion

Perturbative amplitude ▸ full tree level

$$V(s,t,u) = \frac{C_0}{4f_\pi^2}(s-u) + \frac{2C_1}{f_\pi^2}h_1 + \frac{2C_2}{f_\pi^2}h_3(k_2\cdot k_3) + \frac{2C_3}{f_\pi^2}h_5[(k\cdot k_3)(k_1\cdot k_2)+(k\cdot k_2)(k_1\cdot k_3)]$$

f_π: pion decay constant
Isospin coefficients: fixed by symmetry
Low-energy constants: fixed by experiment or by underlying theory
Z.-H. Guo *et al.* Eur. Phys. J.C79, 1, 13 (2019)



Amplitude accounts for elastic scatterings:
$D\pi$, DK, $D\bar{K}$, $D\eta$
$D_s\pi$, D_sK, $D_s\bar{K}$, $D_s\eta$ and their inelastic channels.

Caveat: Exact $\mathcal{S}$-matrix unitarity is lost in the truncation of the EFT
Solution: We impose exact unitarity to the amplitude

Unitarization: Bethe-Salpeter equation

$$T(s) = V(s) + \int VGT \ (s)$$

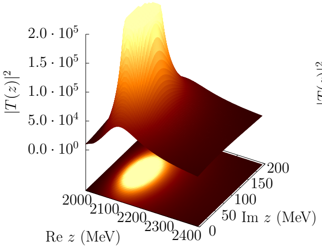
The equation can be reduced using the "on-shell" factorization,
J.A.Oller and E. Oset *Nucl.Phys.A620 (1997) 438*
L. Roca, E. Oset and J. Singh *Phys.Rev.D72 (2005) 014002*

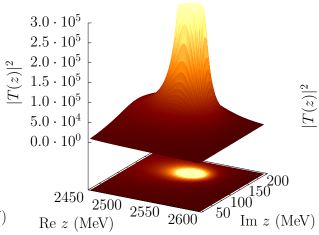
Unitarized scattering amplitude (s-wave)

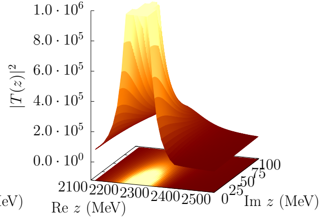
$$T(s) = \frac{V(s)}{1 - G(s)V(s)} \ ; \ G^{l}_{D\Phi}(s) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_D^2 + i\epsilon} \frac{1}{(p-k)^2 - m_\Phi^2 + i\epsilon}$$

Unitarized amplitude (coupled channels)

$$T_{ij}(s) = [1 - G(s)V(s)]^{-1}_{ik} V_{kj}(s)$$







$z = \sqrt{s}$

D_0 (2400) $D^*_{s0}(2317)$

Resonances and Bound states are generated as poles in the complex energy plane

$$m_R = \mathrm{Re}\ z_R\ , \qquad \Gamma_R = 2\mathrm{Im}\ z_R$$

Dynamically generated states at $T = 0$

$J = 0$ ←Heavy-quark spin partners→ $J = 1$

	(S,I)	RS	M_R (MeV)	$\Gamma_R/2$ (MeV)	$\lvert g_i \rvert$ (GeV)	χ_i
	$(-1,0)$	$(-)$	2261.5	102.9	$\lvert g_{D\bar{K}}\rvert = 11.6$	$\chi_{D\bar{K}} = 0.43$
$D_0^*(2300)$	$(0,\frac{1}{2})$	$(-,+,+)$	2081.9	86.0	$\lvert g_{D\pi}\rvert = 8.9$	$\chi_{D\pi} = 0.40$
					$\lvert g_{D\eta}\rvert = 0.4$	$\chi_{D\eta} = 0.00$
					$\lvert g_{D_s\bar{K}}\rvert = 5.4$	$\chi_{D_s\bar{K}} = 0.05$
		$(-,-,+)$	2529.3	145.4	$\lvert g_{D\pi}\rvert = 6.7$	$\chi_{D\pi} = 0.10$
					$\lvert g_{D\eta}\rvert = 9.9$	$\chi_{D\eta} = 0.40$
					$\lvert g_{D_s\bar{K}}\rvert = 19.4$	$\chi_{D_s\bar{K}} = 1.63$
$D_{s0}^*(2317)$	$(1,0)$	$(+,+)$	2252.5	0.0	$\lvert g_{DK}\rvert = 13.3$	$\chi_{DK} = 0.66$
					$\lvert g_{D_s\eta}\rvert = 9.2$	$\chi_{D_s\eta} = 0.17$
	$(1,1)$	$(-,+)$	2264.6	200.9	$\lvert g_{D_s\pi}\rvert = 7.3$	$\chi_{D_s\pi} = 0.21$
					$\lvert g_{DK}\rvert = 5.9$	$\chi_{DK} = 0.08$

	(S,I)	RS	M_R (MeV)	$\Gamma_R/2$ (MeV)	$\lvert g_i \rvert$ (GeV)	χ_i
	$(-1,0)$	$(-)$	2404.9	87.8	$\lvert g_{D^*\bar{K}}\rvert = 13.2$	$\chi_{D^*\bar{K}} = 0.53$
$D_1^*(2430)$	$(0,\frac{1}{2})$	$(-,+,+)$	2222.3	84.7	$\lvert g_{D^*\pi}\rvert = 9.5$	$\chi_{D^*\pi} = 0.40$
					$\lvert g_{D^*\eta}\rvert = 0.4$	$\chi_{D^*\eta} = 0.00$
					$\lvert g_{D_s^*\bar{K}}\rvert = 5.7$	$\chi_{D_s^*\bar{K}} = 0.05$
		$(-,-,+)$	2654.6	117.3	$\lvert g_{D^*\pi}\rvert = 6.5$	$\chi_{D^*\pi} = 0.09$
					$\lvert g_{D^*\eta}\rvert = 10.0$	$\chi_{D^*\eta} = 0.40$
					$\lvert g_{D_s^*\bar{K}}\rvert = 18.5$	$\chi_{D_s^*\bar{K}} = 1.47$
$D_{s1}^*(2460)$	$(1,0)$	$(+,+)$	2393.3	0.0	$\lvert g_{D^*K}\rvert = 14.2$	$\chi_{D^*K} = 0.68$
					$\lvert g_{D_s^*\eta}\rvert = 9.7$	$\chi_{D_s^*\eta} = 0.17$
	$(1,1)$	$(-,-)$	2392.2	193.0	$\lvert g_{D_s^*\pi}\rvert = 7.9$	$\chi_{D_s^*\pi} = 0.22$
					$\lvert g_{D^*K}\rvert = 6.3$	$\chi_{D^*K} = 0.08$

G. Montaña, À. Ramos, L. Tolos and JMT-R, Phys.Rev.D 102 (2020) 9, 096020

Double pole structure of $D_0^*(2300)$ (and $D_1^*(2430)$)

M. Albadalejo *et al.* Phys.Lett.B 767 (2017) 465

Z.-H. Guo *et al.* Eur.Phys.J.C79 (2019)13

U. Meissner, Symmetry 12 (2020) 6, 981

Finite temperature

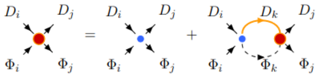
At $T \neq 0$ we use the imaginary time formalism
(energies become Matsubara frequencies, $q^0 \to \omega_n = i2\pi T n$)

$$G_{D\Phi}(E,\vec{p};T) = \int \frac{d^3k}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega,\vec{k};T) S_\Phi(\omega',\vec{p}-\vec{k};T)}{E-\omega-\omega'+i\varepsilon} [1+f(\omega,T)+f(\omega',T)]$$

$$S_D(\omega,\vec{k};T) = -\frac{1}{\pi} \mathrm{Im}\, \mathcal{D}_D(\omega,\vec{k};T) = -\frac{1}{\pi} \mathrm{Im} \left(\frac{1}{\omega^2 - \vec{k}^{\,2} - m_D^2 - \Pi_D(\omega,\vec{k};T)} \right)$$

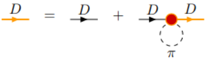
$$\Pi_D(\omega_n,\vec{k};T) = T \int \frac{d^3p}{(2\pi)^3} \sum_m \mathcal{D}_\pi(\omega_m - \omega_n, \vec{p}-\vec{k})\, T_{D\pi}(\omega_m,\vec{p})$$

$$T_{ij} = [1 - G_{D\Phi} V]^{-1}_{ik} V_{kj}$$

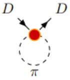


(a)

Self-consistency is required at $T \neq 0$

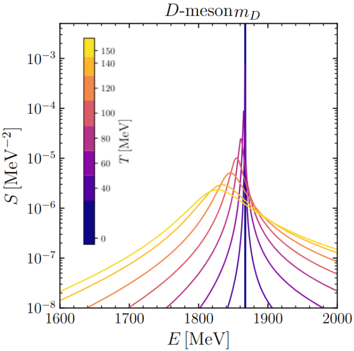


(b)



(c)

We use vacuum propagators for Φ (thermal m_π gives small effect)



spectral function:

$$S_D(E, \mathbf{k}) = -\frac{1}{\pi} \mathrm{Im} \left(\frac{1}{E^2 - \mathbf{k}^2 - m^2 - \Pi^R(E, \mathbf{k})} \right)$$

quasiparticle peak position:

$$E_k^2 - \mathbf{k}^2 - m^2 - \mathrm{Re}\ \Pi^R(E_k, \mathbf{k}) = 0$$

thermal width:

$$\Gamma_k = -\frac{1}{E_k} \mathrm{Im}\ \Pi^R(E_k, \mathbf{k})$$

G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

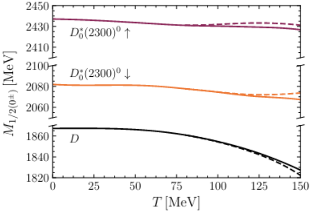
D meson gets lighter and broader with increasing temperature

see other spectral functions

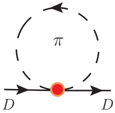
Thermal masses and widths

Chiral parity partners

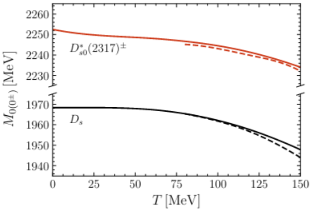
$D(1867)$
$\leftrightarrow$
$D_0^*(2300)$



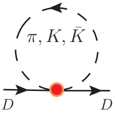
solid line:



$D_s(1968)$
$\leftrightarrow$
$D_{s0}^*(2317)$



dashed line:



G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

No evidence of chiral partner degeneracy due to chiral symmetry restoration

- **PART II:** Kinetic Theory for D mesons
  - Transport equation from real-time dynamics
  - Off-shell Fokker-Planck equation
  - Heavy-flavor transport coefficients

JMTR, G. Montaña, À. Ramos, L. Tolos, to appear

- Previous calculations using Fokker-Planck (or Boltzmann) equation together with some microscopic model:
  G.D. Moore and D. Teaney, Phys.Rev. C71, 064904 (2005)
  H. van Hees and R. Rapp, Phys.Rev.C71, 034907 (2005)
  H. van Hees *et al*, Phys. Rev. Lett.100, 192301 (2008)
  A. Beraudo *et al.*, Nucl. Phys. A831, 59 (2009)
  M. He, R. J. Fries, and R. Rapp, Phys.Lett. B701,445 (2011)
  S. Ghosh it et al., Phys.Rev. D84, 011503 (2011)
  L.M. Abreu *et al.*, Annals Phys.326, 2737 (2011)
  M. He, R. J. Fries, and R. Rapp, Phys. Rev. C86, 014903 (2012)
  L. Tolos and JMTR, Phys. Rev.D88, 074019 (2013)
  V. Ozvenchuk *et al.*, Phys. Rev.C90, 054909 (2014)
  H. Berrehrah *et al.*,Phys. Rev. C89, 054901 (2014)
  S. K. Das *et al.* Phys. Rev. C90, 044901 (2014)
  ...
  Why addressing the derivation of the transport equation?

- **1. Consistency:** Both interactions and kinetic equation come from the SAME microscopic model
- **2. Missing terms?** Thermal corrections to the collision rate

Kadanoff-Baym approach and transport equation

Implement off-shell effects → **Kadanoff-Baym equations**

For the derivation we follow classical approaches : L. Kadanoff, G.Baym, "Quantum statistical mechanics" 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990), J.-P. Blaizot and E. Iancu, Nucl. Phys. B557, 183 (1999), J. Rammer "Quantum field theory of non-equilibrium states" (2007), W. Cassing, Eur. Phys. J.168, 3 (2009)

We exploit the good quasiparticle description of D meson, and adopt a gradient expansion.

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We exploit the good quasiparticle description of D meson, and adopt a gradient expansion.

$$\overbrace{\left(k^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\ \Pi^R(X,k)}{\partial k_\mu}\right)\frac{\partial i G_D^<(X,k)}{\partial X^\mu}}^{\text{Advective term}} = \overbrace{\underbrace{\frac{1}{2} i\Pi^<(X,k) i G_D^>(X,k)}_{\text{Gain term}} - \underbrace{\frac{1}{2} i\Pi^>(X,k) i G_D^<(X,k)}_{\text{Loss term}}}^{\text{Collision term}}$$

Kadanoff-Baym ansatz incorporates the D-meson distribution function

$$iG_D^<(X,k) = 2\pi\ S_D(X,k)\ f_D(X,k^0)$$

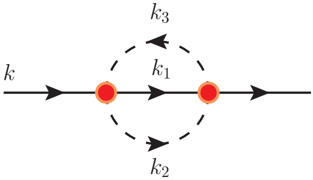
$$iG_D^>(X,k) = 2\pi\ S_D(X,k)\ [1 + f_D(X,k^0)]$$

Still need to model self-energies $\Pi^<(X,k), \Pi^>(X,k), \Pi^R(X,k)$...

To close the kinetic equation we employ the T**-matrix approximation** (L. Kadanoff, G.Baym, "Quantum statistical mechanics" 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990))

T-matrix approximation

$$i\Pi^<(X,k) = \sum_{\{a,b,c\}} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1+k_2-k_3-k)$$
$$\times |T(k_1^0+k_2^0+i\epsilon, \mathbf{k}_1+\mathbf{k}_2)|^2 \, iG^<_{D_a}(X,k_1) iG^<_{\Phi_b}(X,k_2) iG^>_{\Phi_c}(X,k_3)$$



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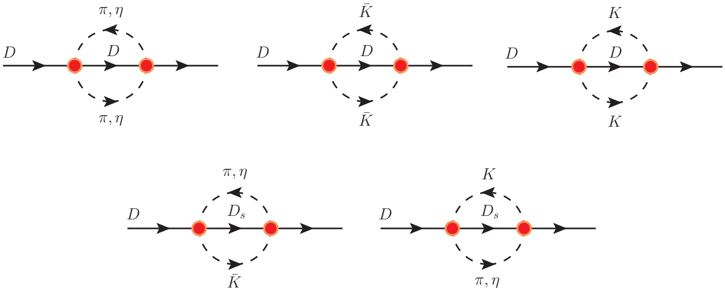
$$i\Pi^<(X,k) = \sum_{\{a,b,c\}} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1+k_2-k_3-k)$$
$$\times \, |T(k_1^0+k_2^0+i\epsilon, \mathbf{k}_1+\mathbf{k}_2)|^2 \; iG^<_{D_a}(X,k_1) iG^<_{\Phi_b}(X,k_2) iG^>_{\Phi_c}(X,k_3)$$



Assuming narrow quasiparticles $S(k_i^0, \mathbf{k}_i) = \frac{1}{2E_i} \sum_{\lambda=\pm} \lambda \delta(k_i^0 - \lambda E_i)$ we can write an "on-shell" version:

$$\left[\frac{\partial}{\partial t} - \frac{\mathbf{k}}{E_k} \cdot \nabla_X\right] f_k = \frac{1}{2E_k} \int \left(\prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3 2E_i}\right) (2\pi)^4 \delta^{(3)}(\mathbf{k} + \mathbf{k}_3 - \mathbf{k}_1 - \mathbf{k}_2)$$
$$\times \left\{\delta(E_k + E_3 - E_1 - E_2)\, |T(E_k + E_3, \mathbf{k} + \mathbf{k}_3)|^2 \left(f_1 f_2 \tilde{f}_3 \tilde{f}_k - \tilde{f}_1 \tilde{f}_2 f_3 f_k\right)\right.$$
$$\left. + \delta(E_k - E_3 - E_1 + E_2)\, |T(E_k - E_3, \mathbf{k} + \mathbf{k}_3)|^2 \left(f_1 \tilde{f}_2 f_3 \tilde{f}_k - \tilde{f}_1 f_2 \tilde{f}_3 f_k\right)\right\}$$

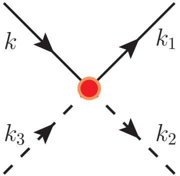
- Contribution above 2-particle threshold: *Unitary cut of G propagator*
- Contribution below 2-particle threshold: *Landau cut of G propagator*

Landau contribution vanishes when $T \to 0$

H. A. Weldon, Phys.Rev.D28, 2007 (1983)

T. Kunihiro, Nucl.Phys.B351, 593 (1991)

A. Das "Finite Temperature Field Theory" (1997)



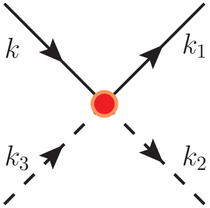
At finite T the Landau contribution is important

$$\left(k^{\mu} - \frac{1}{2}\frac{\partial \mathrm{Re}\ \Pi^R(X,k)}{\partial k_{\mu}}\right)\frac{\partial iG_D^{<}(X,k)}{\partial X^{\mu}} = \frac{1}{2} i\Pi^{<}(X,k) iG_D^{>}(X,k) - \frac{1}{2} i\Pi^{>}(X,k) iG_D^{<}(X,k)$$

It is standard to simplify the Boltzmann equation by using the scale separation,

$$m_D \gg m_\Phi, T$$

where the typical transferred momentum $\mathbf{q} = \mathbf{k} - \mathbf{k_1} \sim T$
(compare with thermal $\mathbf{k} \gtrsim \sqrt{M_H T}$)



Boltzmann Eq. $\rightarrow$ Fokker-Planck Eq.

E.M. Lifshitz and L. P. Pitaevskii, "Physical Kinetics", (vol.10 of Landau and Lifshitz course)
B. Svetitsky, Phys. Rev. D37, 2484 (1988)
L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada, and JMTR, Annals Phys. 326 (2011) 2737-2772

Fokker-Planck equation reduction

Off-shell Fokker-Planck equation

see derivation

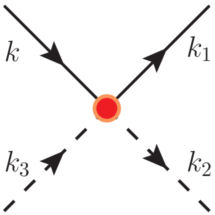
$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i} \left\{ \hat{A}(k;T) k^i G_D^<(t,k) + \frac{\partial}{\partial k^j} \left[\hat{B}_0(k;T)\Delta^{ij} + \hat{B}_1(k;T)\frac{k^i k^j}{\mathbf{k}^2} \right] G_D^<(t,k) \right\}$$

where $\Delta^{ij} = \delta^{ij} - k^i k^j / \mathbf{k}^2$

$$\hat{A}(k^0,\mathbf{k};T) \equiv \left\langle 1 - \frac{\mathbf{k}\cdot\mathbf{k}_1}{\mathbf{k}^2} \right\rangle$$

$$\hat{B}_0(k^0,\mathbf{k};T) \equiv \frac{1}{4}\left\langle \mathbf{k}_1^2 - \frac{(\mathbf{k}\cdot\mathbf{k}_1)^2}{\mathbf{k}^2} \right\rangle$$

$$\hat{B}_1(k^0,\mathbf{k};T) \equiv \frac{1}{2}\left\langle \frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}_1)]^2}{\mathbf{k}^2} \right\rangle$$

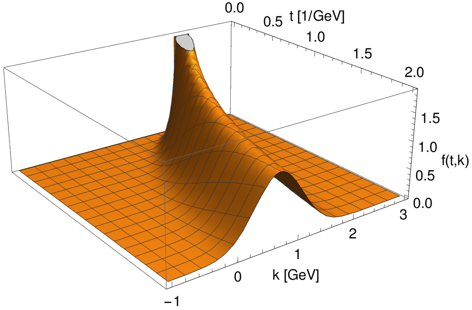


with

$$\langle \cdot \rangle \equiv \frac{1}{2k^0} \sum_{\lambda,\lambda'=\pm} \lambda\lambda' \int_{-\infty}^{\infty} dk_1^0 \int \prod_{i=1}^{3} \frac{d^3 k_i}{(2\pi)^3} \frac{1}{2E_2 2E_3} S_D(k_1^0,\mathbf{k}_1)(2\pi)^3 \delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)$$
$$\times (2\pi)\delta(k^0+\lambda' E_3 - \lambda E_2 - k_1^0) |T(k^0+\lambda' E_3, \mathbf{k}+\mathbf{k}_3)|^2 f^{(0)}(\lambda' E_3)\tilde{f}^{(0)}(\lambda E_2)$$

Interpretation of A, B_0 and B_1

Consider 1+1D evolution, with constant A and B_0



A: Relaxation rate for average momentum

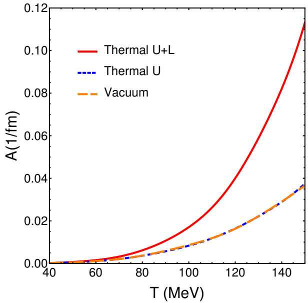
B_0: Controls broadening in momentum distribution

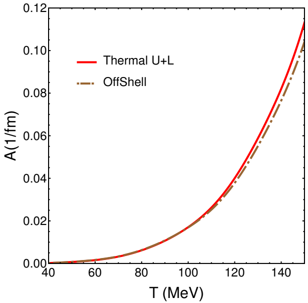
Einstein relation at $k \to 0$

$$A = \frac{B_0}{m_D T} = \frac{B_1}{m_D T} \quad \to \quad \left\langle \frac{k^2}{2m_D} \right\rangle_T = \frac{3k_B T}{2}$$

Drag force A

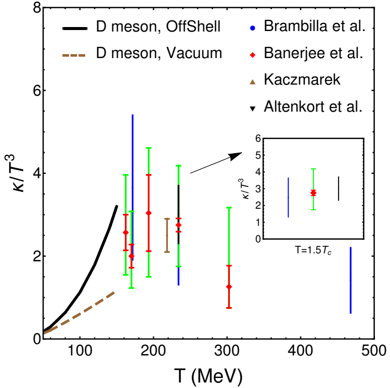
Drag coefficient $A(k;T)$ in the $k \to 0$ limit. Off-shell case at $k^0 = E_k$





- A (relaxation rate, $\tau_R \sim 1/A$) increases (decreases) with T
- Tiny effect of temperature in $\overline{|T|^2}$ and m_D
- Landau contribution very important at finite T
- Off-shell effects on D meson are small: good quasiparticle picture

I plot $\kappa = 2B_0(k \to 0) = 2B_1(k \to 0)$



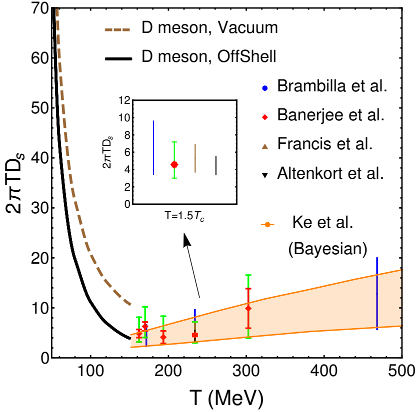
Lattice-QCD calculations

- D. Banerjee *et al.* Phys. Rev. D85, 014510 (2012)
- O. Kaczmarek Nucl. Phys. A931, 633 (2014)
- N. Brambilla *et al.* Phys. Rev. D102, 074503 (2020)
- L. Altenkort *et al.* Phys. Rev. D103, 014511 (2021)

Our result (all effects included) is compatible with lattice-QCD calculations

Spatial diffusion coefficient

$$2\pi T D_s(T) = \frac{2\pi T^3}{B_0(k \to 0, T)}$$



Lattice-QCD calculations

- N. Brambilla *et al.*
  Phys. Rev. D102, 074503 (2020)
- D. Banerjee *et al.*
  Phys. Rev. D85, 014510 (2012)
- A. Francis *et al.*
  Phys. Rev. D92, 116003 (2015)
- L. Altenkort *et al.*
  Phys. Rev. D103, 014511 (2021)

Bayesian study of RHICs

- W. Ke *et al.*
  Phys. Rev. C98, 064901 (2018)

Conclusions

1. We have extended the EFT description of D mesons to finite temperature in a self-consistent fashion
2. We described the thermal dependence of masses and widths of ground states, bound states, and resonances
3. We have revisited the D-meson kinetic theory from QFT, applying the Kadanoff-Baym equations and arrived to an off-shell Fokker-Planck equation
4. We have analyzed several new effects: inelastic channels, thermal dependence of amplitudes, off-shell effects, and the Landau contribution. The relevant importance of the latter is our main result.
5. We have computed heavy-flavor transport coefficients below T_c. Agreement with lattice-QCD and Bayesian analyses above T_c is very good.

D mesons in a thermal environment: interactions, generated states and transport properties

Juan M. Torres-Rincon
(Goethe University Frankfurt)

in collaboration with
G. Montaña, L. Tolos and À. Ramos

Transport Meeting at ITP
Goethe University Frankfurt
May 27, 2021

Effective Lagrangian at NLO

L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

$$\mathcal{L}_{\rm LO} = Tr[\nabla^\mu D \nabla_\mu D^\dagger] - m_D^2 Tr[DD^\dagger] - Tr[\nabla^\mu D^{*\nu} \nabla_\mu D^{*\dagger}_\nu] + m^2_{D^*} Tr[D^{*\mu} D^{*\dagger}_\mu]$$

$$+igTr\left[\left(D^{*\mu} u_\mu D^\dagger - D u^\mu D^{*\dagger}_\mu\right)\right] + \frac{g}{2m_D} Tr\left[\left(D^*_\mu u_\alpha \nabla_\beta D^{*\dagger}_\nu - \nabla_\beta D^*_\mu u_\alpha D^{*\dagger}_\nu\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$\mathcal{L}_{\rm NLO} = -h_0 Tr[DD^\dagger] Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger] Tr[u^\mu u_\mu] + h_3 Tr[D u^\mu u_\mu D^\dagger]$$

$$+h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger] Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger] \ + \ \{D \to D^\mu\}$$

$$D = (D^0, D^+, D^+_s)$$

$$\nabla^\mu = \partial^\mu - \frac{1}{2}(u^\dagger \partial^\mu u + u \partial^\mu u^\dagger)$$

$$u^\mu = i(u^\dagger \partial^\mu u - u \partial^\mu u^\dagger)$$

$$u = \exp\left[\frac{i}{\sqrt{2}F}\Phi\right] = \exp\left[\frac{i}{\sqrt{2}F}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}\right]$$

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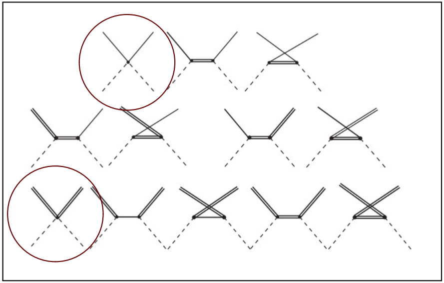
Heavy meson—light meson interaction



Tree-level diagrams for $H^{(*)} - l$ scattering (elastic and inelastic).

Solid line: H meson, Double solid line: H^* meson, Dashed line: light meson $(\pi, K, \bar{K}, \eta)$

- Born exchanges are suppressed by $1/m_H$.
  In particular, spin-flip processes vanish in the HQ limit.
- Only contact terms survive at lowest order!

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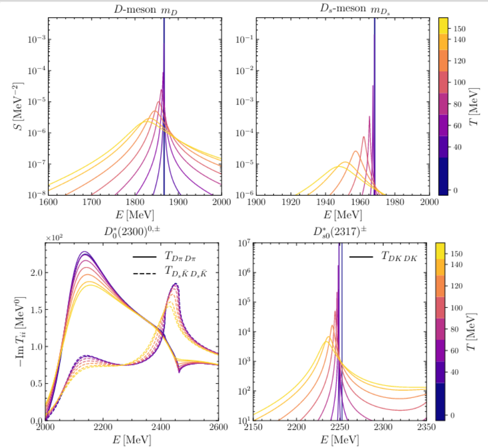

$$S_D(E,\mathbf{q}) = -\frac{1}{\pi}\mathrm{Im}\left(\frac{1}{E^2-\mathbf{q}^2-m^2-\Pi^R(E,\mathbf{q})}\right)$$

quasiparticle peak

$$E_q^2 - \mathbf{q}^2 - m^2 - \mathrm{Re}\,\Pi^R(E_q,\mathbf{q}) = 0$$

$$-\mathrm{Im}\,T_{ii}(E,\mathbf{q}) = -\mathrm{Im}\left(\frac{V(E,\mathbf{q})}{1-G(E,\mathbf{q})V(E,\mathbf{q})}\right)$$

G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

Ground and bound states reduce their mass and acquire a width.
Resonant states remain stable with temperature.

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Chiral parity partners

$D(1867)$

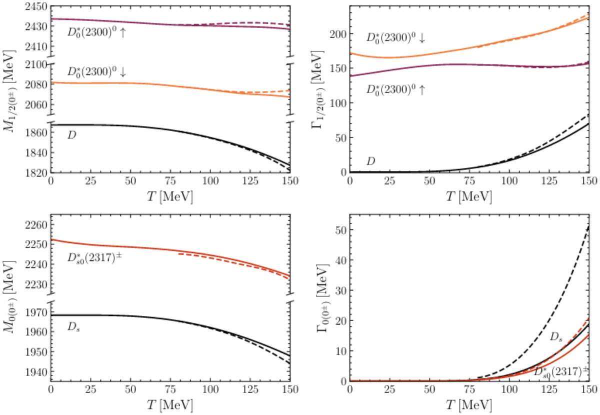
$\leftrightarrow$

$D_0^*(2300)$

$D_s(1968)$

$\leftrightarrow$

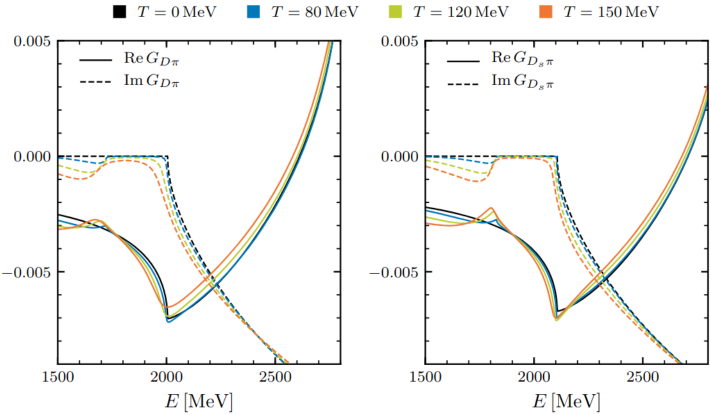
$D_{s0}^*(2317)$



G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

No evidence of chiral partner degeneracy due to chiral symmetry restoration

Loop function: Unitary and Landau cuts



$$G_{D\Phi}(i\omega_m, \mathbf{p}; T) = -T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \mathbf{k}^2 + m_D^2} \frac{1}{(\omega_m - \omega_n)^2 + (\mathbf{p} - \mathbf{k})^2 + m_\Phi^2}$$

Thermal width (damping rate) of D mesons

On-shell D meson with momentum k at equilibrium

$$\Gamma_k = -\frac{1}{E_k} \mathrm{Im}\ \Pi^R(E_k, k) \qquad \Gamma_k = -\frac{1}{2E_k}[\Pi^>(E_k, k) - \Pi^<(E_k, k)]$$

Thermal width (damping rate) of D mesons

On-shell D meson with momentum k at equilibrium

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$$\Gamma_k^{(U)} = \frac{1}{2E_k}\frac{1}{\tilde{f}_k^{(0)}} \sum_{\lambda=\pm} \lambda \int dk_1^0 \int \prod_{i=1}^{3} \frac{d^3k_i}{(2\pi)^3} \frac{1}{2E_2}\frac{1}{2E_3} |T(E_k + E_3, \mathbf{k} + \mathbf{k}_3)|^2 \, S_D(k_1^0, \mathbf{k}_1)$$
$$\times (2\pi)^4 \delta^{(3)}(\mathbf{k} + \mathbf{k}_3 - \mathbf{k}_1 - \mathbf{k}_2)\delta(E_k + E_3 - k_1^0 - \lambda E_2)\tilde{f}^{(0)}(k_1^0) f^{(0)}(E_3)\tilde{f}^{(0)}(\lambda E_2)$$
$$\Gamma_k^{(L)} = \frac{1}{2E_k}\frac{1}{\tilde{f}_k^{(0)}} \sum_{\lambda=\pm} \lambda \int dk_1^0 \int \prod_{i=1}^{3} \frac{d^3k_i}{(2\pi)^3} \frac{1}{2E_2}\frac{1}{2E_3} |T(E_k - E_3, \mathbf{k} + \mathbf{k}_3)|^2 \, S_D(k_1^0, \mathbf{k}_1)$$
$$\times (2\pi)^4 \delta^{(3)}(\mathbf{k} + \mathbf{k}_3 - \mathbf{k}_1 - \mathbf{k}_2)\delta(E_k - E_3 - k_1^0 - \lambda E_2)\tilde{f}^{(0)}(k_1^0)\tilde{f}^{(0)}(E_3)\tilde{f}^{(0)}(\lambda E_2)$$

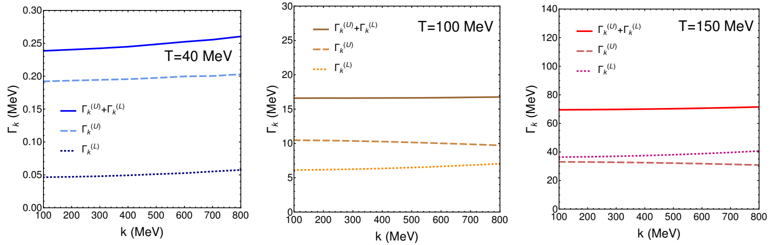
Thermal width (damping rate) of D mesons

On-shell D meson with momentum k at equilibrium

$$\Gamma_k = -\frac{1}{E_k}\,\mathrm{Im}\,\Pi^R(E_k,k) \qquad\qquad \Gamma_k = -\frac{1}{2E_k}\left[\Pi^>(E_k,k) - \Pi^<(E_k,k)\right]$$

$$\Gamma_k^{(U)} = \frac{1}{2E_k}\frac{1}{\tilde f_k^{(0)}}\sum_{\lambda=\pm}\lambda\int dk_1^0\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2}\frac{1}{2E_3}\,|T(E_k+E_3,\mathbf{k}+\mathbf{k}_3)|^2\,S_D(k_1^0,\mathbf{k}_1)$$
$$\times (2\pi)^4\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)\,\delta(E_k+E_3-k_1^0-\lambda E_2)\,\tilde f^{(0)}(k_1^0)\,f^{(0)}(E_3)\,\tilde f^{(0)}(\lambda E_2)$$

$$\Gamma_k^{(L)} = \frac{1}{2E_k}\frac{1}{\tilde f_k^{(0)}}\sum_{\lambda=\pm}\lambda\int dk_1^0\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2}\frac{1}{2E_3}\,|T(E_k-E_3,\mathbf{k}+\mathbf{k}_3)|^2\,S_D(k_1^0,\mathbf{k}_1)$$
$$\times (2\pi)^4\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)\,\delta(E_k-E_3-k_1^0-\lambda E_2)\,\tilde f^{(0)}(k_1^0)\,\tilde f^{(0)}(E_3)\,\tilde f^{(0)}(\lambda E_2)$$



Langevin equation

It is an alternative (but equivalent) description to the Fokker-Planck equation.

$$\begin{cases} dx^i = k^i dt/E_k \,, \\ dk^i = -A(k)k^i dt + C^{ij}(k)\rho^j \sqrt{dt} \,, \end{cases}$$

where ($\Delta^{ij} = \delta^{ij} - k^i k^j/k^2$)

$$C^{ij} = \sqrt{2B_0(k)}\Delta^{ij} + \sqrt{2B_1(k)}\,\frac{k^i k^j}{k^2}$$

and ρ^i a stochastic Gaussian noise

$$\langle \rho^i(t) \rangle = 0$$
$$\langle \rho^i(t)\rho^j(t') \rangle = \delta(t-t')$$

On-shell Fokker-Planck equation

Narrow quasiparticle limit

$$G_D^<(t,k^0,\mathbf{k}) = 2\pi S_D(k^0,\mathbf{k}) f_D(t,k^0)$$

$$S_D(k^0,\mathbf{k}) = \frac{1}{2E_k}\left[\delta(k^0-E_k)+\delta(k^0+E_k)\right]$$

On-shell Fokker-Planck equation

$$\frac{\partial}{\partial t} f_D(t,E_k) = \frac{\partial}{\partial k^i}\left\{ A(\mathbf{k}) k^i f_D(t,E_k) + \frac{\partial}{\partial k^j}\left[B_0(\mathbf{k})\Delta^{ij} + B_1(\mathbf{k})\frac{k^i k^j}{k^2}\right] f_D(t,E_k)\right\}$$

where $\Delta^{ij} = \delta^{ij} - k^i k^j/\mathbf{k}^2$

with

$$\langle\cdot\rangle = \frac{1}{2E_k}\int \frac{d^3k_1}{(2\pi)^4 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}\frac{d^3k_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^{(4)}(k_1+k_2-k_3-k)$$
$$\times\left(|T(E_k+E_3,\mathbf{k}+\mathbf{k}_3)|^2 + |T(E_k-E_2,\mathbf{k}-\mathbf{k}_2)|^2\right) f^{(0)}(E_3)\tilde{f}^{(0)}(E_2)$$

We recover standard formula, **but** with Landau contribution

Average momentum loss

$$\left\langle \frac{dk^i}{dt} \right\rangle = -A(k)\, k^i$$

Assuming constant A one can solve the equation for $k(t)$

$$\langle k(t) \rangle = k(0)\, e^{-At}$$

The inverse of A plays the role of a relaxation time τ_R for the average heavy-hadron momentum

$$\tau_R = \frac{1}{A}$$

Fluctuation-Dissipation Theorem

$A(k)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.

The fluctuation-dissipation theorem relates the 3 coefficients:

Fluctuation-Dissipation Theorem

$$A(k) + \frac{1}{k}\frac{\partial B_1(k)}{\partial k} + \frac{2}{k^2}\left[B_1(k) - B_0(k)\right] = \frac{B_1(k)}{m_D T}$$

In the static limit, i.e. when $k \to 0$ the two diffusion coefficients become degenerate and the Einstein relation is recovered

Einstein Relation

$$A = \frac{B}{m_D T}$$

Off-shell Fokker-Planck equation

$$\left(k^\mu - \frac{1}{2} \frac{\partial \mathrm{Re}\ \Pi^R(X,k)}{\partial k_\mu} \right) \frac{\partial i G_D^<(X,k)}{\partial X^\mu} = \frac{1}{2} i\Pi^<(X,k) i G_D^>(X,k) + -\frac{1}{2} i\Pi^>(X,k) i G_D^<(X,k)$$

Off-shell transport equation can be rewritten as a master equation:

$$2 \left(k^\mu - \frac{1}{2} \frac{\partial \mathrm{Re}\Pi^R}{\partial k_\mu} \right) \frac{\partial}{\partial X^\mu} G_D^<(X,k)$$
$$= \int \frac{dk_1^0}{2\pi} \frac{d^3 q}{(2\pi)^3} [W(k^0, \mathbf{k}+\mathbf{q}, k_1^0, \mathbf{q}) G_D^<(X, k^0, \mathbf{k}+\mathbf{q}) - W(k^0, \mathbf{k}, k_1^0, \mathbf{q}) G_D^<(X, k^0, \mathbf{k})]$$

with transition probability rate

$$W(k^0, \mathbf{k}, k_1^0, \mathbf{q}) \equiv \int \frac{d^4 k_3}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} (2\pi)^4 \delta(k_1^0 + k_2^0 - k_3^0 - k^0) \delta^{(3)}(\mathbf{k}_2 - \mathbf{k}_3 - \mathbf{q})$$
$$\times |T(k_1^0 + k_2^0 + i\epsilon, \mathbf{k} - \mathbf{q} + \mathbf{k}_2)|^2 G_\Phi^>(X, k_2) G_\Phi^<(X, k_3) G_D^>(X, k_1^0, \mathbf{k} - \mathbf{q})$$

Off-shell Fokker-Planck equation

Using $\mathbf{k} \gg \mathbf{q}$ one can Taylor expand

$$f(\mathbf{k}+\mathbf{q}) \simeq f(\mathbf{k}) + q^i \frac{\partial f(\mathbf{k})}{\partial k^i} + \frac{1}{2} q^i q^j \frac{\partial^2 f(\mathbf{k})}{\partial k^i \partial k^j}$$

for the combination

$$f(\mathbf{k}+\mathbf{q}) \equiv W(k^0, \mathbf{k}+\mathbf{q}, k_1^0, \mathbf{q}) G_D^<(X, k^0, \mathbf{k}+\mathbf{q})$$

One gets:

$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i} \left\{ \hat{A}^i(k;T) G_D^<(t,k) + \frac{\partial}{\partial k^j} \hat{B}_0^{ij}(k;T) G_D^<(t,k) \right\}$$

with

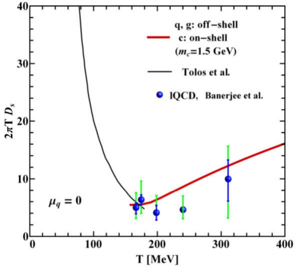
$$A^i(k;T) \equiv \frac{1}{2k^0} \int \frac{dk_1^0}{2\pi} \frac{d^3q}{(2\pi)^3} W(k^0, \mathbf{k}, k_1^0, \mathbf{q})\, q^i$$

$$B^{ij}(k;T) \equiv \frac{1}{2} \frac{1}{2k^0} \int \frac{dk_1^0}{2\pi} \frac{d^3q}{(2\pi)^3} W(k^0, \mathbf{k}, k_1^0, \mathbf{q})\, q^i q^j$$

back

Spatial diffusion coefficient

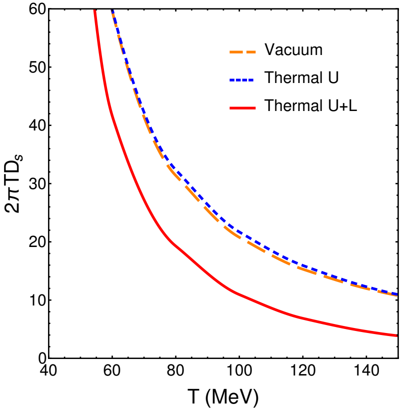
$$2\pi T D_s(T) = \frac{2\pi T^3}{B_0(k \to 0, T)}$$



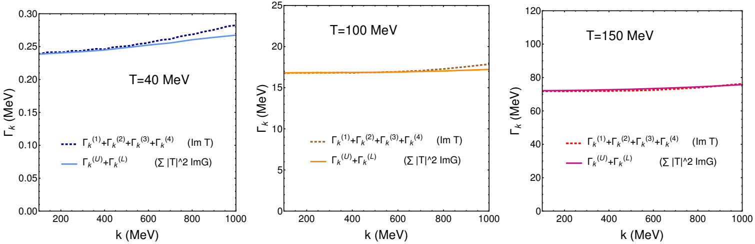
$T > T_c$: DQPM for c quarks.

$T < T_c$: Our result for D meson.

T. Song *et al.*, Phys. Rev. C 96, 014905 (2017)



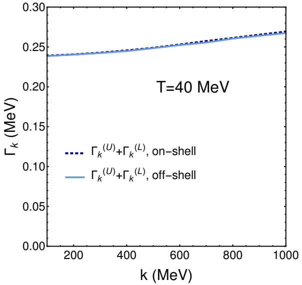
Updated results (still preliminary)

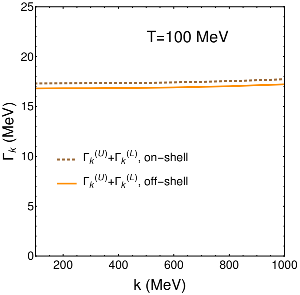


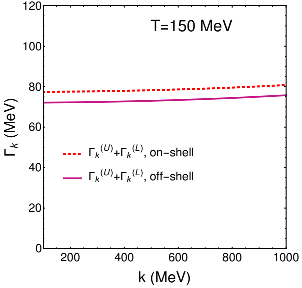
$$\Gamma_k \propto \mathrm{Im}\ \Pi_D^R \propto \mathrm{Im}\ T_{D\Phi\to D\Phi}$$

$$\mathrm{Im}\ T_{D\pi\to D\pi}(E,\mathbf{p}) = \sum_a T^*_{D\pi\to a}(E,\mathbf{p})\ \mathrm{Im}\ G_a^R(E,\mathbf{p})\, T_{a\to D\pi}(E,\mathbf{p})$$

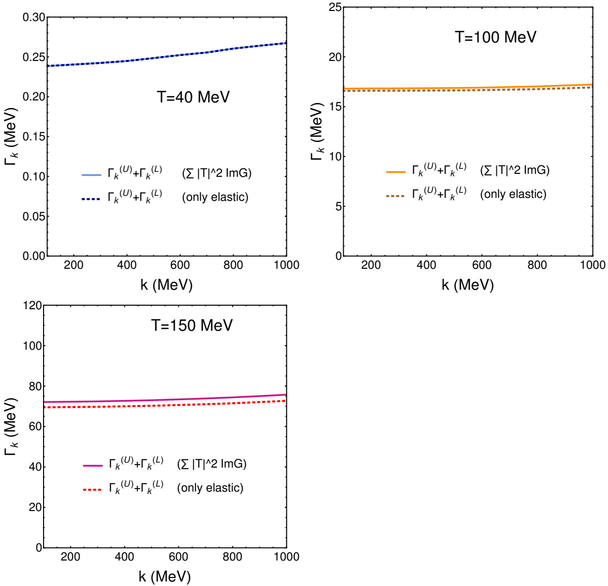
Off-shell effects

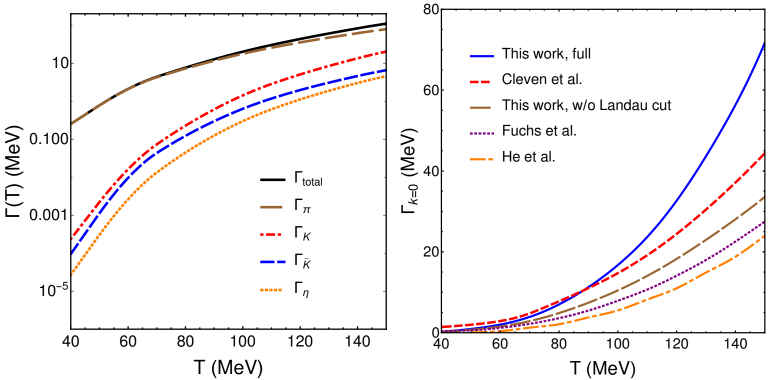






Inelastic channels





Γ(T) (MeV)
10
0.100
0.001
10^-5
40
60
80
100
120
140
T (MeV)
Γ_total
Γ_π
Γ_K
Γ_K̄
Γ_η
80
60
40
20
0
Γ_k=0 (MeV)
This work, full
Cleven et al.
This work, w/o Landau cut
Fuchs et al.
He et al.

Quasiparticle properties