My adventures with thermal model

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Transport meeting

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multiplicities of identified particles in high energy collisions

$$\frac{dN}{dy} \quad \text{of} \quad \pi,\ K,\ p,\ \bar{p},\ \Lambda, \ldots$$

well reproduced by statistical *i.e.* thermal models

Thermal model

Particles emitted from

- **single source**
- **in chemical equilibrium**
- **characterised by V and T, μ_B, μ_S, μ_I**

Yields given by

$$N_i = V n_i = -T \frac{\partial Z_i(T, \mu_i)}{\partial \mu}$$

Thermal model

Dashen-Ma-Berstein theorem: If interactions mediated by *narrow* resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas ⇒ Hadron resonance gas model

Partition function of noninteracting hadron resonance gas:

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 \, \mathrm{d}p \, \ln(1 \pm \exp(-(E_i - \mu_i)/T))$$

Stable particle yields must include contribution from resonance decays:

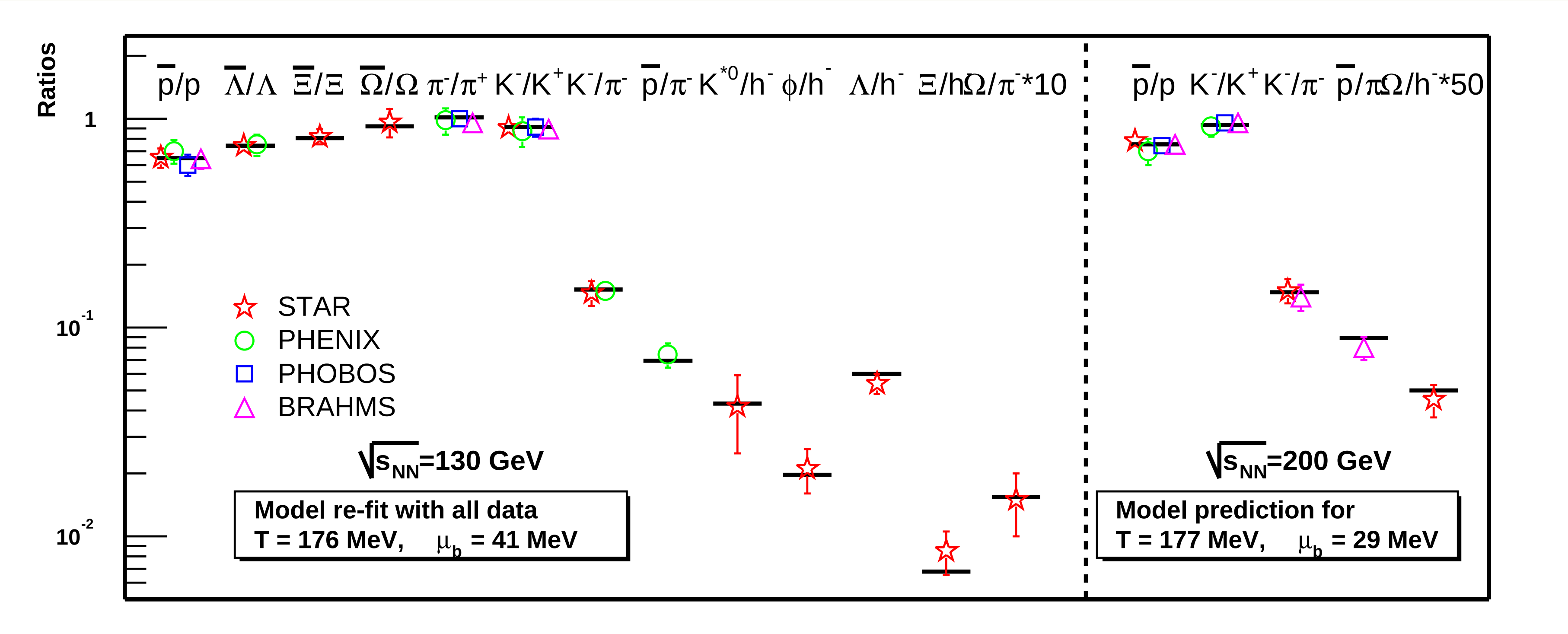
$$N_\pi = V(n_\pi + 2n_\rho + n_\Delta + \cdots)$$

Conservation laws constrain μ_S, μ_I
- **at LHC $\mu_B = \mu_S = \mu_I = 0$**

Fit at each collision energy provides values for V, T, and μ_B

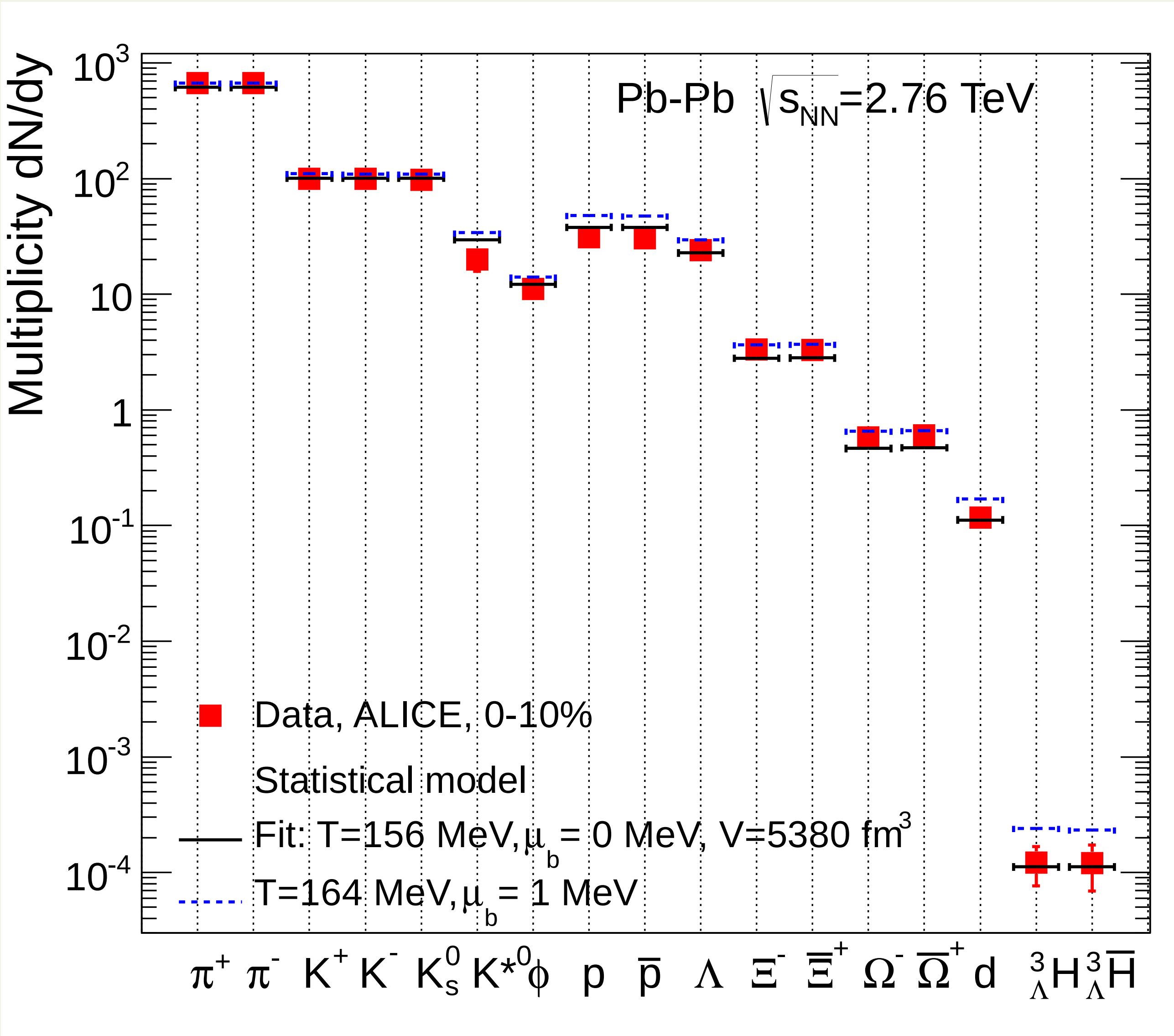
Fit to RHIC data

Braun-Munzinger *et al.*, PLB518, 41 (2001)



Fit to ALICE data

J. Stachel @ sQM13, arXiv:1311.4662



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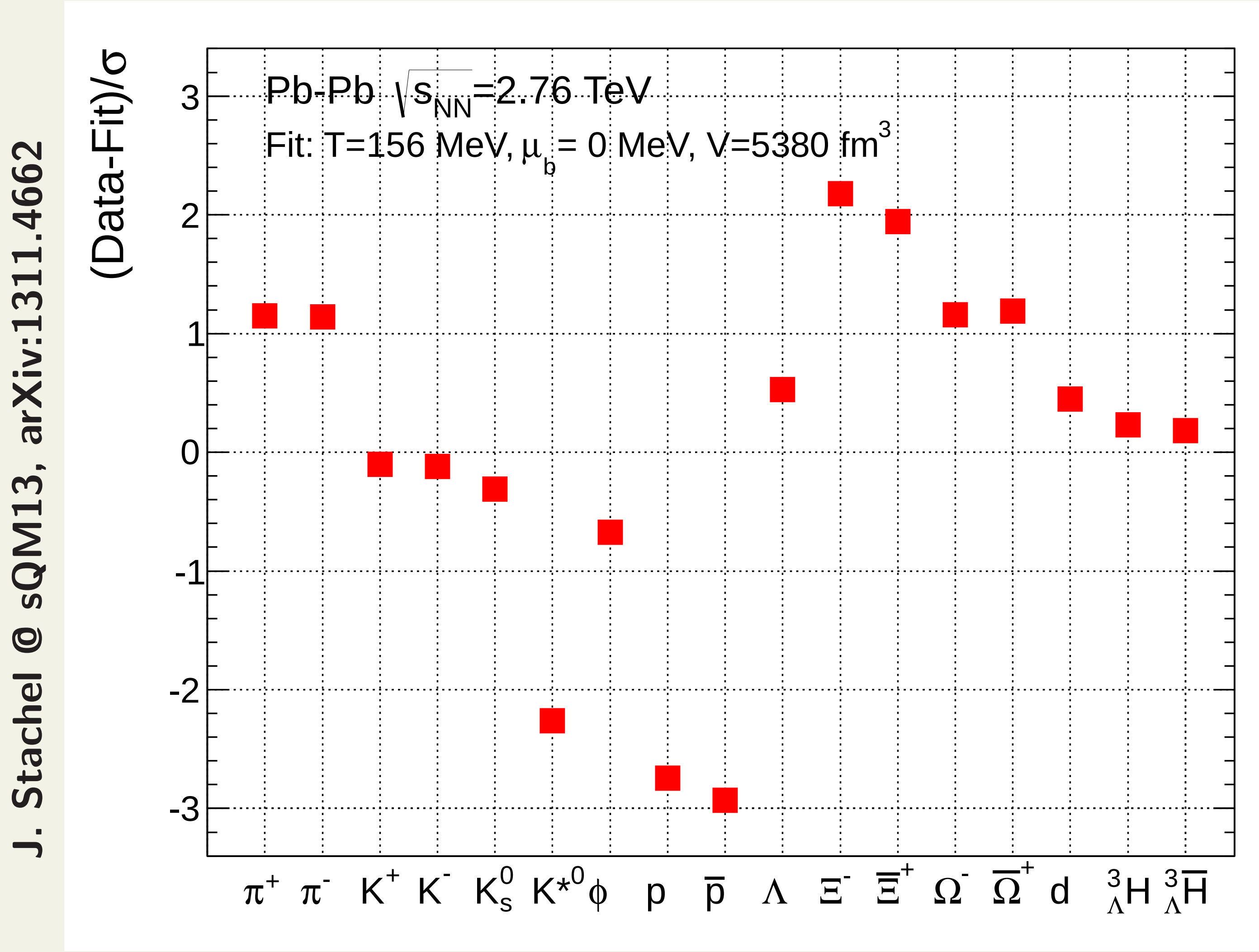


- **more protons/antiprotons than in the data, $\sim 3\sigma$ difference**
- **slightly too few cascades, $\sim 2\sigma$ difference**

usual explanation:

- $N\bar{N}$ annihilations continue after chemical equilibrium is lost

$\Rightarrow$ hybrid models (UrQMD)

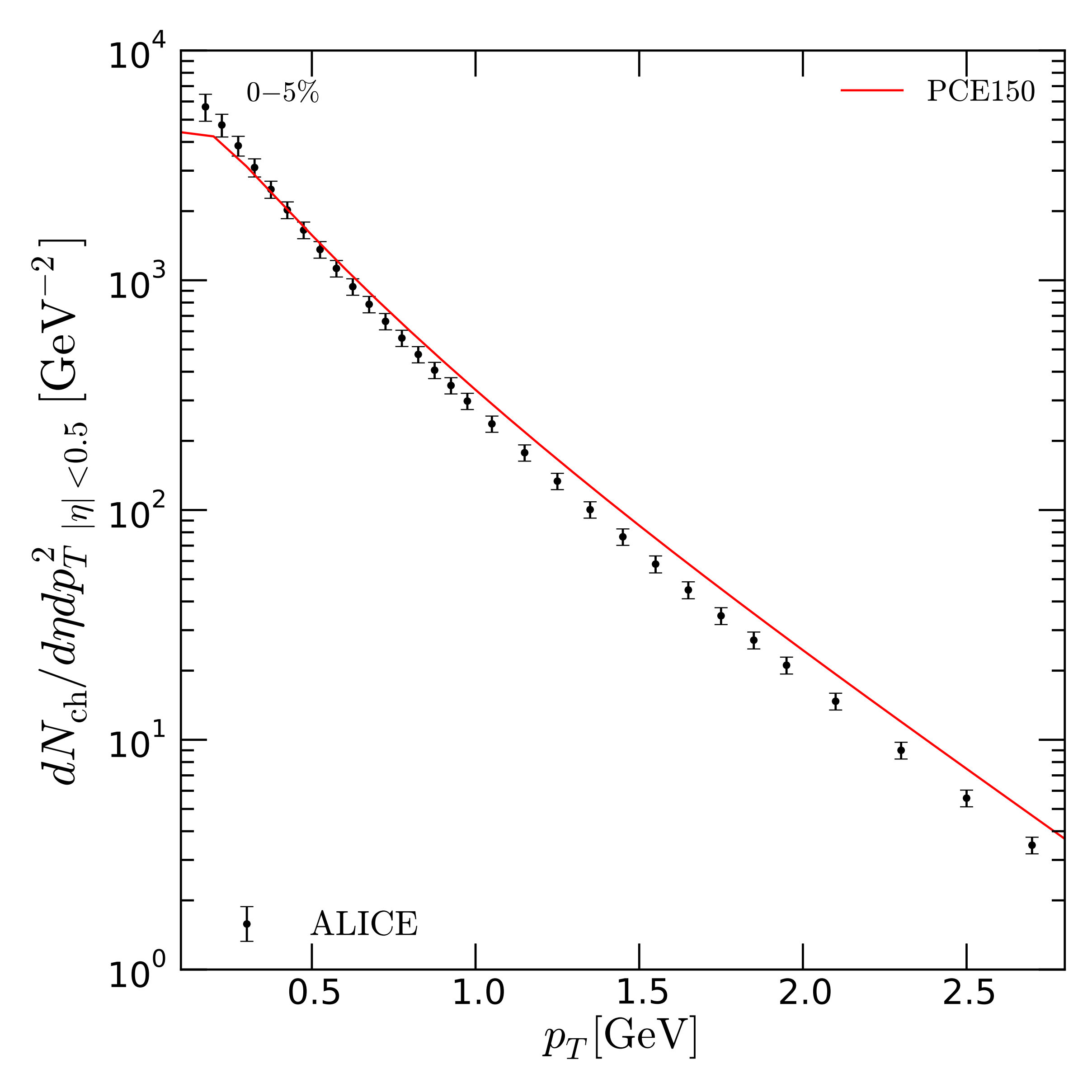
BUT

- hyperons?
- detailed balance?

$$N + \bar{N} \leftrightarrow n\pi$$

- how to do fits?
- pure hydro?

Charged hadron p_T spectrum at LHC



- **viscous hydro**
- **initial state:**

pQCD+saturation

- $\tau_0 \approx 0.2\text{fm}/c$

PCE150:

fit to π, K, p yields

no fit to spectrum

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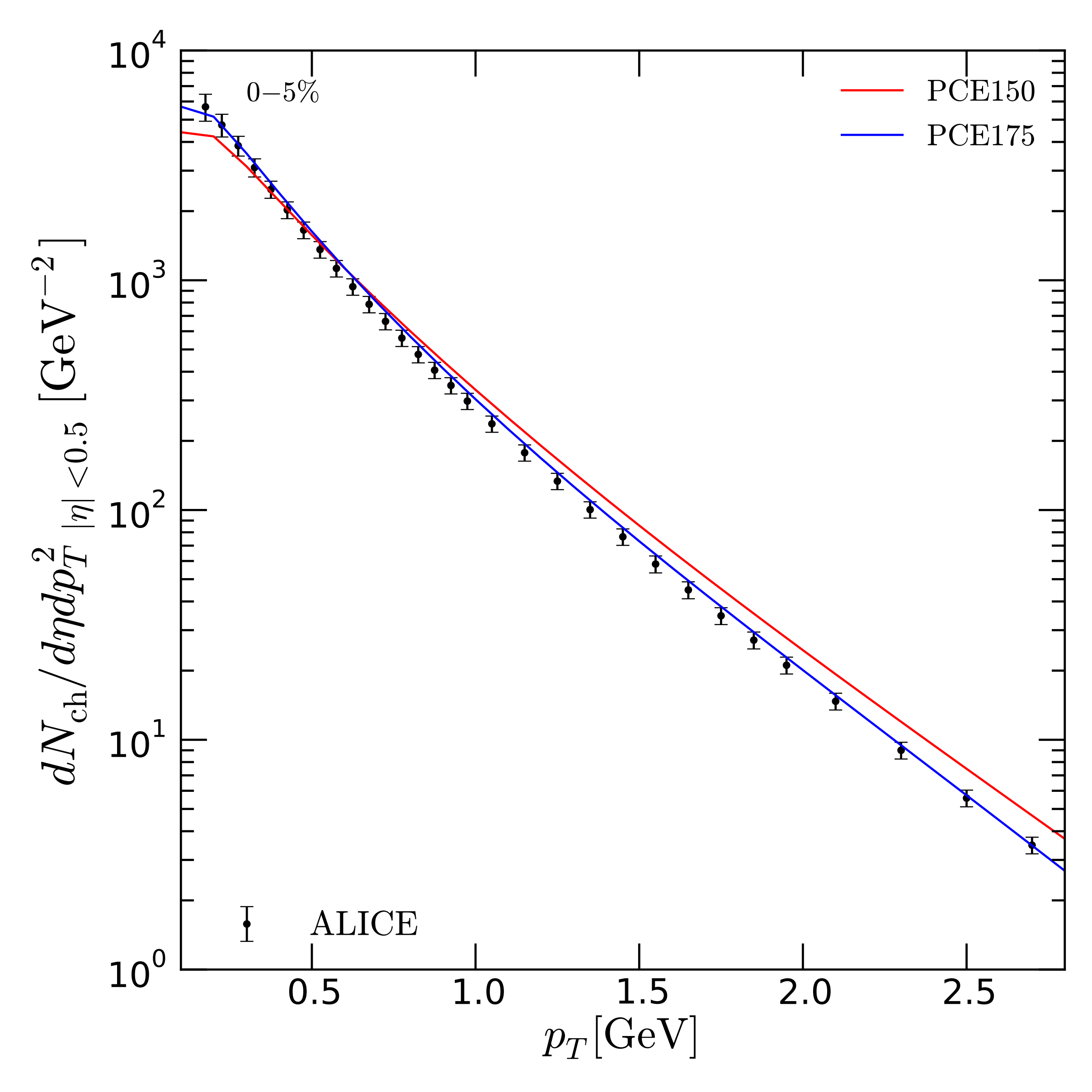
Charged hadron p_T spectrum at LHC



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fit to π, K, p yields

no fit to spectrum

PCE175:

no fit to yields

fits the spectrum

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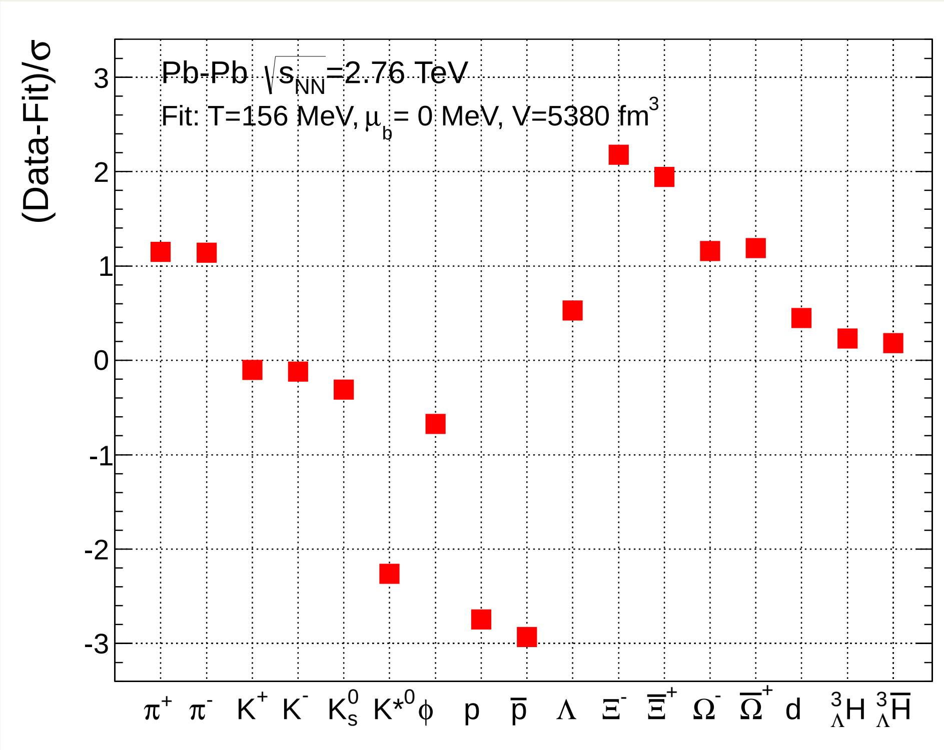
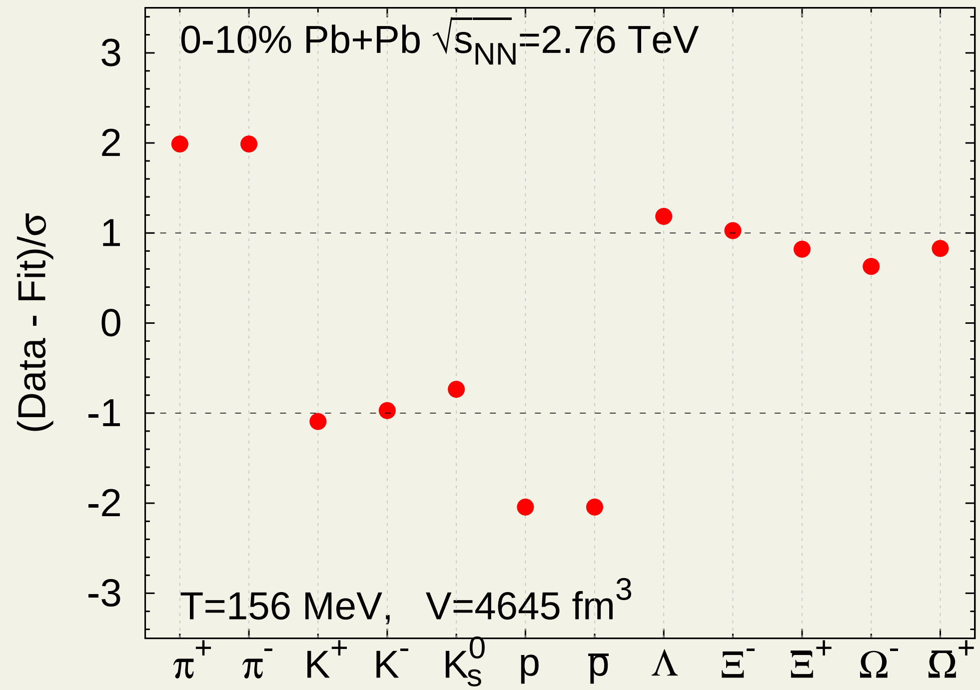
Fit to ALICE data

J. Stachel @ sQM13, arXiv:1311.4662

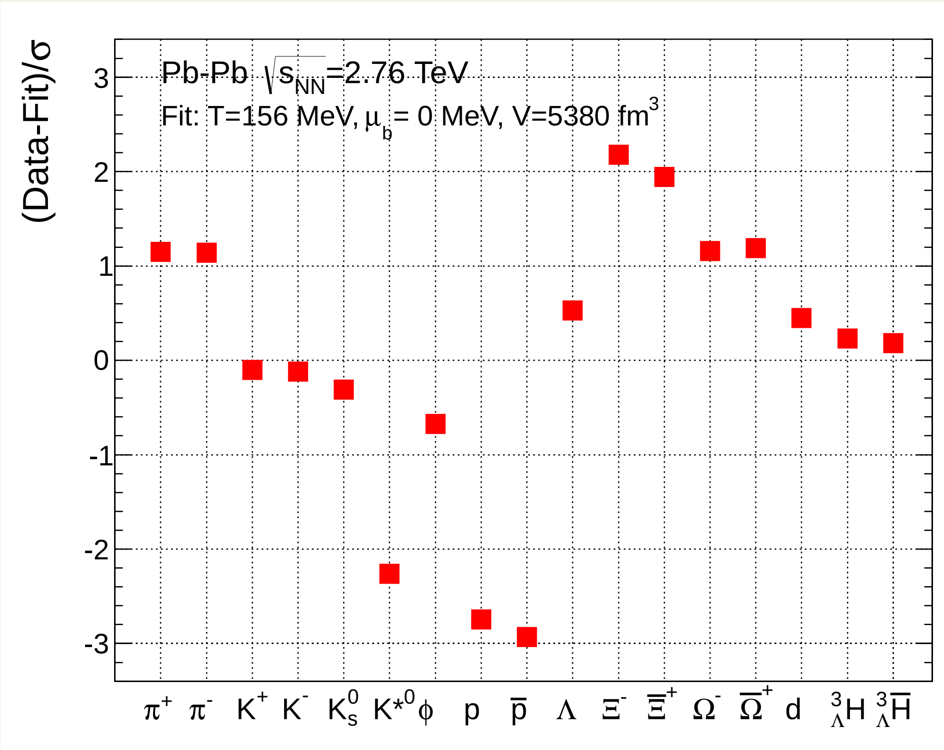


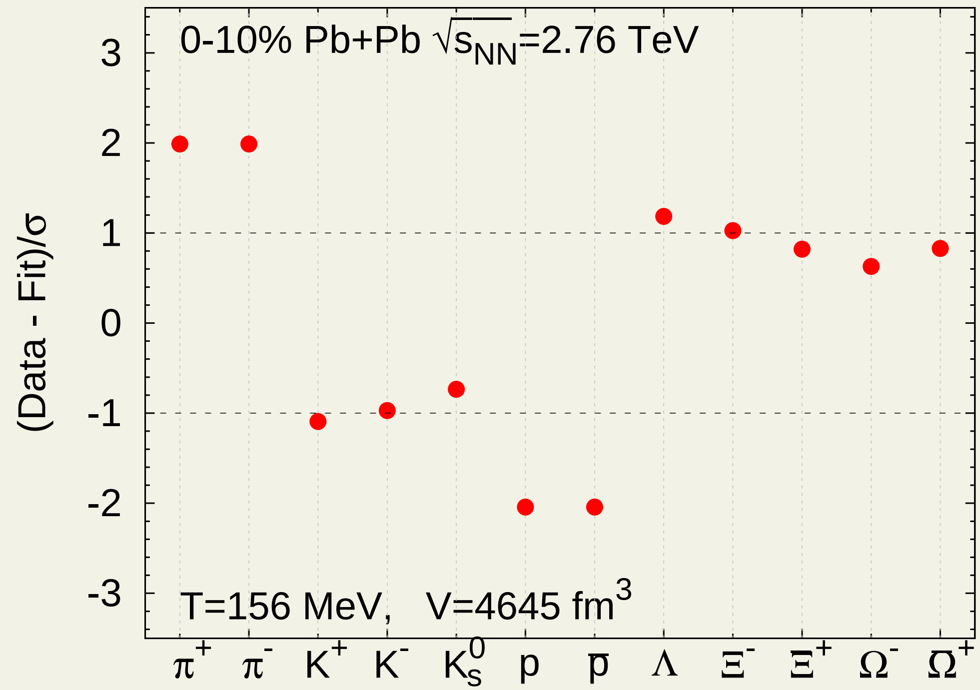


- **Why the difference?**

Fit to ALICE data

J. Stachel @ sQM13, arXiv:1311.4662





- **Why the difference?**
  - **Excluded volume correction does not affect ratios**

Fit to ALICE data

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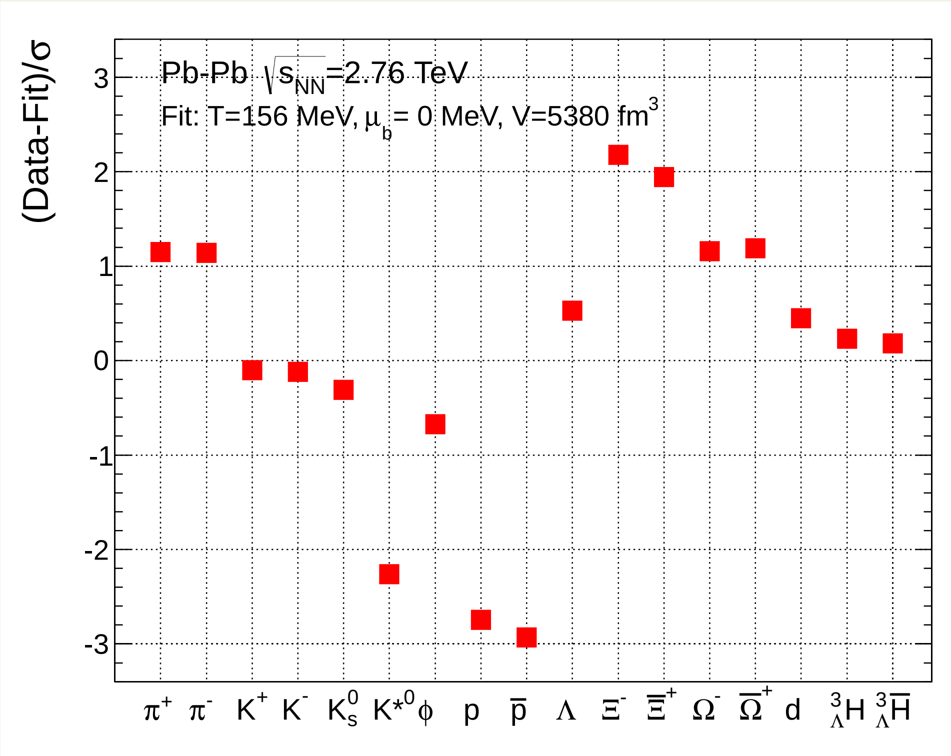
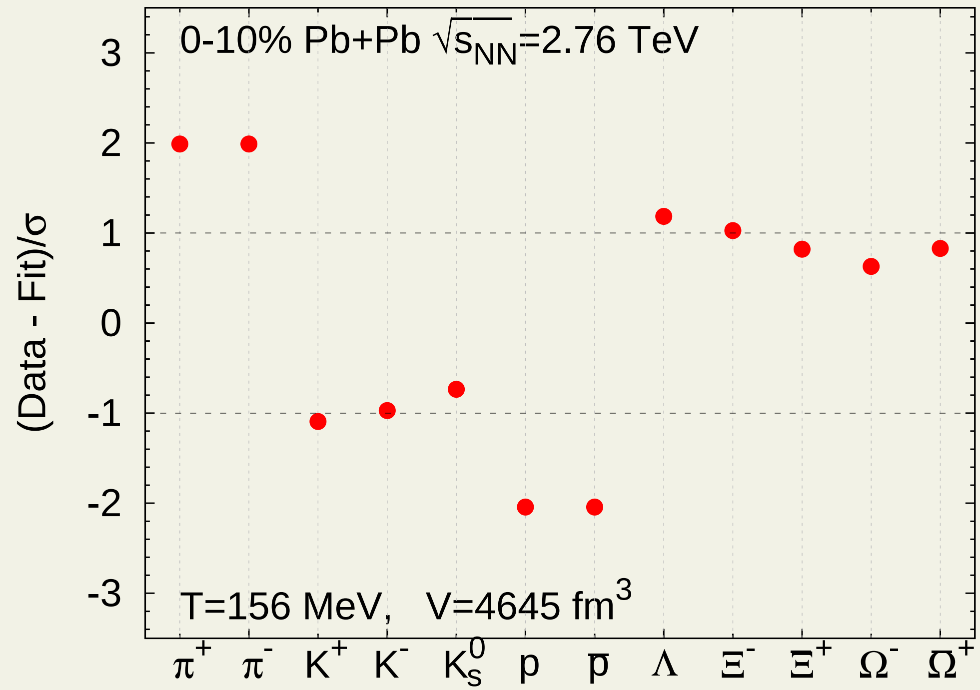
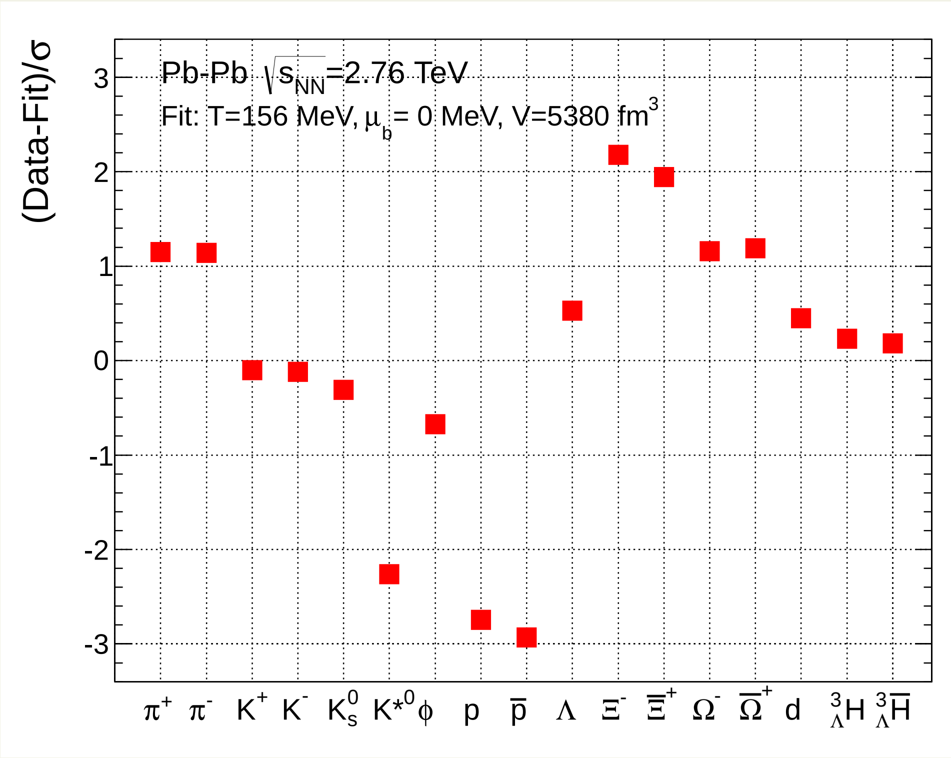


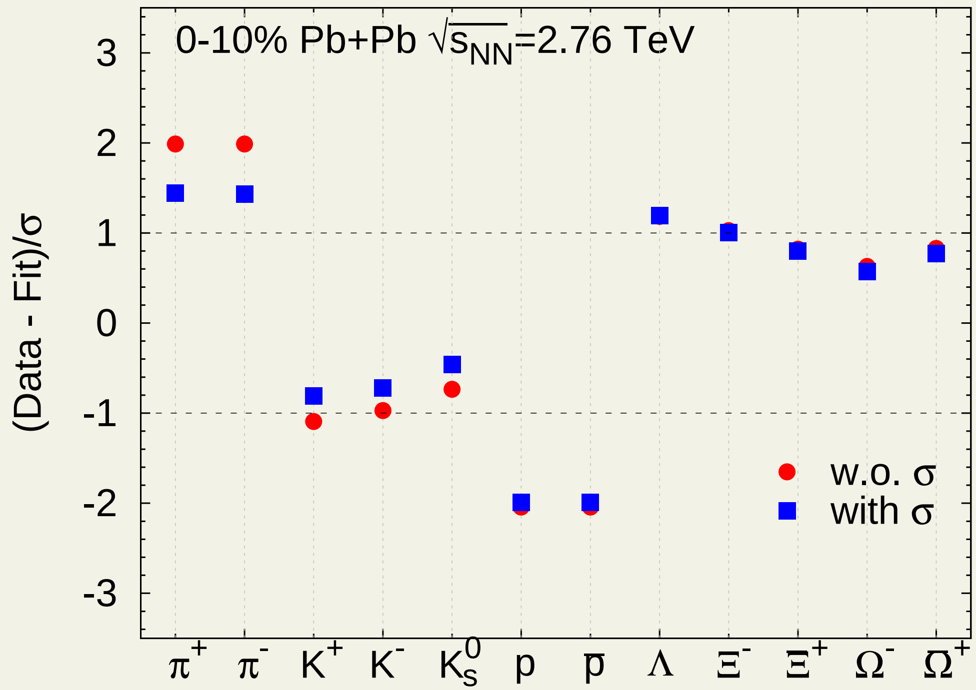


- Why the difference?
  - Excluded volume correction does not affect ratios
  - sigma-meson!

Fit to ALICE data

J. Stachel @ sQM13, arXiv:1311.4662





- **Inclusion of σ-meson improves the fit**
  - **but. . .**

$f_0(500)$ a.k.a. σ-meson

- mass 400-550 MeV
- width 400-700 MeV

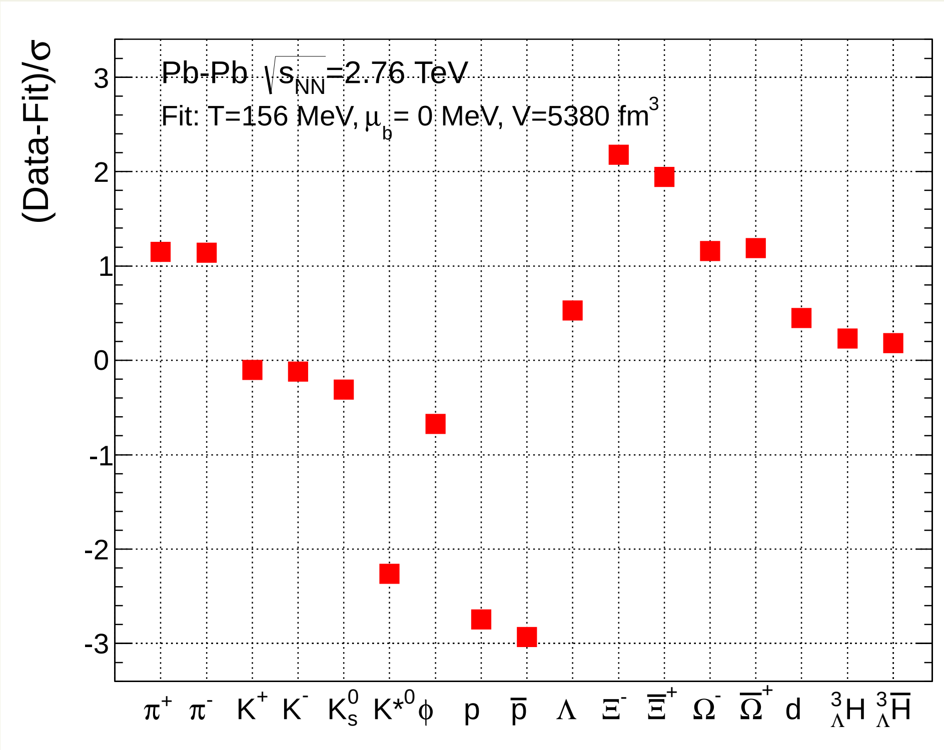
Dashen-Ma-Bernstein: resonances with narrow widths. . .

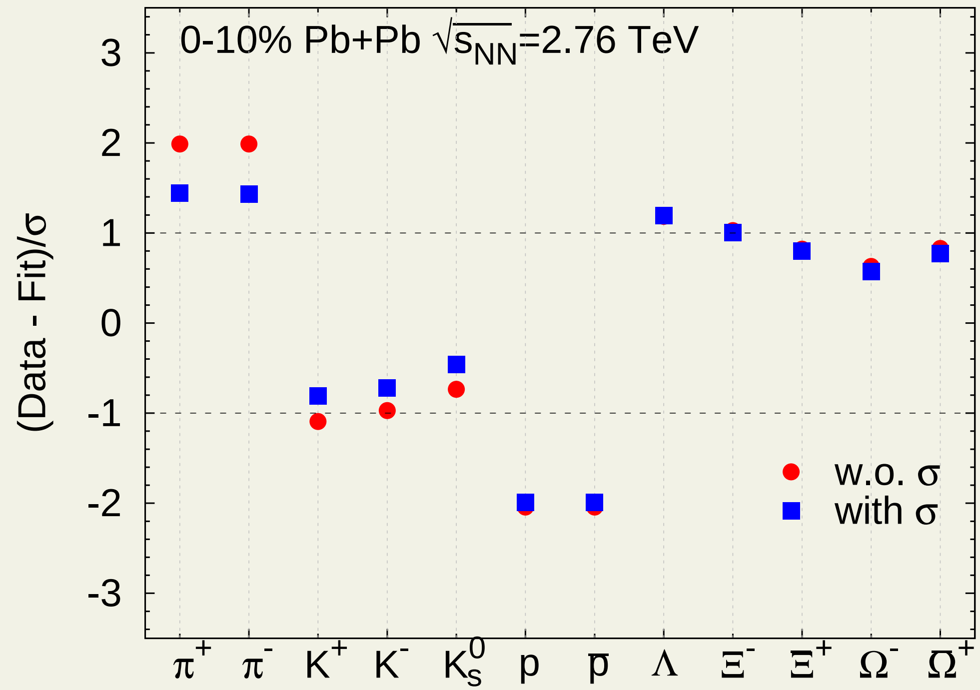
Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts
⇒ HRG good approximation at low temperatures
- phase shifts contain σ
- their HRG did not

⟹ Does σ-meson belong to HRG??

Fit to ALICE data

J. Stachel @ sQM13, arXiv:1311.4662





- **Still fairly large difference. . .**
  - **Finite resonance widths!**

Particle densities:

$$n_i = \frac{g_i}{2\pi^2} \int_0^\infty \mathrm{d}p \, \frac{p^2}{\exp\{(\sqrt{m_i^2 + p^2} - \mu_i)/T\} \pm 1}$$

Finite widths can be taken into account by integrating over mass with Breit-Wigner distribution as weight:

$$n_i = \frac{g_i}{2\pi^2} \frac{1}{N_{\mathrm{BW}}} \int_{M_0}^\infty \mathrm{d}m \, \frac{\Gamma_i^2}{(m - m_i)^2 + \Gamma_i/4} \int_0^\infty \mathrm{d}p \, \frac{p^2}{\exp\{(\sqrt{m_i^2 + p^2} - \mu_i)/T\} \pm 1}$$

Fit to ALICE data

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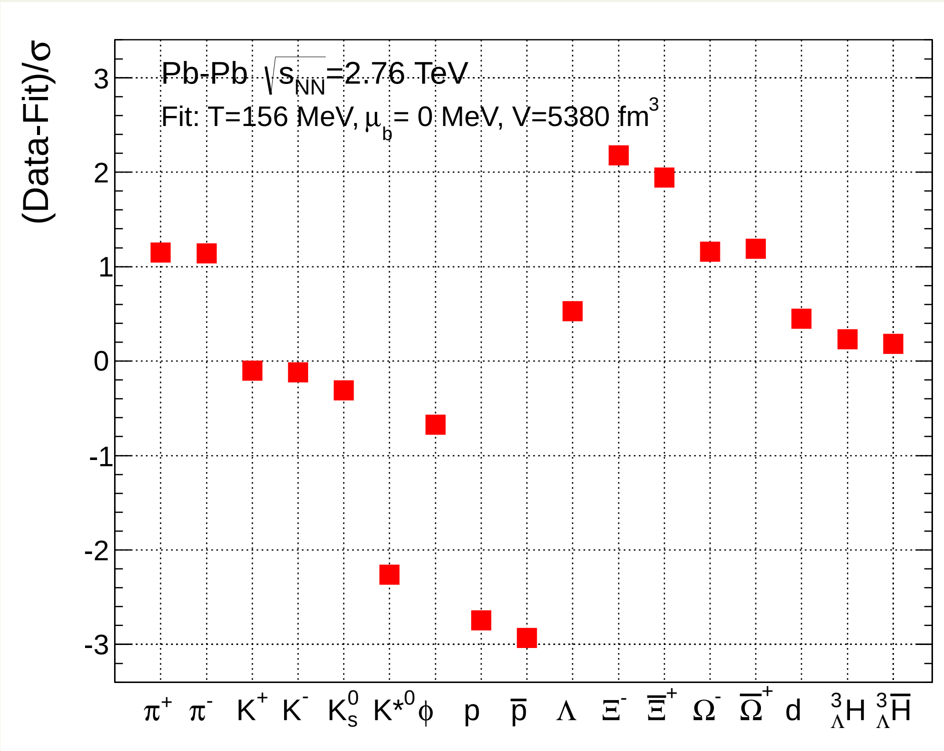
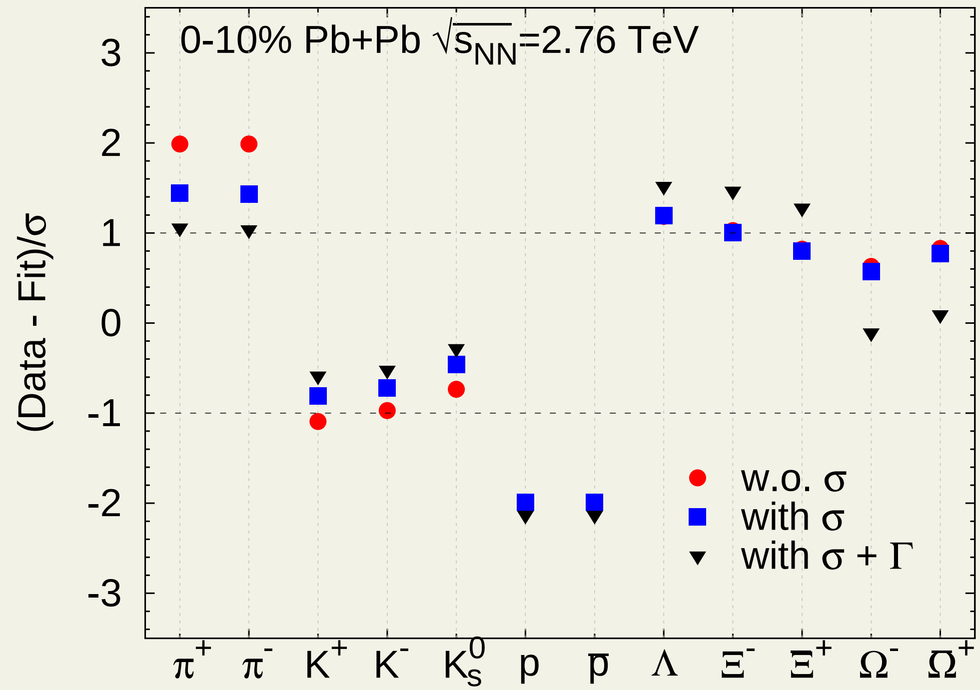
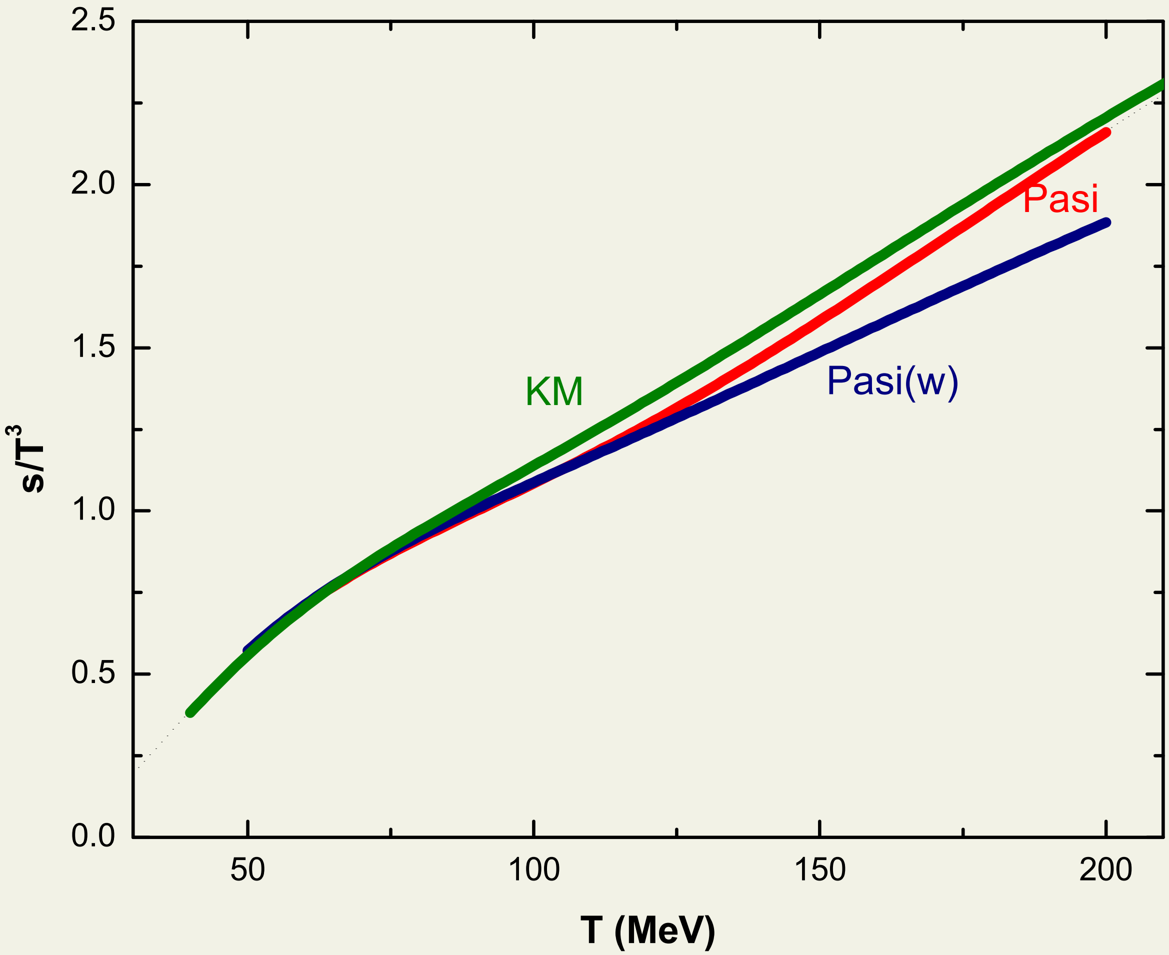




- **differences due to different resonance lists**

Entropy in interacting pion gas



KM: Anton Wiranata's K-matrix calculation

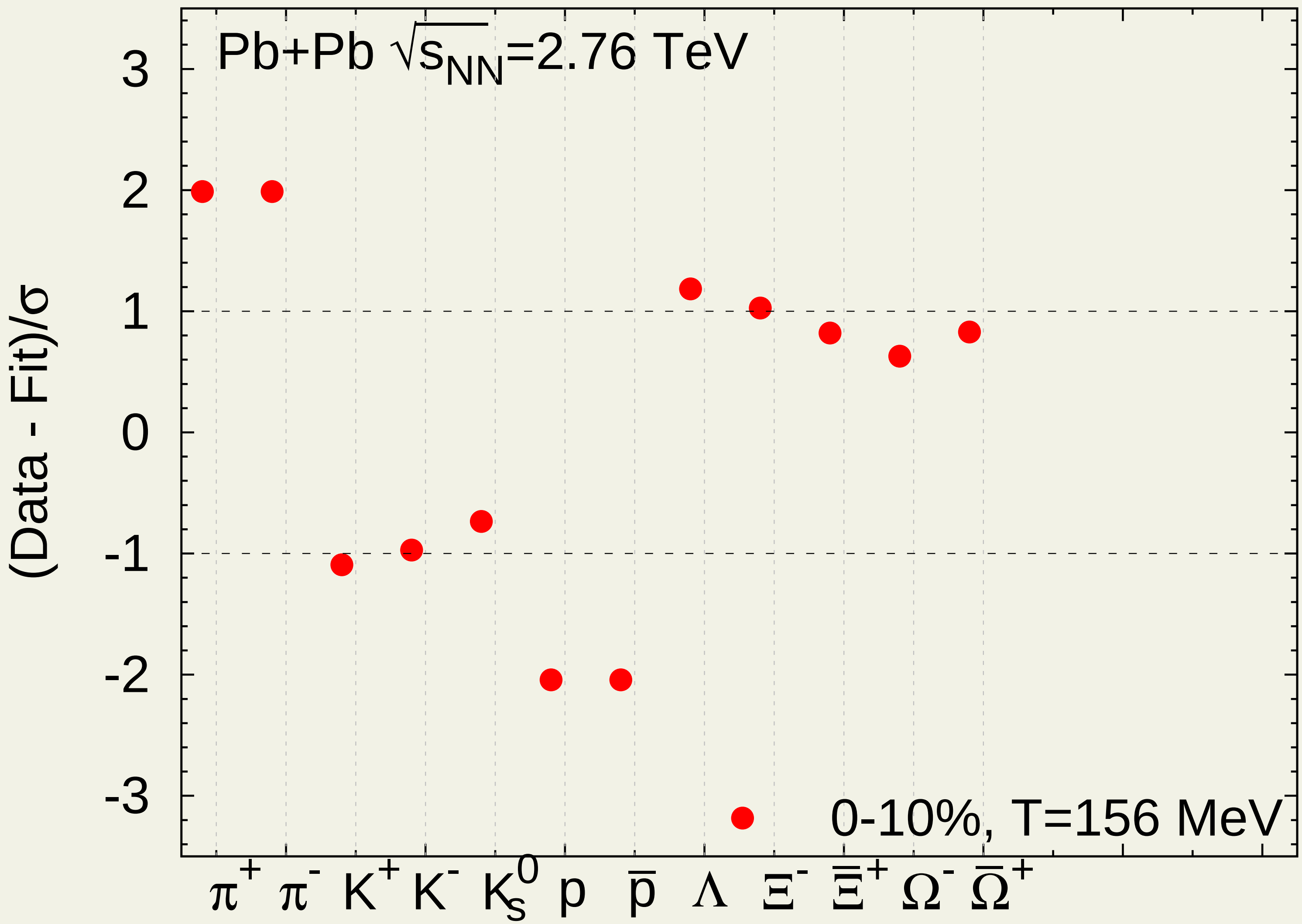
Pasi: $\pi+\rho$-gas, no widhts

Pasi(w): $\pi+\rho$-gas, finite width

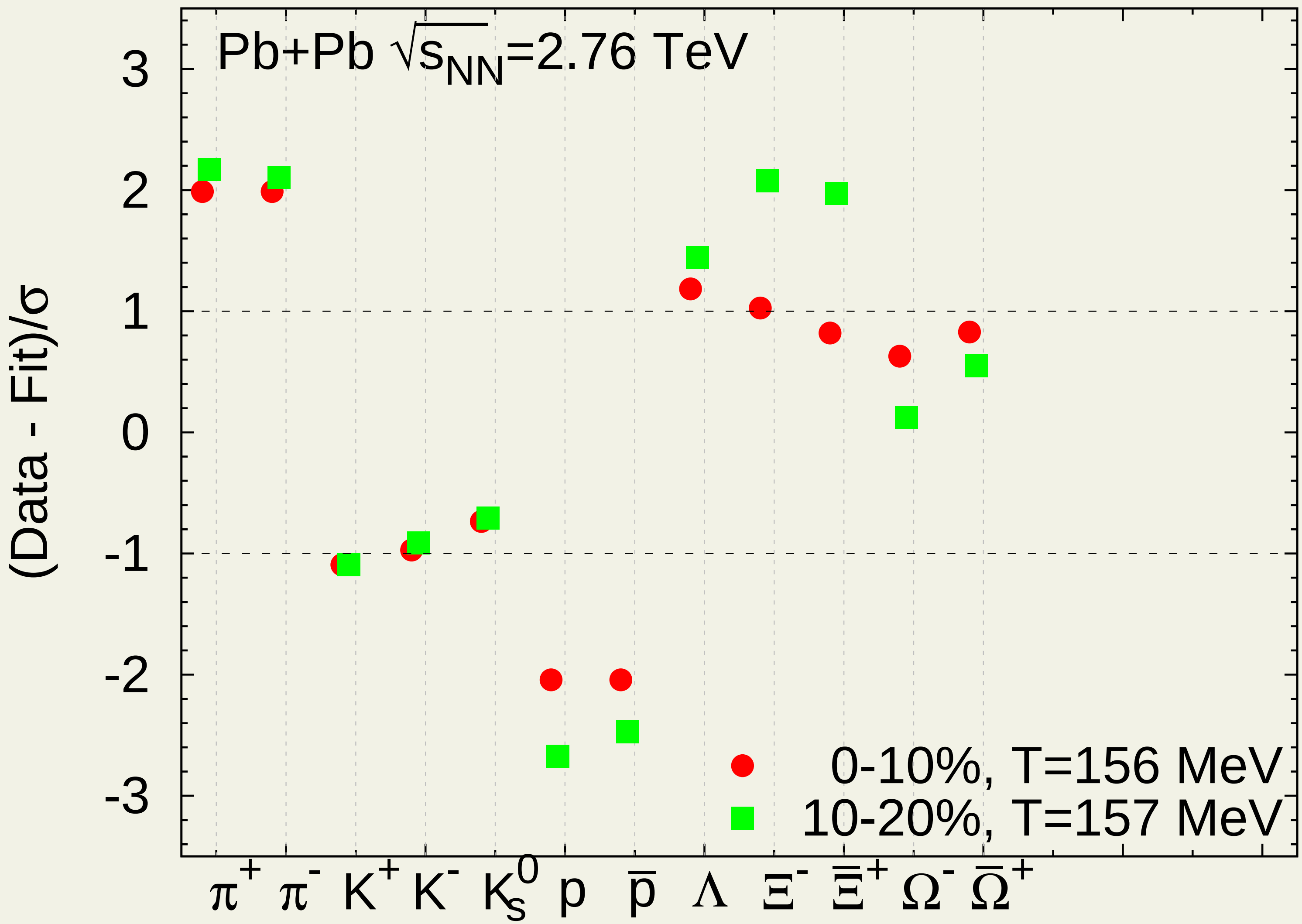
- zero width gives a better approximation!
- spectra calculation very tedious

$\Rightarrow$ no widths nor σ-meson in the following

Centrality dependence



Centrality dependence



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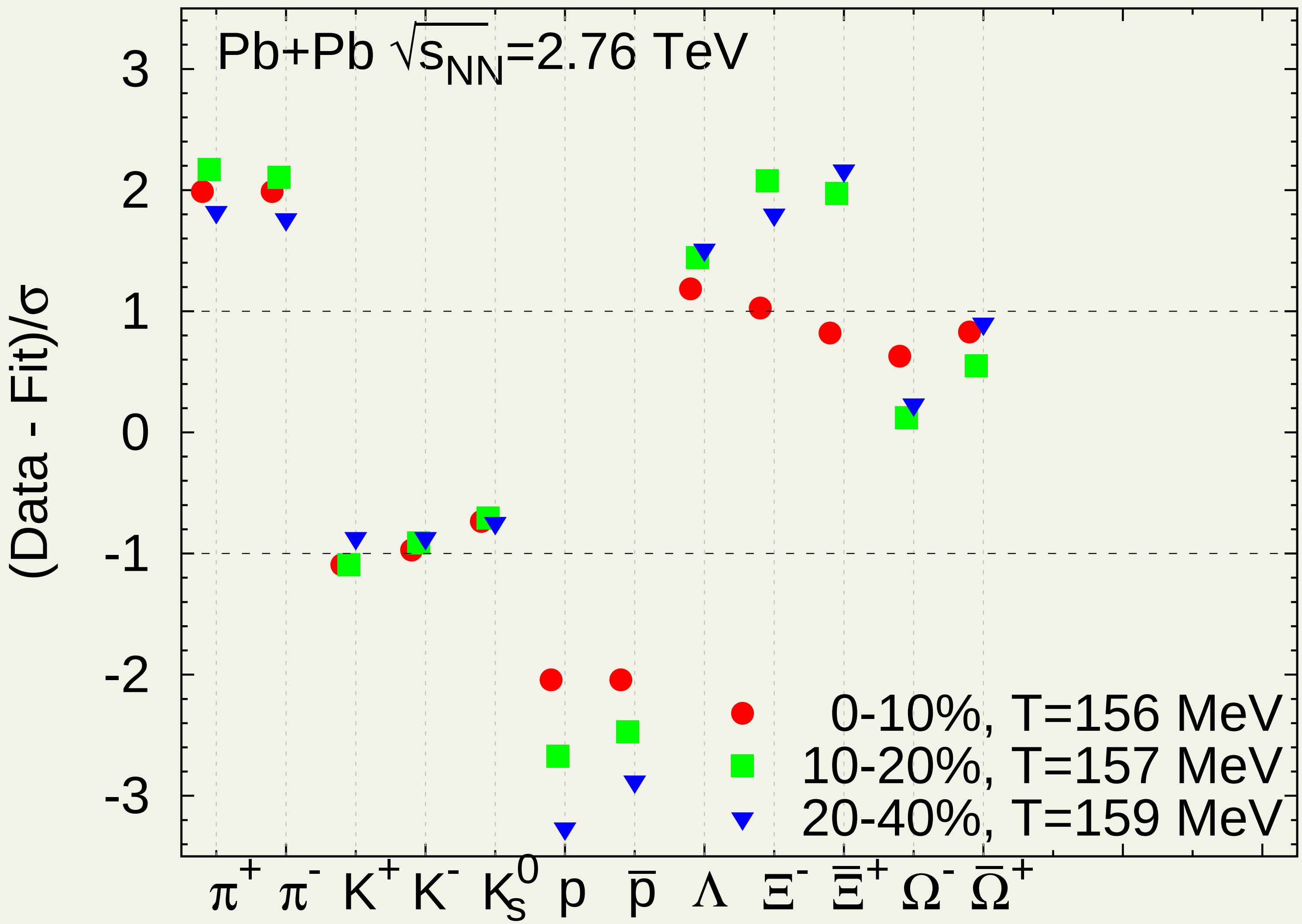

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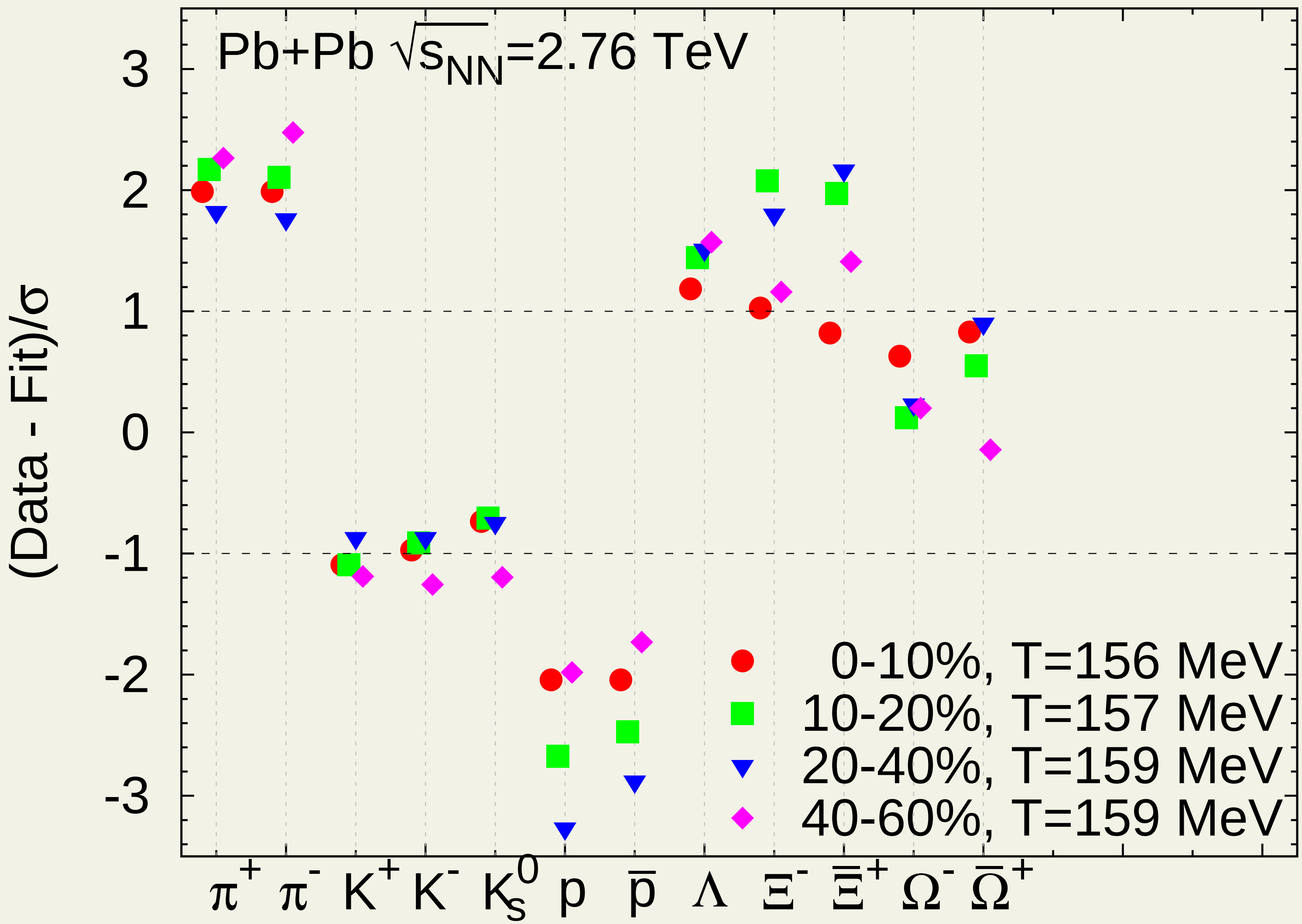

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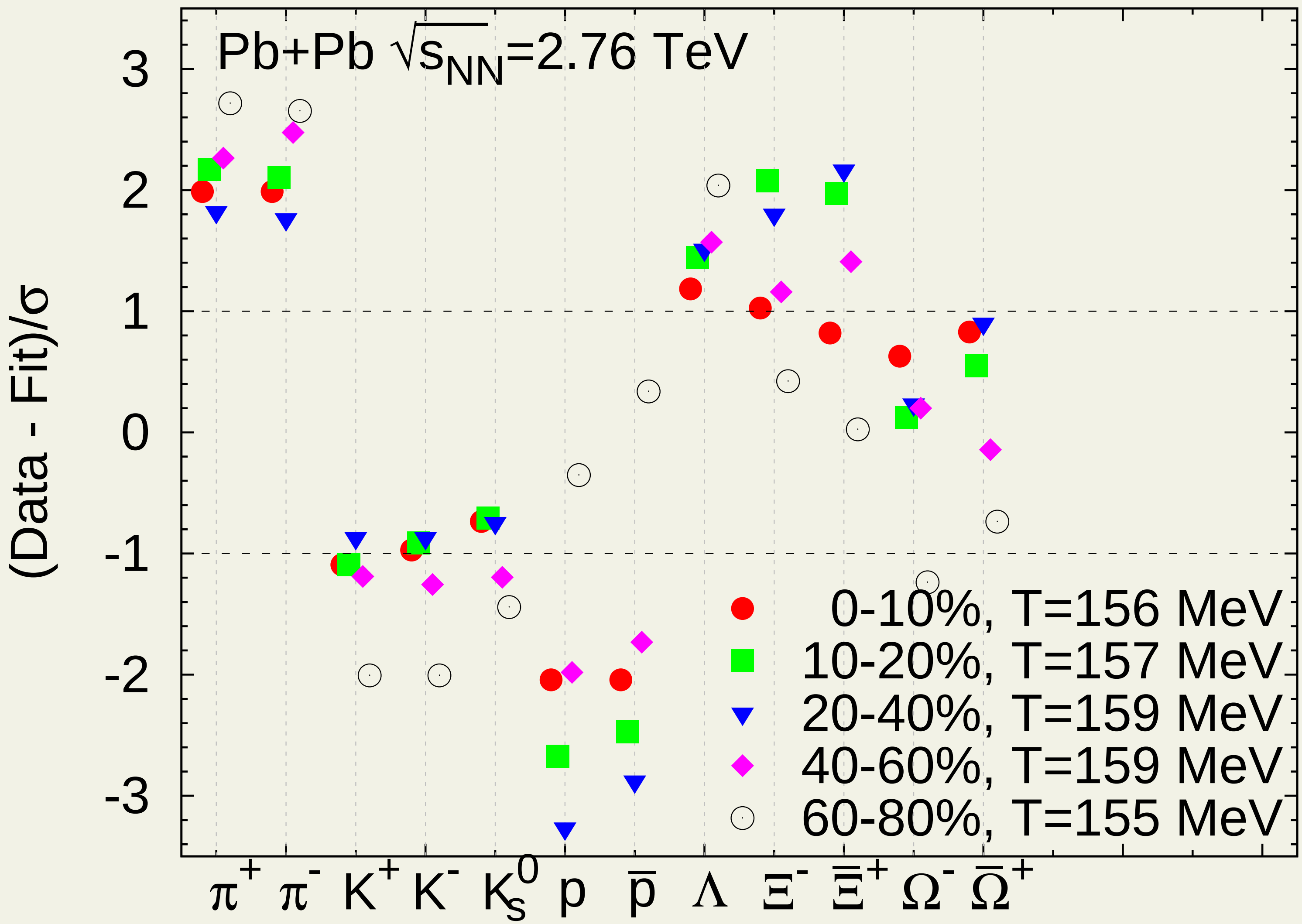
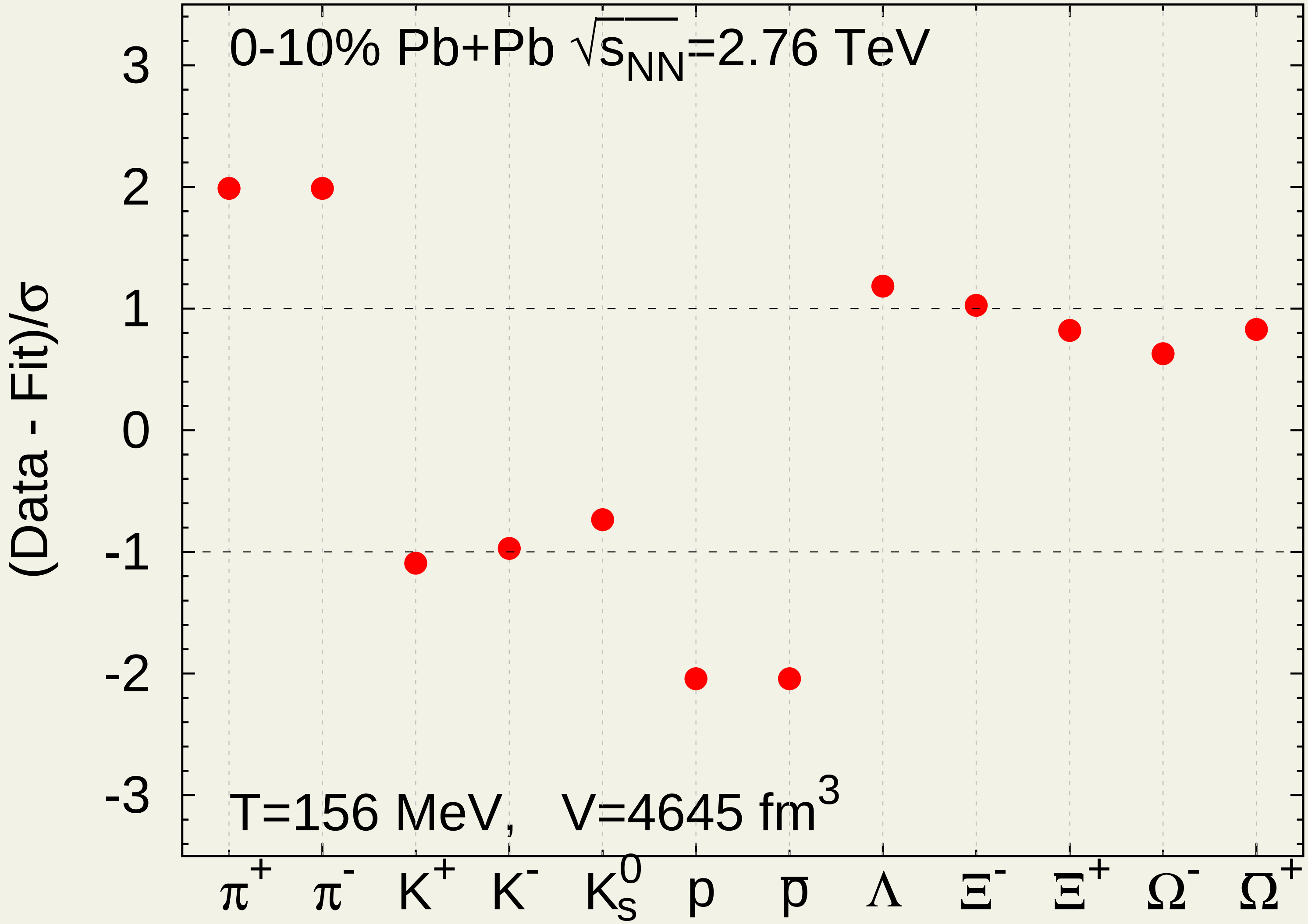


Fit to ALICE data



- how to include nonequilibrium effects in thermal model?

Partial chemical equilibrium

Bebie *et al.*, NPB378, 95 (1992)
After loss of chemical equilibrium at T_1, resonance formation/decay continues:

$$\pi + \pi \leftrightarrow \rho, \qquad \pi + N \leftrightarrow \Delta, \qquad \pi + K \leftrightarrow K^*, \qquad \text{etc.}$$

These processes in equilibrium if

$$\mu_\rho = 2\mu_\pi, \qquad \mu_\Delta = \mu_\pi + \mu_N, \qquad \mu_{K^*} = \mu_K + \mu_\pi, \ldots$$

Conservation of particle # $\Rightarrow$ buildup of chemical potentials

Entropy conservation $\Rightarrow$

$$\frac{\hat{n}_i(T,\mu_i)}{s(T,\{\mu_i\})} = \frac{\hat{n}_i(T_1)}{s(T_1)}$$

fixes the chemical potentials
Note that $\hat{n}_\pi = n_\pi + 2n_\rho + n_\Delta + n_{K^*} + \cdots$

Partial chemical equilibrium

After loss of chemical equilibrium:

System characterised by

- T_1, **temperature where equilibrium is lost**
- T_2, **temperature of final freeze-out**
- V, **volume at** T_2

Partial chemical equilibrium

Assume $N\bar{N}$-annihilations continue after loss of equilibrium at T_1:

$$p + \bar{p} \leftrightarrow 5\pi$$

Equilibrium requires

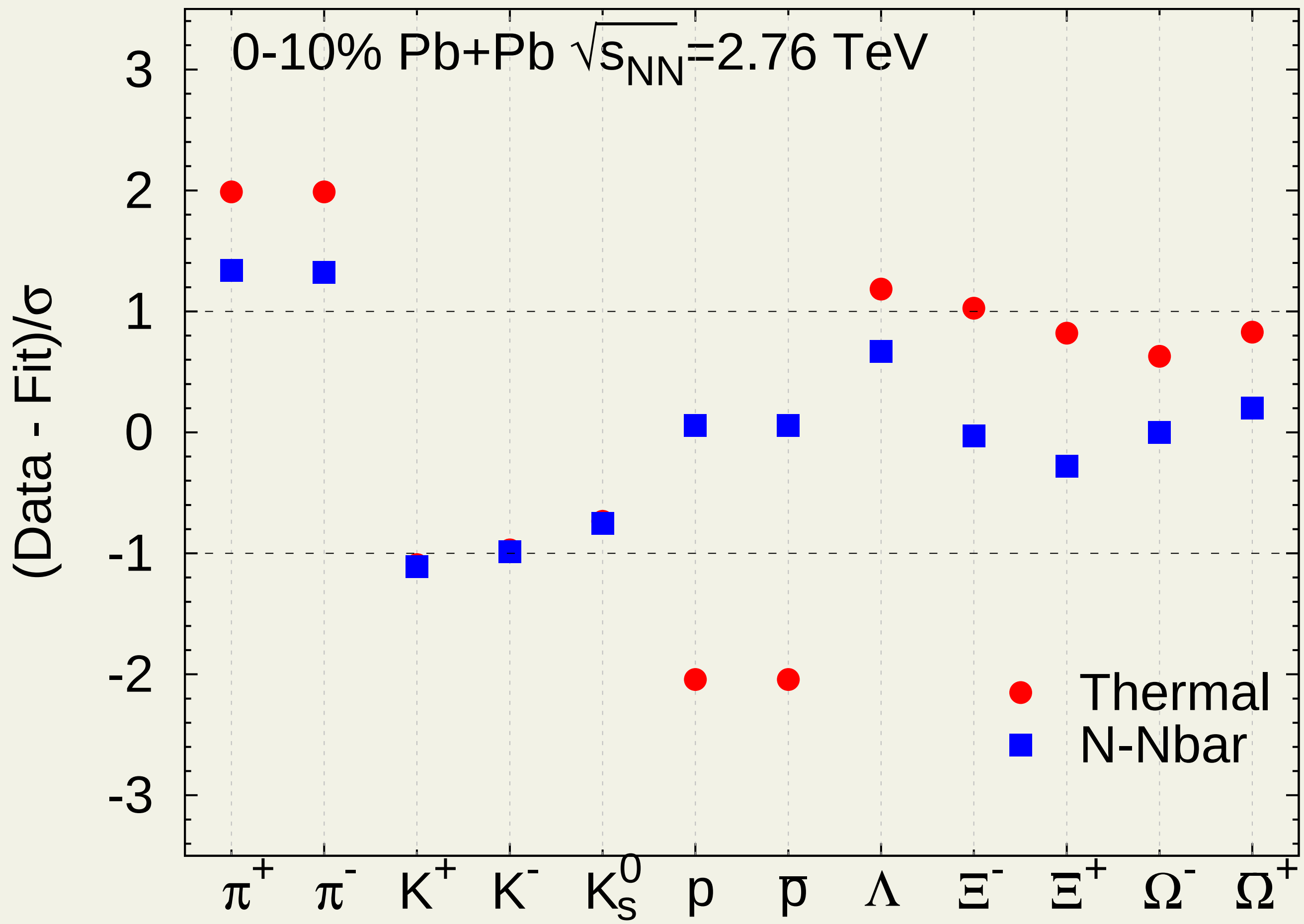
$$\mu_p + \mu_{\bar{p}} = 5\mu_\pi \quad \Leftrightarrow \quad \mu_p = \mu_{\bar{p}} = 2.5\mu_\pi$$

- $\hat{N}_N$, $\hat{N}_{\bar{N}}$ **or** $\hat{N}_\pi$ **not conserved**
- $N_\pi + 2.5N_N + 2.5N_{\bar{N}}$ **is conserved**

$$\frac{\hat{n}_\pi(T,\mu_\pi) + 2.5\,\hat{n}_N(T,\mu_\pi) + 2.5\,\hat{n}_{\bar{N}}(T,\mu_\pi)}{s(T,\{\mu_i\})} = \text{const.}$$

gives $\mu_\pi(T)$

Fit allowing $N\bar{N}$-annihilations



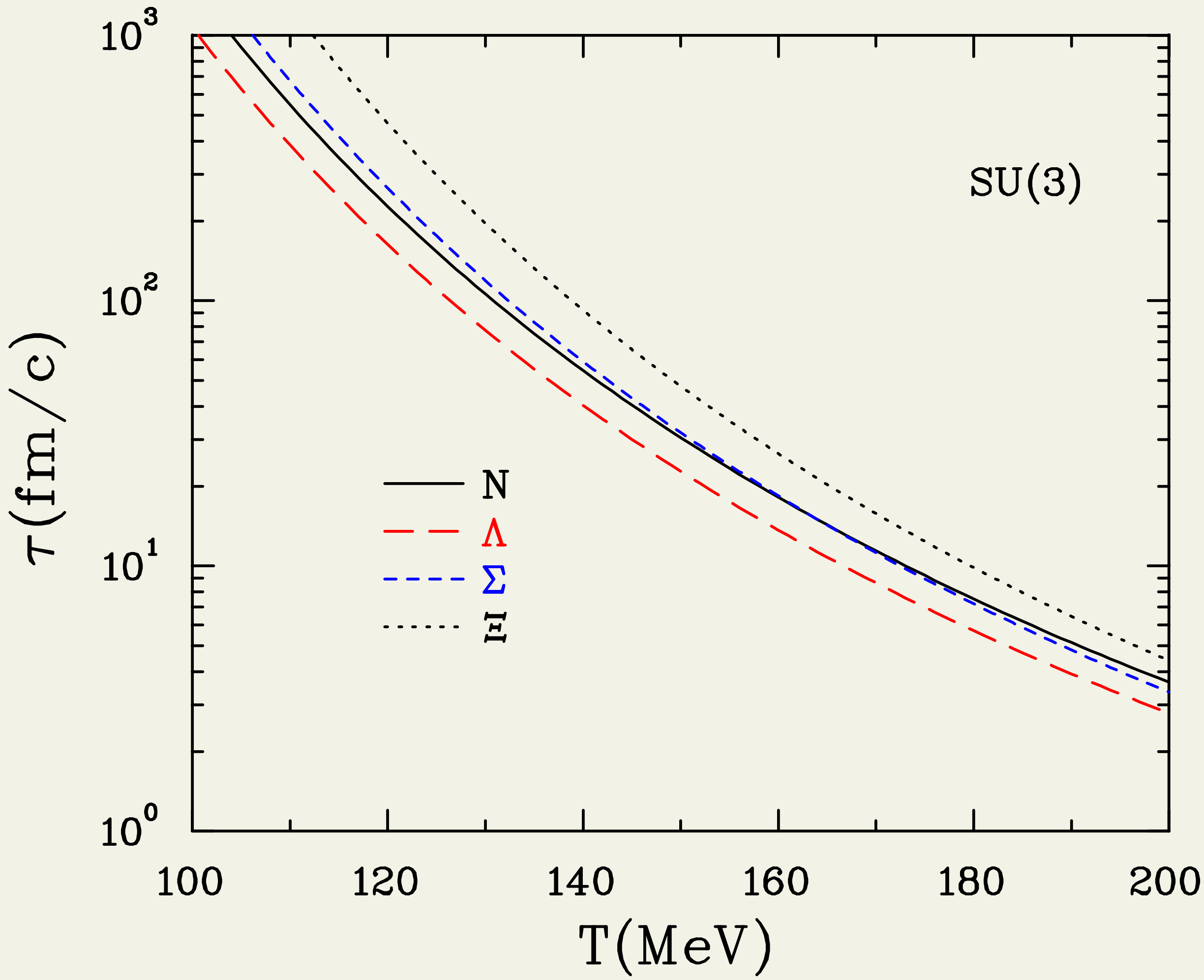
- **Thermal**
  $T = 156\,\mathrm{MeV}$
  $V = 4645\,\mathrm{fm}^3$

- **N-Nbar**
  $T_1 = 160\,\mathrm{MeV}$
  $T_2 = 125\,\mathrm{MeV}$
  $V = 7760\,\mathrm{fm}^3$

- **very good fit!**
- **but. . .**

Chemical equilibration time for nucleons

Kapusta & Shovkovy, Phys. Rev. C68, 014901 (2003)



- **Λ's equilibrate faster than nucleons**
- **Σ's equilibrate as fast as nucleons**

$\Rightarrow$ **include Λ and Σ annihilations**

Λ- and Σ-annihilations

Processes

$$\Lambda + \bar{\Lambda} \leftrightarrow 5\pi \qquad \text{and} \qquad \Sigma + \bar{\Sigma} \leftrightarrow 5\pi$$

in equilibrium when

$$\mu_\Lambda = \mu_\Sigma = 2.5\mu_\pi$$

But

$$\Lambda + \bar{N} \leftrightarrow 4\pi + \bar{K} \qquad \text{and} \qquad \Sigma + \bar{N} \leftrightarrow 4\pi + \bar{K}$$

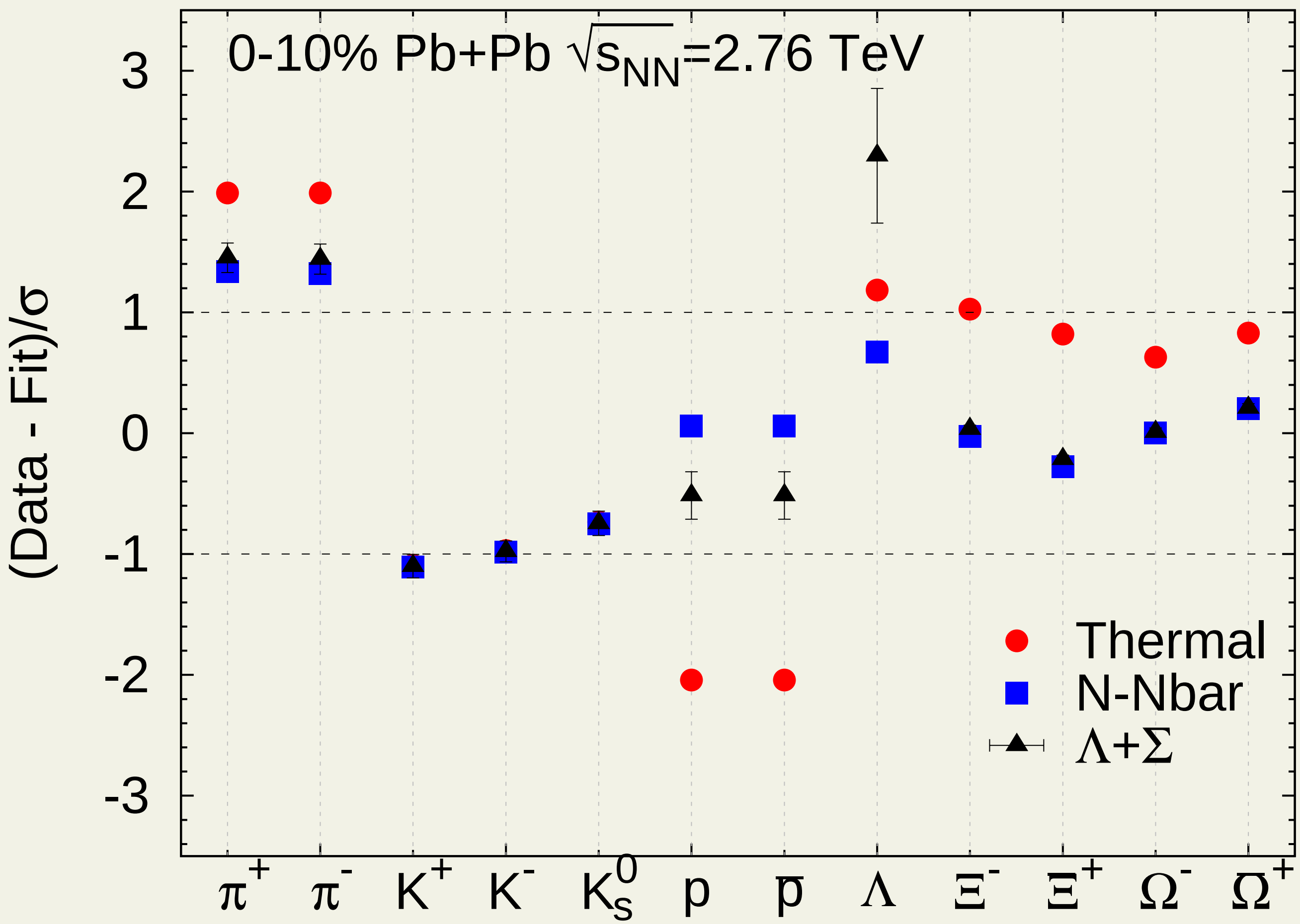
in equilibrium when

$$\mu_\Lambda = \mu_\Sigma = 1.5\mu_\pi + \mu_{\bar{K}}$$

$\Rightarrow$ **Weighted average with rates as weight**

rates unknown $\Rightarrow$ **upper and lower limit**

Fit including Λ- and Σ-annihilations



● **Thermal**
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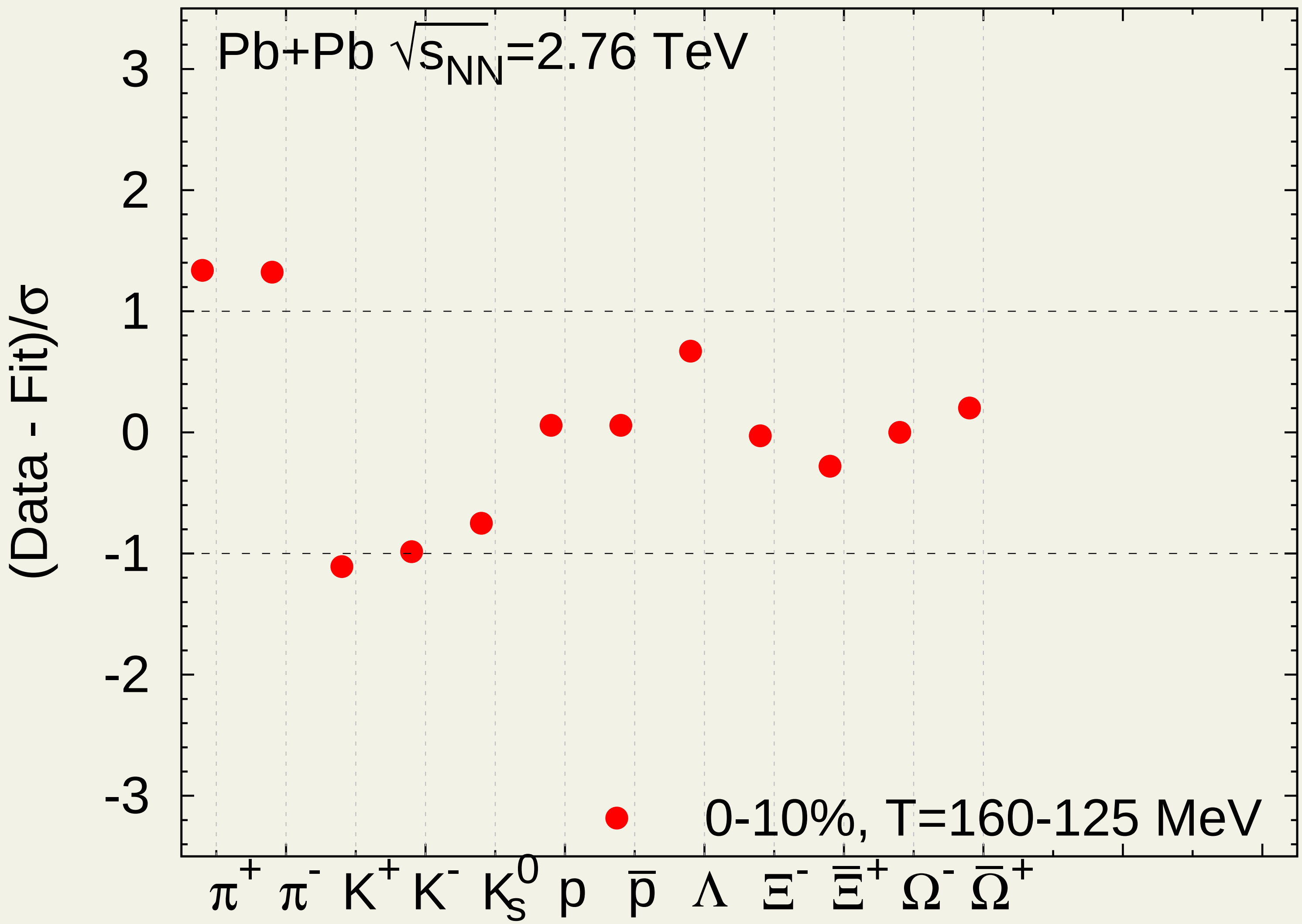
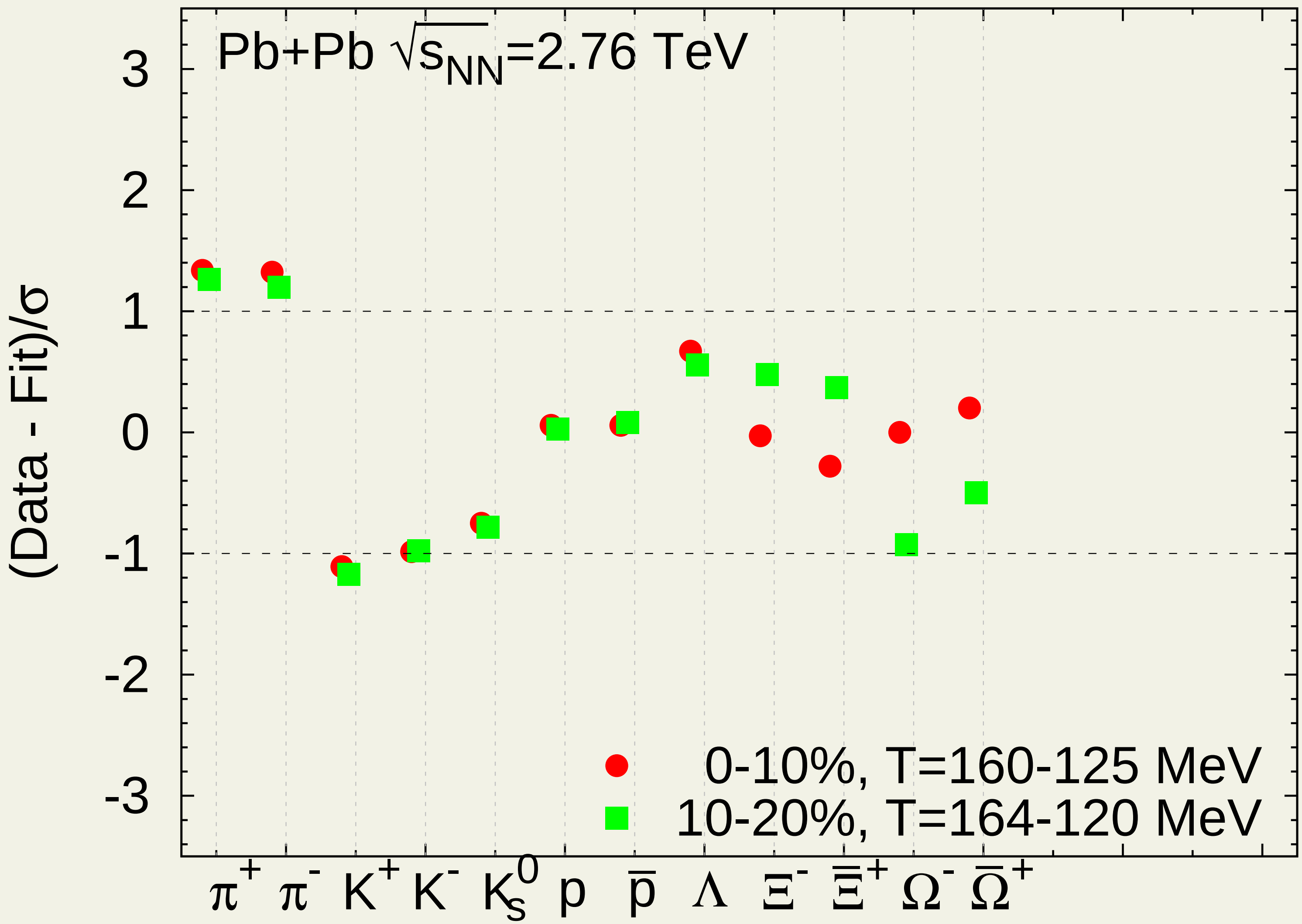
▲ $\Lambda+\Sigma$
$T_1 = 159 - 161\,\mathrm{MeV}$
$T_2 = 133 - 130\,\mathrm{MeV}$
$V = 6750 - 6650\,\mathrm{fm}^3$

- **serious lack of Λ's!**

Centrality dependence



Centrality dependence



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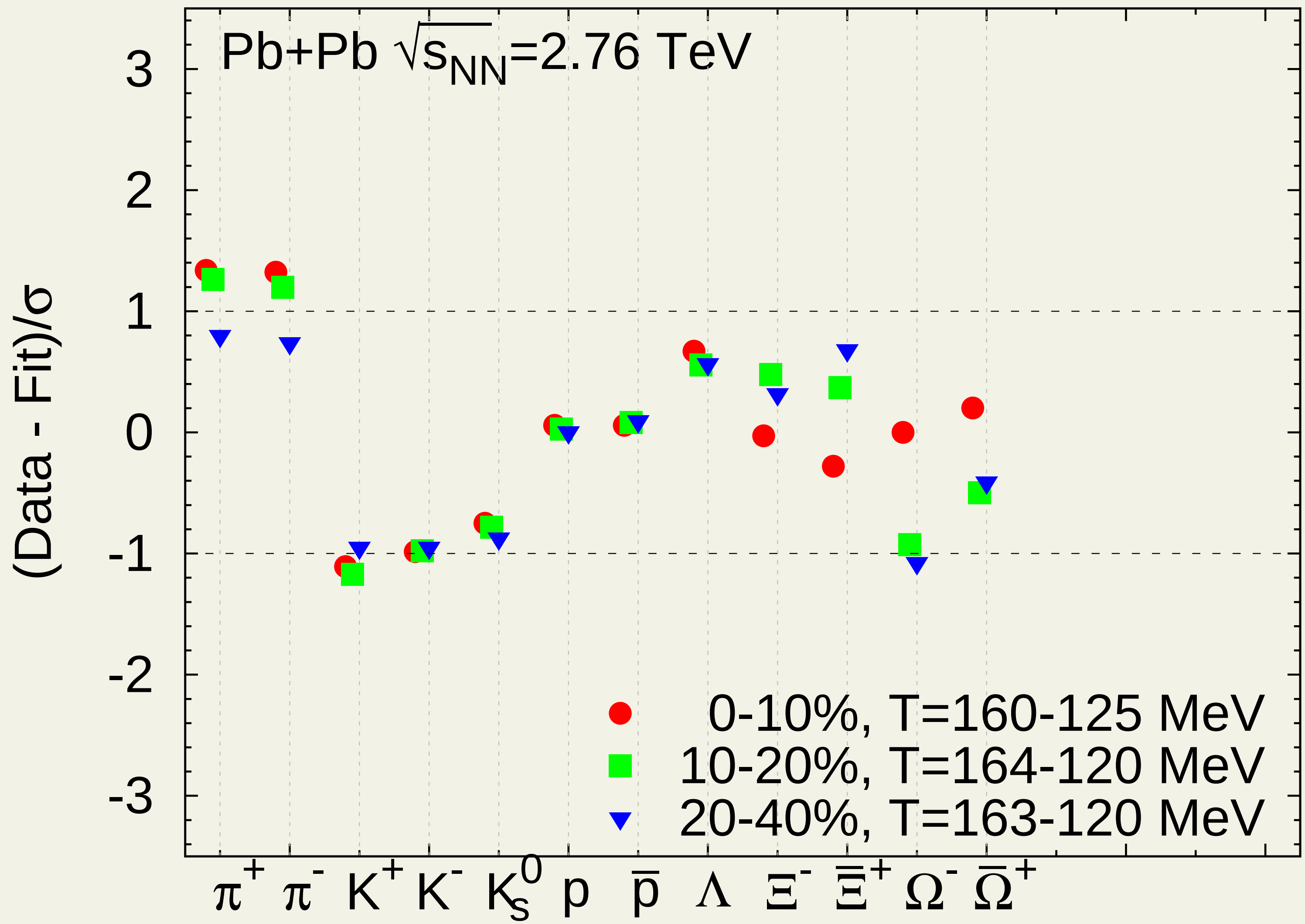

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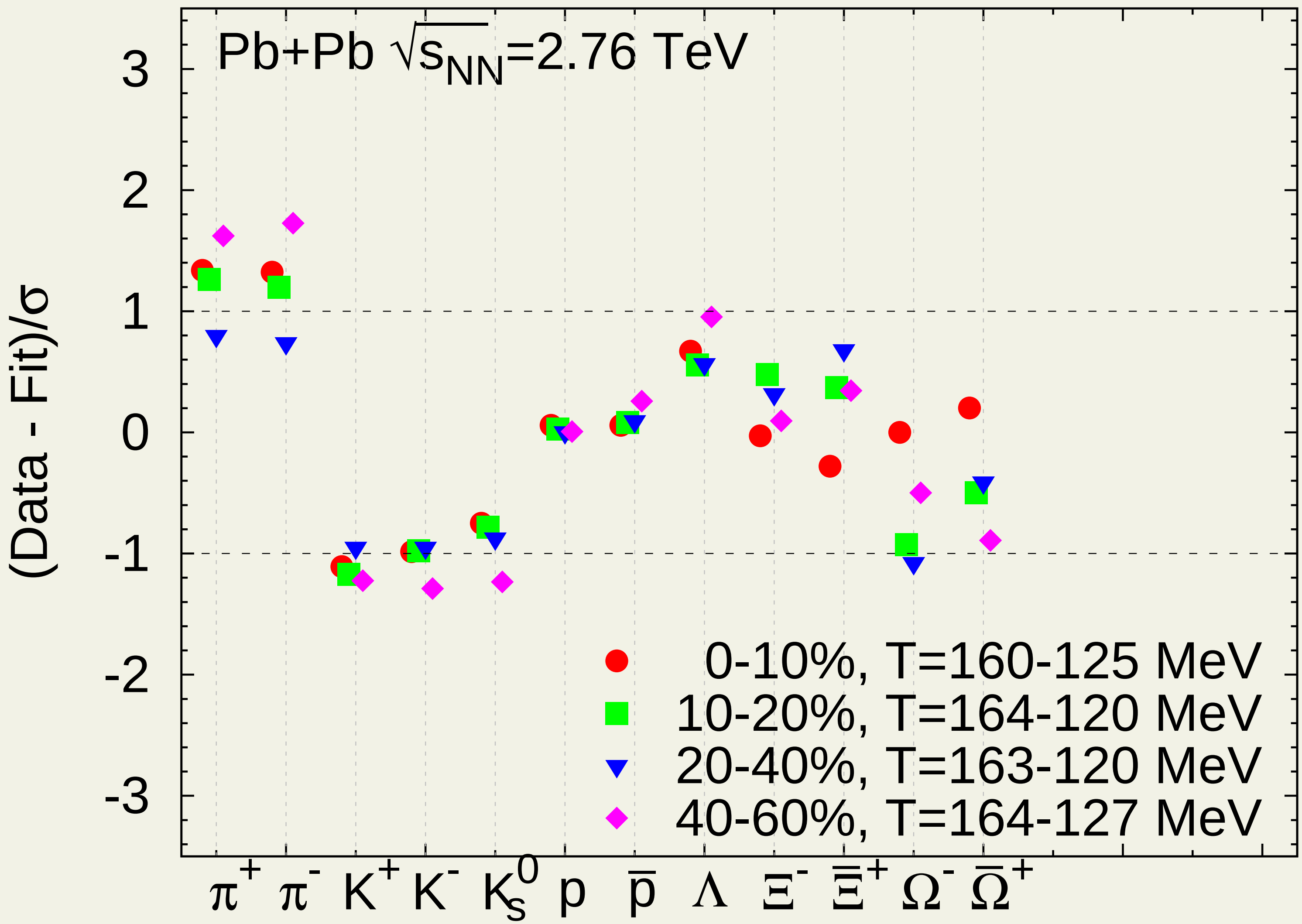
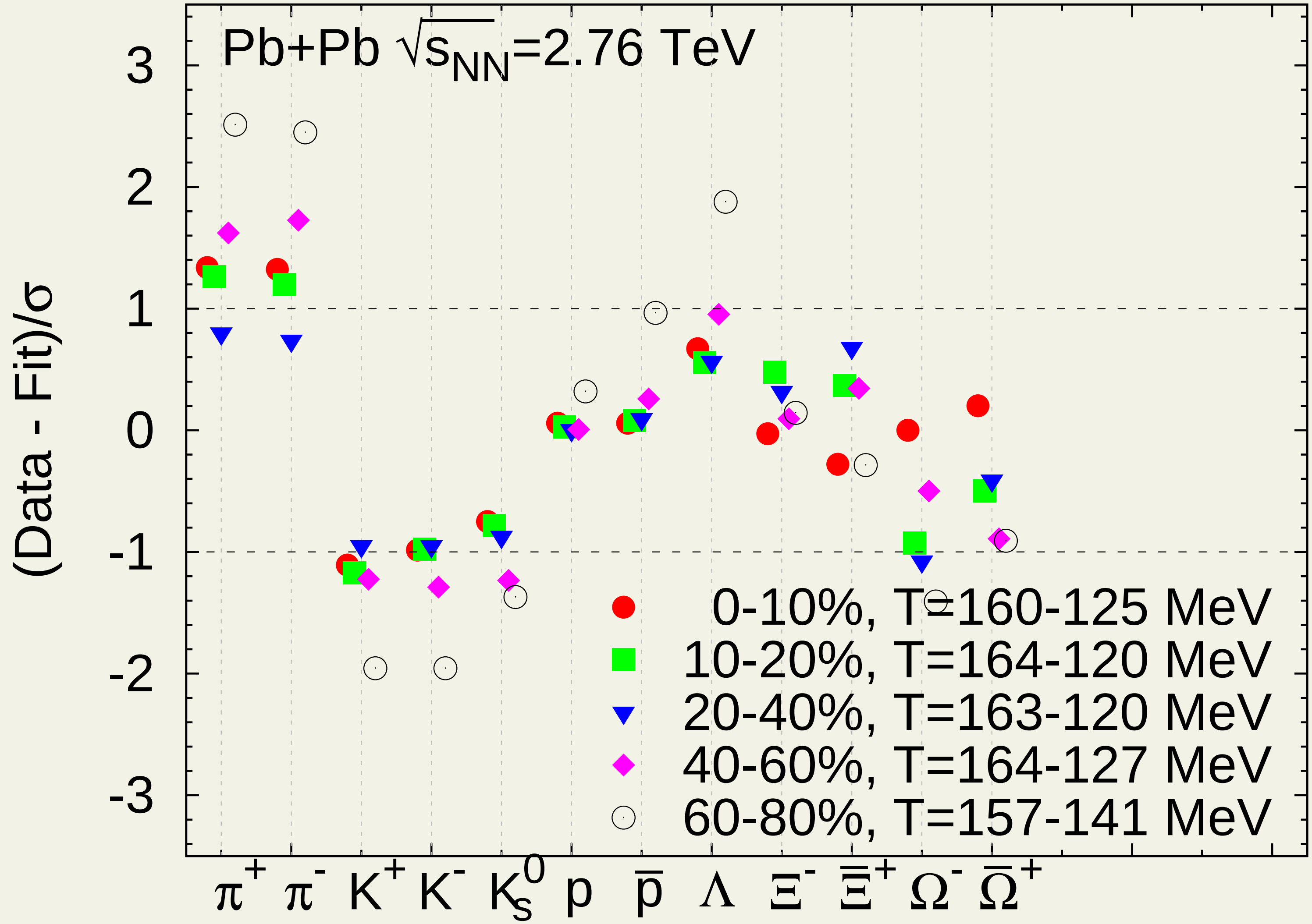
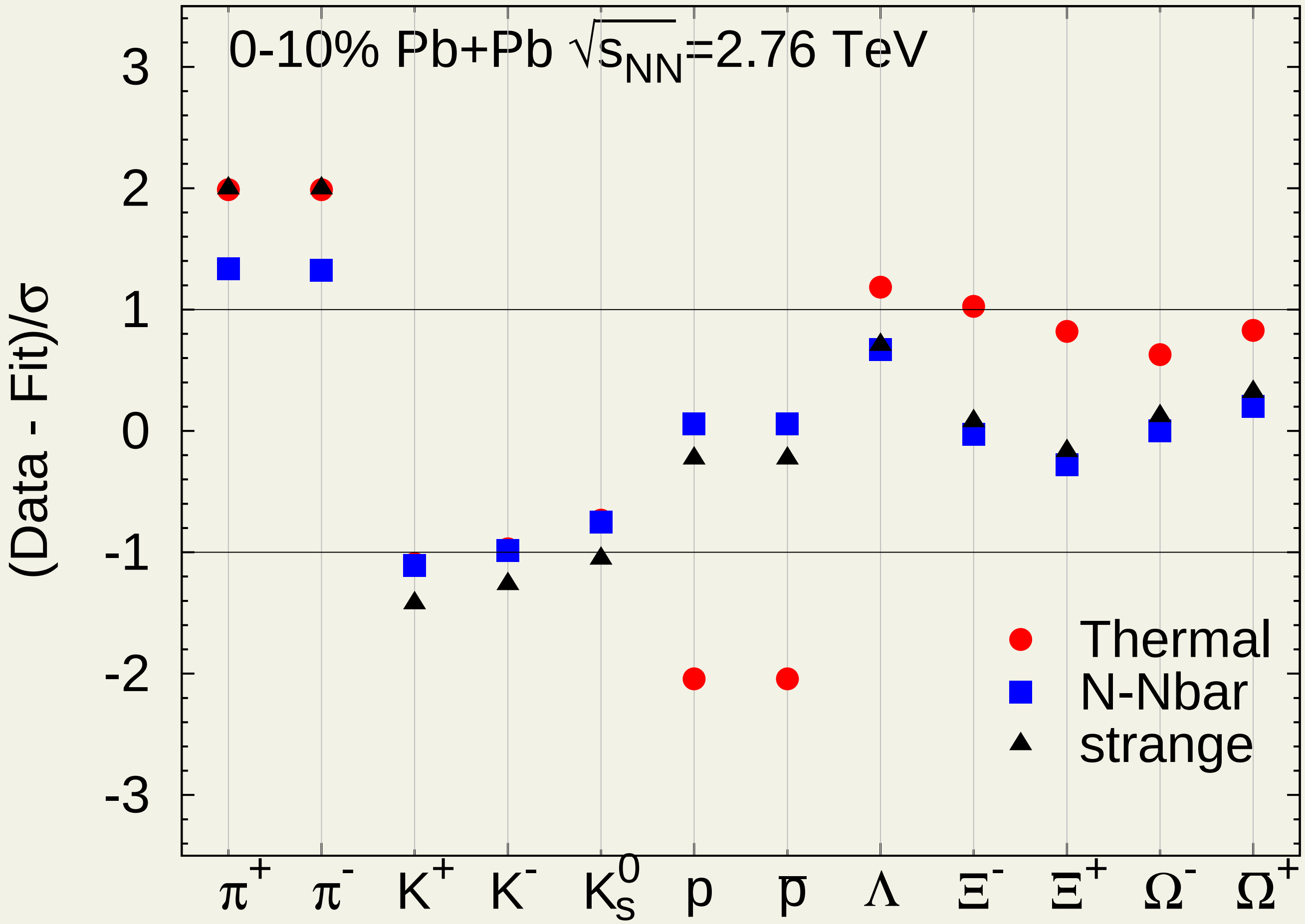


Centrality dependence



What if strange particles freeze-out before non-strange?



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  $T_1 = 160\,\mathrm{MeV}$
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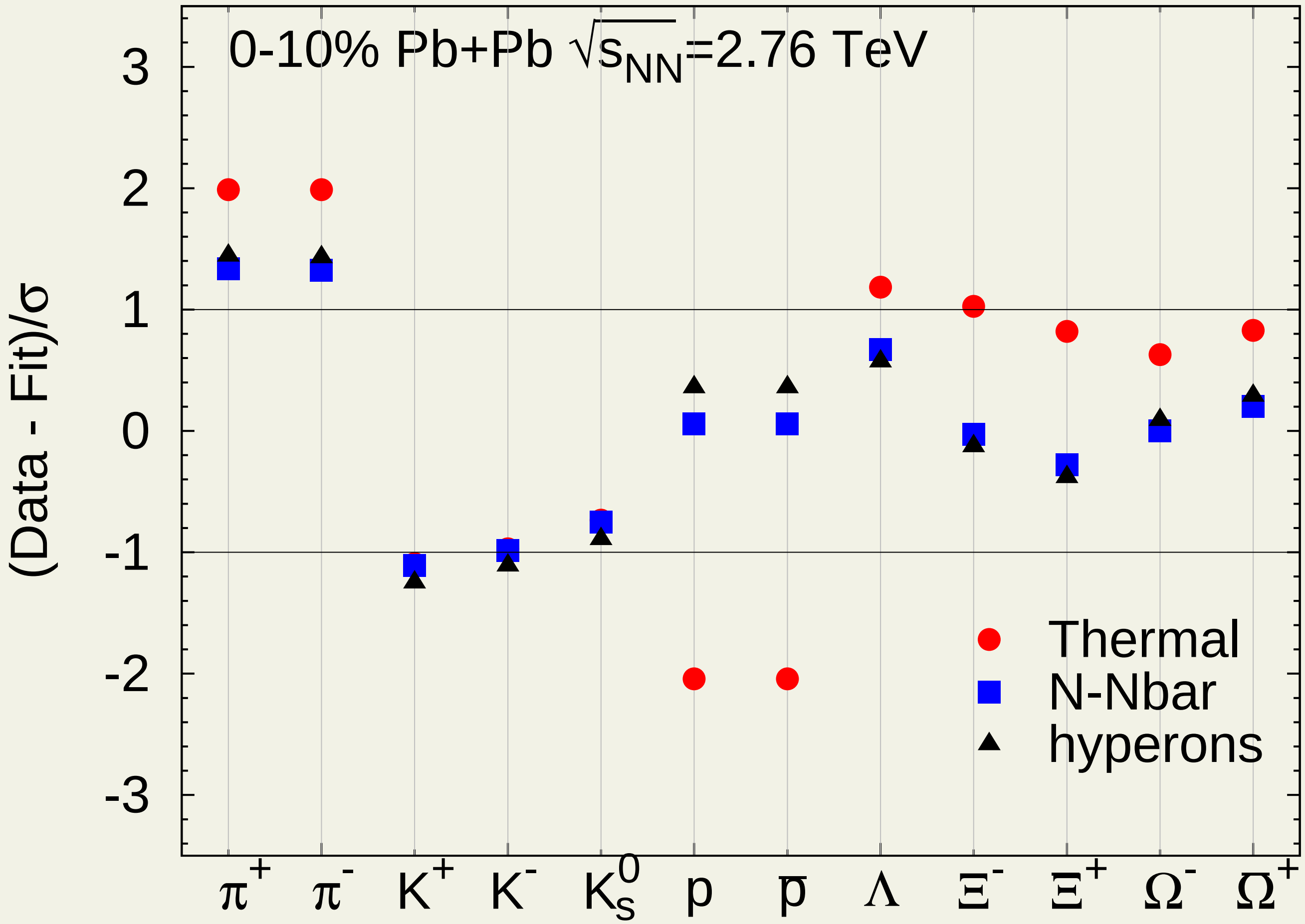
  $V = 7760\,\mathrm{fm}^3$

- **strange**
  $T_{\mathrm{strange}} = 158\,\mathrm{MeV}$
  $T_{\mathrm{rest}} = 149\,\mathrm{MeV}$
  $V = 6230\,\mathrm{fm}^3$

- K/π**-ratio ?**

What if hyperons freeze-out before the rest?



• **Thermal**

$T = 156\,\text{MeV}$

$V = 4645\,\text{fm}^3$

■ **N-Nbar**

$T_1 = 160\,\text{MeV}$

$T_2 = 125\,\text{MeV}$

$V = 7760\,\text{fm}^3$

▲ **hyperons**

$T_{\text{hyperon}} = 157\,\text{MeV}$

$T_{\text{rest}} = 144\,\text{MeV}$

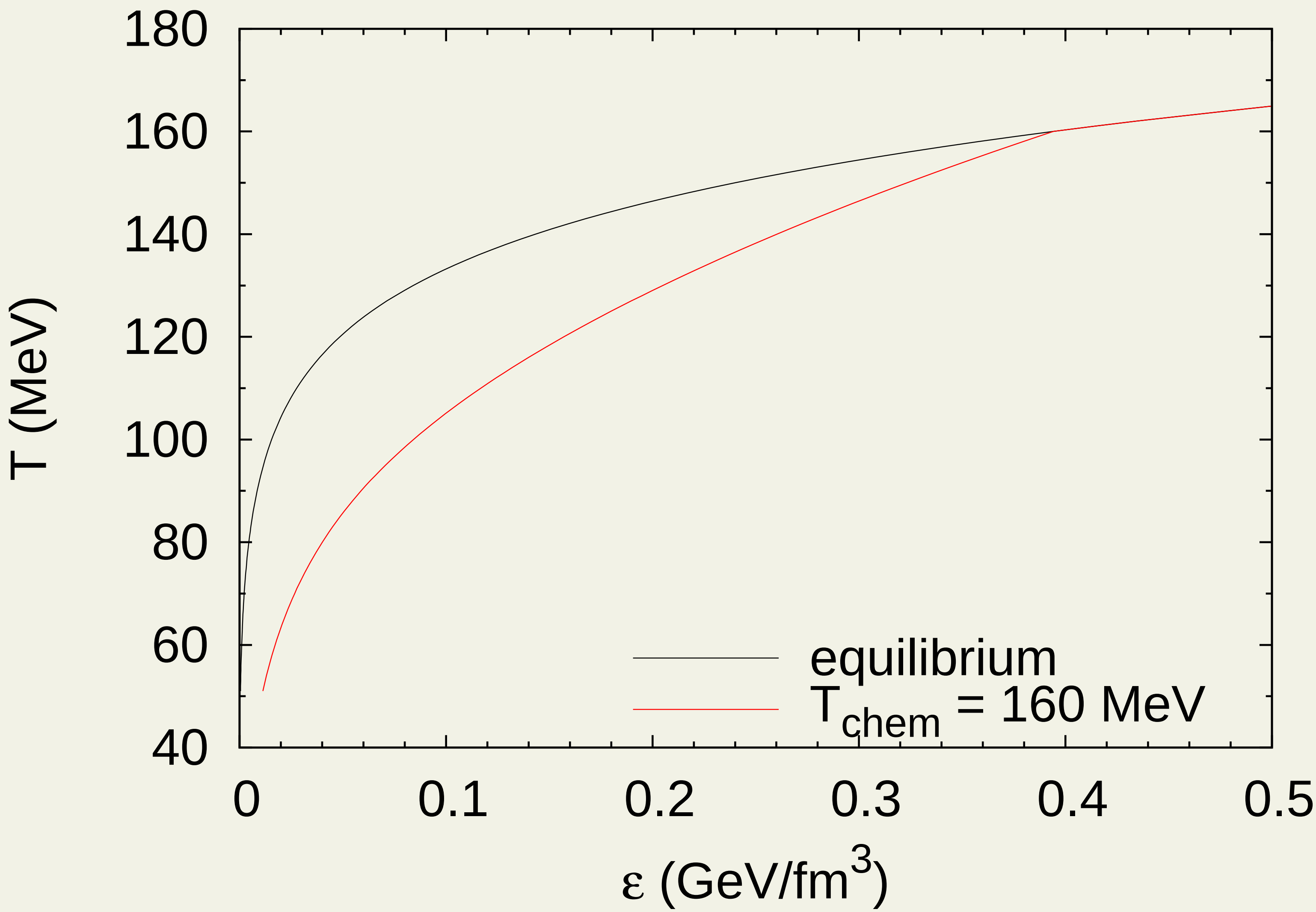
$V = 7890\,\text{fm}^3$

- **equal to N-Nbar fit**

Equation of state

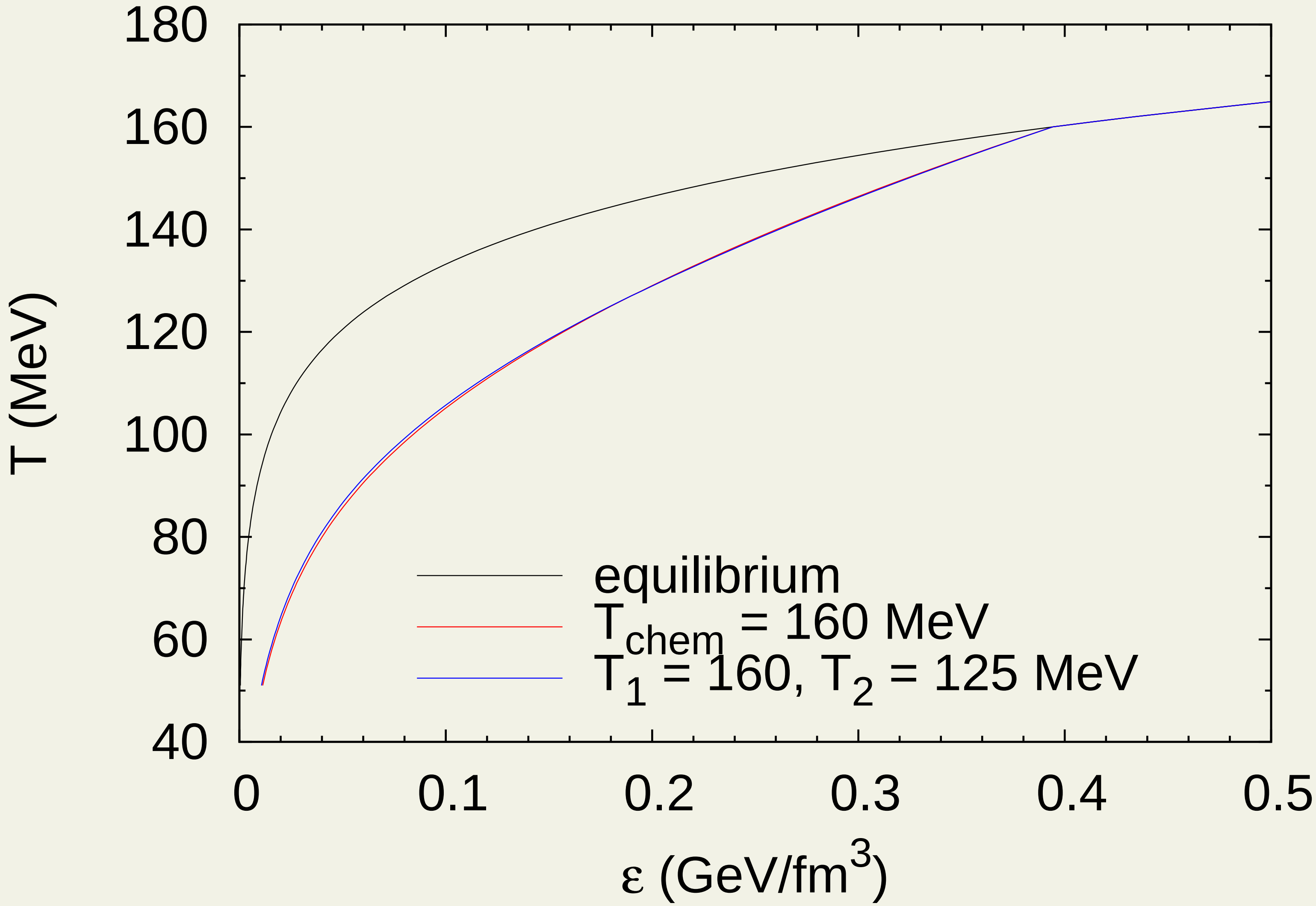
- **built as conventional PCE EoS**
- **loss of chemical equilibrium at** T_1
- **conserved species change at** T_2

Equation of state



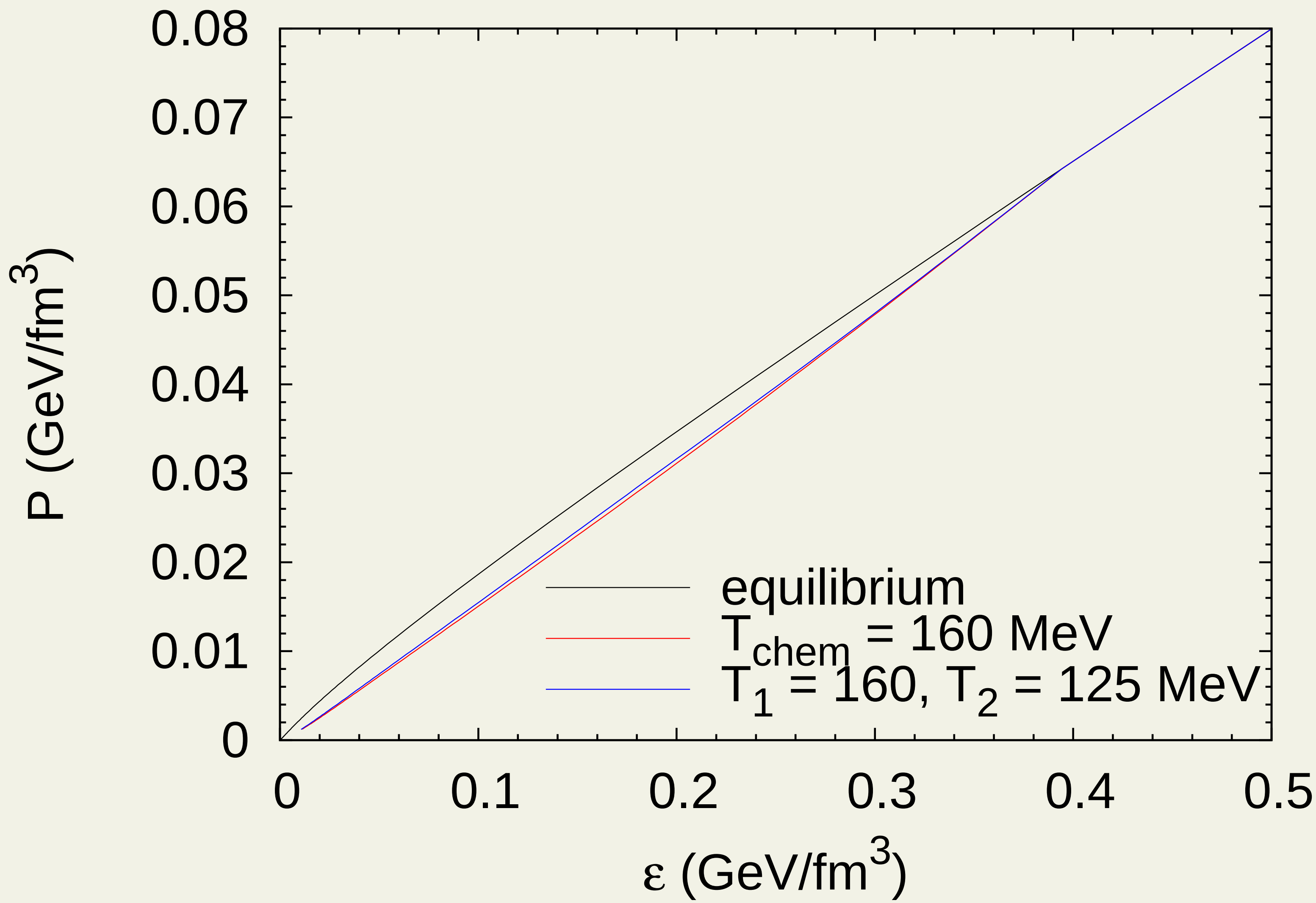
chemical freeze-out ⇒ main effect via temperature

Equation of state



$N\bar{N}$-**annihilations hardly change the EoS**

Equation of state



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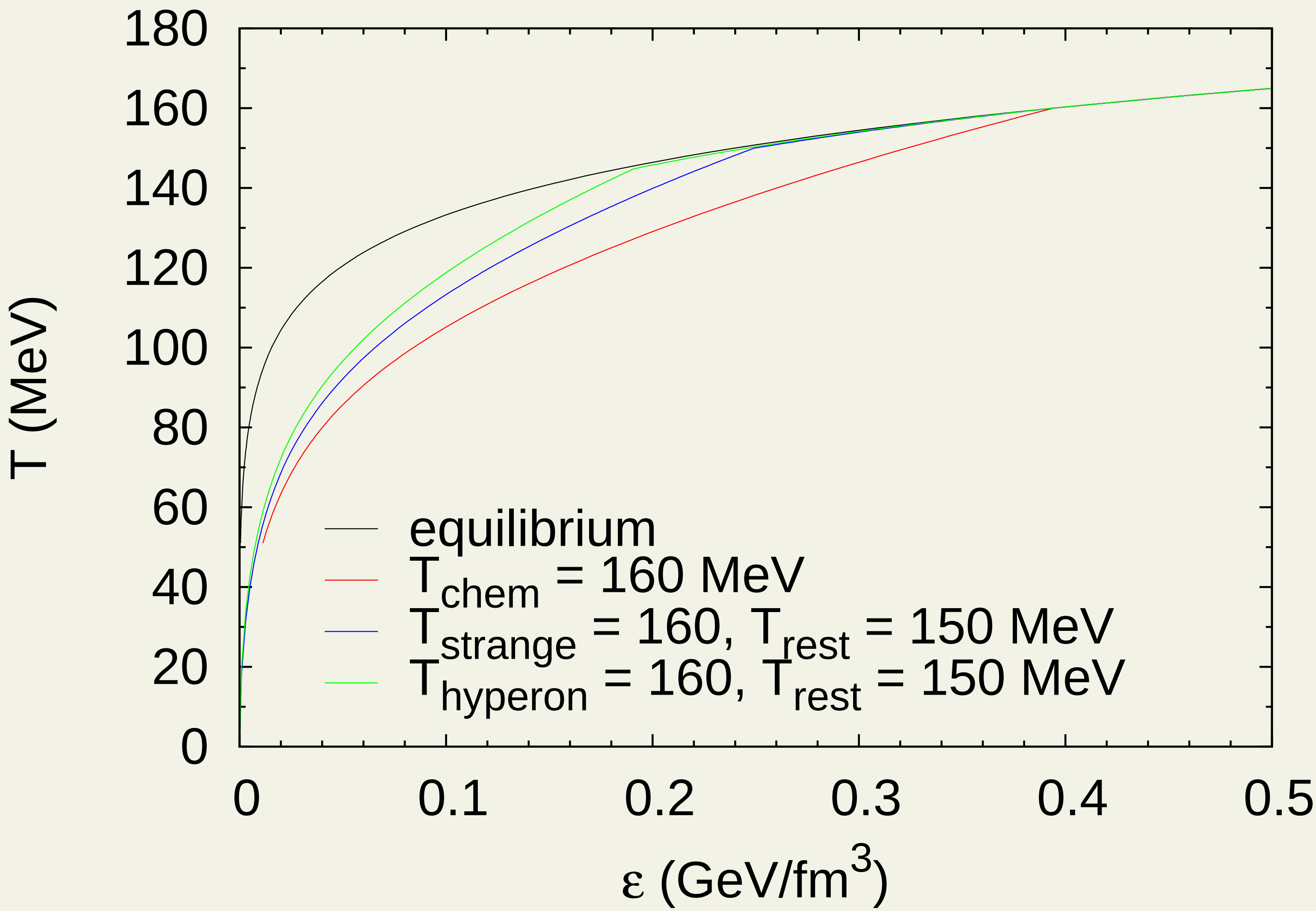
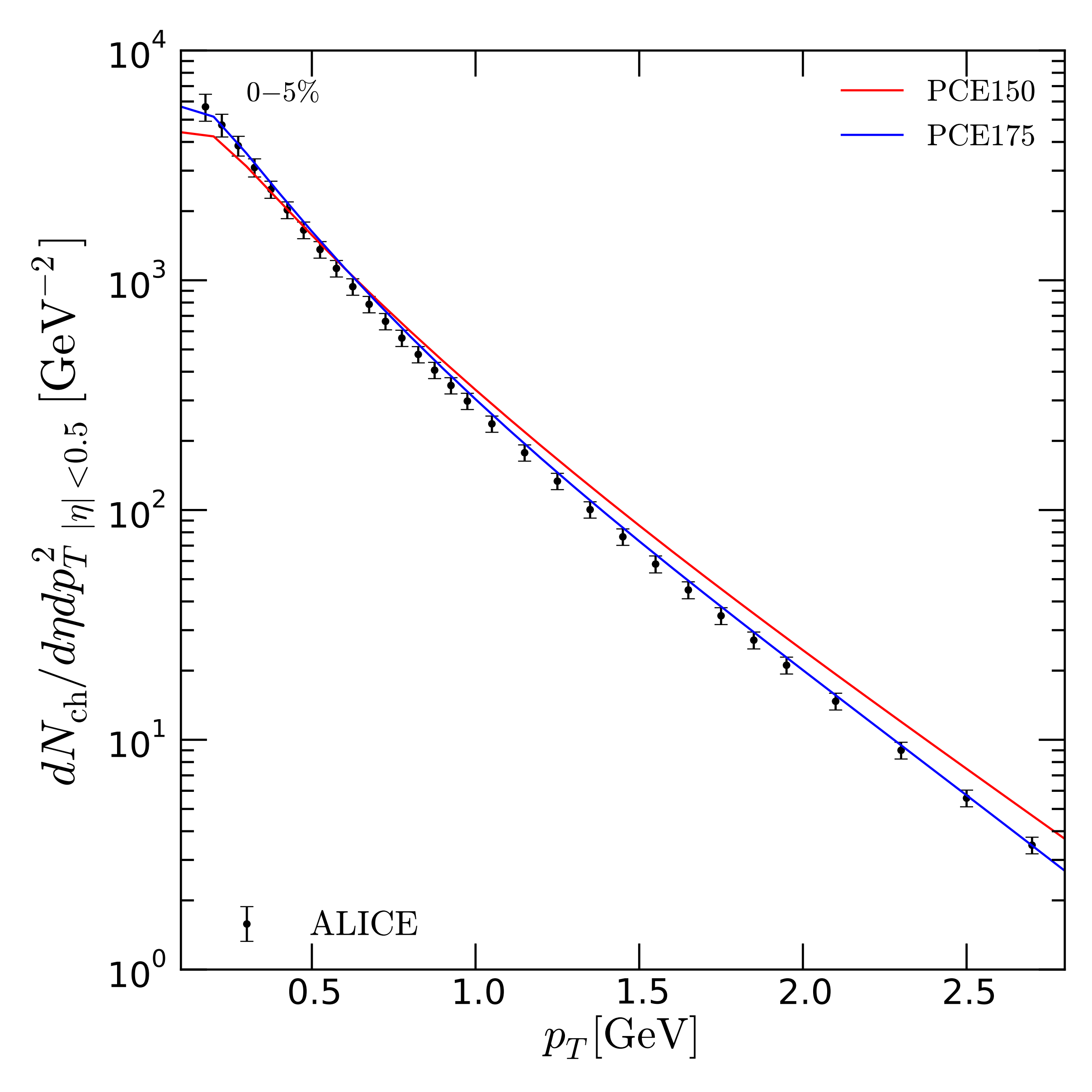


separate strange particle or hyperon freeze-out affects the EoS!

Charged hadron p_T spectrum at LHC



PCE150:
fit to π, K, p yields
no fit to spectrum

PCE175:
no fit to yields
fits the spectrum

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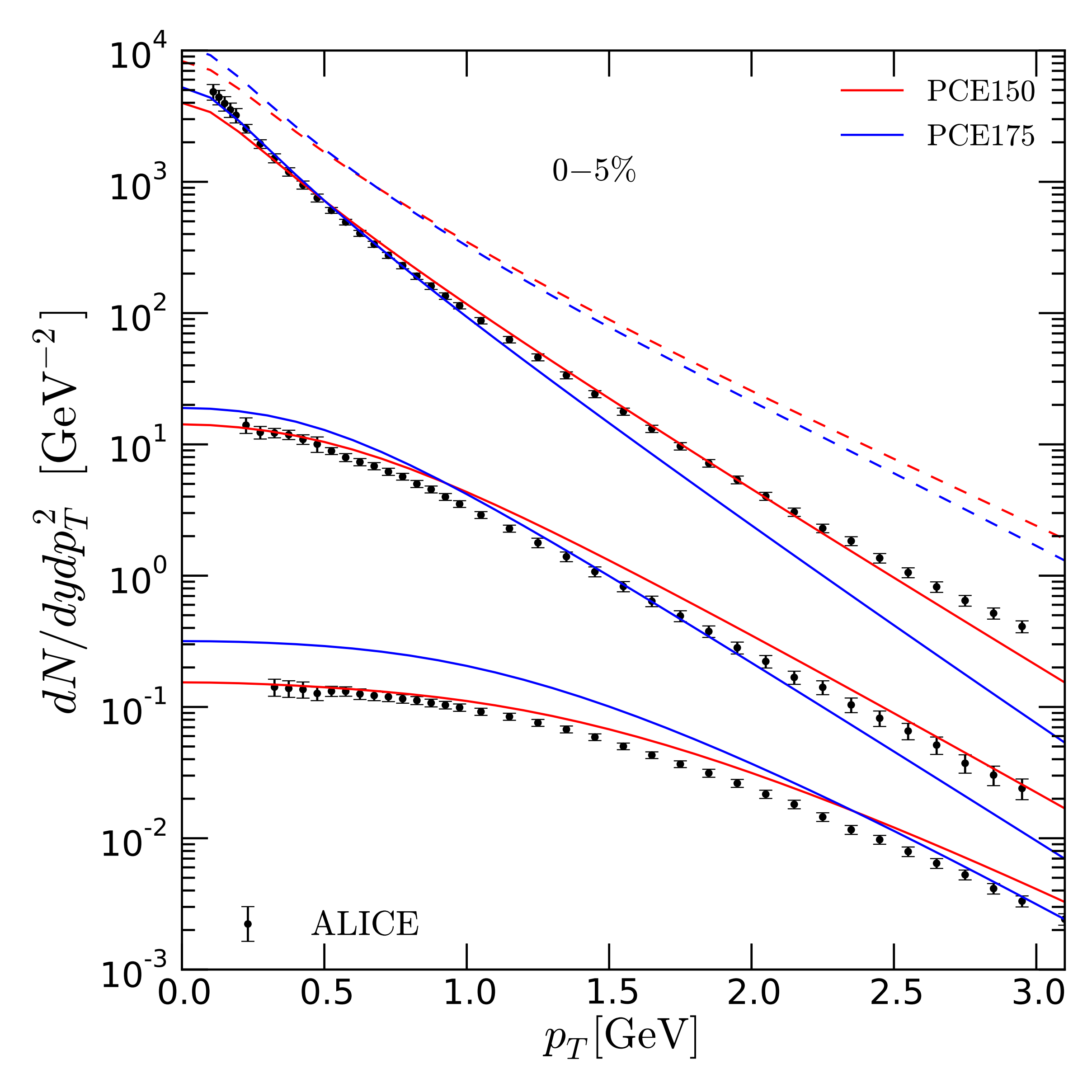
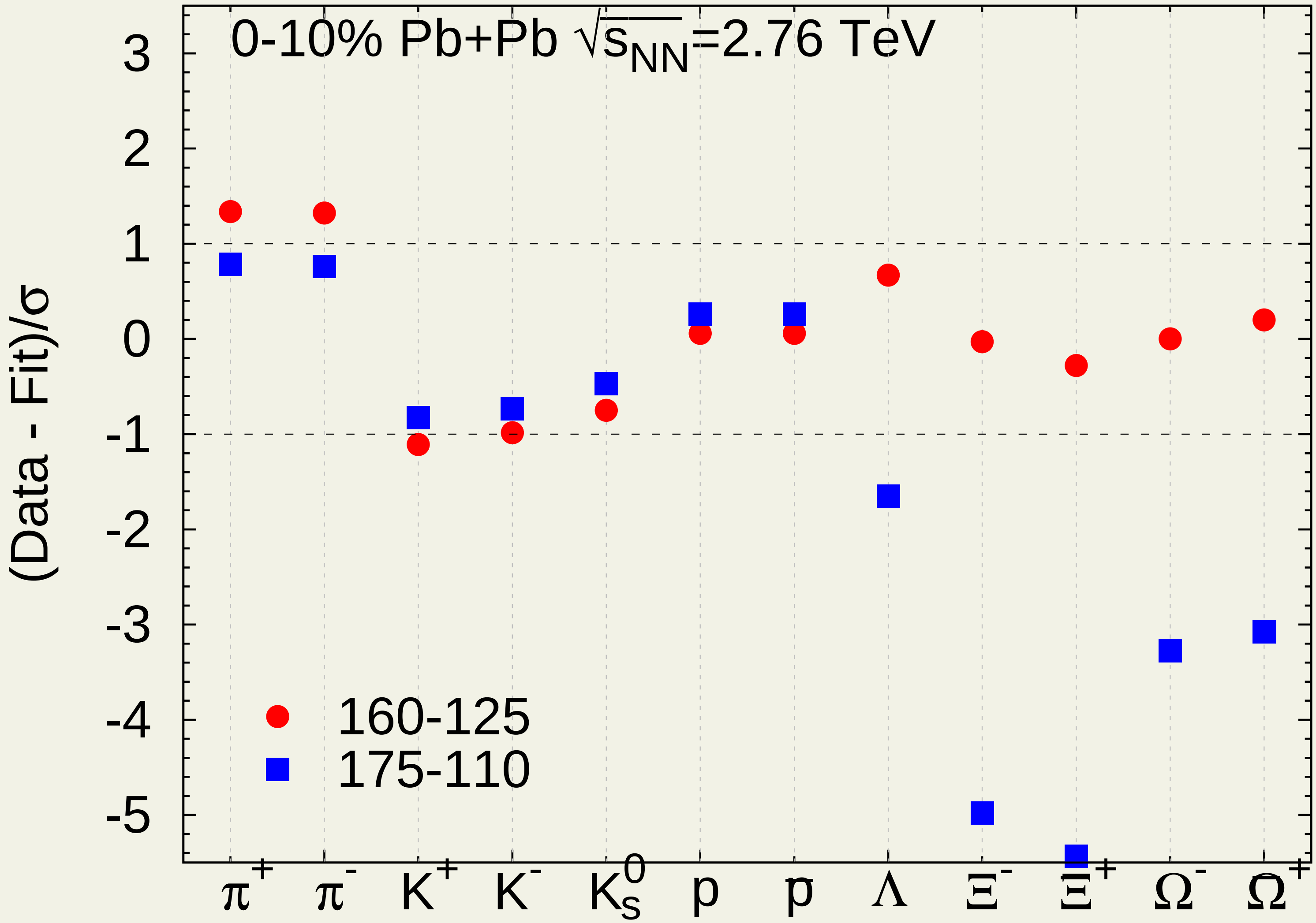
Identified particle p_T spectrum at LHC



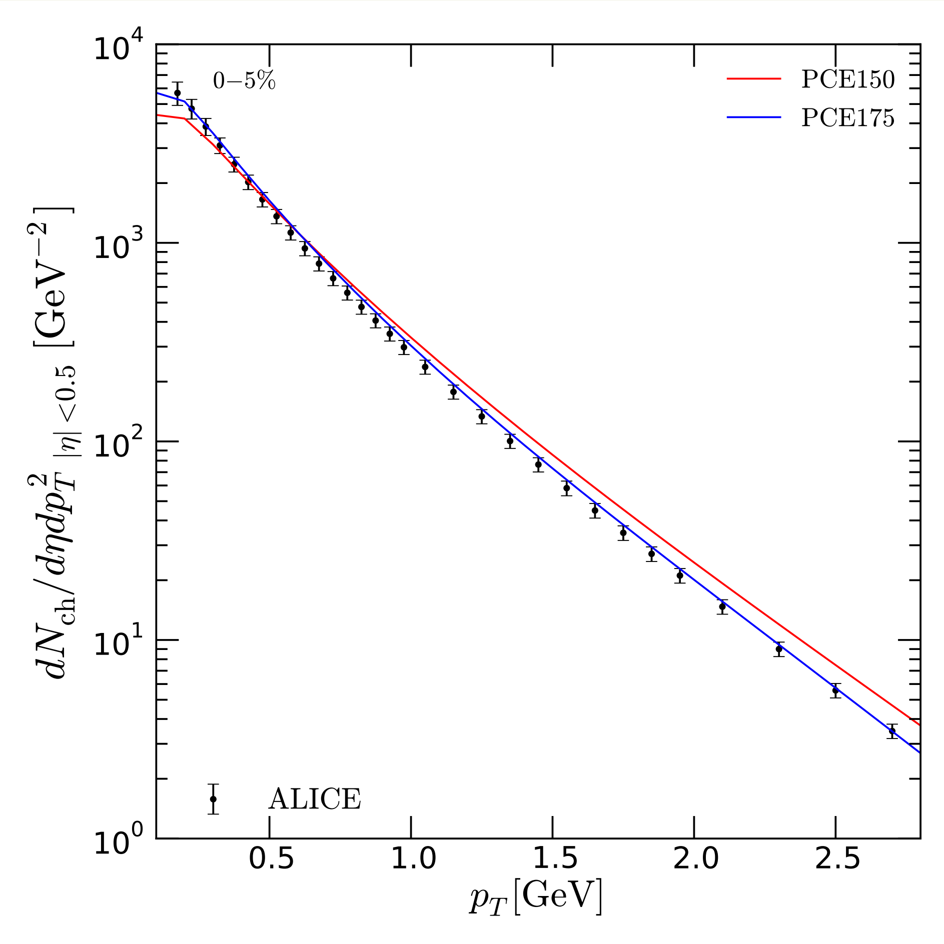
PCE150:
fits pions at high p_T

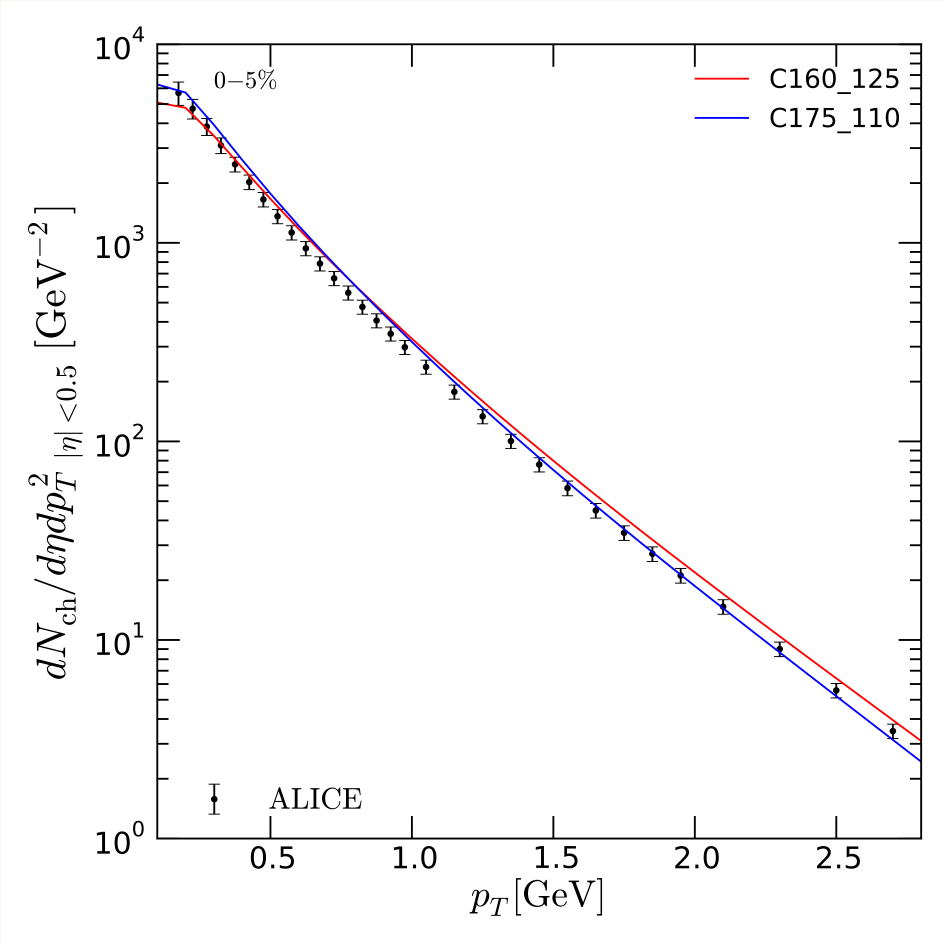
PCE175:
fits pions at low p_T

©H. Niemi


0-10% Pb+Pb $\sqrt{s_{NN}}$=2.76 TeV
(Data - Fit)/σ
3
2
1
0
-1
-2
-3
-4
-5
160-125
175-110
π^+ π^- K^+ K^- K_S^0 p $\bar{p}$ Λ Ξ^- $\bar{\Xi}^+$ Ω^- $\bar{\Omega}^+$

Charged hadron p_T spectrum at LHC

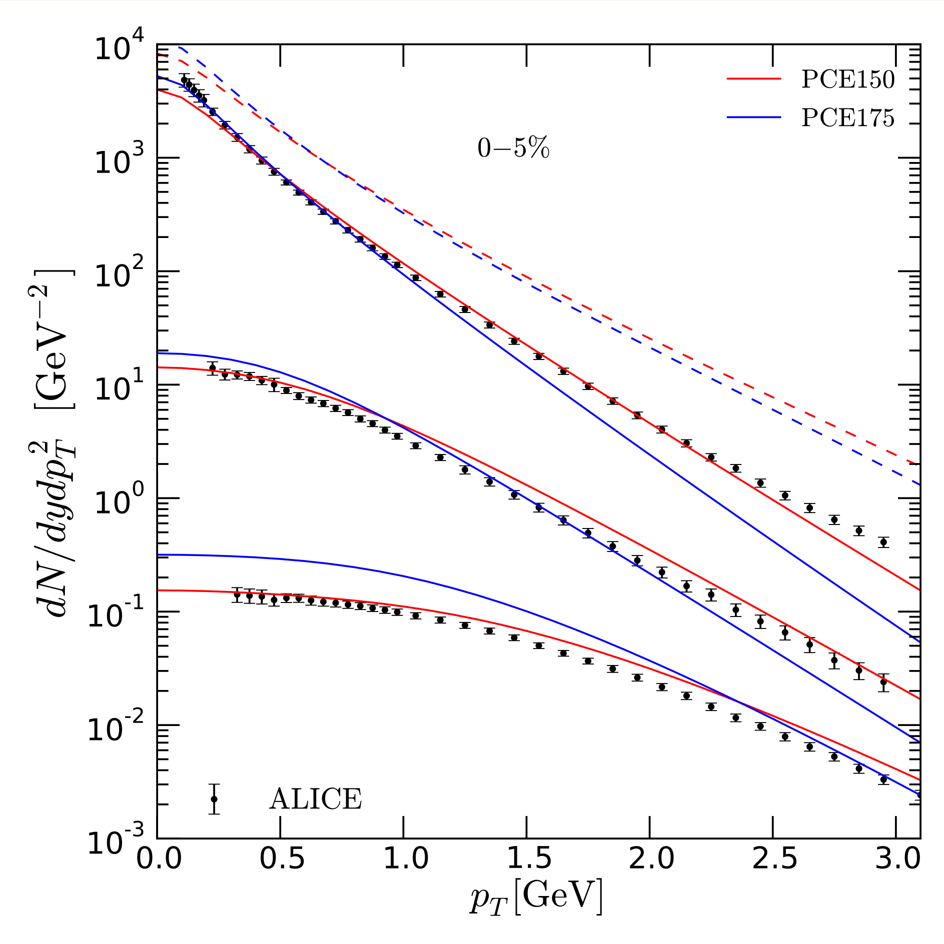


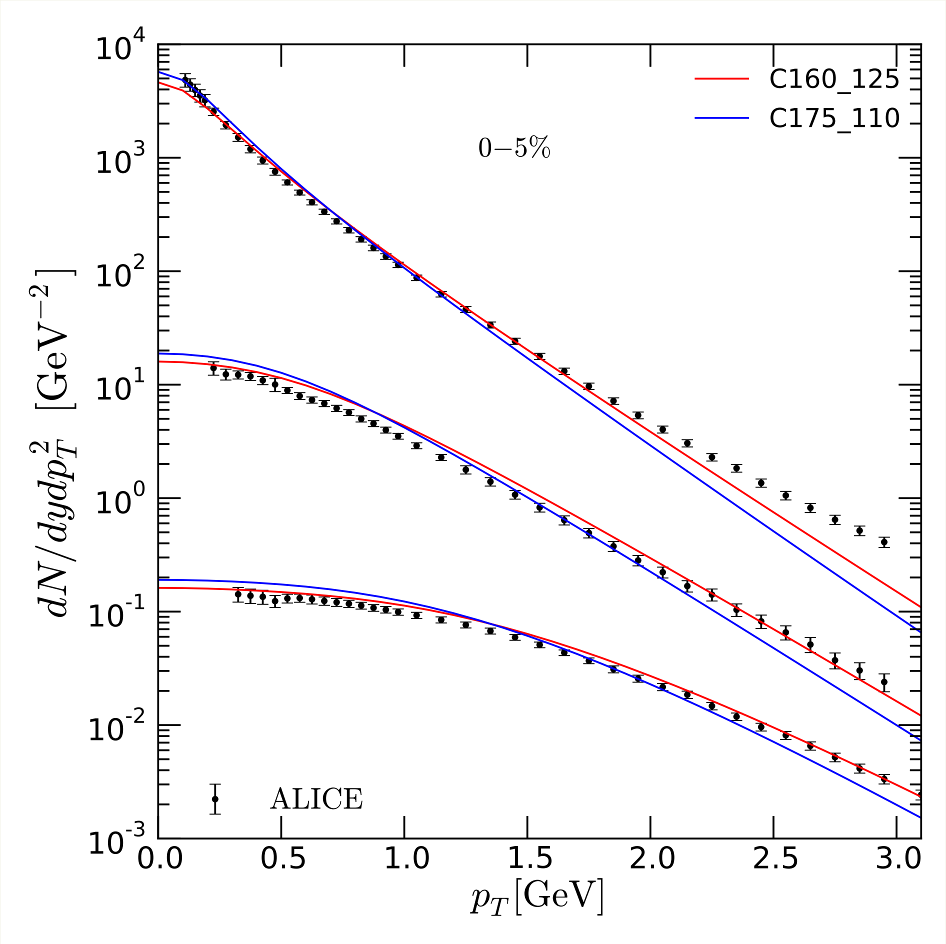


PCE150: $T_{\rm chem} = 150$ **MeV**
PCE175: $T_{\rm chem} = 175$ **MeV**

C160_125: $T_1 = 160$ **MeV**, $T_2 = 125$ **MeV**
C175_110: $T_1 = 175$ **MeV**, $T_2 = 110$ **MeV**

Identified particle p_T spectrum at LHC





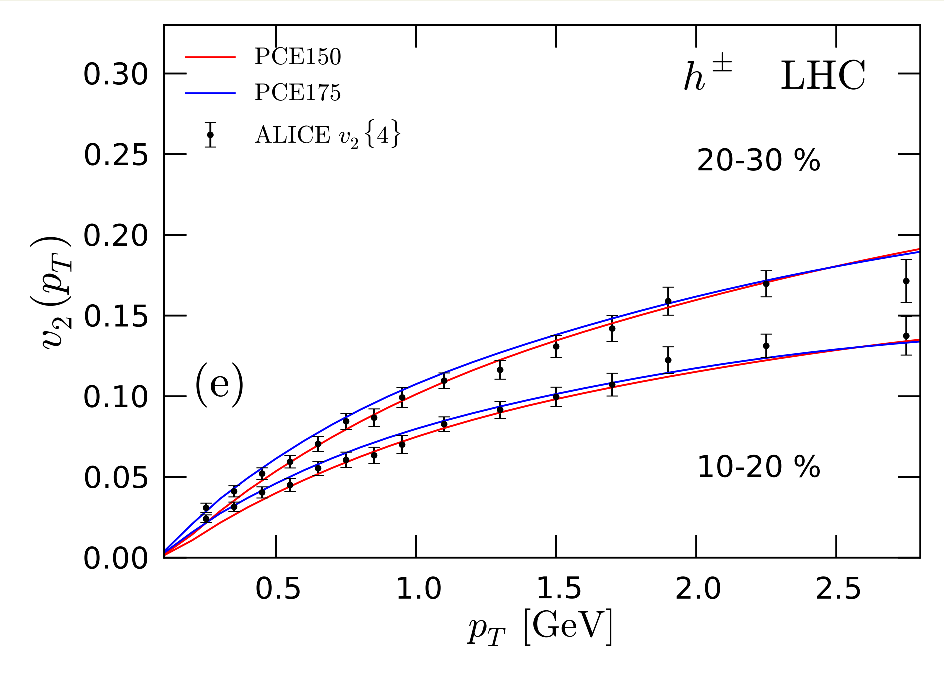
PCE150: $T_{\rm chem} = 150$ **MeV**

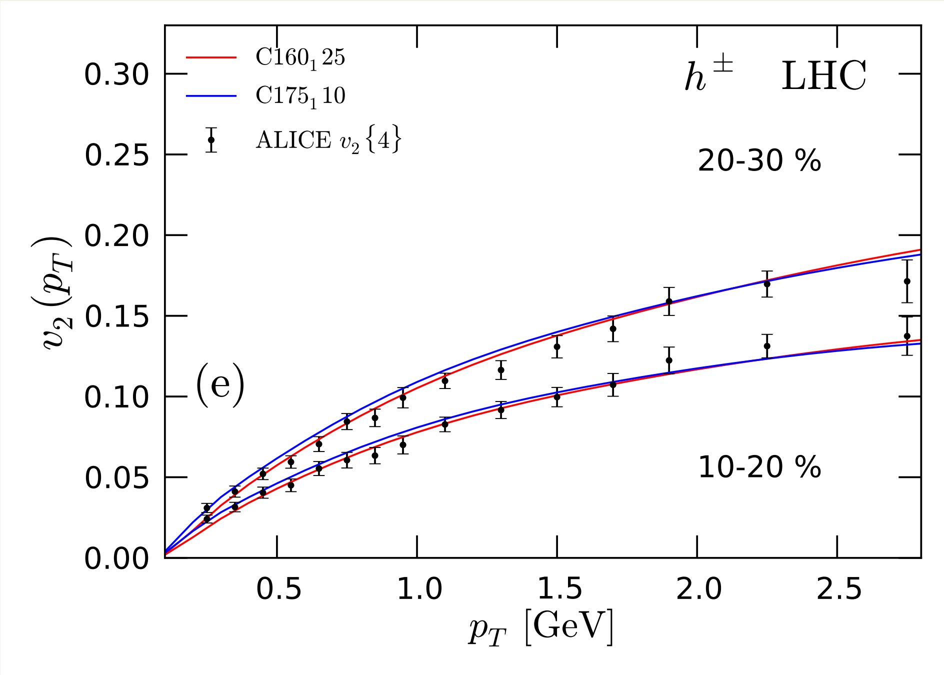
PCE175: $T_{\rm chem} = 175$ **MeV**

C160_125: $T_1 = 160$ **MeV,** $T_2 = 125$ **MeV**

C175_110: $T_1 = 175$ **MeV,** $T_2 = 110$ **MeV**

$v_2(p_T)$ at LHC





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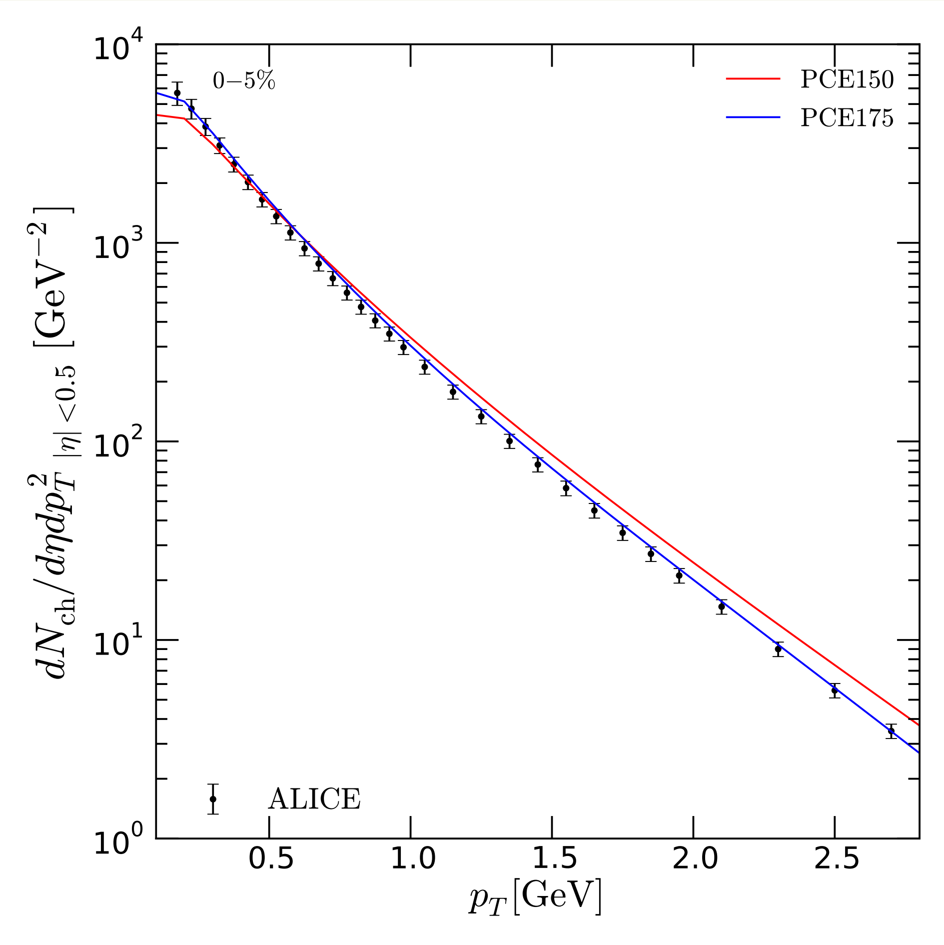
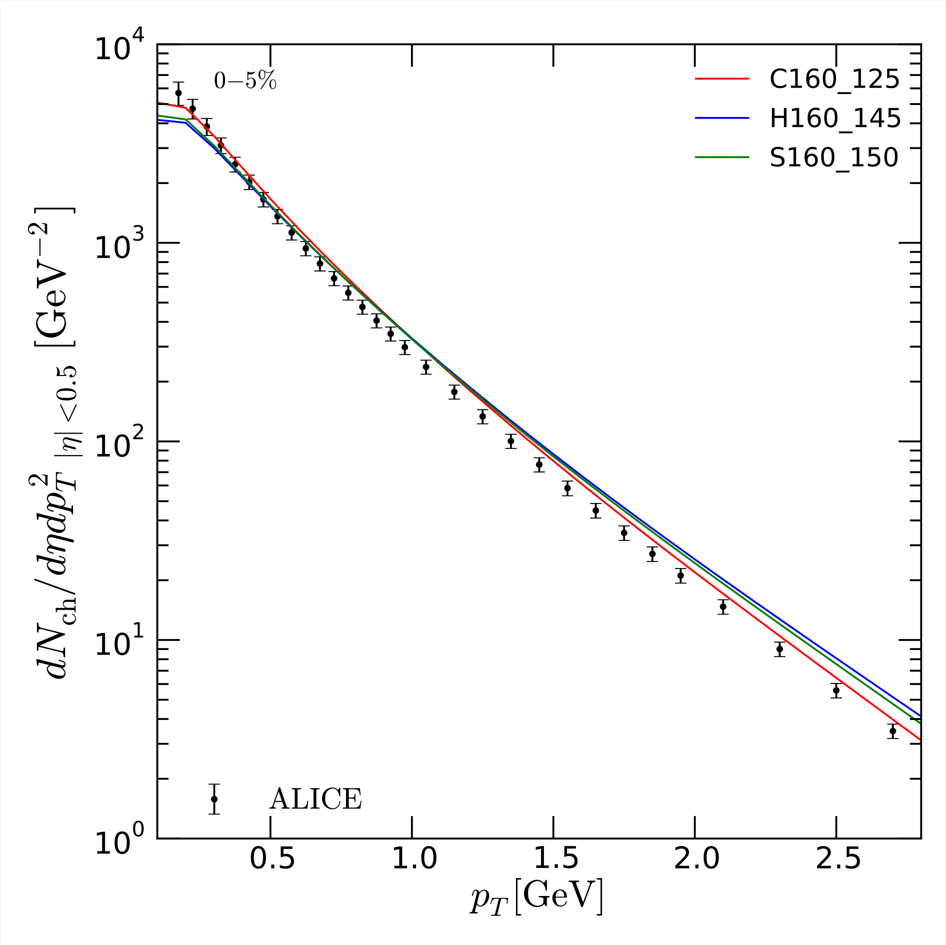
Charged hadron p_T spectrum at LHC





PCE150: $T_{\mathrm{chem}} = 150$ **MeV**

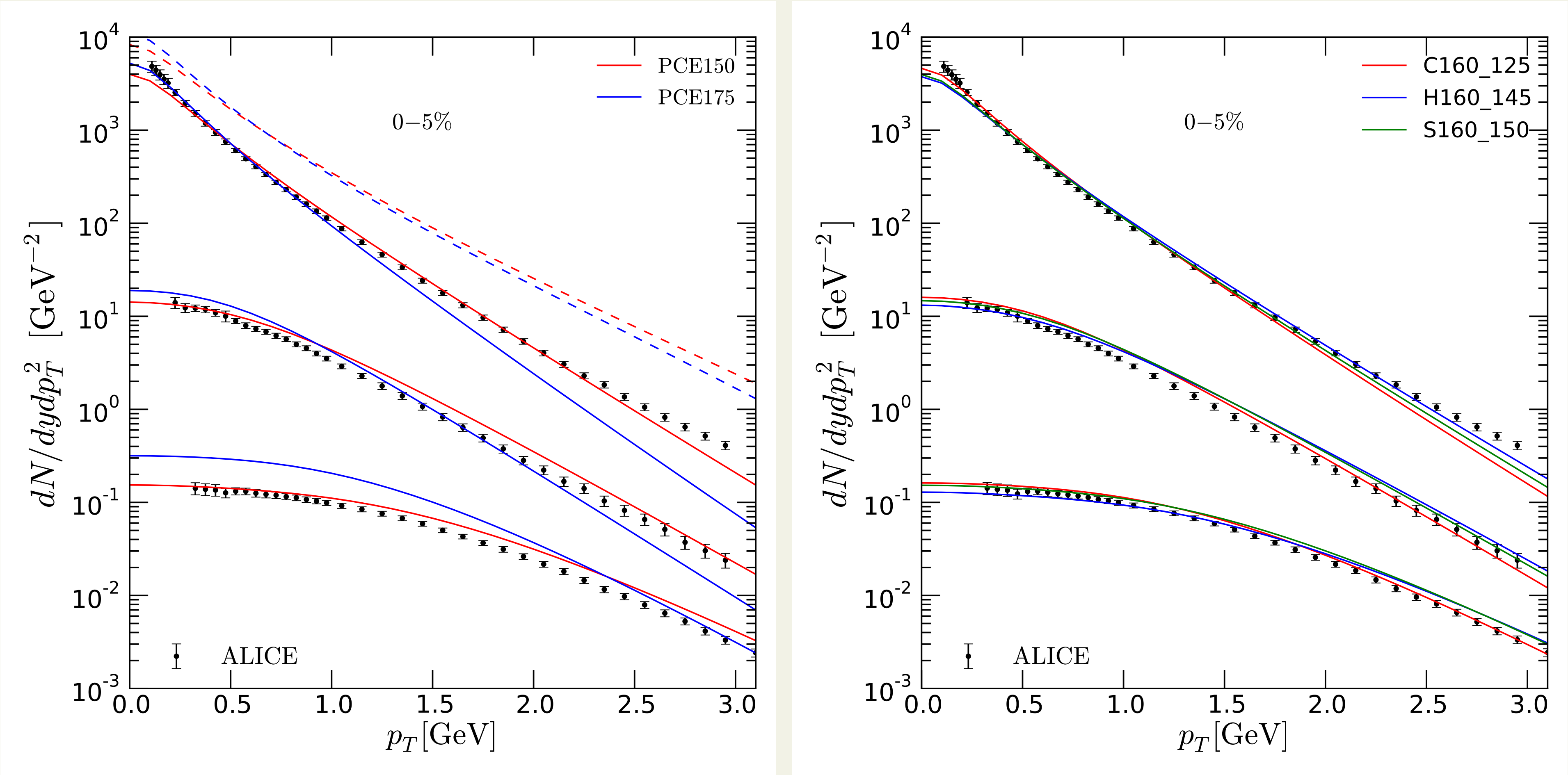
PCE175: $T_{\mathrm{chem}} = 175$ **MeV**

C160_125: $T_1 = 160$ **MeV**, $T_2 = 125$ **MeV**

H160_145: $T_{\mathrm{hyperon}} = 160$ **MeV**, $T_{\mathrm{rest}} = 145$ **MeV**

S160_150: $T_{\mathrm{strange}} = 160$ **MeV**, $T_{\mathrm{rest}} = 150$ **MeV**

Identified particle p_T spectrum at LHC



PCE150: $T_{\rm chem} = 150$ **MeV**

PCE175: $T_{\rm chem} = 175$ **MeV**

C160_125: $T_1 = 160$ **MeV**, $T_2 = 125$ **MeV**

H160_145: $T_{\rm hyperon} = 160$ **MeV**, $T_{\rm rest} = 145$ **MeV**

S160_150: $T_{\rm strange} = 160$ **MeV**, $T_{\rm rest} = 150$ **MeV**

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- possible to construct an EoS to fit the observed yields