

Baryon and strangeness diffusion and their cross-talk as a function of temperature in a hadronic transport approach

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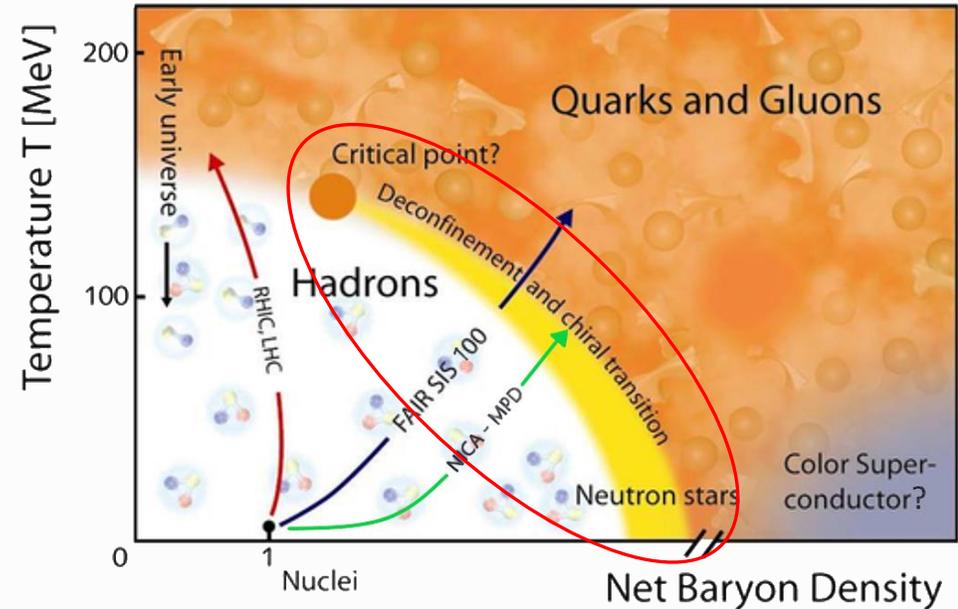
April 29th, 2021



Introduction

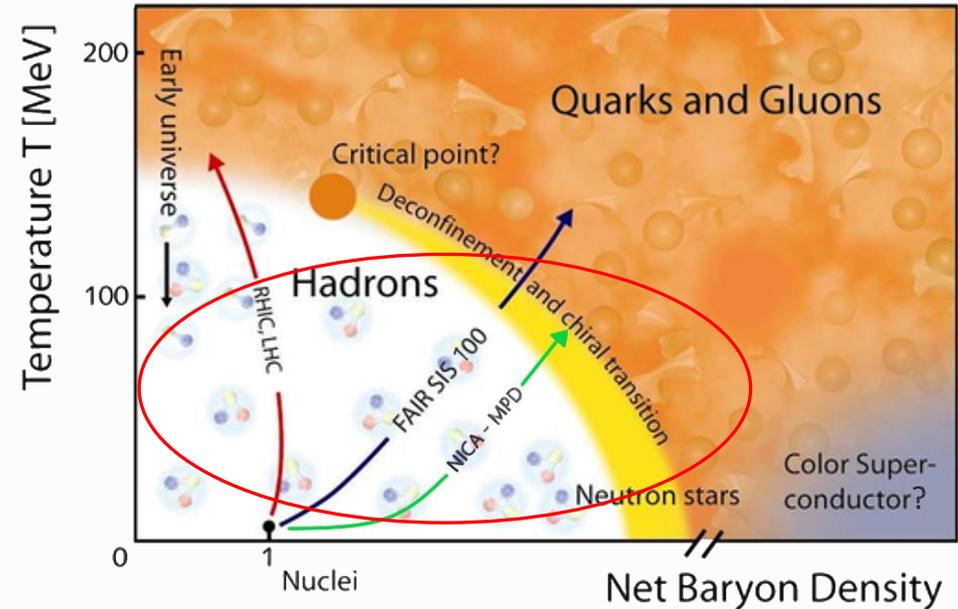
Motivation

- Heavy-ion collisions performed to investigate QCD phase diagram
- Finding expected 1st order phase transition and critical point
- After QGP phase: hadronization
- Low-energy heavy-ion collisions: hadron gas



Motivation

- Heavy-ion collisions performed to investigate QCD phase diagram
 - Finding expected 1st order phase transition and critical point
- After QGP phase: hadronization
- Low-energy heavy-ion collisions: only hadron gas



Motivation

- Low-energy heavy-ion collisions:

- RHIC BES program @ BNL

- FAIR @ GSI

- NICA @ JINR

- Hydrodynamic and transport approaches (or hybrids)

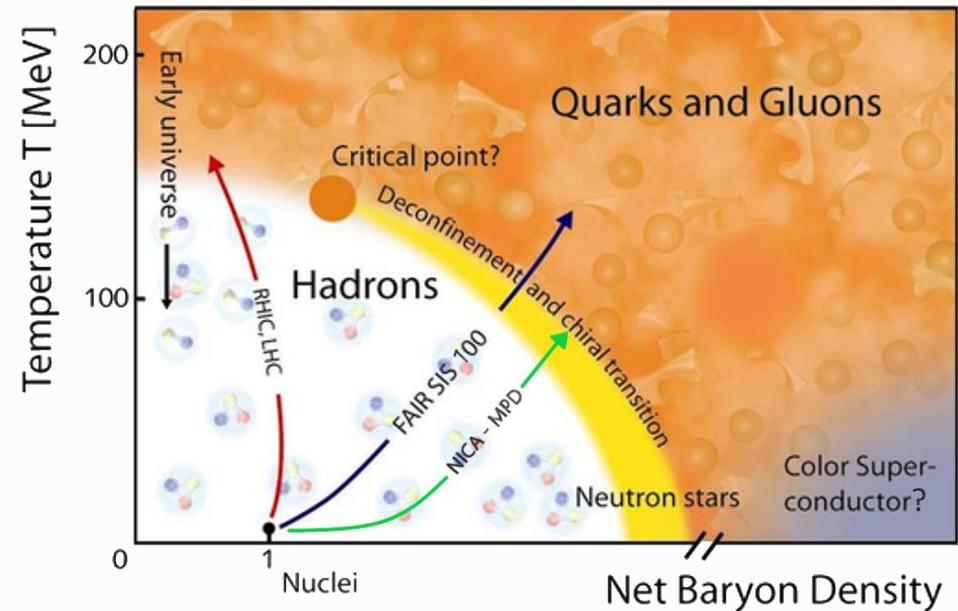
- UrQMD

- BAMPS

- PHSD

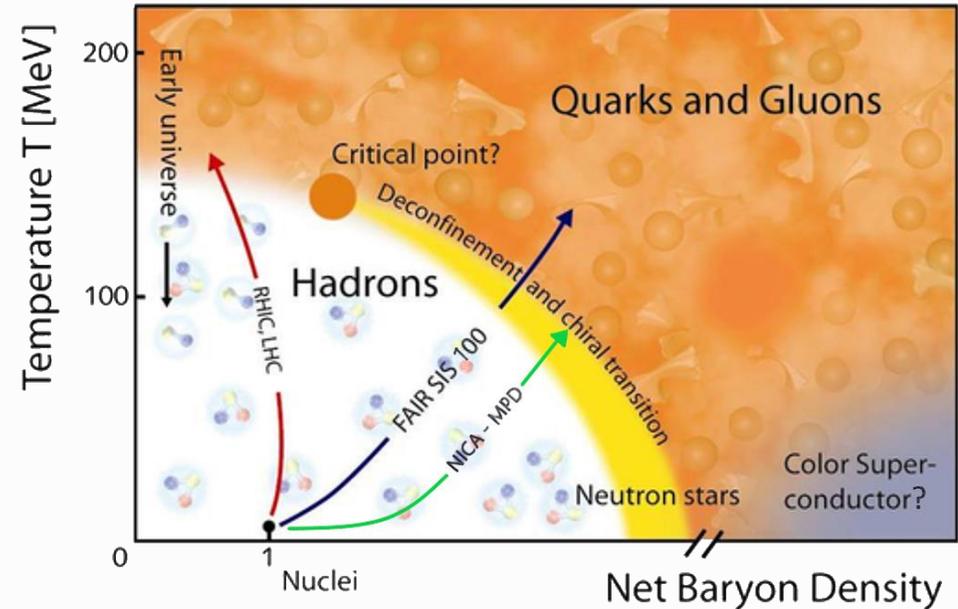
- SMASH

- ...



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 - ...



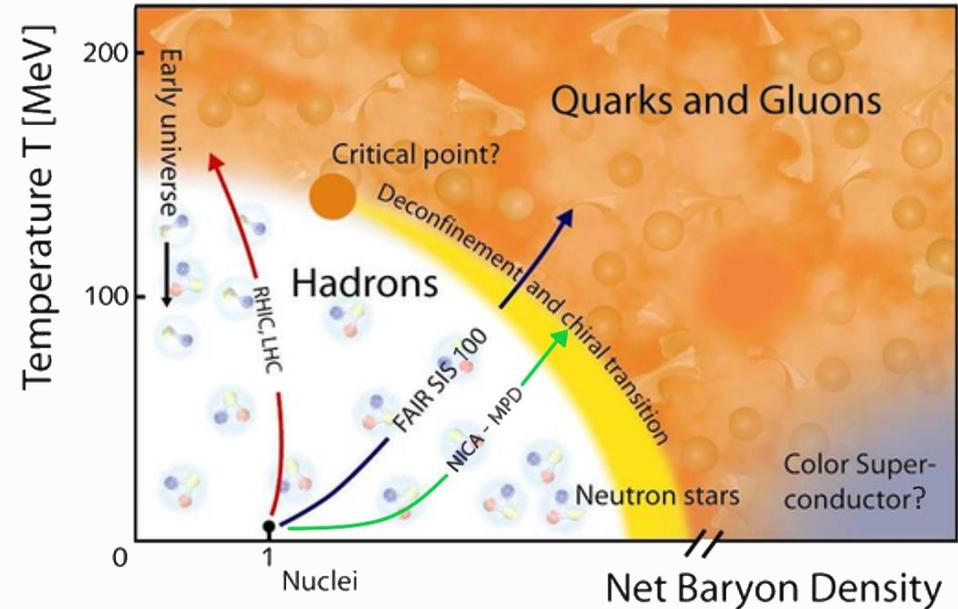
Motivation

- Linear response transport coefficients can be used to characterize hadronic matter
- Shear and bulk viscosities: η, ζ
- Conductivities / diffusion coefficients related to baryon, electric and strangeness charge: σ_{ij}, κ_{ij}

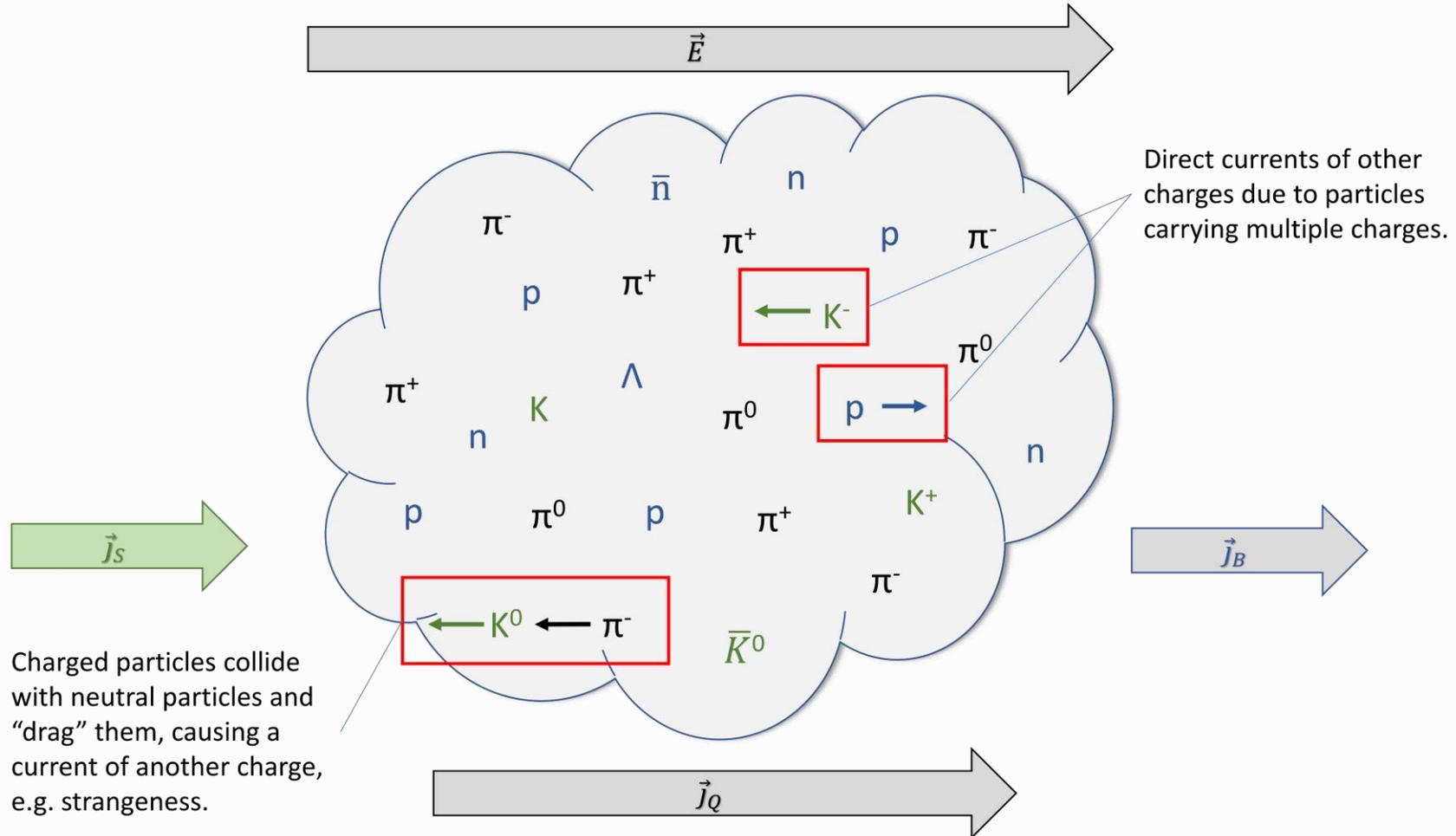
current $\longrightarrow \vec{J}_k = v_k \vec{X}_k \longleftarrow$ gradient

↑

transport coefficient



Charge diffusion



Charge diffusion

- Diffusion processes associated with conserved currents (B,Q,S) do not occur independently from each other

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

charged current

diffusion coefficient matrix (symmetric)
 $\kappa_{ij} = \sigma_{ij}/T$

gradients of charged thermal potentials
 $\alpha_i = \mu_i/T$

Goals

- Calculate full diffusion coefficient matrix of the hadron gas using SMASH -> specifically, baryon and strangeness related coefficients

$$\begin{pmatrix} j_B^\mu \\ j_Q^\mu \\ j_S^\mu \end{pmatrix} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Compare results to previous calculations by J.-B. Rose (SMASH) and kinetic theory by M. Greif & J. Fotakis *et.al.*

The model: SMASH

SMASH (Simulating Many Accelerated Strongly-interacting Hadrons)

- Hadronic transport approach using an effective solution of the relativistic Boltzmann equation:

$$p^\mu \partial_\mu f_a(\vec{x}, \vec{p}, t) + m_a F^\alpha \partial_\alpha^p f_a(\vec{x}, \vec{p}, t) = C_a^{coll}$$

- Geometric collision criterion:

$$d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Spectral functions of resonances are described by relativistic Breit-Wigner functions, with resonance life-time $\tau = 1/\Gamma(m)$

SMASH (Simulating Many Accelerated Strongly-interacting Hadrons)

- Elastic collision via resonances, fully parametrized cross-sections, or scaled from πp cross-sections using the Additive Quark Model
- Inelastic 2 \rightarrow 2 processes:
 - $NN \leftrightarrow NR$
 - $NN \leftrightarrow \Delta R$
 - $KN \leftrightarrow KN$
 - $KN \leftrightarrow K\Delta$
 - $KN \leftrightarrow \pi Y, \quad Y = \{\Lambda, \Sigma, \Xi\}$
- Processes that violate detailed balance are turned off, e.g. string excitations

SMASH – Infinite matter simulations

- 3D cube with periodic boundary conditions
- Initialized with temperature T and chemical potentials μ_i with $i = \{B, Q, S\} \rightarrow$ thermal momenta and multiplicities:

$$N_a = g_a \frac{VT^3}{2\pi^2} \exp\left(\frac{\sum_i \mu_a^i q_a^i}{T}\right) \left(\frac{m_a}{T}\right)^2 K_2\left(\frac{m_a}{T}\right)$$

↑
Isospin degeneracy of particle species a

- All chemical potentials are set to 0 in this work

Degrees of freedom

DOF: Simplified hadron gas ($\sigma = 30$ mb)

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1318}	Ω_{1672}	π_{138}	$f_0 980$	$f_2 1276$	$\rho_3 1690$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1533}	Ω_{2252}	π_{1300}	$f_0 1370$	$f_2' 1525$		K_{892}^*
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_0 1500$	$f_2 1950$	$\phi_3 1850$	$K_1 1270$
N_{1535}	Δ_{1900}	Λ_{1600}	Σ_{1670}	Ξ_{1823}			$f_0 1710$	$f_2 2010$		$K_1 1400$
N_{1650}	Δ_{1905}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_2 2300$	$a_4 2040$	K_{1420}^*
N_{1675}	Δ_{1910}	Λ_{1690}	Σ_{1775}	Ξ_{2025}		η'_{958}	$a_0 980$	$f_2 2340$		$K_0^* 1430$
N_{1680}	Δ_{1920}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_0 1450$		$f_4 2050$	$K_2^* 1430$
N_{1700}	Δ_{1930}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_1 1285$		K_{1680}^*
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N_{1875}		Λ_{1830}	Σ_{2250}				ϕ_{1680}			$K_3^* 1780$
N_{1880}		Λ_{1890}				σ_{800}		$a_2 1320$		$K_2 1820$
N_{1895}		Λ_{2100}					$h_1 1170$			$K_4^* 2045$
N_{1900}		Λ_{2110}				ρ_{776}		$\pi_1 1400$		
N_{1990}		Λ_{2350}				ρ_{1450}	$b_1 1235$	$\pi_1 1600$		
N_{2060}						ρ_{1700}				
N_{2080}							$a_1 1260$	$\eta_2 1645$		
N_{2100}						ω_{783}				
N_{2120}						ω_{1420}		$\omega_3 1670$		
N_{2190}						ω_{1650}			$\pi_2 1670$	
N_{2220}										
N_{2250}										

DOI:10.5281/zenodo.4336358

DOF: Simple hadron gas with resonances

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1318}	Ω_{1672}	π_{138}	$f_0 980$	$f_2 1276$	$\rho_3 1690$	K_{494}
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N_{2220}								$\pi_2 1670$		
N_{2250}										

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DOF: Full hadron gas

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1318}	Ω_{1672}	π_{138}	$f_0 980$	$f_2 1276$	$\rho_3 1690$	K_{494}
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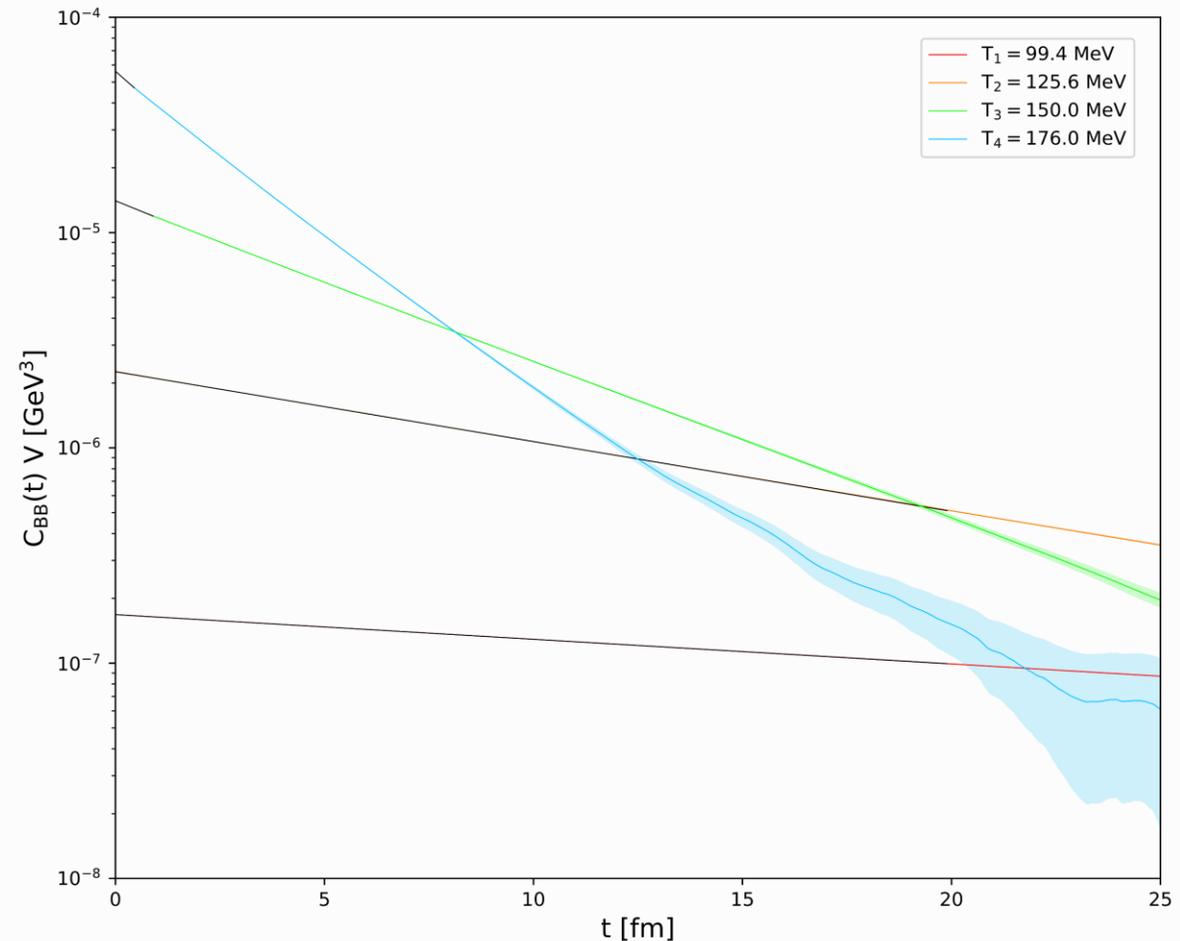
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Methods

Linear response theory + Green-Kubo

- Allows to describe transport coefficients by equilibrium state correlation functions ($i, j = \{B, Q, S\}$):

$$\begin{aligned} C_x^{ij}(t) &\equiv \langle J_x^i(t) J_x^j(0) \rangle \\ &= \frac{1}{V} \frac{\kappa^{ij}}{\tau} \exp\left(-\frac{t}{\tau^{ij}}\right) \\ &= C_x^{ij}(0) \cdot \exp\left(-\frac{t}{\tau^{ij}}\right) \end{aligned}$$



Equilibrium of the hadron gas

- After initialization, a box is not in chemical or thermal equilibrium, e.g. due to Poissonian sampling of particles, initializations at pole masses, etc.
- Chemical and thermal equilibrium of a hadron gas is reached if:
 - steady state in terms of its chemical composition and momentum distribution is achieved
 - the temperature saturates at a constant value

$$\frac{d N_a}{d p} \propto p_a^2 \exp\left(-\frac{\sqrt{p_a^2 + m_a^2} - \sum_k \mu_a^k q_a^k}{T}\right)$$

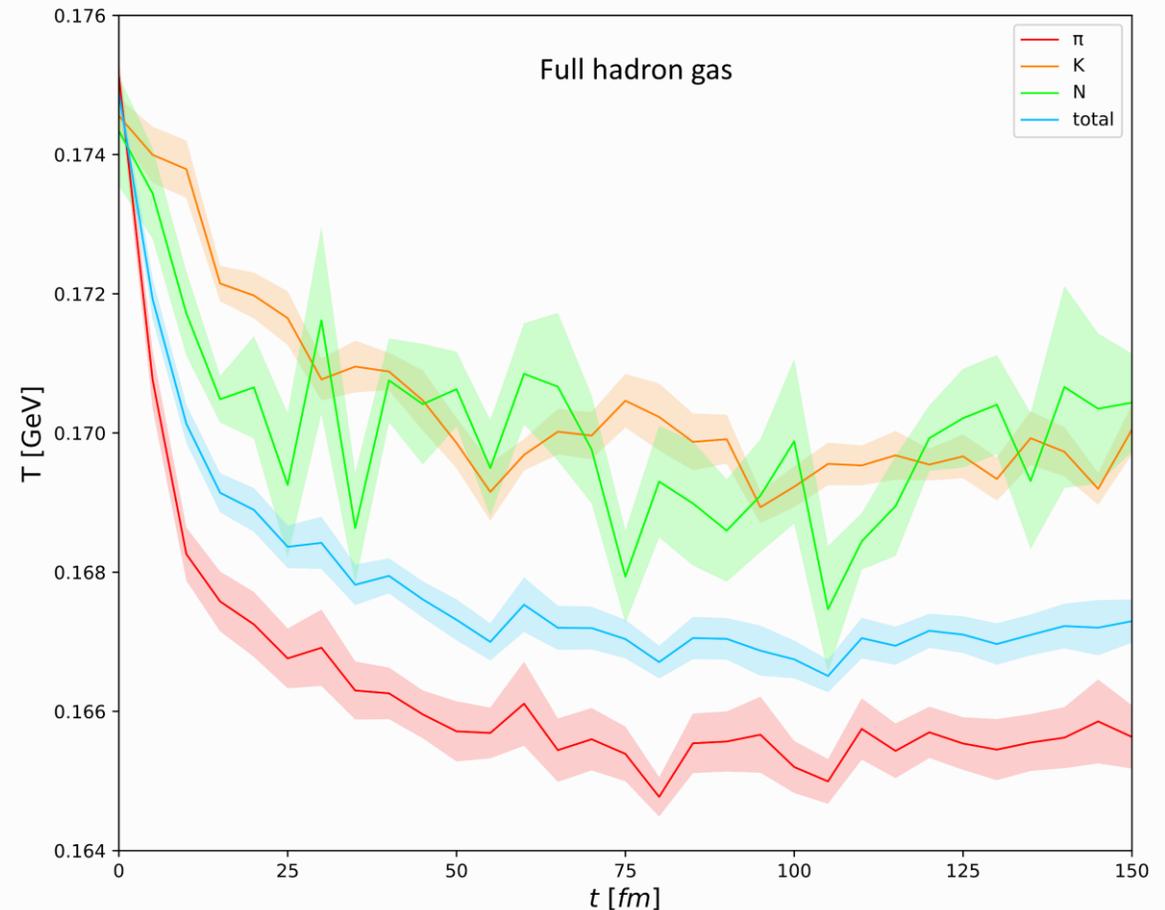
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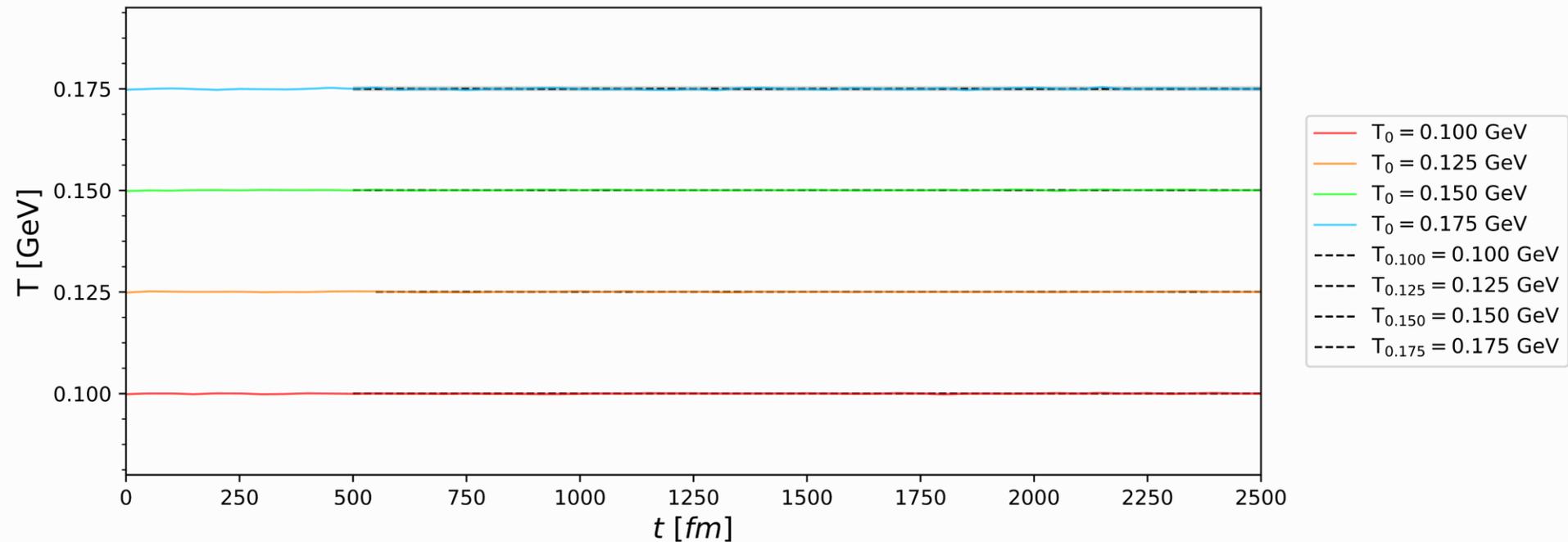
Equilibrium of the hadron gas

- Thermal equilibrium is reached after chemical, so monitoring the temperature is sufficient
- Temperature differs for different particle species
- Weighted average of most abundant particles:
pions + kaons + nucleons

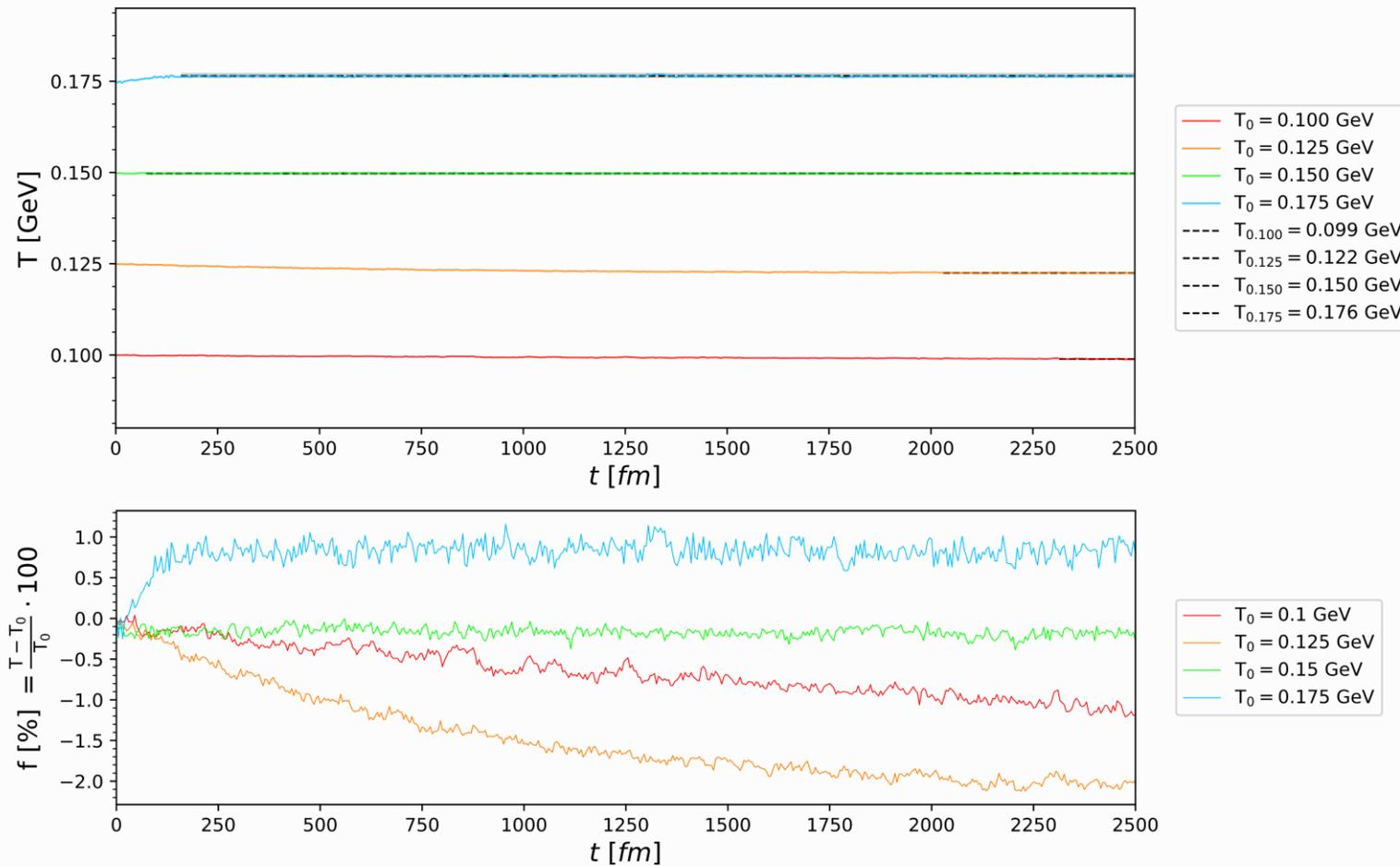


Results

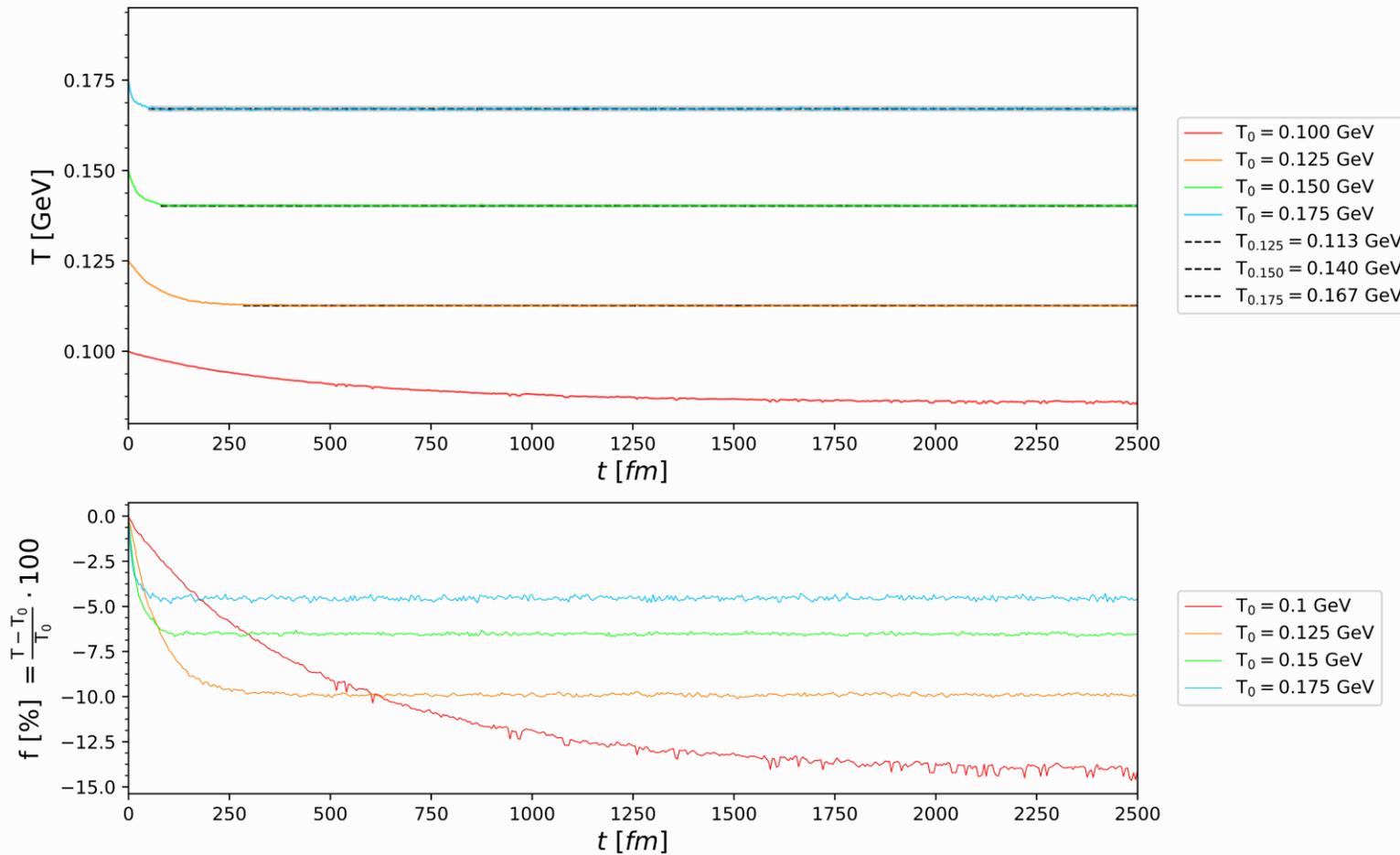
Temperature: Simplified hadron gas



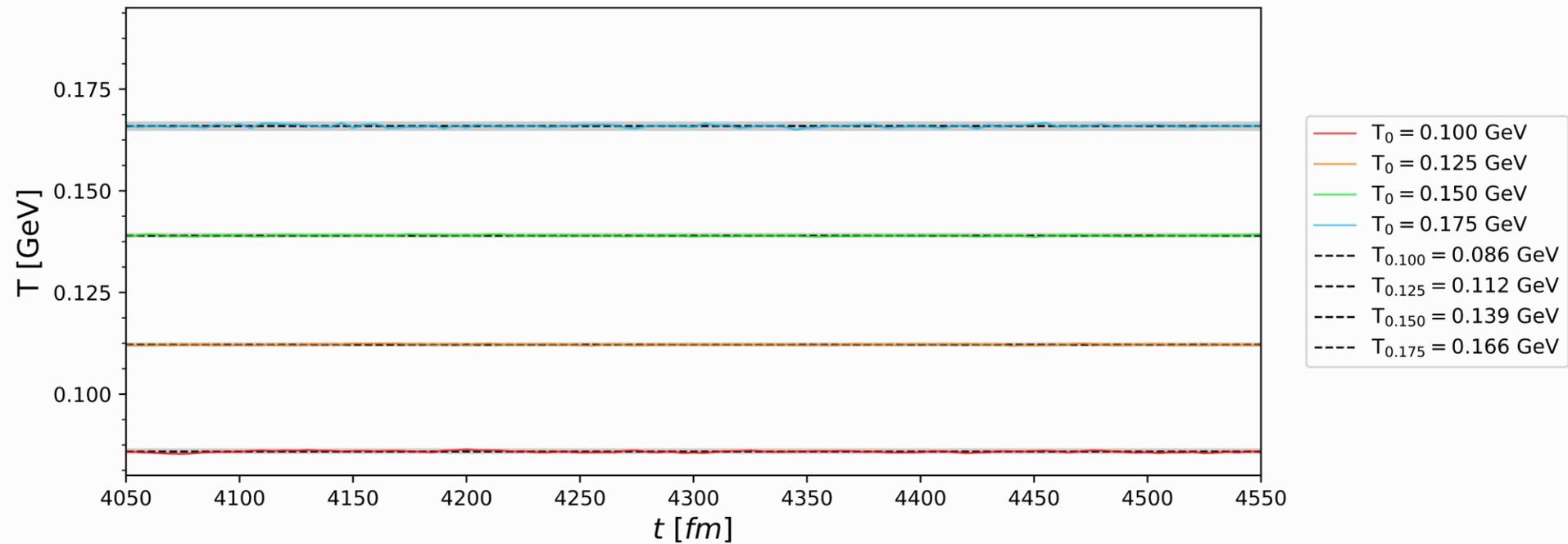
Temperature: Simple hadron gas with resonances



Temperature: Full hadron gas



Temperature: Full hadron gas



Correlation functions in SMASH

- SMASH provides full phase space information of each particle at every time step

$$J_i^\mu(t) = \frac{1}{V} \sum_{a=1}^N q_i^a \frac{p_a^\mu(t)}{p_a^0(t)} \quad q_i^a = \{B^a, Q^a, S^a\}$$

- Correlation function:

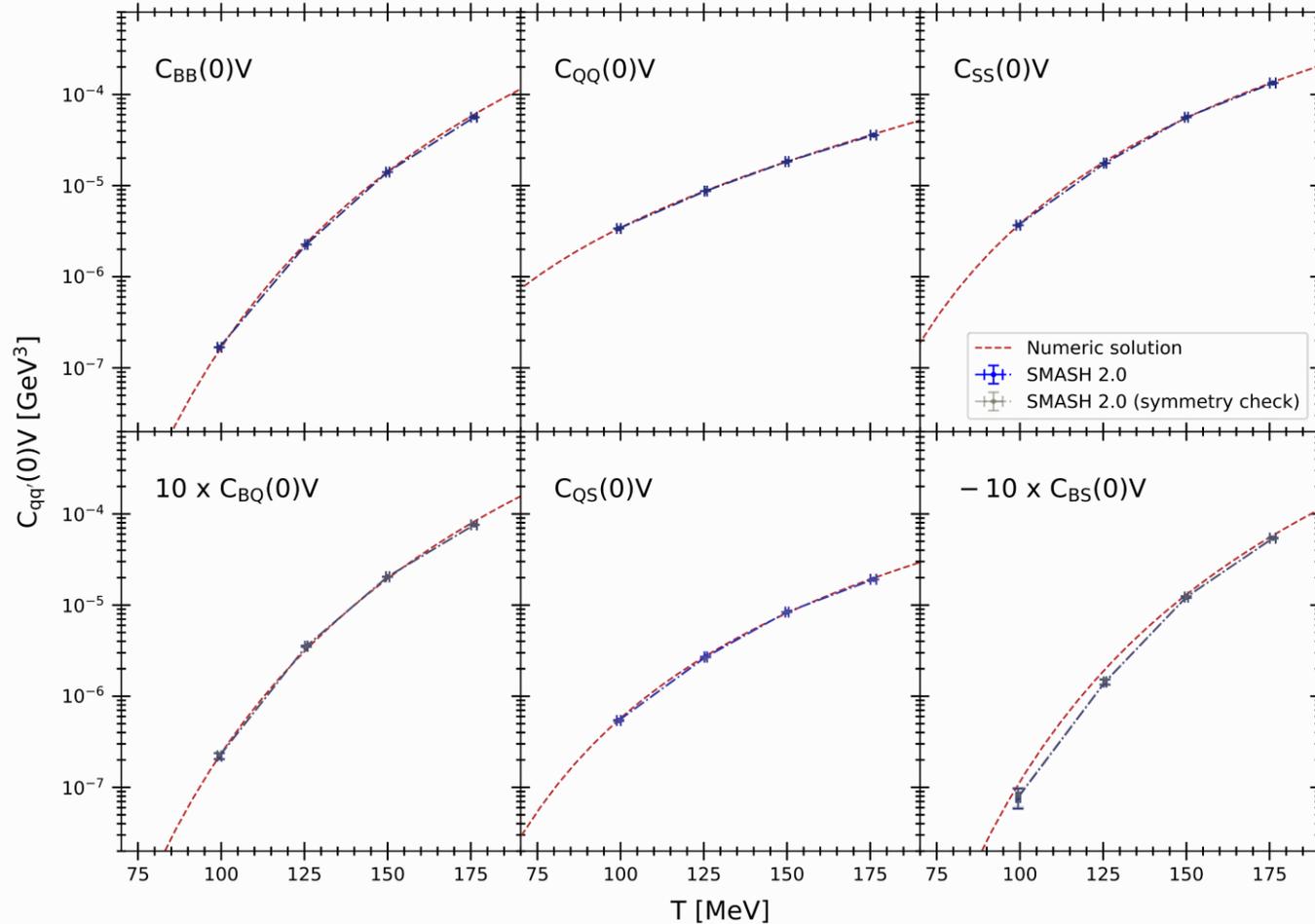
$$C(t) = \langle J(0) J(u\Delta t) \rangle = \lim_{K \rightarrow \infty} \frac{1}{K - u} \sum_{s=0}^{K-u} J(s\Delta t) J(s\Delta t + u\Delta t)$$

Total number of
considered time steps



K=8000, Δt=0.05 fm

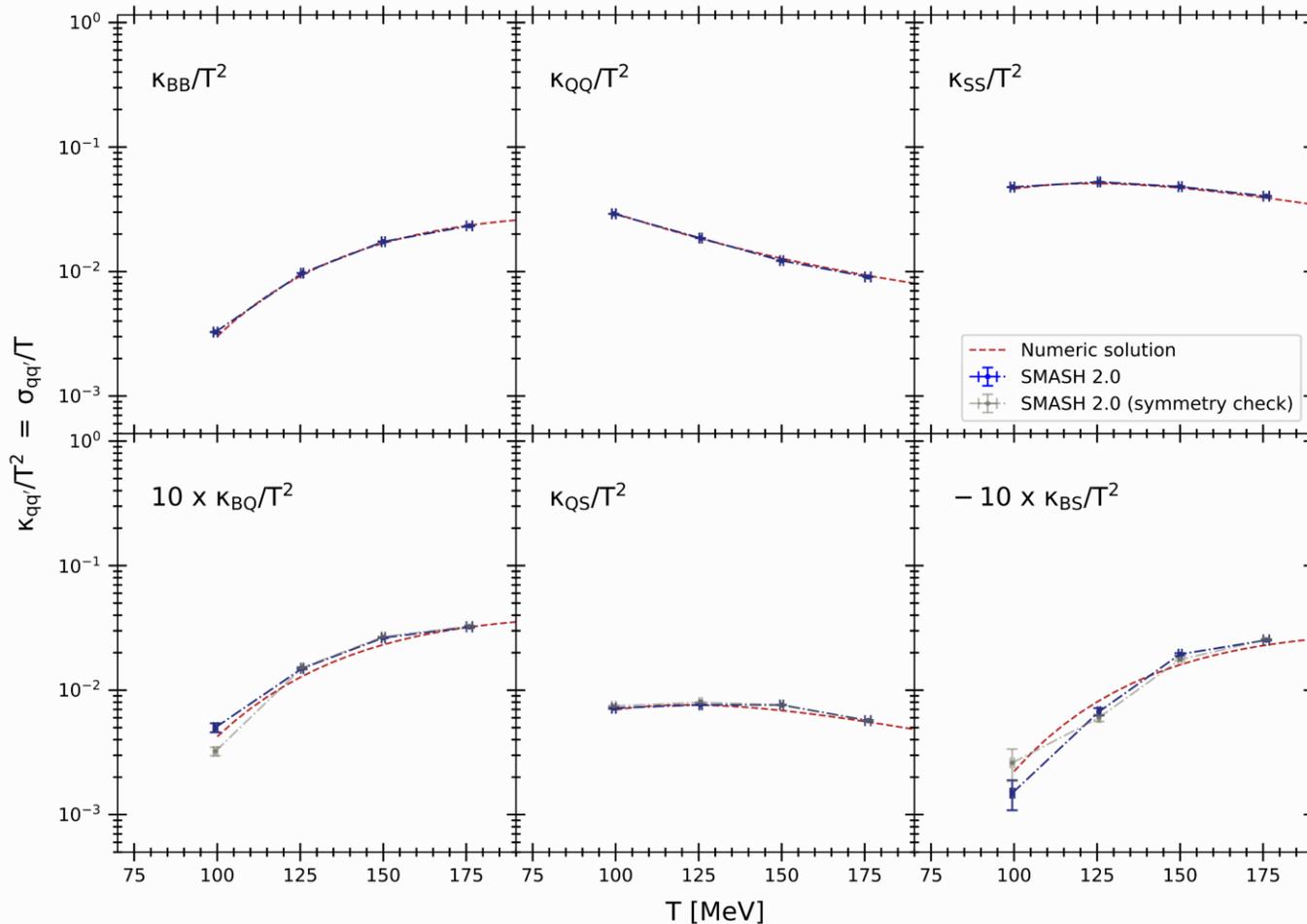
C(0)V: Simplified hadron gas



$$C_{ij}(t) = C_{ij}(0) \cdot \exp\left(-\frac{t}{\tau_{ij}}\right)$$

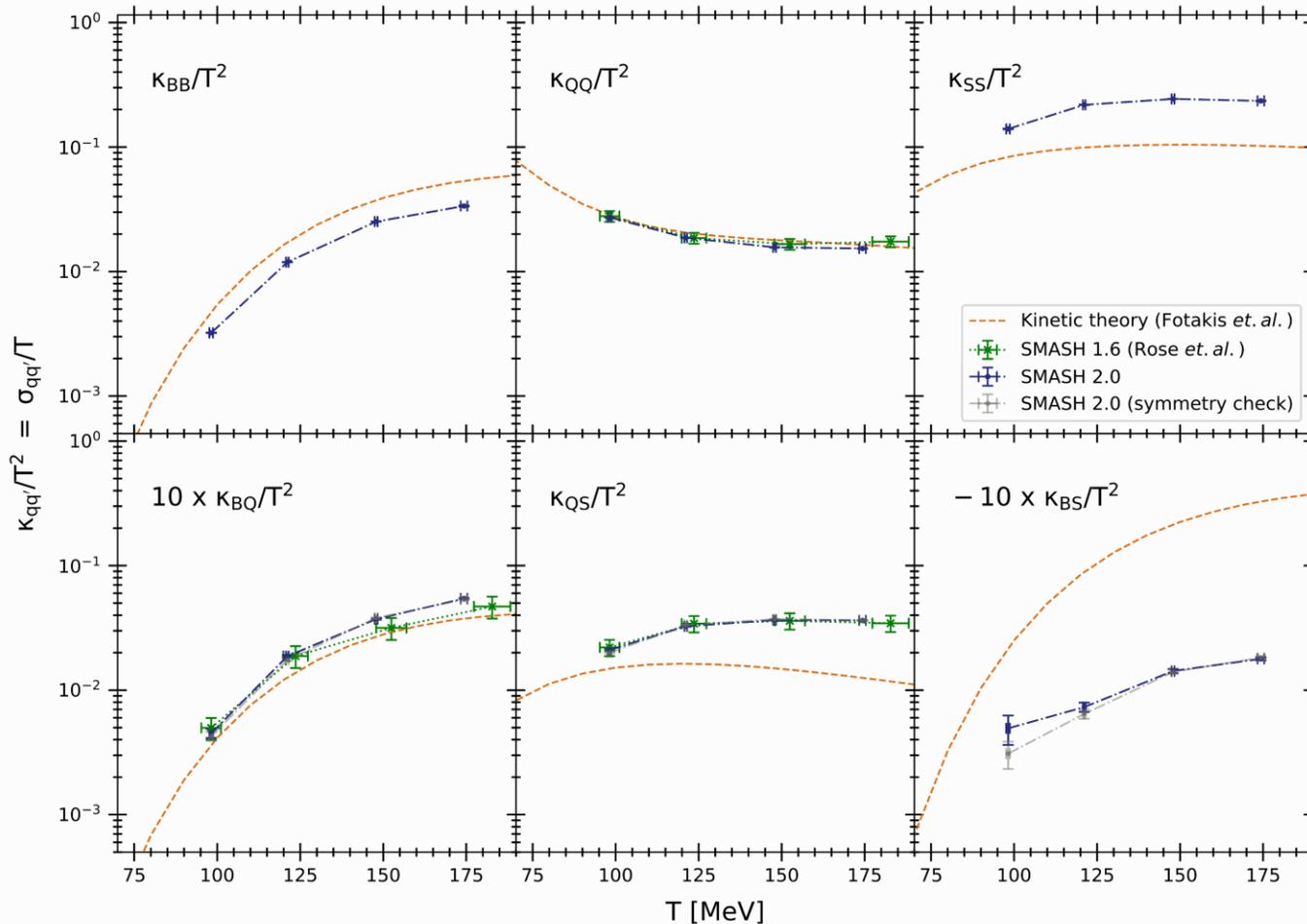
$$\kappa_{ij}(t) = C_{ij}(0)V \cdot \tau_{ij}$$

Diffusion: Simplified hadron gas



- Differences of symmetric quantities serve as an estimation for a systematical error
- All coefficients approximately of the same order of magnitude
- Larger errors for κ_{BQ} and κ_{BS} at lower T

Diffusion: Simple hadron gas with resonances



- Data compared to kinetic theory that uses a hadron gas of different chemical composition with more degrees of freedom
- κ_{QQ} and κ_{BQ} similar to kinetic theory due to contributing particles

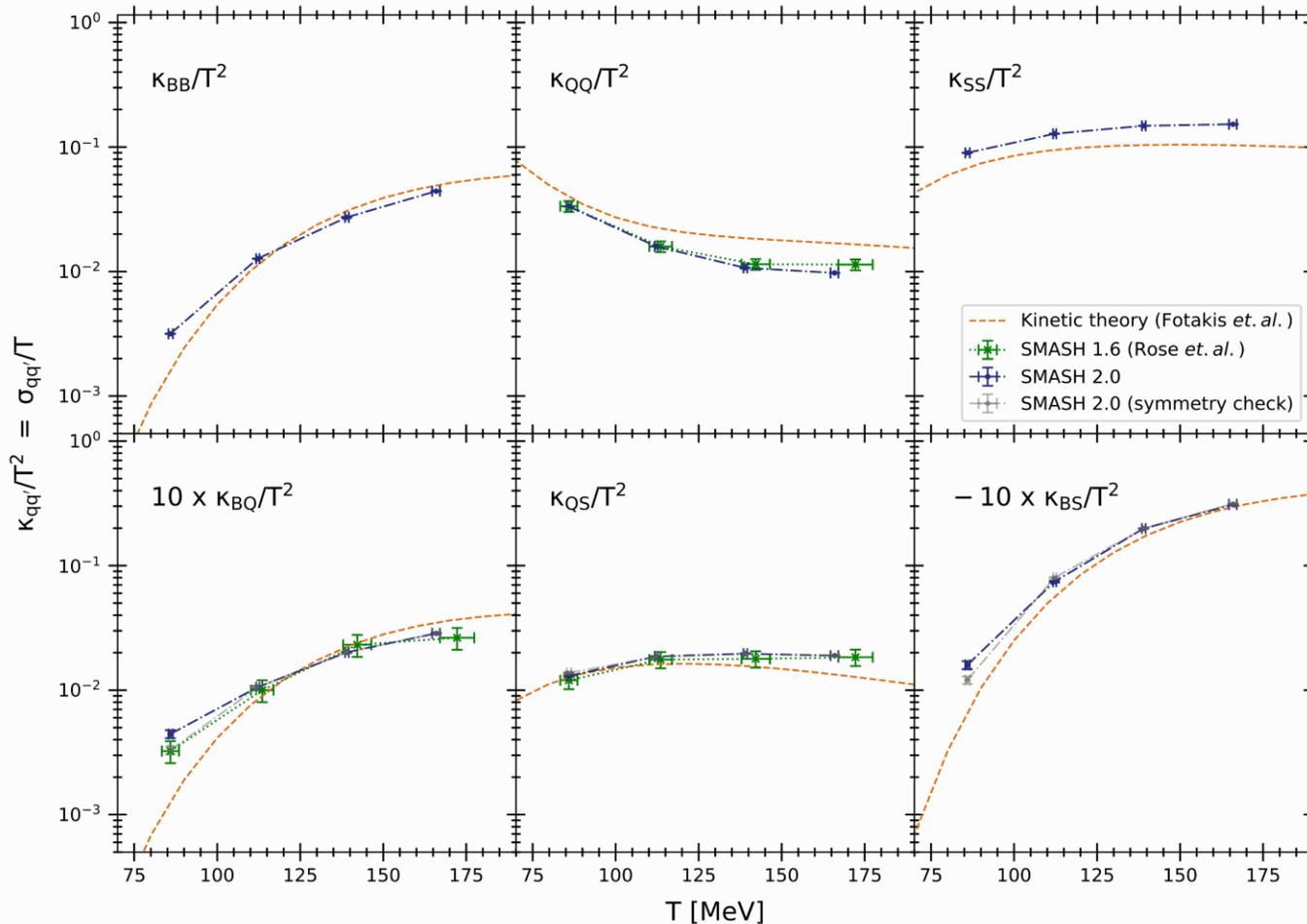
Kinetic theory: Composition

Particle	M_0 [GeV]	g
π	0.138	3
K	0.494	4
N	0.938	4
Λ	1.116	2
Σ^0	1.193	2
Σ^+	1.189	2
Σ^-	1.197	2

- No resonances, only parametrized, energy-dependent or constant cross-sections
- Addition of Σ baryons that can carry all three conserved charges (B,Q,S)

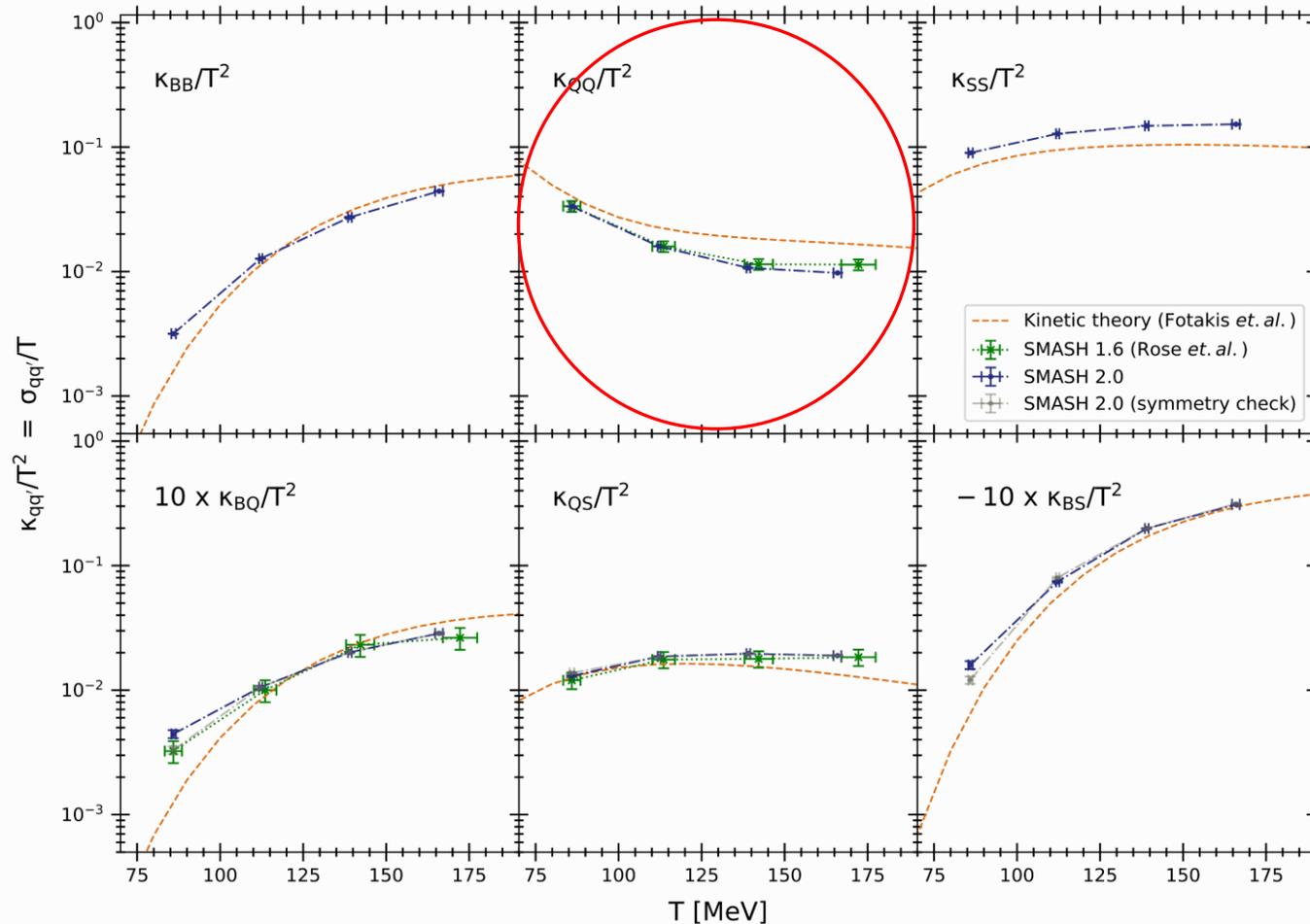
[arXiv:1912.09103[hep-ph]]

Diffusion: Full hadron gas



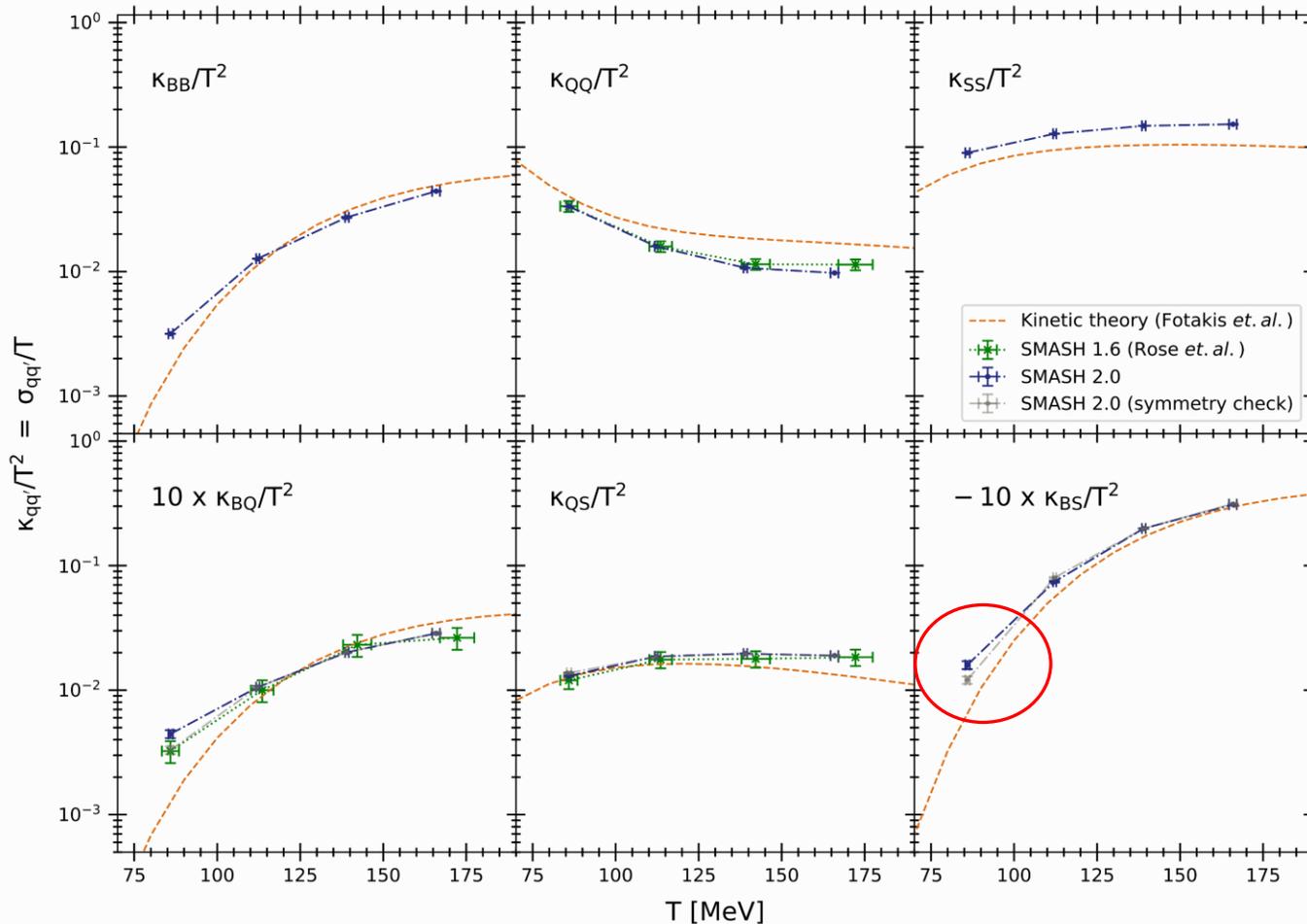
- Large contribution from the Σ baryon
- κ_{BS} errors smaller due to more contributing particles
- Kinetic theory gives good approximation of data, even though a simpler system was used

Diffusion: Full hadron gas



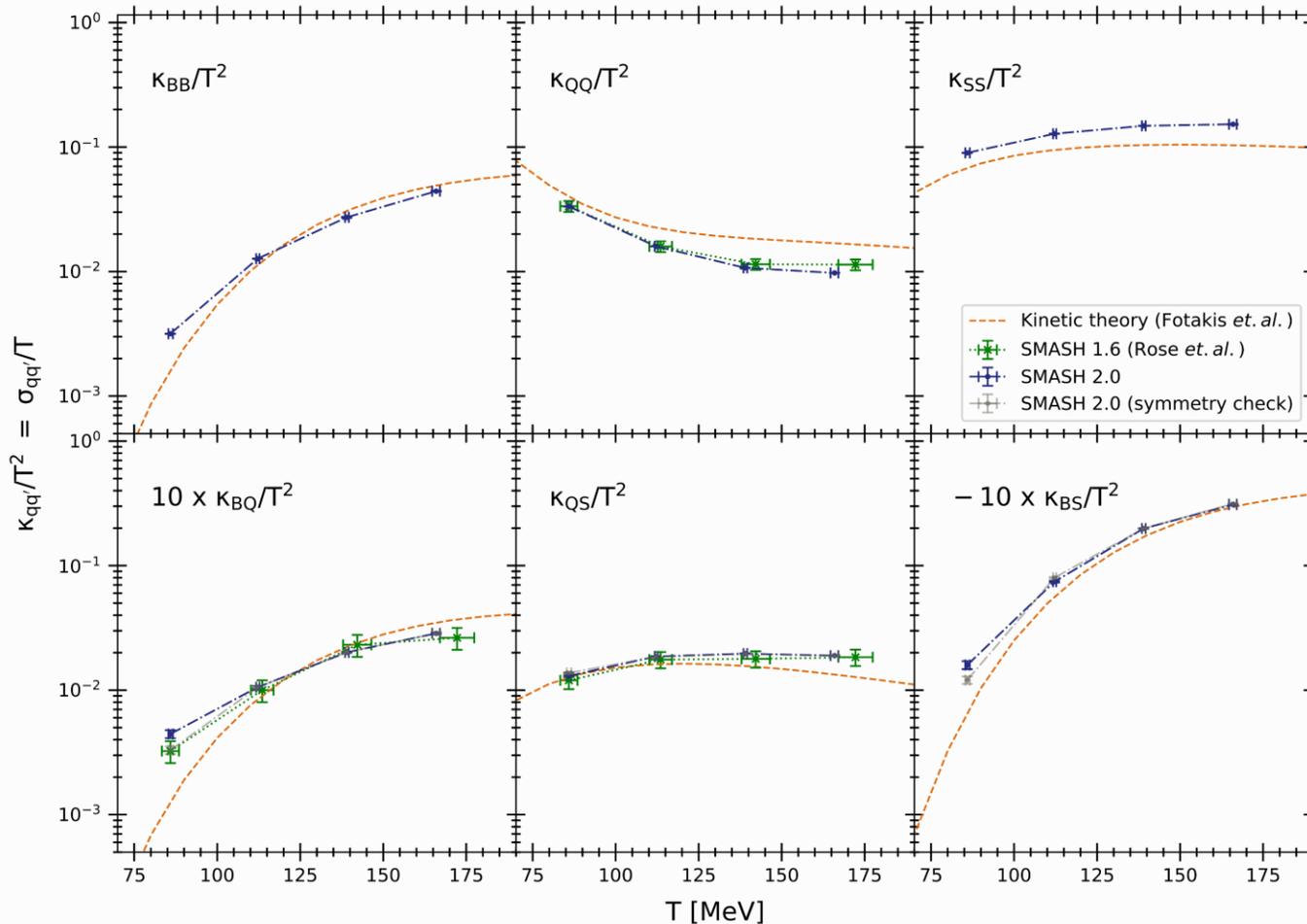
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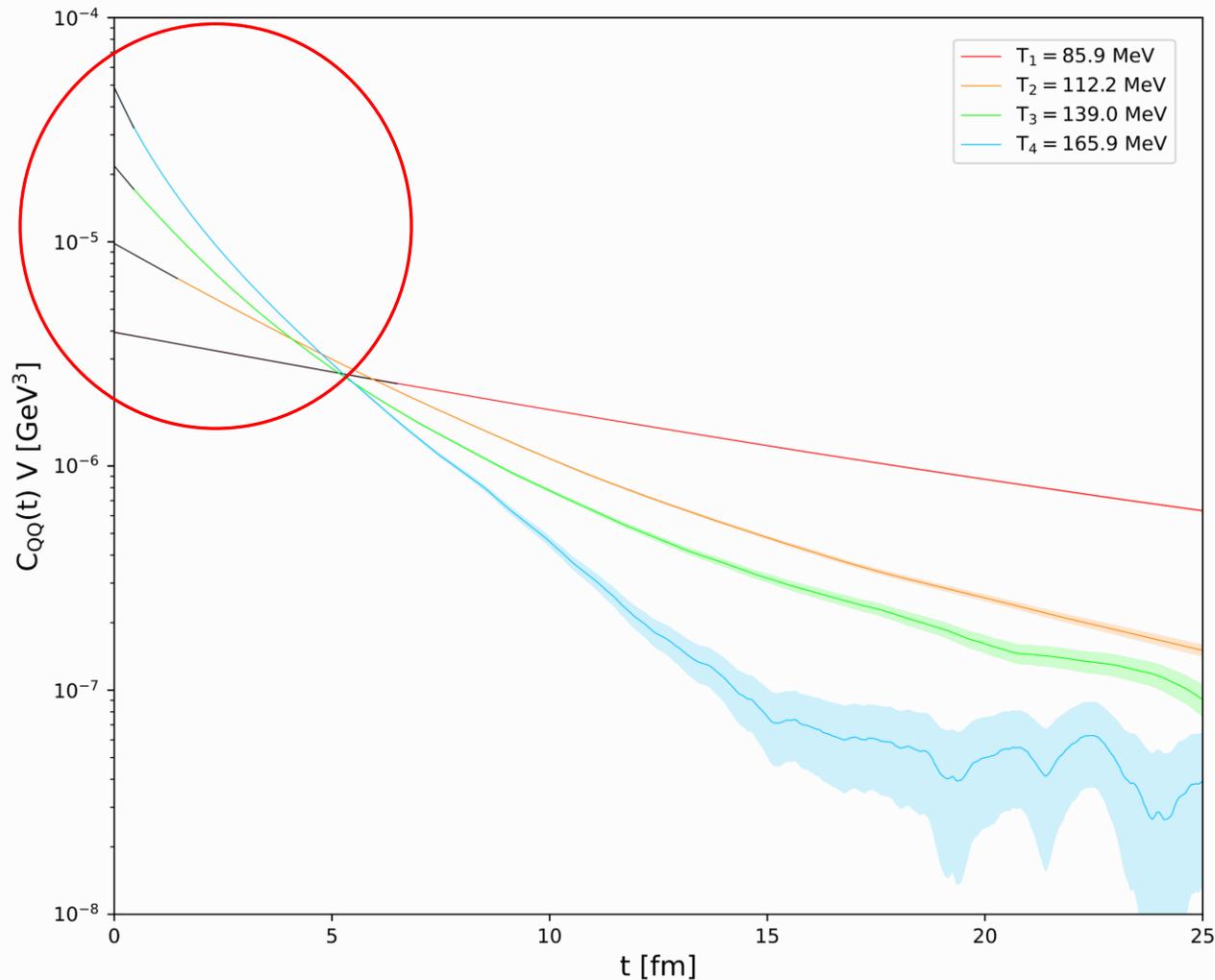
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Diffusion: Full hadron gas



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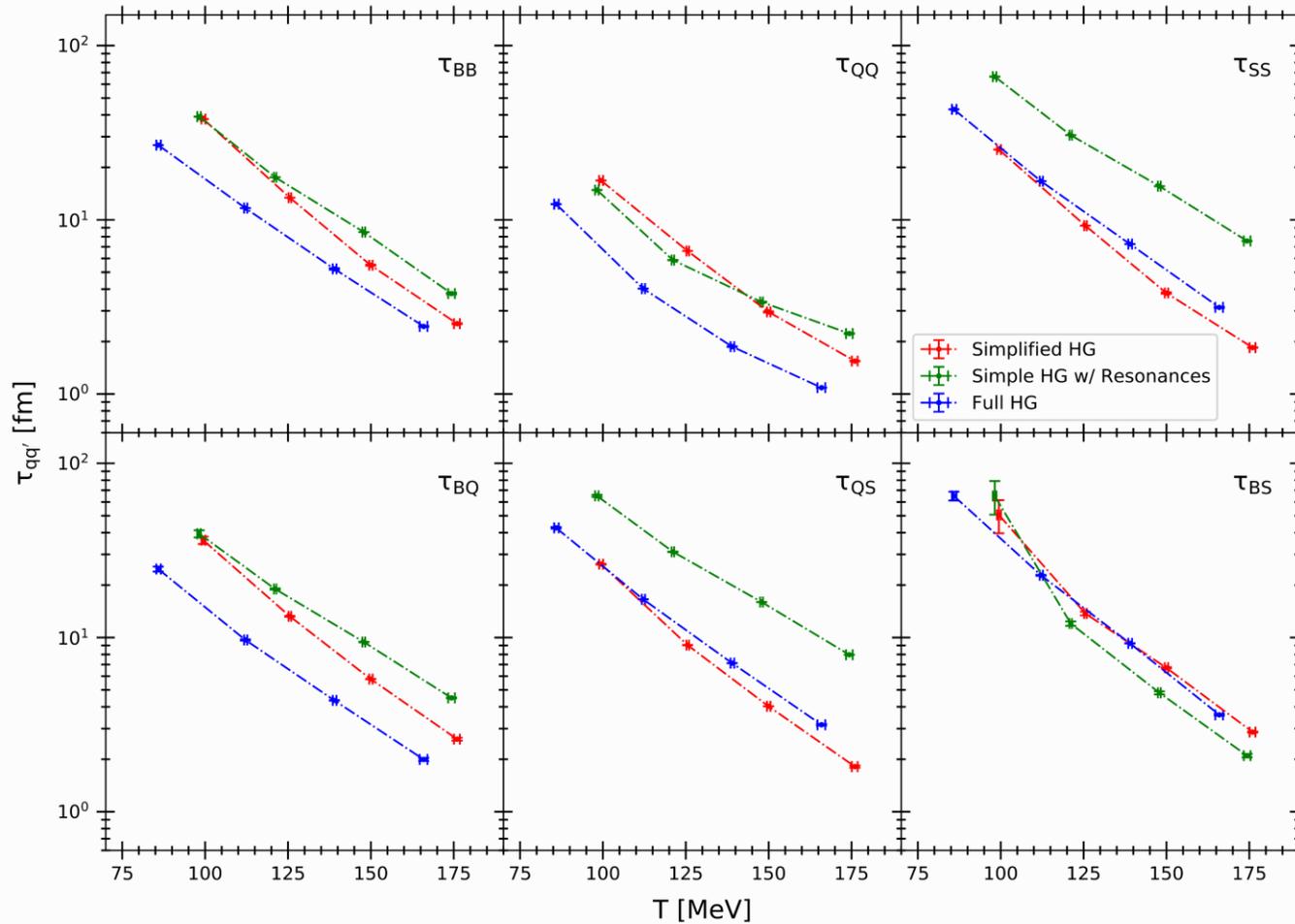
Breakdown of the exponential ansatz



➤ System very dense at high temperatures

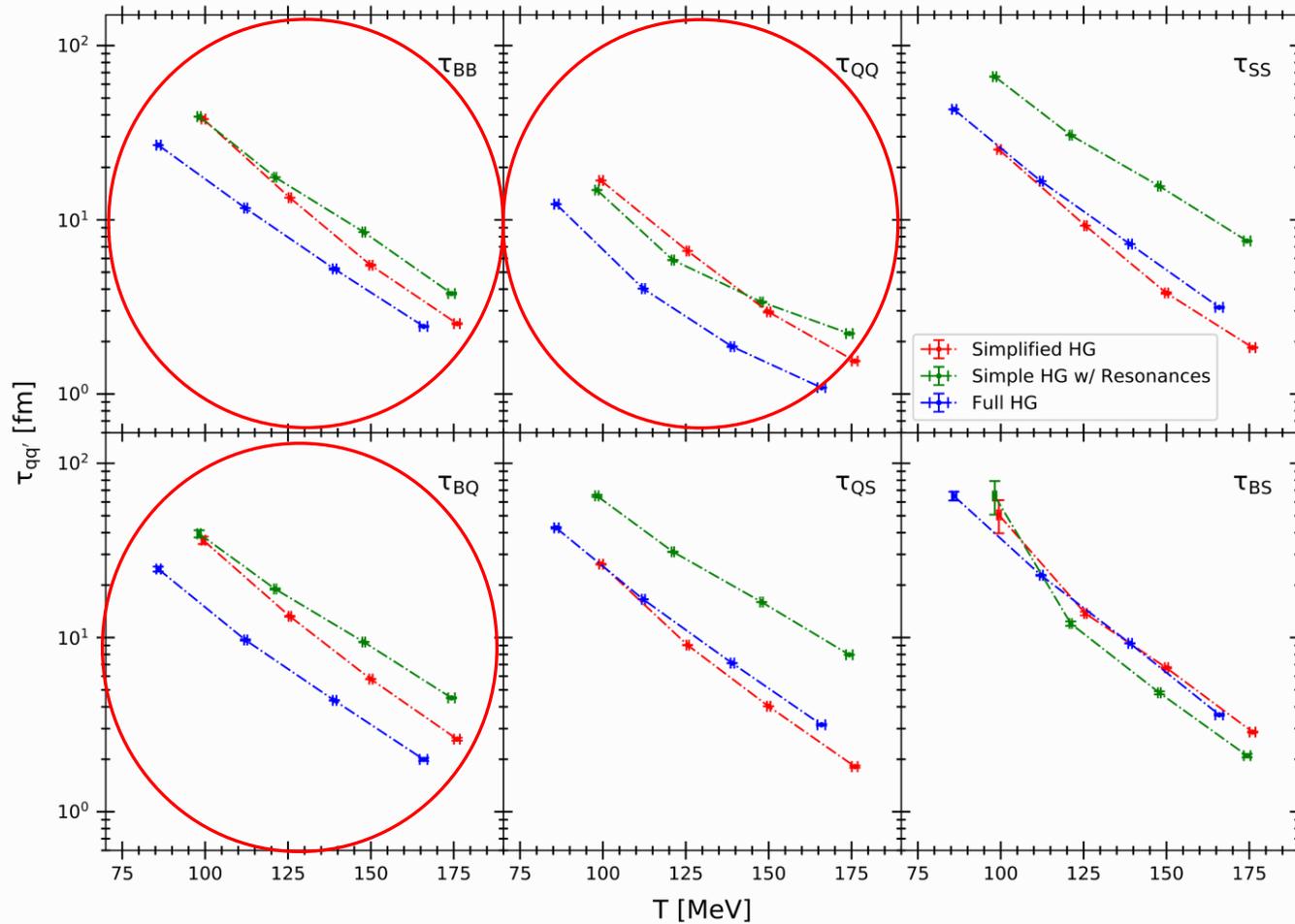
$$C^{ij}(t) \neq C^{ij}(0) \cdot \exp\left(-\frac{t}{\tau}\right)$$

Relaxation times



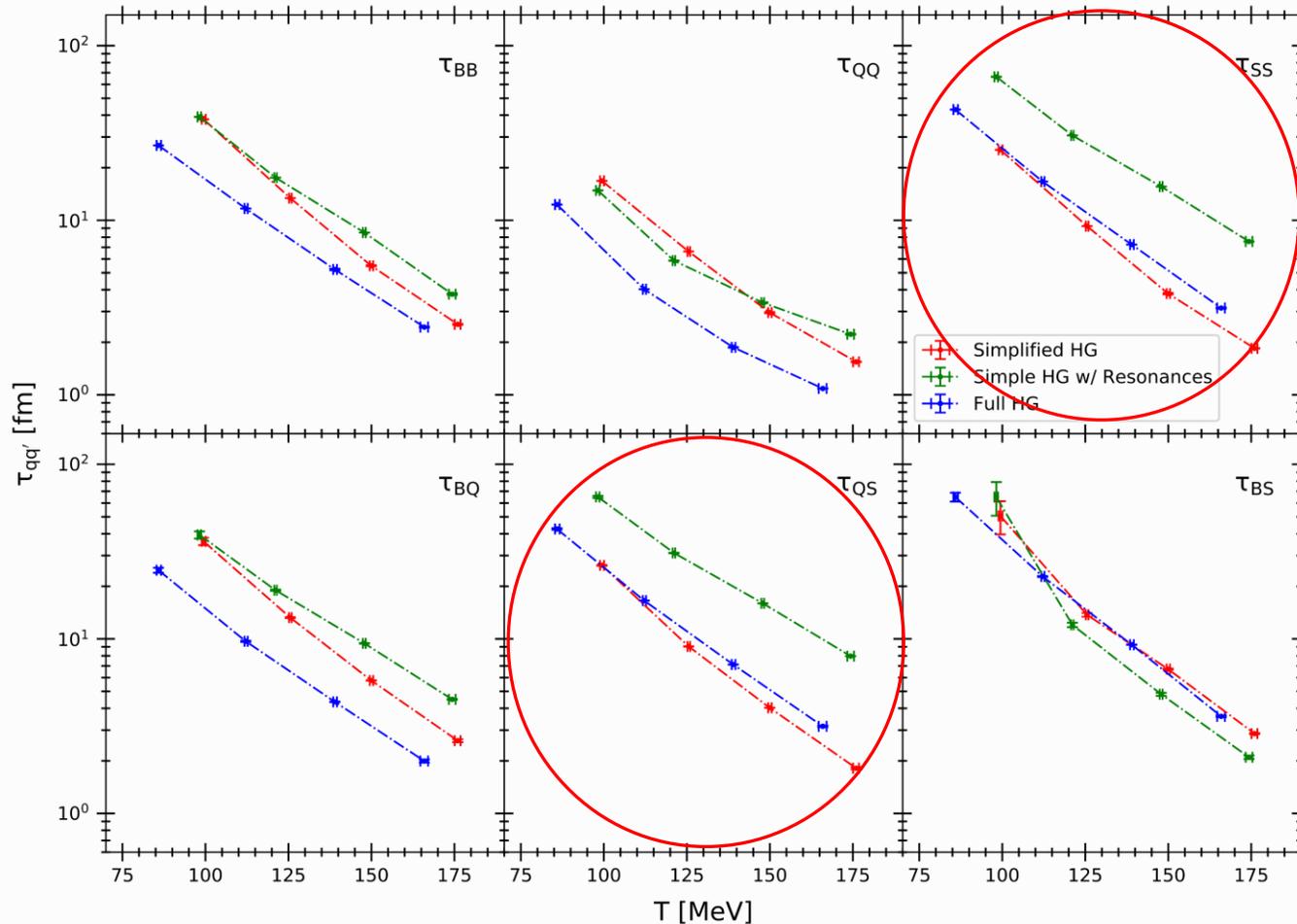
- Time it takes for system to get back to equilibrium after perturbation
- Potential overestimation of contribution in the simplified hadron gas due to fixed cross-sections

Relaxation times



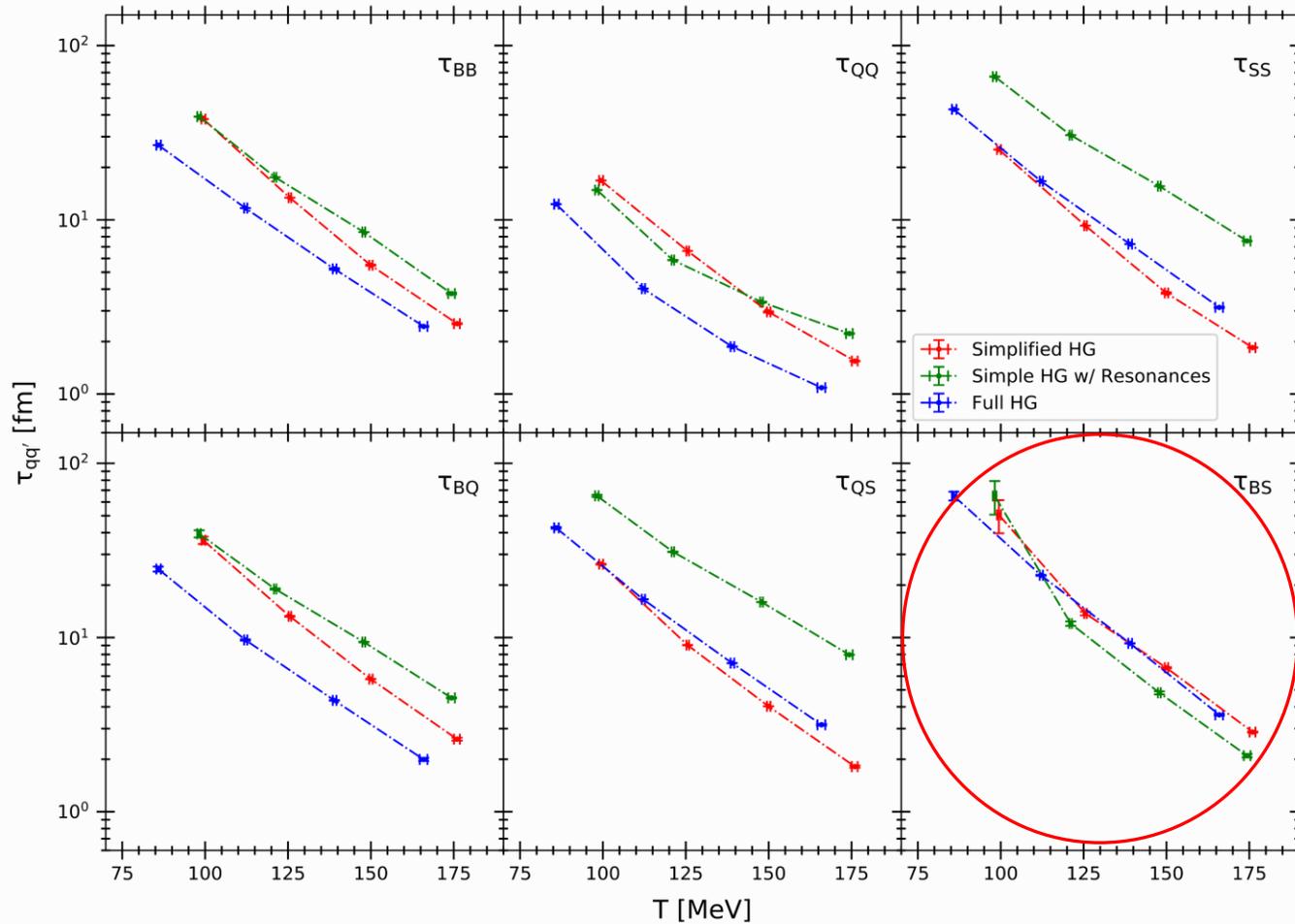
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Conclusion

Summary

- Transport diffusion coefficients as a function of temperature for three hadron gases of different chemical composition have been calculated with SMASH
- Diffusion coefficients are sensitive to an increasing number of degrees of freedom
- Increasing the degrees of freedom has a larger effect on simpler systems than on systems that already have many different particle species
- Kinetic theory calculations from Fotakis *et.al.* serve as a good first approximation of the full hadron gas in SMASH

Outlook

- Investigation of all diffusion coefficients using SMASH at finite baryon, electric and strangeness chemical potentials
- Calculations take very long for complex systems, even longer with finite chemical potentials -> using already equilibrated systems for sampling more data
- Exponential ansatz breaks down -> potentially needing to find other methods for calculation
- QGP calculations have to be compared to those of the hadron gas in the region of the expected phase transition
- Hydrodynamic calculations with diffusion coefficients from SMASH as input



Backup Slides

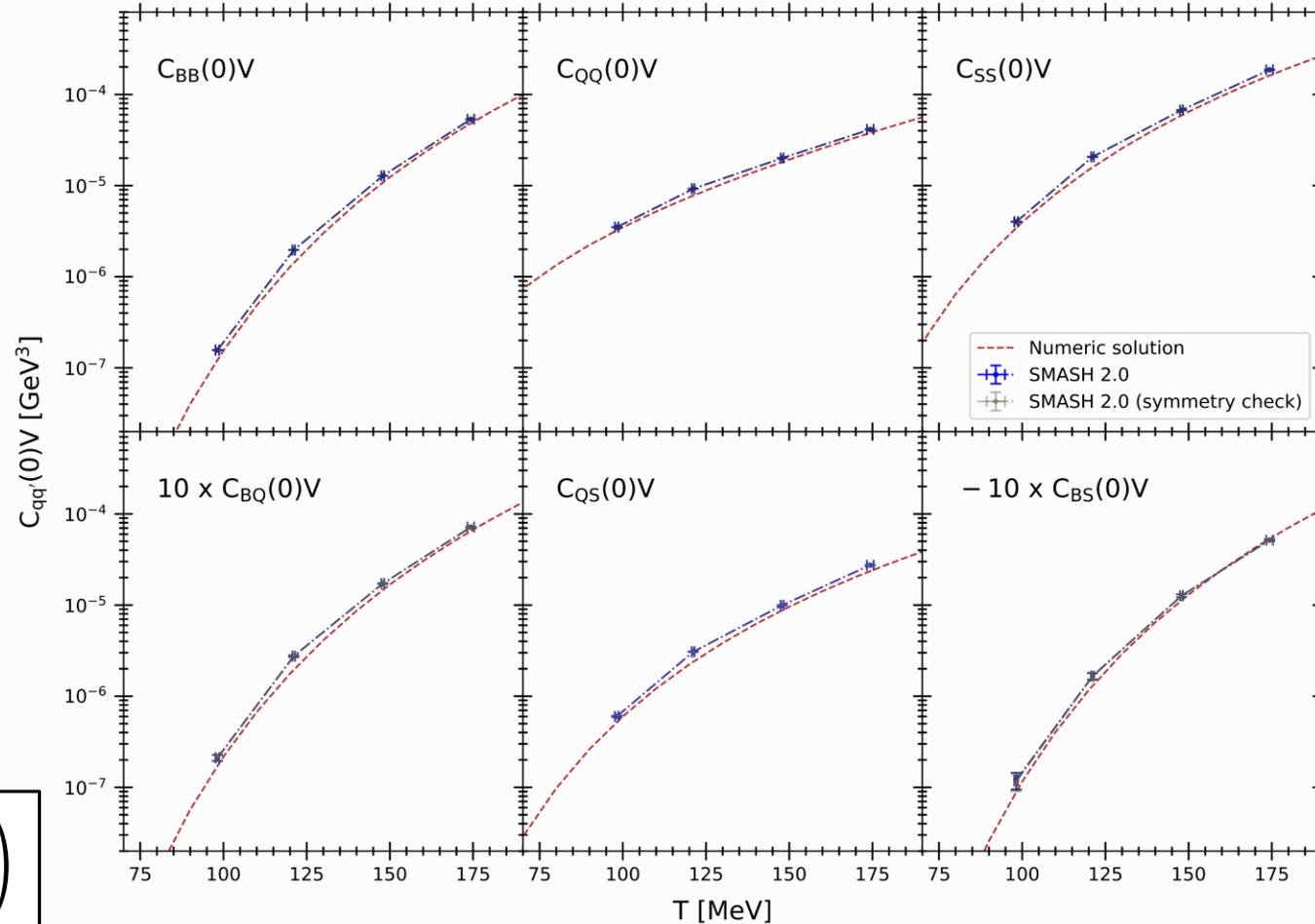
Name	Mass [MeV/c ²]	Spin	Degeneracy	Baryon Number	Electric Charge	Strangeness
π^+	138	0	1	0	+e	0
π^-	138	0	1	0	-e	0
π^0	138	0	1	0	0	0
K^+	496	0	1	0	+e	+1
K^-	496	0	1	0	-e	-1
K^0	496	0	1	0	0	+1
\bar{K}^0	496	0	1	0	0	-1
p	938	1/2	2	+1	+e	0
\bar{p}	938	1/2	2	-1	+e	0
n	938	1/2	2	+1	0	0
\bar{n}	938	1/2	2	-1	0	0
Λ^0	1116	1/2	2	+1	0	-1
$\bar{\Lambda}^0$	1116	1/2	2	-1	0	+1
Σ^0	1193	1/2	2	+1	0	-1
$\bar{\Sigma}^0$	1193	1/2	2	-1	0	+1
Σ^+	1189	1/2	2	+1	+e	-1
$\bar{\Sigma}^+$	1189	1/2	2	-1	-e	+1
Σ^-	1197	1/2	2	+1	-e	-1
$\bar{\Sigma}^-$	1197	1/2	2	-1	+e	+1

[arXiv:1912.09103[hep-ph]]

	π^+	π^-	π^0	K^+	K^-	K^0	\bar{K}^0	p	n	\bar{p}	\bar{n}	Λ^0	$\bar{\Lambda}^0$	Σ^0	$\bar{\Sigma}^0$	Σ^+	$\bar{\Sigma}^+$	Σ^-	$\bar{\Sigma}^-$
π^+	10	res	res	10	10	res	10	res	10	10	res	23.1	23.1	5	5	5	5	5	5
π^-		10	res	res	10	10	res	res	res	res	10	23.1	23.1	5	5	5	5	5	5
π^0			5	res	10	res	res	res	res	res	res	23.1	23.1	5	5	5	5	5	5
K^+				10	10	10	50	res	10	20	10	18.5	18.5	3	3	3	3	3	3
K^-					10	50	10	res	res	6	10	18.5	18.5	3	3	3	3	3	3
K^0						10	50	6	6	20	20	18.5	18.5	3	3	3	3	3	3
\bar{K}^0							10	8	20	6	6	18.5	18.5	3	3	3	3	3	3
p								res	res	res	20	34.7	34.7	10	10	10	10	10	10
n									20	res	100	34.7	34.7	10	10	10	10	10	10
\bar{p}										10	10	34.7	34.7	10	10	10	10	10	10
\bar{n}											10	34.7	34.7	10	10	10	10	10	10
Λ^0												30	30	10	10	10	10	10	10
$\bar{\Lambda}^0$													30	10	10	10	10	10	10
Σ^0														10	10	10	10	10	10
$\bar{\Sigma}^0$															10	10	10	10	10
Σ^+																10	10	10	10
$\bar{\Sigma}^+$																	10	10	10
Σ^-																		10	10
$\bar{\Sigma}^-$																			10

[arXiv:1912.09103[hep-ph]]

C(0)V: Simple hadron gas with resonances



$$C_{ij}(t) = C_{ij}(0) \cdot \exp\left(-\frac{t}{\tau_{ij}}\right)$$

$$\kappa_{ij}(t) = C_{ij}(0)V \cdot \tau_{ij}$$