Constraints on neutron star properties from GW170817

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GW170817

LIGO Lab MIT/Caltech

and LIGO-Livingston data. The signal is not visible
summarized in
ration of the detectors at the time of GW170817 is

to contribute to the analysis of the inspiral. The configu-
also operating at the time, but its sensitivity was insufficient

2.73 and

0

the horizon was 58 Mpc. The GEO600 detector

[32,62,63]

BNS system (SNR
Livingston and LIGO-Hanford detectors could detect a

was composed of two neutron stars.

black holes in galactic binary systems, suggests the source

binaries, and their inconsistency with the masses of known

dynamically measured masses of known neutron stars in

ruled out, the consistency of the mass estimates with the

Moreover, although a neutron star

magnetic emission demonstrates the presence of matter.

The detection of GRB 170817A and subsequent electro-

combination of the masses is the chirp mass

BNS systems are between 2.57 and

gravitational-wave signal, as the total masses of known

uncertainties. This suggests a BNS as the source of the

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Can we translate GW170817 to constraints on the EOS?
Ingredients to constrain EOS

**Numerical relativity**

**Physics modelling**

**EOS CONSTRAINTS**

**Observation**
Maximum mass constraints from GW170817
Maximum mass constraints

CONSTRAINING THE MAXIMUM MASS OF NEUTRON STARS FROM MULTI-MESSENGER OBSERVATIONS OF GW170817

Ben Margalit & Brian D. Metzger

GW170817: Modeling based on numerical relativity and its implications

Masaru Shibata,1 Sho Fujibayashi,1 Kenta Hotokezaka,2,1 Kenta Kiuchi,1 Koutarou Kyutoku,3,1 Yuichiro Sekiguchi,4,1 and Masaomi Tanaka5

USING GRAVITATIONAL-WAVE OBSERVATIONS AND QUASI-UNIVERSAL RELATIONS TO CONSTRAIN THE MAXIMUM MASS OF NEUTRON STARS

Luciano Rezzolla1,2, Elias R. Most1, and Lukas R. Weih1

GW170817, General Relativistic Magnetohydrodynamic Simulations, and the Neutron Star Maximum Mass

Milton Ruiz,1 Stuart L. Shapiro,1,2 and Antonios Tsokaros1
The outcome of GW170817

• The product of GW170817 was likely a hypermassive star, i.e. a differentially rotating object with initial gravitational mass

\[ M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot \]

• Sequences of equilibrium models of nonrotating stars will have a maximum mass: \( M_{\text{TOV}} \)
The outcome of GW170817

- The product of GW170817 was likely a hypermassive star, i.e. a differentially rotating object with initial gravitational mass

\[ M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot \]

- Sequences of equilibrium models of nonrotating stars will have a maximum mass: \( M_{\text{TOV}} \)

- This is true also for uniformly rotating stars at mass shedding limit: \( M_{\text{max}} \)

- \( M_{\text{max}} \) simple and quasi-universal function of \( M_{\text{TOV}} \)

(Breu & Rezzolla 2016)

\[ M_{\text{max}} = (1.20^{+0.02}_{-0.05}) M_{\text{TOV}} \]
The outcome of GW170817

- The product of GW170817 was likely a hypermassive star, i.e. a differentially rotating object with initial \textit{gravitational} mass $M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_{\odot}$

- \textbf{Green} region is for \textit{uniformly} rotating equilibrium models.

- \textbf{Salmon} region is for \textit{differentially} rotating equilibrium models.
The outcome of GW170817

- The product of GW170817 was likely a hypermassive star, i.e. a differentially rotating object with initial gravitational mass $M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot$

- **Green** region is for uniformly rotating equilibrium models.
- **Salmon** region is for differentially rotating equilibrium models.
- **Supramassive** stars have $M > M_{\text{Tov}}$
- **Hypermassive** stars have $M > M_{\text{max}}$

![Graph showing the outcomes of GW170817 with regions labeled for uniformly and differentially rotating models, as well as regions for supramassive and hypermassive stars.]
The outcome of GW170817

- Merger product in GW170817 could have followed two possible tracks in diagram: fast (2) and slow (1).

- It rapidly produced a BH when still differentially rotating (2).

- It lost differential rotation leading to a uniformly rotating core (1).

- (1) is more likely because of large ejected mass (long lived).

- Final mass is near $M_{\text{max}}$ and we know this is universal!
The outcome of GW170817

• Merger product in GW170817 could have followed two possible tracks in diagram: fast (2) and slow (1)

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• It lost differential rotation leading to a uniformly rotating core (1).
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Cho, Bicknell, Science 2018
Maximum mass constraint

- The merger product of GW170817 was initially differentially rotating but collapsed as uniformly rotating object.

- HMNS core has about 95% gravitational mass of $M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot$

- Ejected rest mass deduced from kilonova emission $M_{\text{ej}}^{\text{blue}} = 0.014^{+0.010}_{-0.010} M_\odot$

- Use universal relations and account errors to obtain pulsar timing $2.01^{+0.04}_{-0.04} \leq M_{\text{TOV}} / M_\odot \lesssim 2.16^{+0.17}_{-0.15}$ universal relations and GW170817; similar estimates by other groups
### Overview of different results

<table>
<thead>
<tr>
<th>Group</th>
<th>Methodology</th>
<th>Mass Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARGALIT+</td>
<td>Baysian analysis + threshold mass</td>
<td>&lt; 2.17 $M_{\odot}$</td>
</tr>
<tr>
<td>SHIBATA+</td>
<td>Numerical simulations</td>
<td>&lt; 2.25 $M_{\odot}$</td>
</tr>
<tr>
<td>REZZOLLA, ERM, LW</td>
<td>Universal relations</td>
<td>&lt; 2.16 $M_{\odot}$</td>
</tr>
<tr>
<td>RUIZ+</td>
<td>Ruffini-Treves mass limit</td>
<td>&lt; 2.17 $M_{\odot}$</td>
</tr>
</tbody>
</table>

**Note:** All groups use input from kilonova modelling

**Bottom line:**

$M_{\text{max}} \sim 2.2 \, M_{\odot}$
Radius constraints from GW170817: A Frankfurt perspective
GW170817: What do we know?

\[ M_1 + M_2 = 2.74^{+0.04}_{-0.01} M_\odot \]

\[ M_1 = 1.36 - 1.60 M_\odot \]

\[ M_2 = 1.17 - 1.36 M_\odot \]

Abbott et al 2017

\[ \tilde{\Lambda}_{1.4} < 800 \]
How is BH-BH different from NS-NS?

Read et al. (2013)
How is BH-BH different from NS-NS?

Neutron stars in binary are tidally deformed by companion.

\[ U = G \frac{M}{r} + G \frac{Q}{r^3} P_2 (\cos \theta) \]

\( Q \): quadrupole moment
What is the quadrupole moment?

\[ GQ = 2\kappa_2 R^5 \frac{GM'}{b^3} \]

Tidal field of companion star

Tidal Love number

\[ \Lambda = \frac{2}{3} \kappa_2 \left( \frac{Rc^2}{GM} \right)^5 \]

Tidal deformability of the isolated neutron star

Imprint on gravitational wave:

\[ \tilde{h}_{GW}(f) = \mathcal{A}_{SPA}(f) \exp i\psi_{SPA}(f) \]

\[ \psi_{SPA} \sim \cdots - \frac{39}{2} \nu^{-2} \tilde{\Lambda} (\pi \mathcal{M} f)^{10/3} \]

Mass weighted average
GW170817: What do we know?

Abbott et al 2017

low-spin case and \((1.0, 0.7)\) in the high-spin case. Further analysis is required to establish the uncertainties of these tighter bounds, and a detailed study of systematics is a subject of ongoing work.

Preliminary comparisons with waveform models under development \([171,173–177]\) also suggest the post-Newtonian model used will systematically overestimate the value of the tidal deformabilities. Therefore, based on our current understanding of the physics of neutron stars, we consider the post-Newtonian results presented in this Letter to be conservative upper limits on tidal deformability. Refinements should be possible as our knowledge and models improve.

V. IMPLICATIONS

A. Astrophysical rate

Our analyses identified GW170817 as the only BNS-mass signal detected in O2 with a false alarm rate below \(1 / 100 \text{yr}\). Using a method derived from \([27,178,179]\), and assuming that the mass distribution of the components of BNS systems is flat between 1 and \(2 M_\odot\) and their dimensionless spins are below 0.4, we are able to infer the local coalescence rate density \(R\) of BNS systems. Incorporating the upper limit of \(12600 \pm 3 \text{Gpc}^3 \text{yr}^{-1}\) from O1 as a prior, \(R = 1540 \pm 3200 \pm 1220 \text{Gpc}^3 \text{yr}^{-3}\). Our findings are consistent with the rate inferred from observations of galactic BNS systems \([19,20,155,180]\).

From this inferred rate, the stochastic background of gravitational wave s produced by unresolved BNS mergers throughout the history of the Universe should be comparable in magnitude to the stochastic background produced by BBH mergers \([181,182]\). As the advanced detector network improves in sensitivity in the coming years, the total stochastic background from BNS and BBH mergers should be detectable \([183]\).

B. Remnant

Binary neutron star mergers may result in a short- or long-lived neutron star remnant that could emit gravitational waves following the merger \([184–190]\). The ringdown of a black hole formed after the coalescence could also produce gravitational waves, at frequencies around 6 kHz, but the reduced interferometer response at high frequencies makes their observation unfeasible. Consequently, searches have been made for short (tens of ms) and intermediate duration (≤500 s) gravitational-wave signals from a neutron star remnant at frequencies up to 4 kHz \([75,191,192]\). For the latter, the data examined start at the time of the coalescence and extend to the end of the observing run on August 25, 2017. With the time scales and methods considered so far \([193]\), there is no evidence of a postmerger signal of...
Light curves

Observations consistent with two component model

Drout et al 2017
Kilonova constraints on the tidal deformability

- Consistency with kilonova modelling (mass ejection) requires lower limit on tidal deformability

\[ \tilde{\Lambda} = \frac{16}{13} \left[ \frac{(M_A + 12M_B)M_A^4A_2^{(A)}}{(M_A + M_B)^5} + (A \leftrightarrow B) \right] , \]

Errors unclear
Might be as low as \( \sim 200 \)
(Coughlin+ 2018)
Limits on radii and deformabilities

• Constraining NS radii of neutron stars is an effort with thousands of papers published over the last 40 years.
• Question is deeply related with EOS of nuclear matter.
• Can new constraints be set by GW170817?

• Ignorance can be parameterised and EOSs can be built arbitrarily as long as they satisfy specific constraints on low and high densities.
Mass-radius relations

• We have produced $10^6$ EOSs with about $10^9$ stellar models.

• Can impose differential constraints from the maximum mass and from the tidal deformability from GW170817

ERM, LW, Rezzolla, Schaffner-Bielich (PRL 2018)
one-dimensional cuts

- Closer look at a mass of $M = 1.40 \, M_\odot$

- Can play with different constraints on maximum mass and tidal deformability.

- Overall distribution is very robust

$12.00 < R_{1.4} / \text{km} < 13.45$

$R_{1.4} = 12.45 \, \text{km}$

ERM, LW, Rezzolla, Schaffner-Bielich (PRL 2018)
Constraining tidal deformability

- Can explore statistics of all properties of our $10^9$ models.
- In particular can study PDF of tidal deformability: $\tilde{\Lambda}$
- LIGO has already set upper limit: 
  $$\tilde{\Lambda}_{1.4} \lesssim 800$$
- Our sample naturally sets a lower limit: 
  $$\tilde{\Lambda}_{1.4} > 375$$
What about phase transitions?

- All EOSs so far are purely hadronic; a conservative but probably reasonable assumption.
- What about the possibility of phase transitions?
- These are not trivial but not too difficult to model.
What about phase transitions?

- All EOSs so far are purely hadronic; a conservative but probably **reasonable** assumption.
- What about the possibility of **phase transitions**?
- These are not trivial but not too difficult to model.
Mass-radius relations

• Presence of a phase transition leads to second stable branch and “twin-star” models.

ERM, LW, Rezzolla, Schaffner-Bielich (PRL 2018)
One-dimensional cuts: PTs

Applying all constraints from GW170817:

\[ 8.53 < R_{1.4} / \text{km} < 13.74 \quad \bar{R}_{1.4} = 13.06 \text{ km} \]
Constraining tidal deformability: PTs

- Can repeat considerations with EOSs having PTs
- Lower limit much weaker: \( \tilde{\Lambda}_{1.4} \gtrsim 35 \)
- Large masses have sharp cut-off on upper limit: \( \tilde{\Lambda}_{1.7} \lesssim 460 \)

GW detection with \( \tilde{\Lambda}_{1.7} \sim 700 \) would rule out twin stars!
Conclusions from Frankfurt

GW170817 provides new limits on maximum mass and radii:

\[ 2.01^{+0.04}_{-0.04} \leq M_{\text{TOV}} / M_\odot \lesssim 2.16^{+0.17}_{-0.15} \]

\[ 12.00 < R_{1.4}/\text{km} < 13.45 \]
\[ \bar{R}_{1.4} = 12.45 \text{ km} \]

hadronic EOS phase transitions

\[ 8.53 < R_{1.4}/\text{km} < 13.74 \]
\[ \bar{R}_{1.4} = 13.06 \text{ km} \]

Upper limit on deformability can rule out twin stars

\[ \tilde{\Lambda}_{1.7} \lesssim 460 \]
A flood of publications
Methods

Statistical
\[ \Lambda < 800 \]

Numerical

Universal relations

---

**Statistical**

\[ \Lambda < 800 \]

**Numerical**

**Universal relations**
Comparison to numerical simulations

No prompt collapse

\[ M_{\text{thres}} > M_{\text{tot}}^{\text{GW170817}} = 2.74^{+0.04}_{-0.01} \, M_\odot \]

Comparison with numerical simulations:

\[ M_{\text{thres}} = \left( -3.606 \frac{G M_{\text{max}}}{c^2 R_{1.6}} + 2.38 \right) \cdot M_{\text{max}} \]

Causality: \( M_{\text{thres}} > 1.22 M_{\text{max}} \)

excluded

Bauswein, Just, Janka, Stergioulas (2017)
Comparison to numerical simulations

Bauswein, Just, Janka, Stergioulas (2017)
Comparison to numerical simulations

\[ M_{\text{thres}} = \left(-3.606 \frac{GM_{\text{max}}}{c^2 R_{1.6}} + 2.38\right) \cdot M_{\text{max}} \]

1. Only purely hadronic EOSs
2. Not derived in full GR: error at least 5%

Need more precise equation

Bauswein, Just, Janka, Stergioulas (2017)
Universal relations (+ statistics)

STEP 1

**Chirp mass:**
\[ M_{\text{chirp}} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = 1.188 M_\odot \]

**Mass ratio:**
\[ q = m_1/m_2 = 0.7 - 1 \]

**Symmetric deformability:**
\[ \Lambda_s = (\Lambda_1 + \Lambda_2)/2 \]

**Asymmetric deformability:**
\[ \Lambda_a = (\Lambda_1 - \Lambda_2)/2 \]

**EOS-independent (universal) relation #1:**
\[ \Lambda_a = \Lambda_a (\Lambda_s, q) \]

**EOS-independent (universal) relation #2:**
\[ \Lambda = \Lambda (C) \text{ with compactness } C = M/R \]

STEP 2

- Sample \( \Lambda_s \in [0, 5000] \)
- Compute \( \Lambda_a (\Lambda_s, q) \) to obtain \( \Lambda_1 \) and \( \Lambda_2 \)
- Compute \( \Lambda_{1,2} \) by inverting \( \Lambda_{1,2} = \Lambda_{1,2} (C_{1,2}) \)


(De, Finstad, Lattimer, Brown, Berger, Biwer arXiv:1804.08583)
Universal relations (+ statistics)

NS1: $9.1 < R_1 < 12.8$ (90%)

NS2: $9.2 < R_2 < 12.8$ (90%)

Include $M_{\text{max}} > 2.01M_{\odot}$ constraint:

$10.5 < R_1 \sim R_2 \sim R_{1.4} < 13.3$ (90%)

Summary

Statistical

\[ 12.0 > R_{1.4} > 13.5 \]
(and many others)

Numerical

\[ 10.6 < R_{1.6} \sim R_{1.4} \]

Universal relations

\[ 10.5 < R_1 \sim R_2 \sim R_{1.4} < 13.3 \]
We don't disagree!

What about phase-transitions?

Not enough studies yet.

How to construct the PT?
So what about the EOS?

All constraints applied.

Outer core determines radius.
Different approaches yield the same results:

- $M_{\text{max}} < 2.2M_{\odot}$
- $10^{(12)} > R_{1.4} > (13.5)^{14}$
- No tight limits for EOS with phase transition
  BUT: could be distinguished via tidal deformability

**To-Do:**
How does this compare to constraints from X-ray observations?
Summary

• GW170817 has helped to improve our knowledge of **maximum masses** and **radii** of neutron stars

• Future **multimessenger observations** will help to even more narrow down uncertainties of neutron star properties and will help to unravel the **EOS**

< 50 per year