CRC-TR211 Transport Meeting

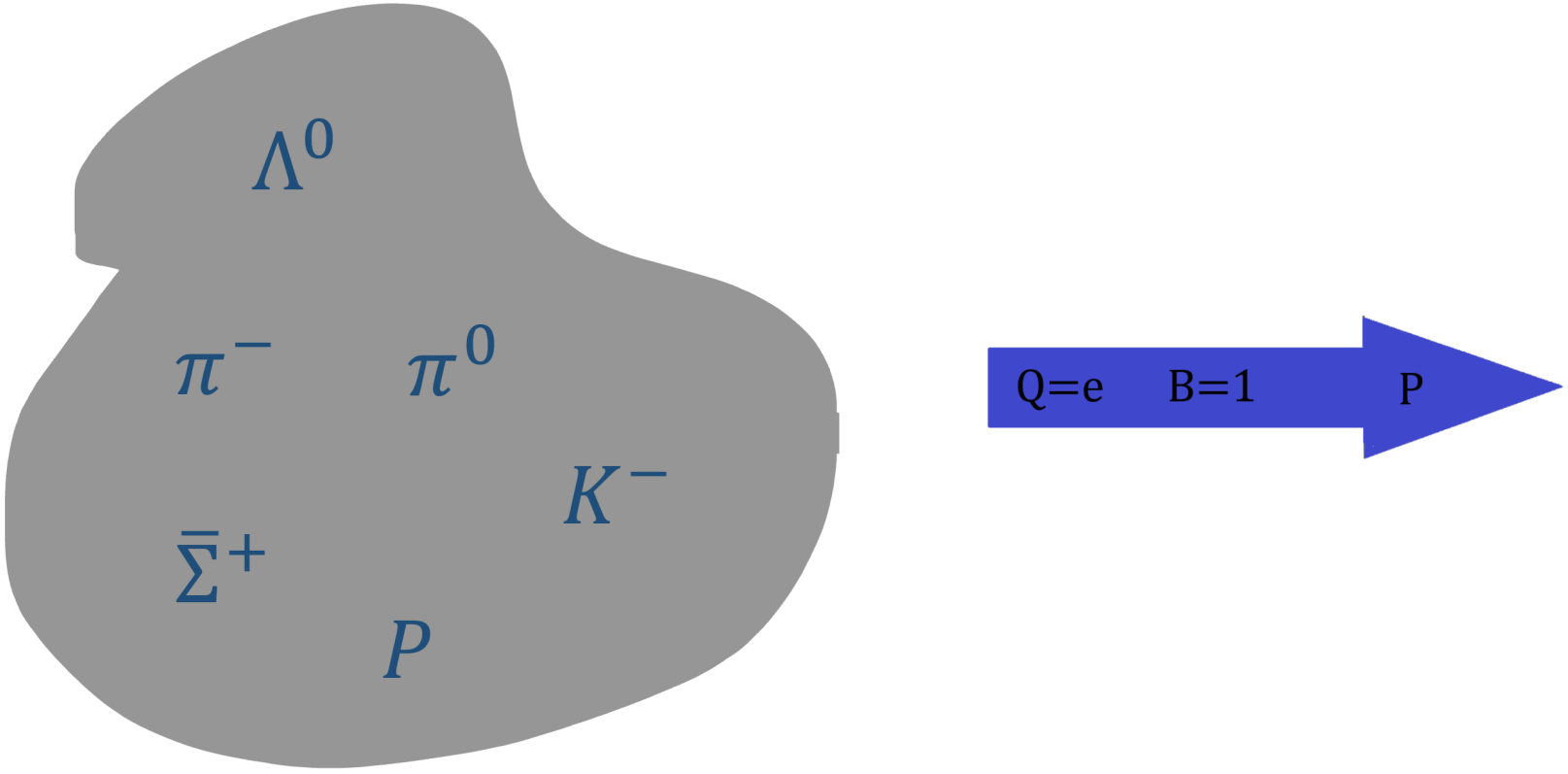
Diffusion Coefficients for Hot Hadron Gases in the Quark Flavor Representation

Jakob Lohr

22. April 2021

Supervisors: Prof. Dr. Carsten Greiner, Jan Fotakis

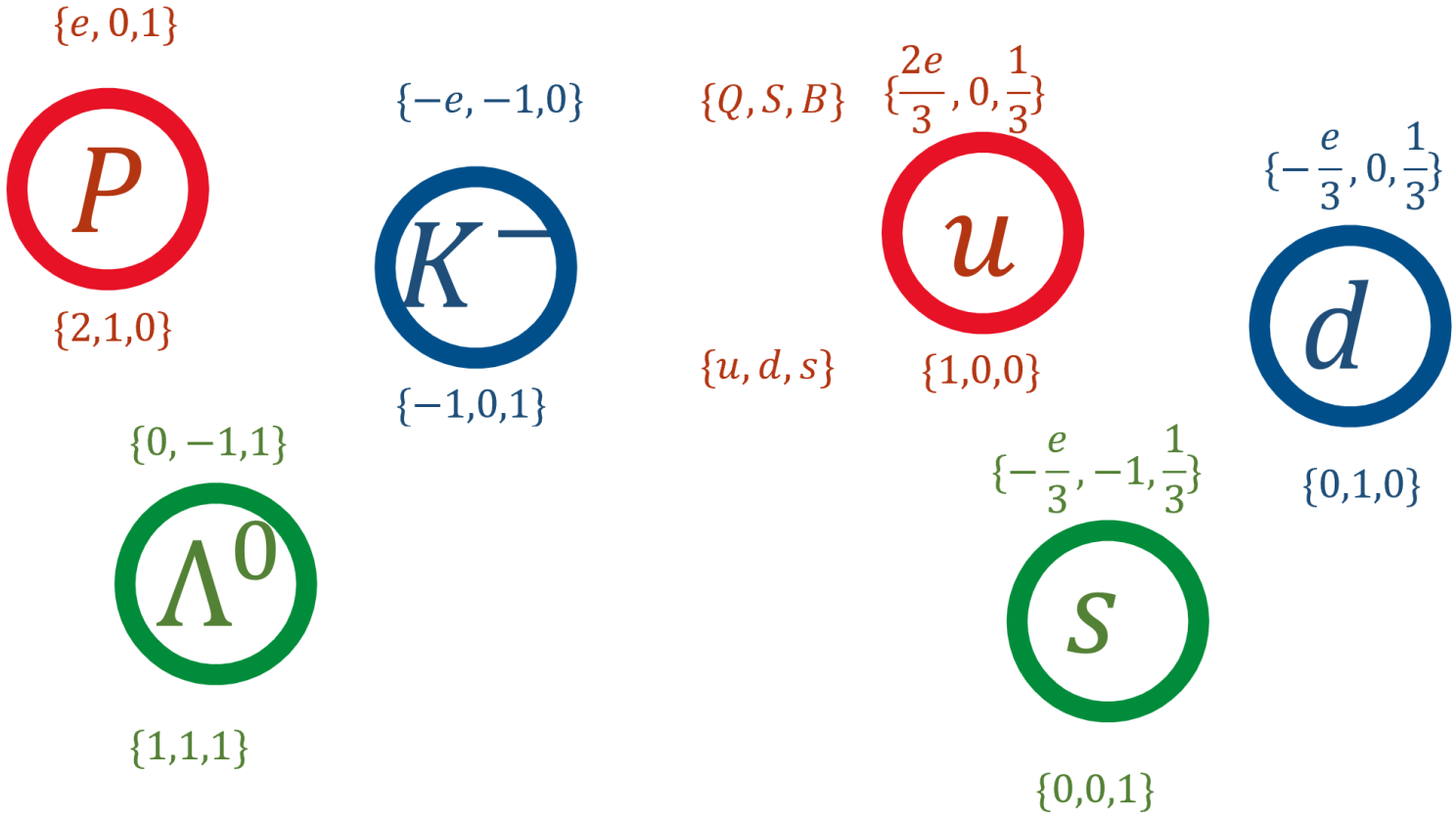
Motivation



Chemical composition

Possible identification of chemical composition through diffusion coefficients proposed by J.-B. Rose et. al., PRD 101, 114028 (2020)

Motivation



Natural description

Description trough quark content is more natural in QCD

Linear response theory in kinetic theory

Boltzmann equation

$$p_i^\mu \partial_\mu f_{i,p} = C[f_{i,p}] \rightarrow p_i^\mu \partial_\mu f_{i,p}^{(0)} = C[f_{i,p}]^{(1)}$$

Expansion of distribution in orders of Knudsen number

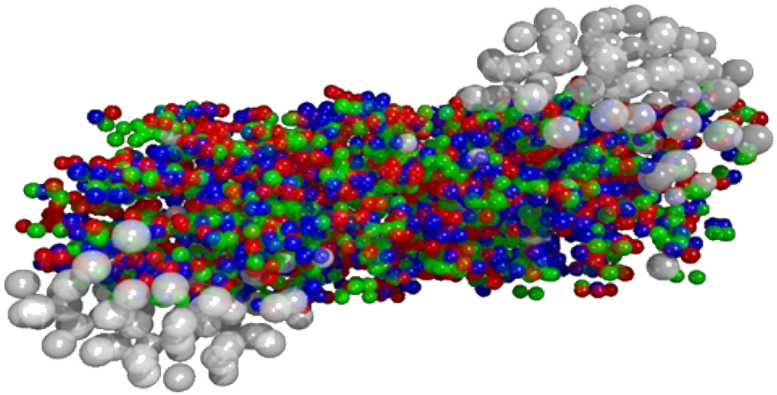
$$f_{i,p} = f_{i,p}^{(0)} + \epsilon f_{i,\mathbf{p}}^{(1)} + \mathcal{O}(\epsilon^2)$$

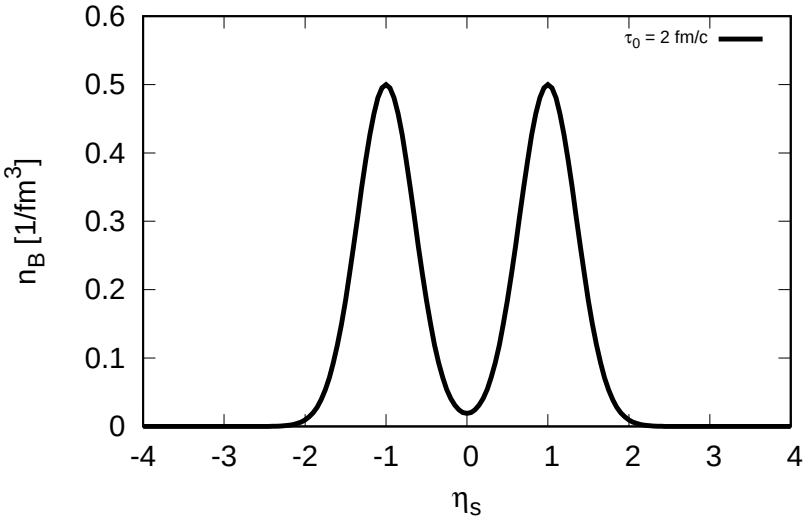
Maxwell-Jüttner distribution

$$f_{i,p}^{(0)} = g_i \exp(-\beta E_{i,p} + \alpha_i)$$

$$\beta = \frac{1}{T} \qquad \alpha_i = \frac{\mu_i}{T}$$

Navier-Stokes theory





cf. Greif et al., PRL 120, 242301 (2018)

Diffusion in Navier-Stokes theory and kinetic theory

$$j_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T} \right) = \sum_i q_i \int \mathrm{d}P_i p_i^{<\mu>} f_{i,\mathbf{p}}^{(1)}$$

$$\kappa = \begin{pmatrix} \kappa_{QQ} & \kappa_{QS} & \kappa_{QB} \\ \kappa_{SQ} & \kappa_{SS} & \kappa_{SB} \\ \kappa_{BQ} & \kappa_{BS} & \kappa_{BB} \end{pmatrix}$$

cf. L. Onsager, Physical Review 37, 405 (1931), L. Onsager, Physical Review 38, 2265 (1931)

Diffusion coefficients

Power series ansatz to linearized Boltzman eq.

$$f^{(1)}_{i,\mathbf{p}} = \sum_{q' \in \{B,S,Q\}} f^{(0)}_{i,p} \, p_i^{<\mu>} \nabla_\mu \alpha_{q'} \sum_{m=0}^{M} a_{i,q',m} E^m_{i,p}$$

Diffusion coefficients

$$\kappa_{qq'} = \frac{1}{3} \sum_{i=1}^{N_s} q_i \sum_{m=0}^{M} a_{i,q',m} \int \mathrm{d}P_i \Delta_{\mu\nu} p_i^\mu p_i^\nu E^m_{i,p} f^{(0)}_{i,p}$$

cf. Greif et al., PRL 120, 242301 (2018),
Fotakis et al., PRD 101, 076007 (2020)

Basis transformation

Basis transformation

$$\sum_f M_{cf} f_i = c_i$$

Invariance of the partition function

$$\rho = \exp\left(\sum_i \left(\beta H_i + \sum_f \beta \mu_f f_i\right)\right) \stackrel{!}{=} \exp\left(\sum_i \left(\beta H_i + \sum_c \beta \mu_c c_i\right)\right)$$

Fick's law for relativistic cases

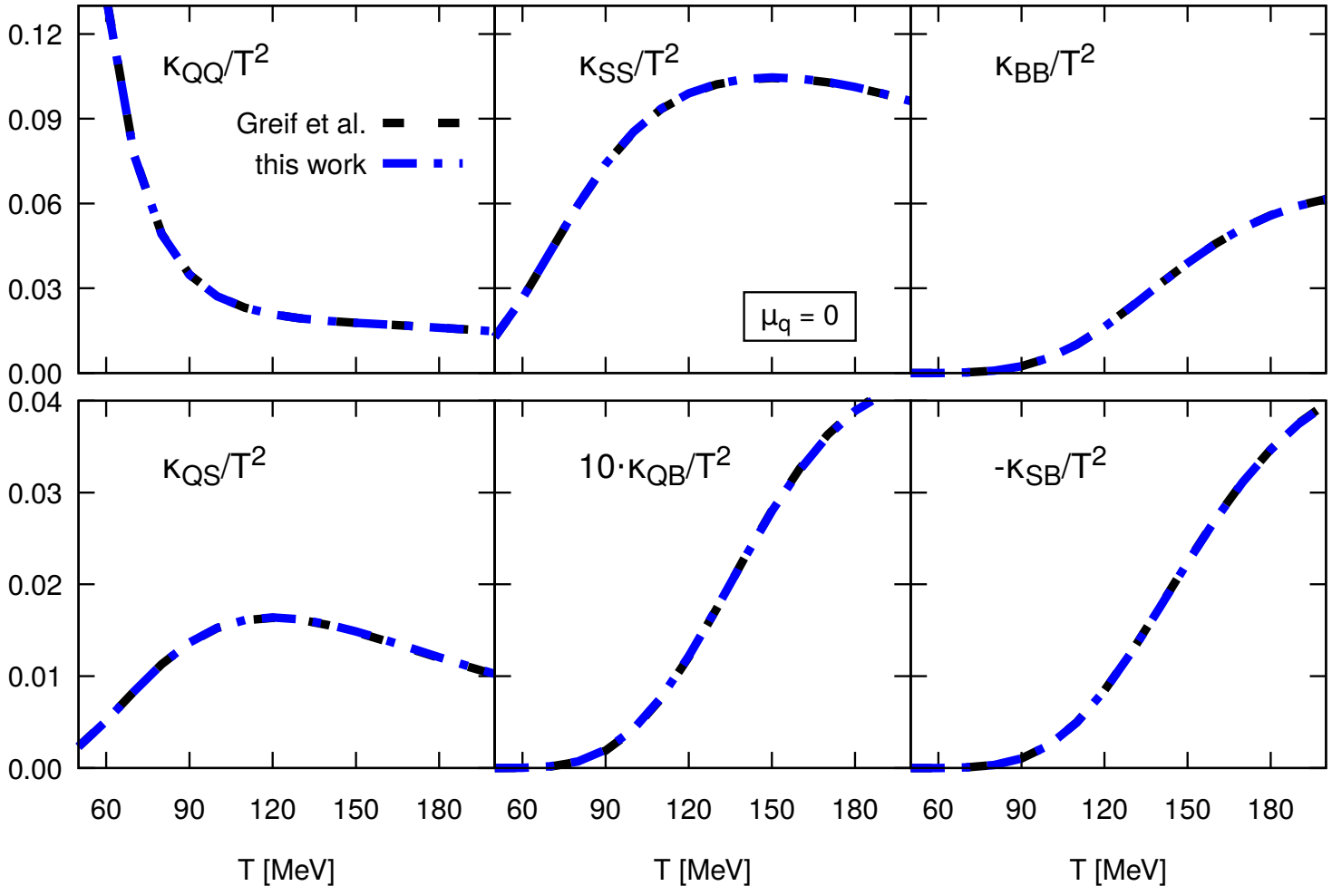
$$j_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

Transformation of coefficients

$$\kappa_{cc'} = \sum_{f,f'} M_{cf} \kappa_{ff'} M_{c'f'}$$

$$M = \begin{pmatrix} \frac{2}{3}e & -\frac{1}{3}e & -\frac{1}{3}e \\ 0 & 0 & -1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$$

Reproduction



System

Assumptions

chemical potential $\mu_q = 0$

classical statistics

isotropic elastic binary cross sections

Used particles

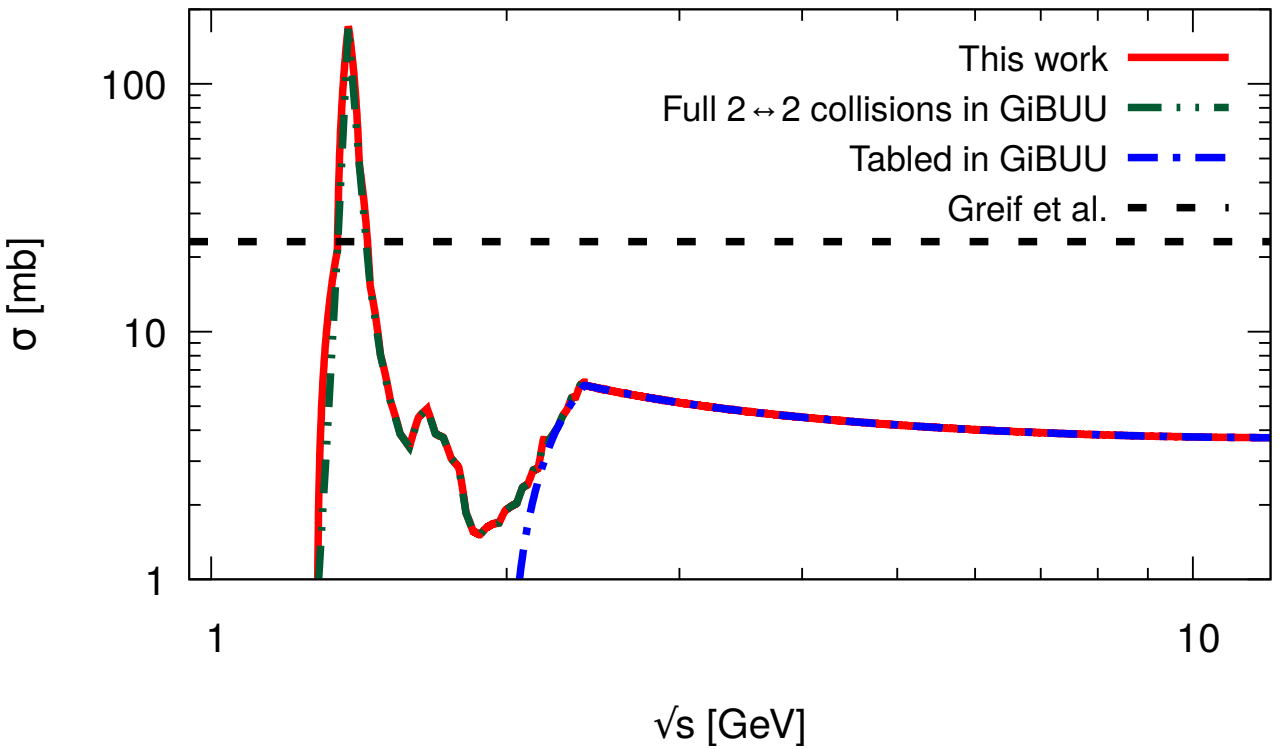
name	mass[Mev]	degeneracy	spin	electric charge	strangeness	baryon number
p	938.0	2	1/2	e	0	1
n	938.0	2	1/2	0	0	1
π^0	138.0	1	0	0	0	0
π^+	138.0	1	0	e	0	0
K^0	496.0	1	0	0	1	0
K^+	496.0	1	0	e	1	0
Λ^0	1116.0	2	1/2	0	-1	1
Σ^0	1192.6	2	1/2	0	-1	1
Σ^+	1189.3	2	1/2	e	-1	1
Σ^-	1197.4	2	1/2	-e	-1	1

Improvements

CP-symmetry restored

New cross sections

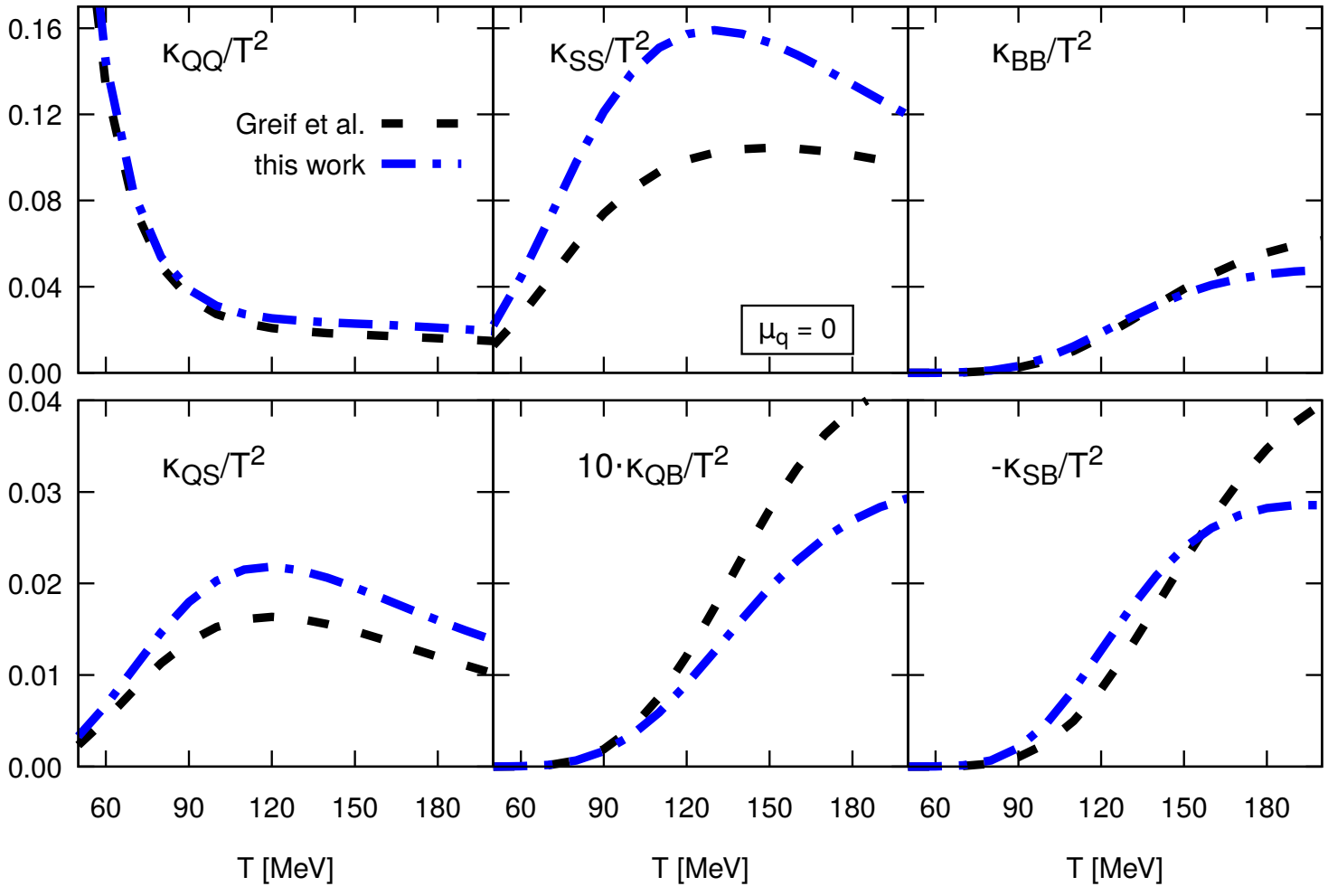
exemplary: $\pi^- - \Sigma^0$ cross section



cf. O. Buss et al., Phys. Rept. 512 (2012)

K. Gallmeister, private communications.

Changes


κ_{QQ}/T^2
Greif et al.
this work
κ_{SS}/T^2
κ_{BB}/T^2
$\mu_q = 0$
κ_{QS}/T^2
$10 \cdot \kappa_{QB}/T^2$
$-\kappa_{SB}/T^2$
0.16
0.12
0.08
0.04
0.00
0.04
0.03
0.02
0.01
0.00
60 90 120 150 180
T [MeV]

System

Assumptions

chemical potential $\mu_q = 0$

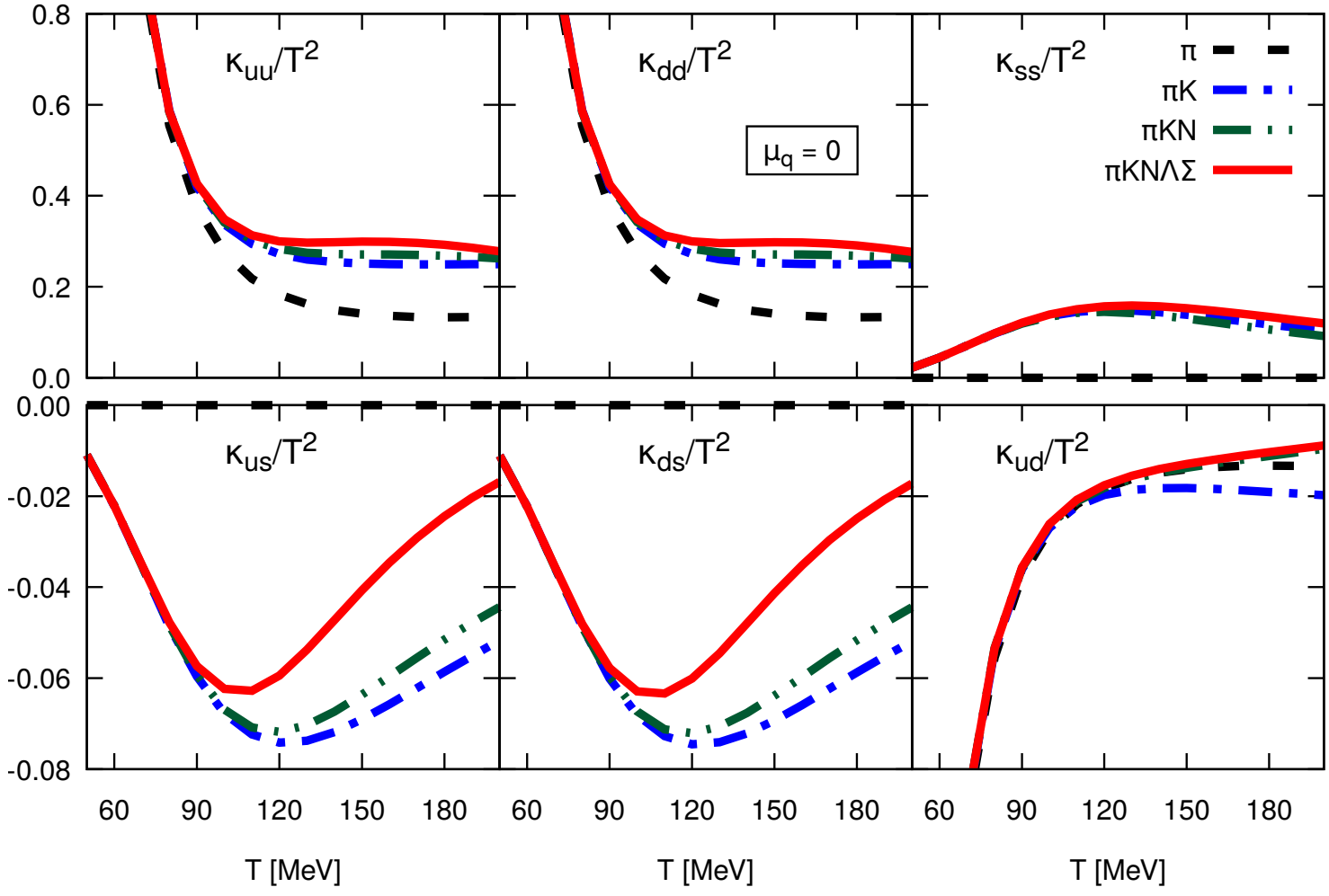
classical statistics

isotropic elastic binary cross sections

Used particles

name	mass[Mev]	degeneracy	spin	up-quarks	down-quarks	strange-quarks
p	938.0	2	1/2	2	1	0
n	938.0	2	1/2	1	2	0
π^0	138.0	1	0	0	0	0
π^+	138.0	1	0	1	-1	0
K^0	496.0	1	0	0	1	-1
K^+	496.0	1	0	1	0	-1
Λ^0	1116.0	2	1/2	1	1	1
Σ^0	1192.0	2	1/2	1	1	1
Σ^+	1189.0	2	1/2	2	0	1
Σ^-	1197.0	2	1/2	0	2	1

Results


κ_{uu}/T^2
κ_{dd}/T^2
κ_{ss}/T^2
$\mu_q = 0$
π
πK
πKN
πKNΛΣ
κ_{us}/T^2
κ_{ds}/T^2
κ_{ud}/T^2
0.8
0.6
0.4
0.2
0.0
0.00
-0.02
-0.04
-0.06
-0.08
60
90
120
150
180
T [MeV]

Conclusion & Outlook

Basis transformation

A basis transformation between charge- and quark content- description has been proposed

Chemical composition and diffusion coefficients

The chemical composition of a gas has an effect on its diffusion coefficients

Outlook:

DQPM

Calculate coefficients with DQPM tune

Comaprison with SMASH

Translate SMASH results into the flavor basis