Dynamical coupling of hydrodynamics and transport for heavy ion collisions

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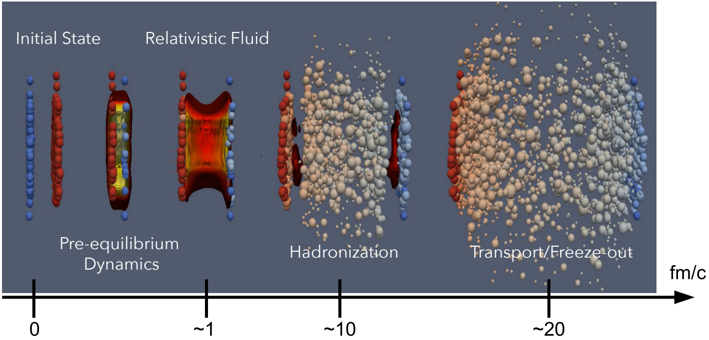
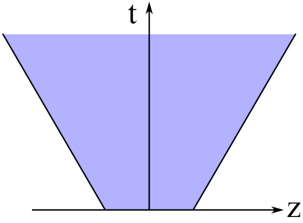
Heavy ion collision in the view of hybrid models



- Hydrodynamics: local thermal equilibrium,
  $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^\mu = 0$, EoS, boundary conditions
  Applicability: $\lambda \simeq (n\sigma)^{-1} \ll L \implies$ high density
- Transport: Monte-Carlo simulation of particle collisions
  Applicability: negligible multi-particle collisions $\implies$ low density
- Hybrid: hydro at high density + transport at low density

Conventional hybrid models



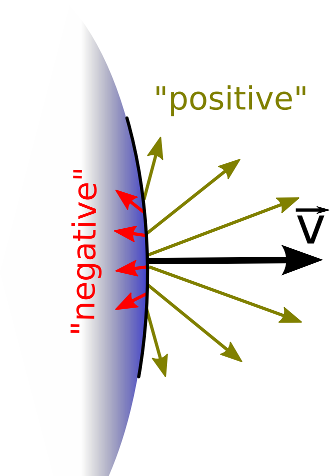
- Solve hydro equations in the light cone
- Find freeze-out hypersurface *aposteriori*
- Particlization (Cooper-Frye formula)
- Particles are *decoupled* from hydro, but can scatter with each other

Conventional approximation breaks (many particles return to hydro)

- at low collision energies
- in event-by-event simulations

DO, HP [PRC **91**, 2, 024906 (2015)]

Particlization and negative contributions



- Particlization
  - know ϵ, p, u_μ on the surface
  - from EoS - T, μ
  - want particles
- "Cooper-Frye formula"

  $$d^3N(p) = f(p)\frac{d^3p}{(2\pi\hbar)^3}\frac{p^\mu}{p^0}d\sigma_\mu$$

  $\frac{p^\mu}{p^0} \cdot d\sigma_\mu$ - analog of $n \cdot V$

  e.g. ideal hydro $f(p) = \left(e^{\frac{p^\mu u_\mu - \mu}{T}} \pm 1\right)^{-1}$
- Negative contribution
  - $p^\mu d\sigma_\mu > 0$: positive contribution, particles fly out
  - $p^\mu d\sigma_\mu < 0$: negative contribution, particles fly in

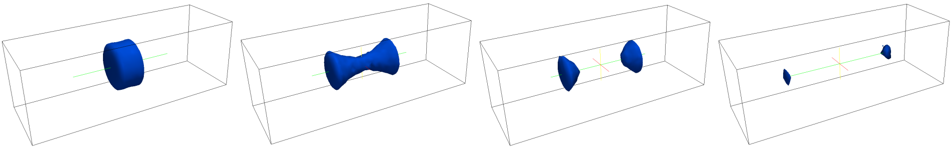
$d\sigma_\mu$ - normal 4-vector
$u_\mu = (\gamma, \gamma\vec{v})$ - 4-velocity
T - temperature
μ - chemical potential

Negative contributions using coarse-grained UrQMD (I)

Hypersurface of constant Landau rest frame energy density:
mimic hybrid model transition surface

- Generate many UrQMD events
- On a (t,x,y,z) grid calculate $T^{\mu\nu} = \left\langle \frac{1}{V_{cell}} \sum_{i \in cell} \frac{p_i^\mu p_i^\nu}{p_i^0} \right\rangle_{\text{event average}}$
- In each cell go to Landau frame: $T_L^{0\nu} = (\epsilon_L, 0, 0, 0)$
- Construct surface $\epsilon_L(t, x, y, z) = \epsilon_0$

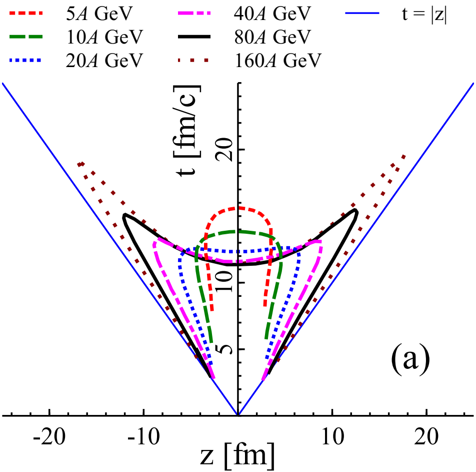
Example: E = 160 AGeV, Au+Au central collision, $\epsilon_0 = 0.3$ GeV/fm^3



t = 4 fm/c | t = 11 fm/c | t = 13 fm/c | t = 18 fm/c

Negative contributions using coarse-grained UrQMD (II)

Au+Au central collisions, $\epsilon = 0.3$ GeV/fm^3 hypersurface projected to t-z.



Definitions for negative contributions

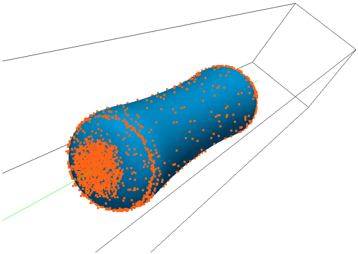
- Hypersurface of constant Landau rest frame energy density
  - A) T and μ from Hadron Gas EoS, Cooper-Frye formula
  - B) Many UrQMD events, count particles crossing hypersurface
- Will coincide if particle distribution from UrQMD is exactly equilibrated

A) Cooper-Frye

$$p^0 \frac{d^3 N^+}{dp^3} = \frac{p^\mu d\sigma_\mu}{exp(p^\nu u_\nu / T) \pm 1} \theta(p^\nu d\sigma_\nu)$$

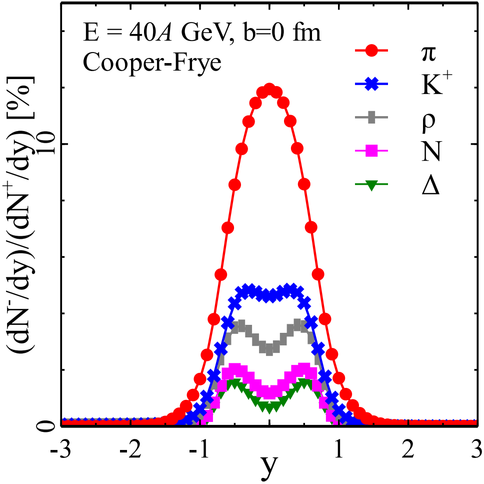
$$p^0 \frac{d^3 N^-}{dp^3} = \frac{p^\mu d\sigma_\mu}{exp(p^\nu u_\nu / T) \pm 1} \theta(-p^\nu d\sigma_\nu)$$

B) "by particles"



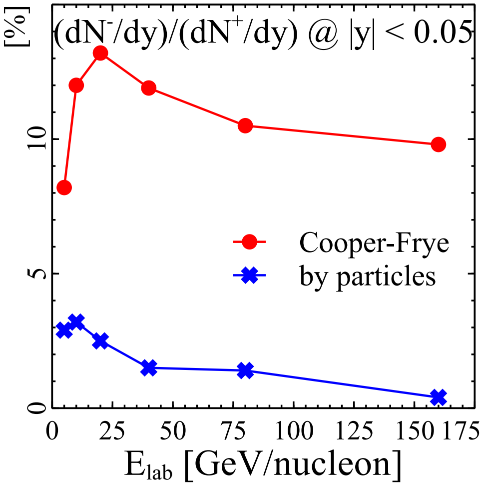
Negative contributions: particle mass dependence

E = 40 AGeV, b = 0, $\epsilon_0 = 0.3$ GeV/fm^3, dN/dy distributions



Smaller mass - larger negative contribution

Negative contributions: energy dependence

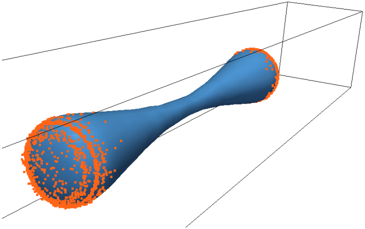


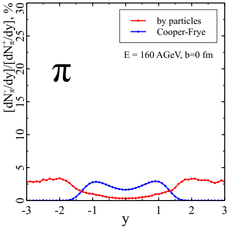
Lower collision energy - slower expansion - larger negative contributions
Non-equilibrium calculation gives much smaller values

Negative contributions: surface lumpiness

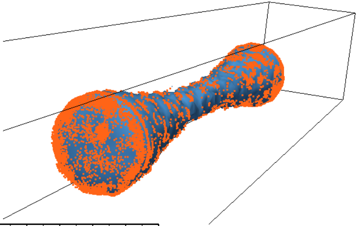
E = 160 AGeV, b = 0

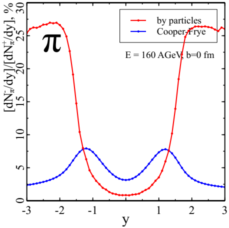
Smooth surface





Lumpy surface

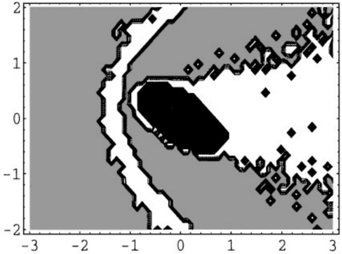




Lumpier surface - larger negative contributions

Hybrid model in a perfect world

- Coupled hydrodynamics and kinetic equations
- Transition surface found dynamically
  (so-called dynamical decomposition)
- Some works in this direction
  K. Bugaev, Phys Rev Lett. 2003; L. Czernai, Acta Phys. Hung., 2005



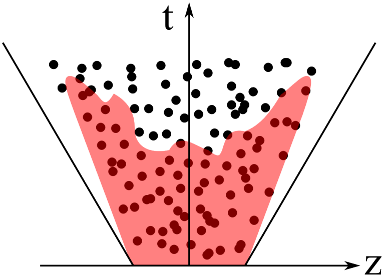
Example from non-relativistic hydrodynamics [Tiwari, J. Comp. Phys. 144, 710726 (1998)]
2D flow of gas around solid ellipse
White domains - Boltzmann equation, grey domains - Euler equation

Summary of introduction

- Hybrid approaches adopt approximations:
  - aposteriori determination of particlization surface
  - particles decouple from hydrodynamics once and cannot get back into it
- These approximations become inadequate for
  - low collision energies
  - large fluctuations (event-by-event/fluctuating hydrodynamics)
- In non-relativistic hydrodynamics there are dynamic decomposition approaches, which go beyond these approximations

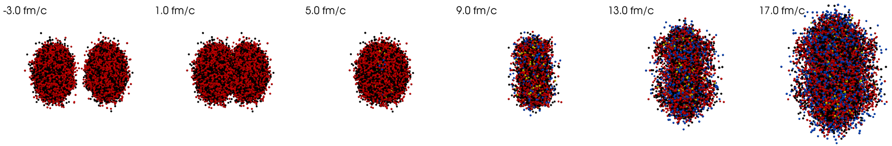
Alternative way: hydro bubbles in transport

- Pure transport
- Force instant local thermalization, where density is high
- Effectively accounts for multiparticle collisions
- Conceptually similar to hybrid model, where
  - "Hydro" region defined dynamically
  - "Hydro" and transport are coupled



SMASH transport model

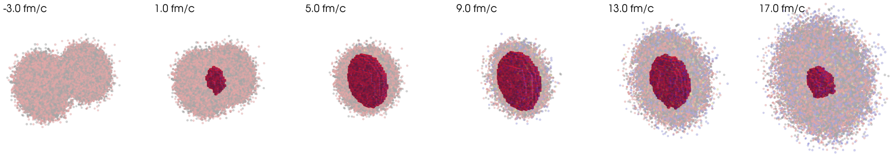
- hadronic cascade, $2 \leftrightarrow 2$ and $2 \leftrightarrow 1$ reactions
  - Mesons: π, ρ, η, ω, ϕ, σ, f_2, K, $K^*(892)$, $K^*(1410)$
  - Baryons: up to $m \simeq$ 2GeV - N, N^*, Δ, Δ^*, Λ, Λ^*, Σ, Σ^*; Ξ, Ω
- simulates AA collision as a sequence of elementary reactions
- timesteps: $\dots \rightarrow$ collide/decay $\rightarrow$ propagate $\rightarrow \dots$
- testparticles ansatz: $N \rightarrow N \cdot N_{test}$, $\sigma \rightarrow \sigma / N_{test}$
- model in active development, no strings yet
  - currently only reliable at low energies



central Au+Au collision, $E_{\rm kin} = 2$ GeV, $N_{test} = 100$
color coding: neutrons, protons, Δ, π

Hydrodynamic bubble using SMASH

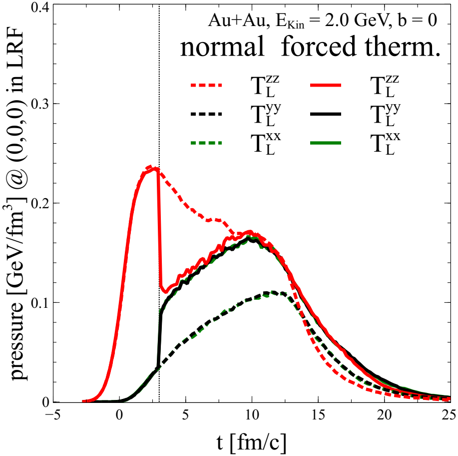
- take a cartesian grid, cells $(0.5\,\mathrm{fm})^3$
- in each cell compute local Landau rest frame energy density ϵ
- $\epsilon > \epsilon_{ft} = 0.3$ GeV/fm^3 $\Longrightarrow$ forced thermalization in cell
- A) forced isotropization
  - reshuffles momenta microcanonically - no need for external EoS
  - conserves total energy and momentum locally
  - does not change hadronic content
- B) forced grand-canonical thermalization
  - forced chemical equilibration
  - allows to set Equation of State (EoS)



central Au+Au collision, $E_{\mathrm{kin}} = 2$ GeV, $N_{test} = 100$, purple region - hydrodynamic bubble, $\epsilon_{ft} = 0.3$ GeV/fm^3:

Effects of forced isotropization: pressure

Forced isotropization at $t > 3$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.
Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$

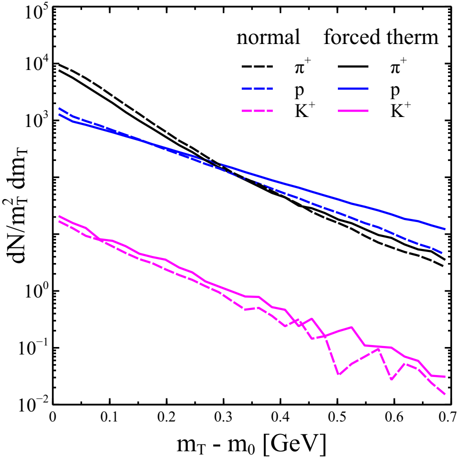


Pressures rapidly equalize

Effects of forced isotropization: m_T spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.

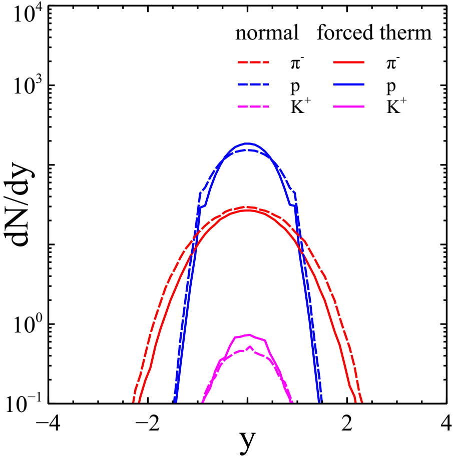
Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$



Particles move from low p_T to high p_T: "transverse push"

Effecs of forced isotropization: y spectra

Forced isotropization at $t > 1$ fm/c, where $\epsilon > 0.3$ GeV/fm^3.
Au+Au, CM frame, $E_{Kin} = 2$ A GeV, $b = 0$, $N_{test} = 100$



Nucleons move to midrapidity, less pions, more kaons

Forced grand-canonical thermalization

Every time interval Δt_{ft}:

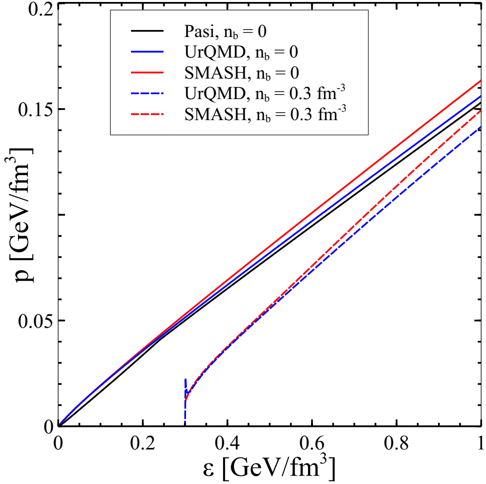
1. Span a lattice and compute $T^{\mu\nu}$, j_B^μ, j_S^μ on it
2. Find rest frame ϵ, n_b, n_S, T, μ from EoS in every cell
   - Assuming ideal hydro form of $T^{\mu\nu}$, j^μ
3. Remove particles from $\epsilon > \epsilon_c$ region
4. Sample new particles in $\epsilon > \epsilon_c$ region
   - Thermal distribution function
   - Isochronous Cooper-Frye formula $\rightarrow$ no negative contributions
   - Conserving charges, energy and momentum globally - "modes sampling" P. Huovinen, HP, Eur.Phys.J. A48 (2012) 171
5. Let particles propagate, collide and decay within transport model until next thermalization

Ways to interpret the procedure:

- changing local distribution function to the thermal one
- effective treatment of multi-particle collisions
- "Zero-time hydro" = fluidization + immediate particlization

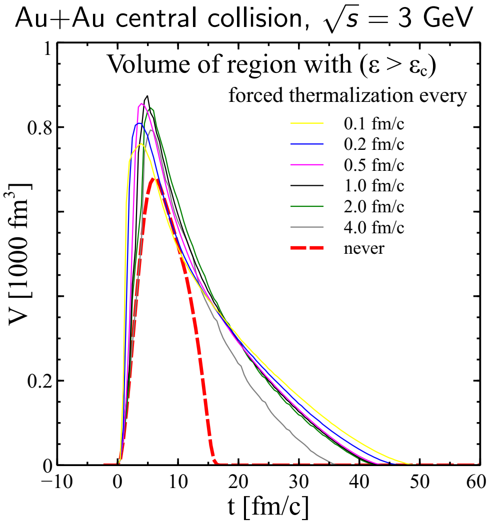
SMASH ideal hadron gas EoS

More hadron sorts - smaller pressure at given energy density



Pasi ≡ Hadron Gas EoS from P. Huovinen, P. Petreczky, Nucl.Phys. A837 (2010) 26-53
UrQMD ≡ Hadron Gas EoS from UrQMD tables by J. Steinheimer

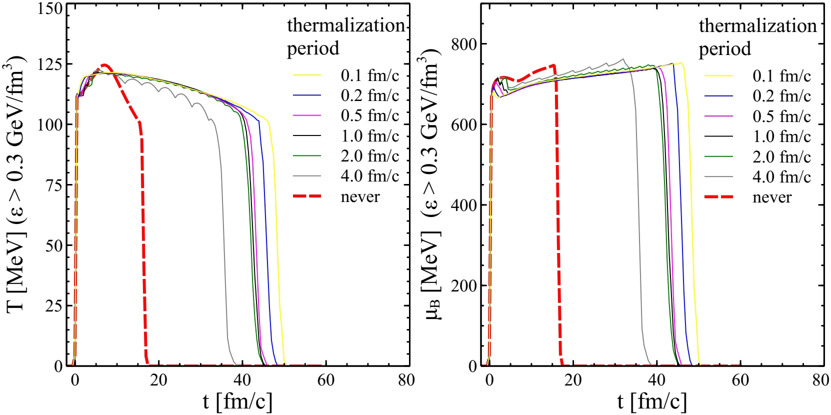
Effects of forced thermalization



High-density region exists longer

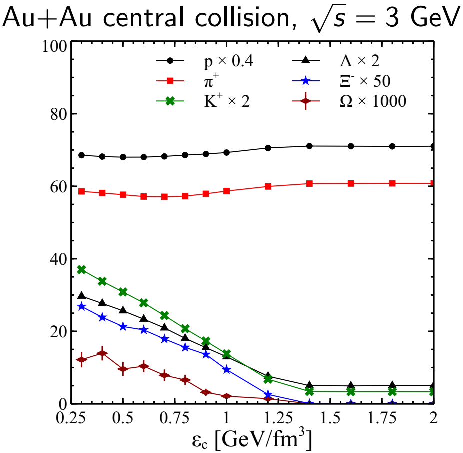
$\langle T \rangle$ and $\langle \mu_B \rangle$ in the thermalization region

Au+Au central collision, $\sqrt{s} = 3$ GeV, $\langle A \rangle = \frac{\sum \epsilon_{ijk} A_{ijk}}{\sum \epsilon_{ijk}}$



- Wiggles at every thermalization time
- Equilibration by reactions and forced thermalization not identical
  - Some particles cannot be produces by reactions, e.g. $\bar{p}$
  - Resonances sampled at pole mass

Effects of forced thermalization: multiplicities

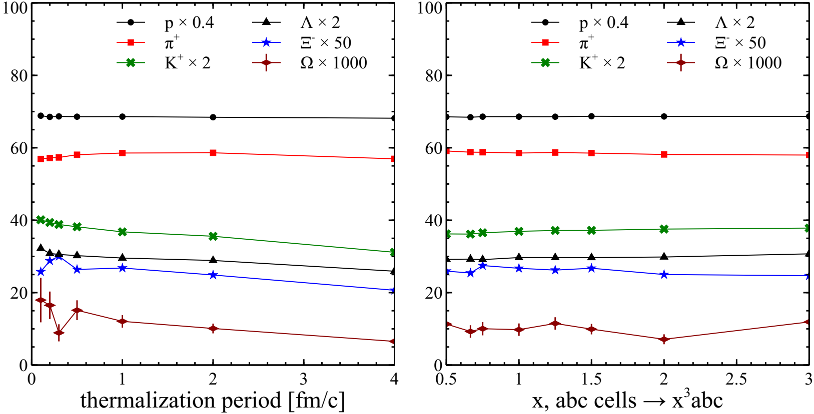


High ϵ_c - no thermalization

Thermalization leads to strangeness enhancement

Effects of forced thermalization: multiplicities

Au+Au central collision, $\sqrt{s} = 3$ GeV, $[1.0 \times 1.0 \times 0.5]$ lattice spacing



Thermalization period and lattice spacing are not important for multiplicities

Summary

- Hybrid approaches adopt approximations:
  - aposteriori determination of particlization surface
  - particles decouple from hydrodynamics once and cannot get back into it
- These approximations become inadequate for
  - low collision energies
  - large fluctuations (event-by-event/fluctuating hydrodynamics)
- Suggestion: pure transport, forced thermalization in regions with high energy density
  - one can plug in arbitrary EoS
  - backflow is automatically taken into account
  - transition hypersurface is determined dynamically
- Tested on SMASH, observed
  - longer lifetime of high-density region
  - more energy transferred to midrapidity
  - strangeness enhancement

Outlook: further testing, plug in EoS with 1st order phase transition

Smearing kernel DO, HP, Phys.Rev. C93 (2016) no.3, 034905

The energy-momentum tensor $T^{\mu\nu}$ is constructed as

$$T^{\mu\nu}(\vec{r}) = \frac{1}{N_{ev}} \sum_{events} \sum_{i} \frac{p_i^{\mu} p_i^{\mu}}{p_i^0} K(\vec{r} - \vec{r}_i, p_i) \quad (1)$$

Smearing kernel $K(r)$ should be such that $K(r)d^3r$ is Lorentz-scalar

$$\Delta x^i = \Lambda^i_j \Delta x'^j \quad (2)$$

$$\Lambda^i_j = \delta^i_j + (u^i u_j)/(1+\gamma) \quad (3)$$

$$(\Delta x^i)^2 = \Lambda^i_j \Delta x'^j \Lambda^i_k \Delta x'^k \quad (4)$$

$$\Lambda^i_j \Lambda^i_k = \delta_{jk} + u_j u_k \quad (5)$$

$$(\Delta \vec{x})^2 = (\Delta \vec{x'})^2 + (\Delta \vec{x'} \cdot \vec{u})^2 \quad (6)$$

$$K(\vec{r}) = \gamma (2\pi\sigma^2)^{-3/2} exp\left(-\frac{\vec{r}^2 + (\vec{r}\cdot\vec{u})^2}{2\sigma^2}\right) \quad (7)$$

Normalization using $\int \left(\prod_{i=1}^n dx_i\right) e^{-x_i A^{ij} x_j} = \pi^{n/2} (detA)^{-1/2}$