Dissipative effects in mixtures

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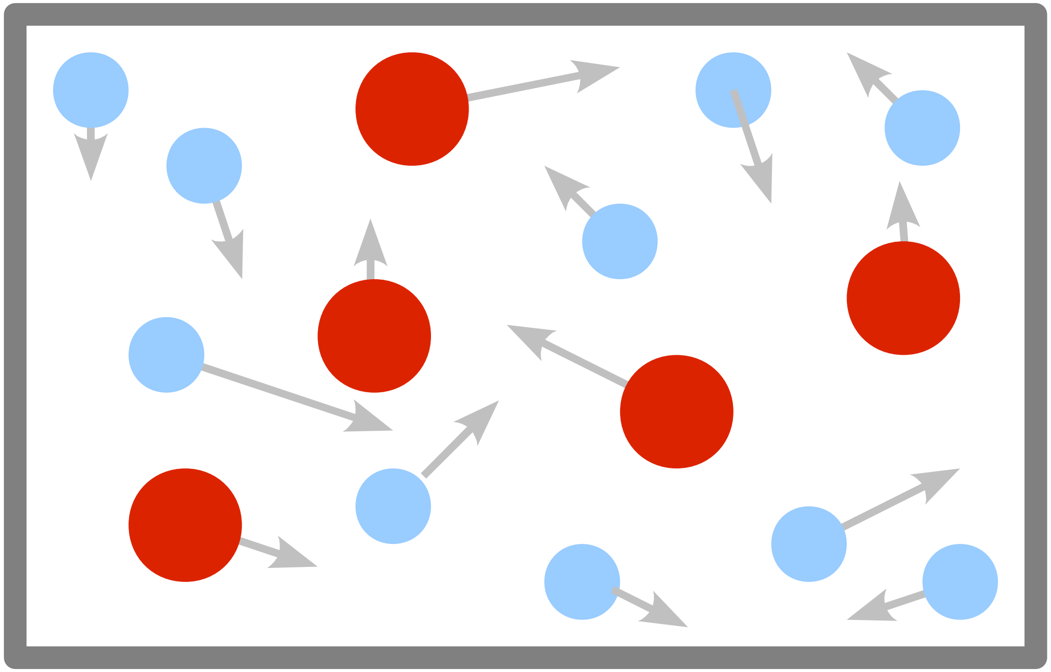
Motivation

Comparison of hydrodynamic calculations with experimental data
$\rightarrow$ extraction of η/s, *EoS* ...

From η/s we learn about the inner dynamics in the medium

Motivation

Can we apply standard, one-component hydrodynamics to describe dissipative effects in a mixture?



Interaction cross sections

σ_{11}, σ_{12}, σ_{22}

Two distinct mean-free path scales

QGP or Hadron Gas are mixtures

Dissipative hydro

Ideal hydrodynamics:

$$\partial_\mu T^{\mu\nu}=0$$
$$\partial_\mu N^\mu=0$$
$$EoS$$

$T^{\mu\nu}$ is isotropic locally ↔ Momentum distribution is isotropic

Israel-Stewart hydrodynamics:

$$\partial_\mu T^{\mu\nu}=0$$
$$\partial_\mu N^\mu=0$$
$$\dot{\pi}^{\mu\nu}=-\frac{\pi^{\mu\nu}}{\tau_\pi}+\frac{\sigma^{\mu\nu}}{\beta_2}+\pi^{\mu\nu}\frac{T}{\beta_2}\partial_\alpha\left(\frac{\beta_2}{2\mathrm{T}}u^\alpha\right)$$

↓

Relaxation time $\tau_\pi=2\beta_2\eta$

$\pi^{\mu\nu}$ *describes anisotropy of the momentum space distribution*

Dissipative hydro in a mixture

$\partial_\mu T^{\mu\nu} = 0$ *total energy is conserved*

$\partial_\mu N^\mu = 0$ *total particle number is conserved*

$\dot{\pi}^{\mu\nu} = \sum \dot{\pi}_i^{\mu\nu}$

↓

Same as the standard Israel-Stewart for each mixture component?
*Then, what is the viscosity of **mixture components**?*

Dissipative hydro in a mixture

For our derivation we assume:

- particle numbers are conserved (no radiative processes)
- $T_1=T_2=T$ (rather strong assumption, but we really need this one)
- the global frame u^μ is not very different from the frames u^μ_i of the components

Need to check these two in transport calculations

Dissipative hydro in a mixture

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- the global frame u^μ is not very different from the frames u^μ_i of the components

- we take isotropic scatterings, $d\sigma/d\Omega$ independent of Ω

↓

can translate cross section to viscosity $\eta = \frac{6}{5}\frac{T}{\sigma}$

Dissipative hydro in a mixture

For two components with a given n_1 / n_2 and cross sections σ_{11}, σ_{12}, σ_{22}

$$\dot{\pi}_1 = -\pi_1 \cdot \left(\frac{5}{9} n_1 \sigma_{11} + \frac{7}{9} n_2 \sigma_{12} \right) + \pi_2 \cdot \frac{2}{9} n_1 \sigma_{12} + gradient \quad terms$$

$$\dot{\pi}_2 = -\pi_2 \cdot \left(\frac{5}{9} n_2 \sigma_{22} + \frac{7}{9} n_1 \sigma_{12} \right) + \pi_1 \cdot \frac{2}{9} n_2 \sigma_{12} + gradient \quad terms$$

Two relaxation times per equation
Compare with the Israel-Stewart Eq.:

$$\dot{\pi} = -\pi \cdot \frac{5}{9} n \sigma + + gradient \quad terms$$

Dynamics in a mixture

Let's check the relaxation part of the equations

$$\dot{\pi}_1 = -\pi_1 \cdot \left(\frac{5}{9} n_1 \sigma_{11} + \frac{7}{9} n_2 \sigma_{12} \right) + \pi_2 \cdot \frac{2}{9} n_1 \sigma_{12} + gradient \quad terms$$

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Dynamics in a mixture

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BAMPS BOX

n_1, n_2

$T_1 = T_2 = T$

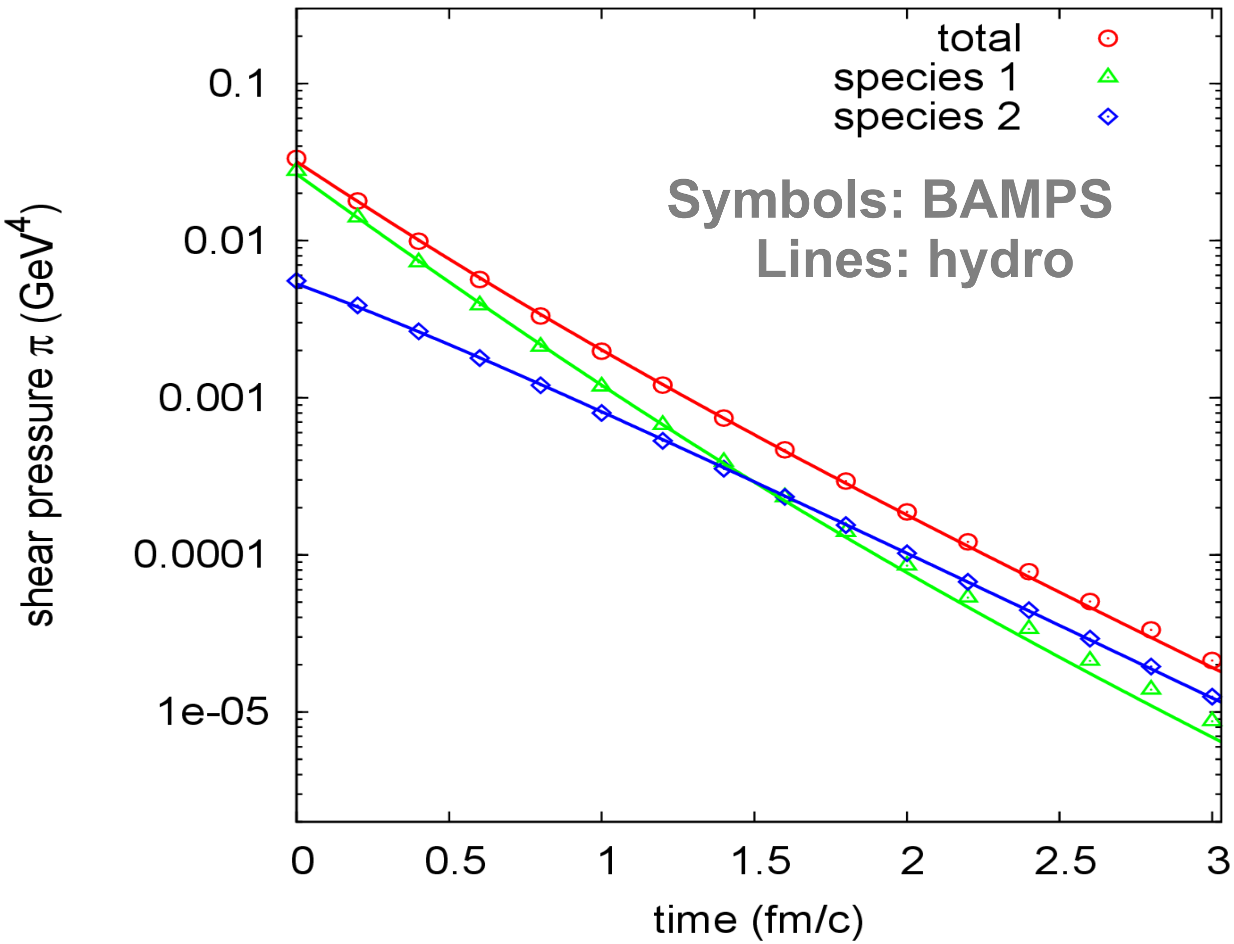
$\sigma_{11}, \sigma_{12}, \sigma_{22}$

π_1, π_2

$$f_i = d \cdot e^{-E/T} \cdot \left(1 + \frac{3}{8 e_i T^2} \cdot \pi_i \cdot \left(\frac{1}{2} p_T^2 - p_z^2 \right) \right)$$

Grad 's Formalism

Mixture in BAMPS



$n_1 / n_2 = 5$

initial $\pi_1 / \pi_2 = n_1 / n_2$

$\sigma_{11} = 4$ *mb*

$\sigma_{12} = 2$ *mb*,

$\sigma_{22} = 1$ *mb*

$T = 0.4$ *GeV*

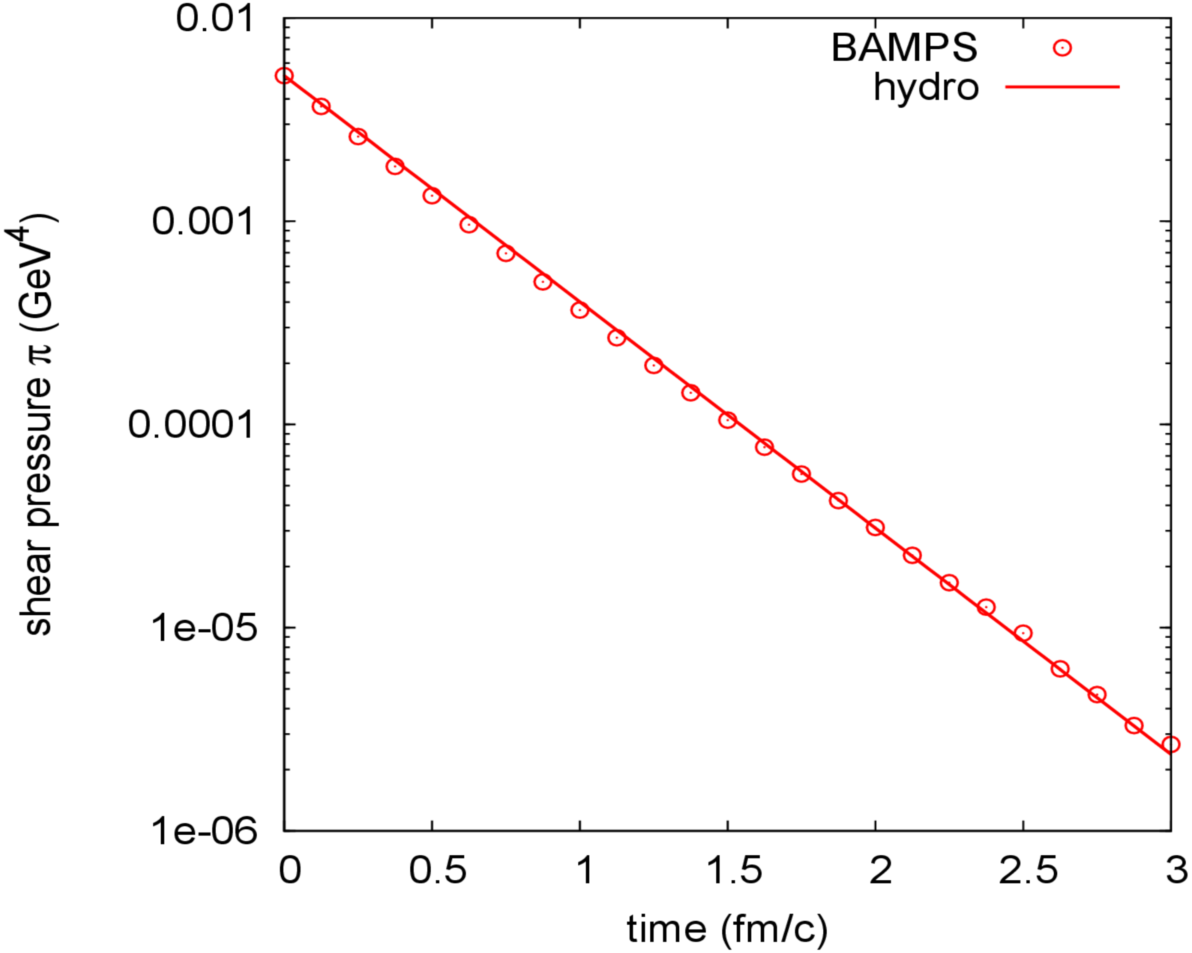
$$\dot{\pi}_1 = -\pi_1 \cdot \left(\frac{5}{9} n_1 \sigma_{11} + \frac{7}{9} n_2 \sigma_{12} \right) + \pi_2 \cdot \frac{2}{9} n_1 \sigma_{12}$$

$$\dot{\pi}_2 = -\pi_2 \cdot \left(\frac{5}{9} n_2 \sigma_{22} + \frac{7}{9} n_1 \sigma_{12} \right) + \pi_1 \cdot \frac{2}{9} n_2 \sigma_{12}$$

Relaxation in BAMPS

In the standard **one-component** Israel-Stewart hydrodynamics:

$$\dot{\pi} = -\frac{\pi}{\tau_\pi} = -\pi \cdot \frac{5}{9} n \sigma \longrightarrow \pi = \pi(0) \cdot e^{-\tau/\tau_\pi}$$



σ= 3.4 mb

T= 0.4 GeV

Green-Kubo

Application of Green-Kubo formula in BAMPS*: C.Wesp et al, arXiv:1106.4306*

$$\eta = \frac{V}{T} \int_0^\infty C(\tau)\, d\tau$$

Auto-correlation function

$$C(\tau) = \frac{1}{3}\left(\langle \pi^{xy}(0)\pi^{xy}(\tau) \rangle + \langle \pi^{xz}(0)\pi^{xz}(\tau) \rangle + \langle \pi^{yz}(0)\pi^{yz}(\tau) \rangle \right)$$

τ = correlation time

Green-Kubo

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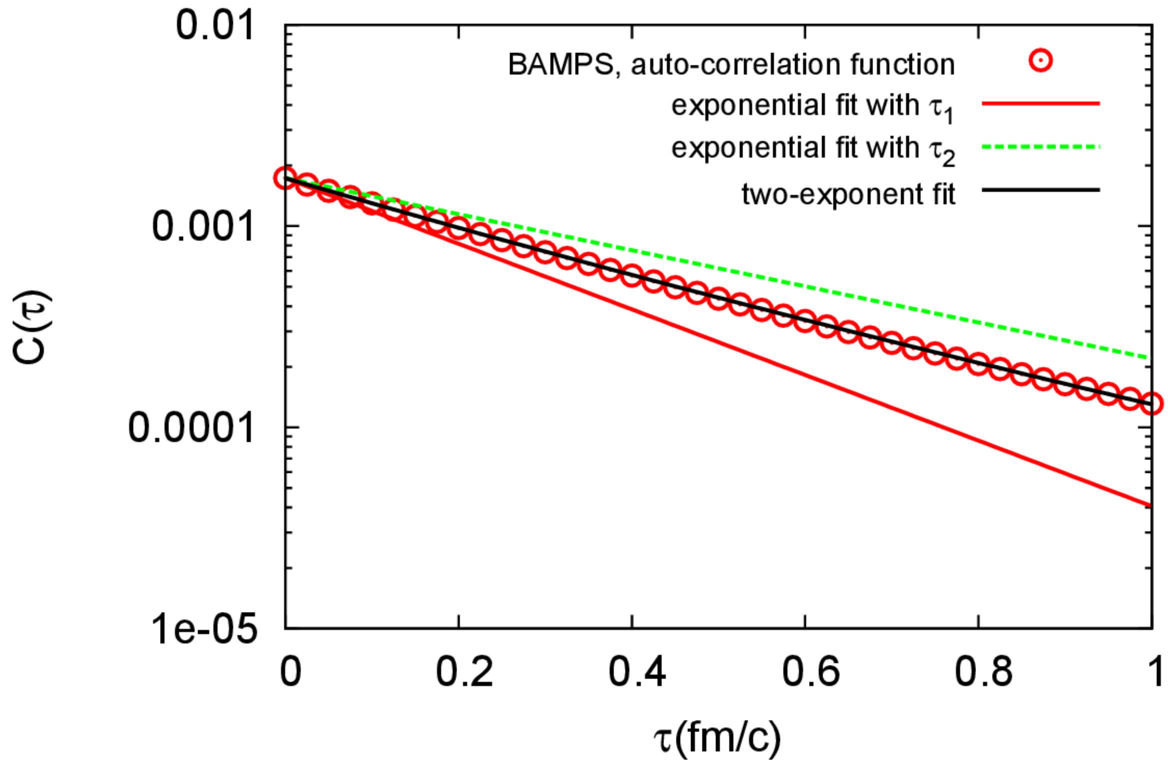
$$\eta = \frac{V}{T}\int_0^\infty C(\tau)\, d\tau$$

Mixture in BAMPS



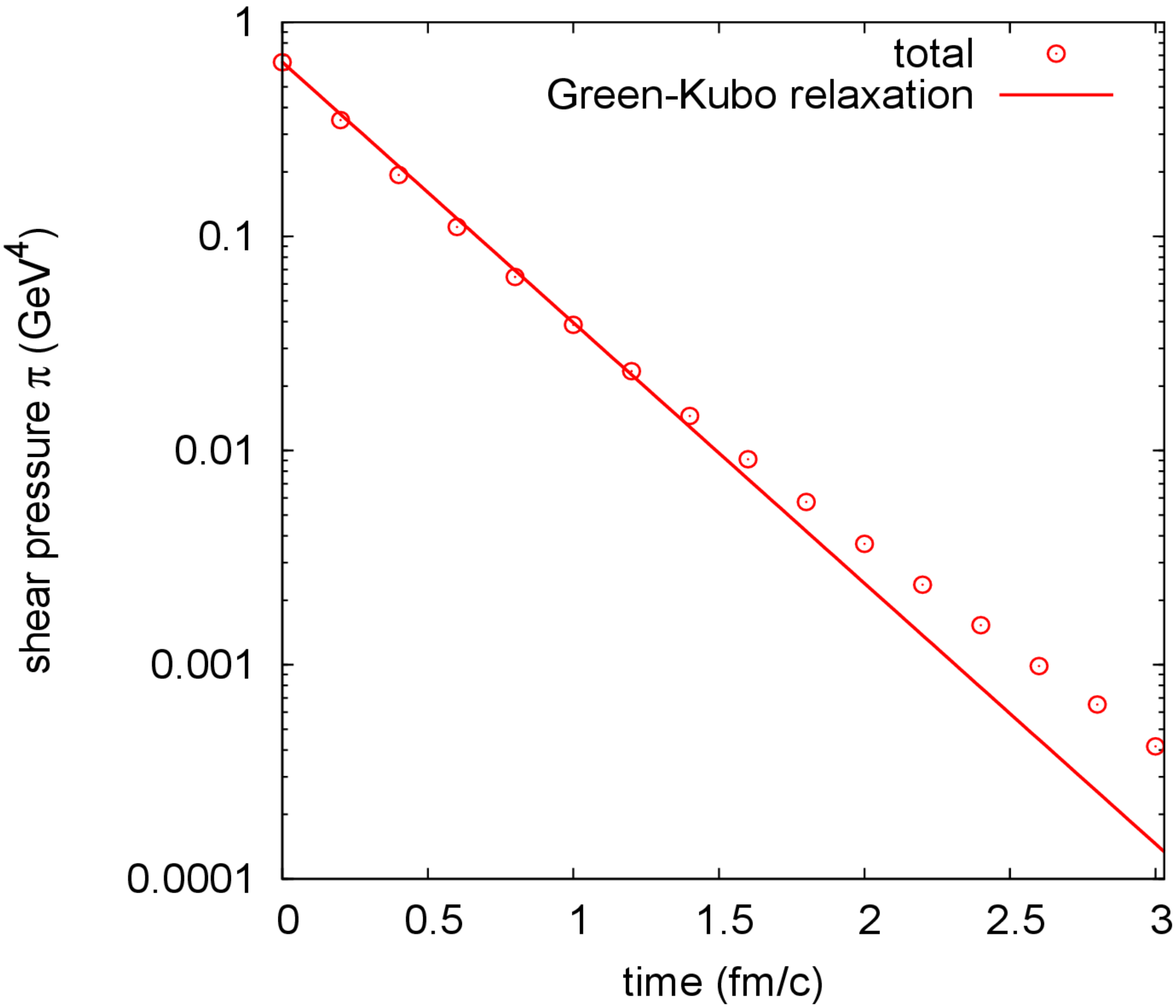
$$C(\tau) = C_1 \cdot e^{-\tau/\tau_1} + C_2 \cdot e^{-\tau/\tau_2}$$

$$\tau_1 \approx 0.2\ fm/c$$

$$\tau_2 \approx 0.4\ fm/c$$

$\eta = 0.062\ GeV^3$

Mixture in BAMPS



$n_1 / n_2 = 5$

initial $\pi_1 / \pi_2 = n_1 / n_2$

$\sigma_{11} = 4\ mb$

$\sigma_{12} = 2\ mb$,

$\sigma_{22} = 1\ mb$

$T = 0.4\ GeV$

Green-Kubo viscosity
$\eta = 0.062\ GeV^3$

Dynamics in a mixture

$$\dot{\pi}_1 = -\pi_1 \cdot \left(\frac{5}{9} n_1 \sigma_{11} + \frac{7}{9} n_2 \sigma_{12} \right) + \pi_2 \cdot \frac{2}{9} n_1 \sigma_{12}$$

$$\dot{\pi}_2 = -\pi_2 \cdot \left(\frac{5}{9} n_2 \sigma_{22} + \frac{7}{9} n_1 \sigma_{12} \right) + \pi_1 \cdot \frac{2}{9} n_2 \sigma_{12}$$

$$\pi(\tau) = A \cdot e^{-\tau/\tau_1} + B \cdot e^{-\tau/\tau_2}$$

$$\pi(\tau) = \pi_0 \cdot e^{-\tau/\tau_\pi}$$

$$\eta(\tau) = \frac{2}{5} \cdot e \cdot \left(\frac{r(\tau)}{1 + r(\tau)} \cdot \lambda_1^{-1} + \frac{1}{1 + r(\tau)} \cdot \lambda_2^{-1} \right)$$

with

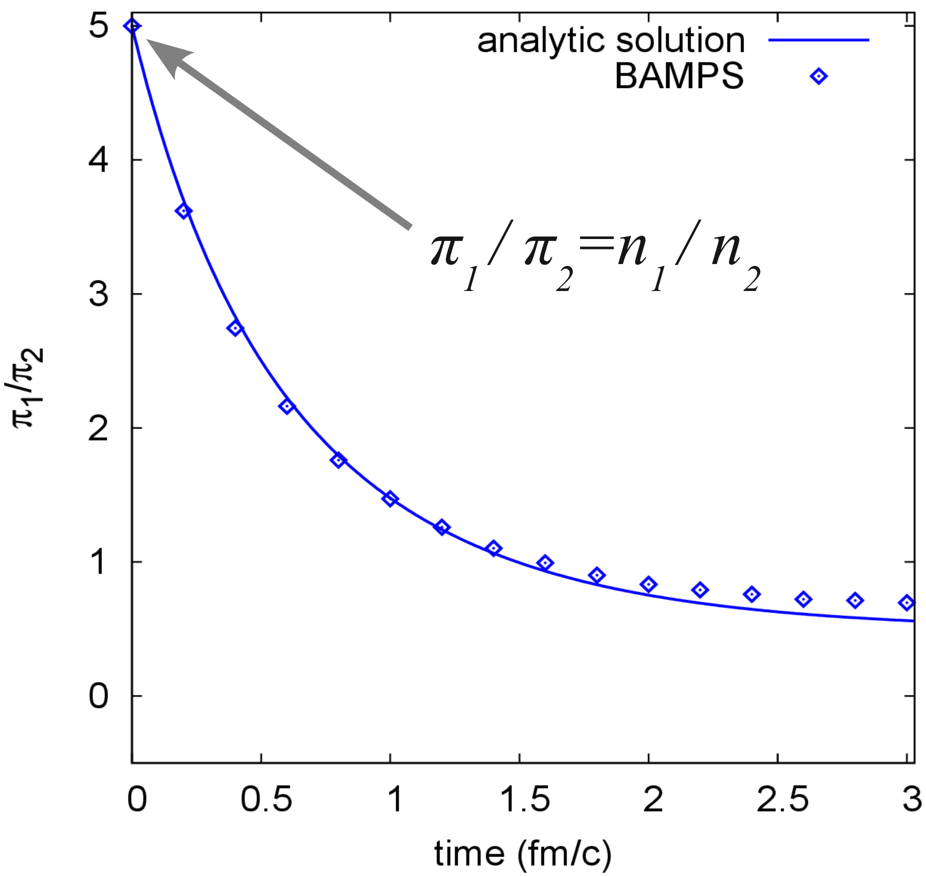
$$r(\tau) = \frac{\pi_1(\tau)}{\pi_2(\tau)} = A(n, \sigma) \cdot \tanh\left(\tau \cdot B(n, \sigma) + C(n, \sigma, \pi_0) \right)$$

$$\lambda_1^{-1} = n_1 \sigma_{11} + n_2 \sigma_{12} \qquad \lambda_2^{-1} = n_2 \sigma_{22} + n_1 \sigma_{12}$$

Mixture in BAMPS

$$\eta(\tau)=\frac{2}{5}\cdot e\cdot\left(\frac{r(\tau)}{1+r(\tau)}\cdot\lambda_1^{-1}+\frac{1}{1+r(\tau)}\cdot\lambda_2^{-1}\right)$$

$$r(\tau)=\frac{\pi_1(\tau)}{\pi_2(\tau)}=A(n,\sigma)\cdot\tanh\left(\tau\cdot B(n,\sigma)+C(n,\sigma,\pi_0)\right)$$

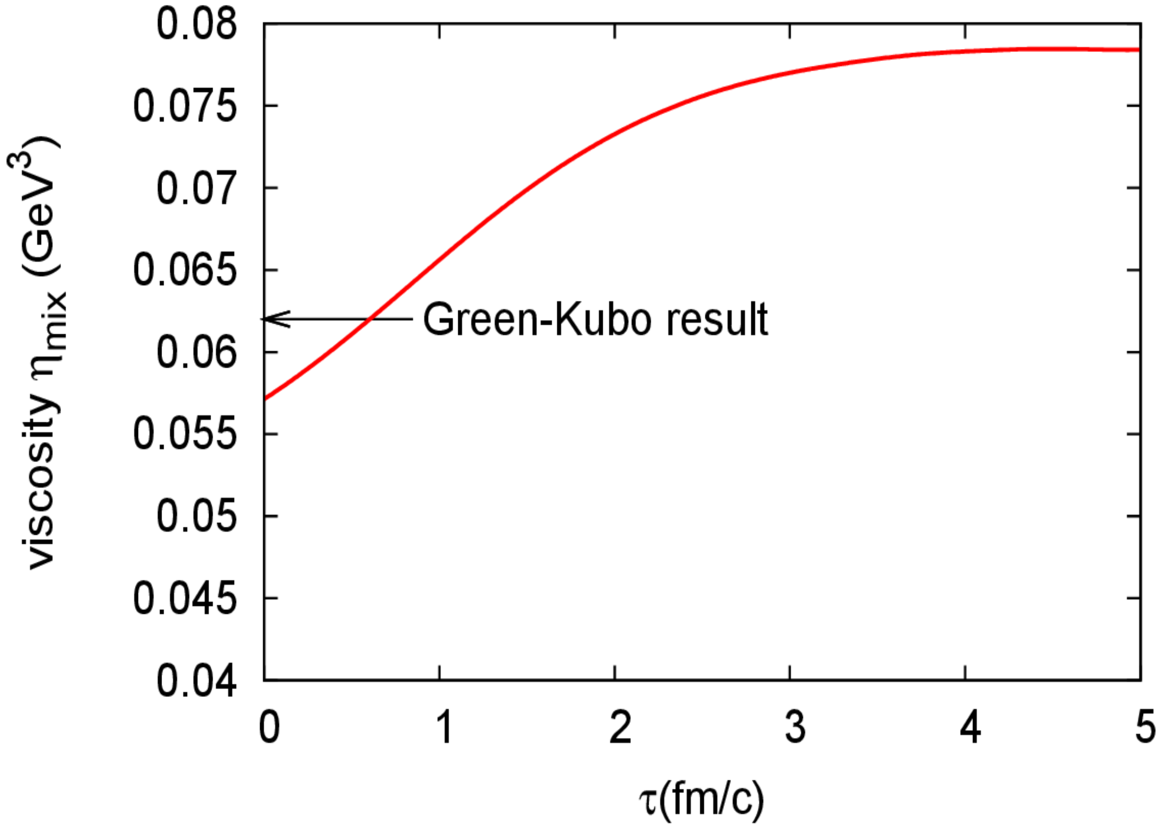


Existence of a characteristic stationary value

Dynamics in mixtures

$$\eta(\tau) = \frac{2}{5} \cdot e \cdot \left(\frac{r(\tau)}{1+r(\tau)} \cdot \lambda_1^{-1} + \frac{1}{1+r(\tau)} \cdot \lambda_2^{-1} \right)$$

$$r(\tau) = \frac{\pi_1(\tau)}{\pi_2(\tau)} = A(n,\sigma) \cdot \tanh\left(\tau \cdot B(n,\sigma) + C(n,\sigma,\pi_0)\right)$$



mixture with mean free path scales λ_1~0.2 fm and λ_2~0.4 fm

Dynamics in mixtures

$$\eta(\tau) = \frac{2}{5} \cdot e \cdot \left(\frac{r(\tau)}{1 + r(\tau)} \cdot \lambda_1^{-1} + \frac{1}{1 + r(\tau)} \cdot \lambda_2^{-1} \right)$$

$$r(\tau) = \frac{\pi_1(\tau)}{\pi_2(\tau)} = A(n, \sigma) \cdot \tanh\left(\tau \cdot B(n, \sigma) + C(n, \sigma, \pi_0)\right)$$

From the results so far we can conclude

> Existence of a characteristic time-dependence of the viscosity in a mixture

> Applicability of one-component hydrodynamics to a mixture depends on the chosen initial conditions

Initializing hydrodynamic calculations

$$\dot{\pi}^{\mu\nu} = -\frac{\pi^{\mu\nu}}{\tau_\pi} + \frac{\sigma^{\mu\nu}}{\beta_2} + \pi^{\mu\nu}\frac{T}{\beta_2}\partial_\alpha\left(\frac{\beta_2}{2\mathrm{T}}u^\alpha\right)$$

In one-component hydrodynamic calculations the standard choices are

$$\pi^{\mu\nu}(\tau_0) = 0 \quad \vee \quad \pi^{\mu\nu}(\tau_0) = 2\eta\sigma^{\mu\nu}$$

...but there is no clear prescription what's the right choice.

For a mixture this would mean

$$\pi_i^{\mu\nu}(\tau_0) = 0 \;\rightarrow\; \dot{\pi}_i^{\mu\nu}(\tau_0) = \frac{\sigma^{\mu\nu}}{\beta_i} \;\rightarrow\; \frac{\pi_1(\tau_0 + d\tau)}{\pi_2(\tau_0 + d\tau)} = \frac{\beta_2}{\beta_1} = \frac{e_1}{e_2} = \frac{n_1}{n_2}$$

Which also means, that the characteristic time-dependence of shear viscosity must be taken into account

Conclusions and Outlook

> standard one-component hydrodynamics in general cannot be applied to describe dissipative effects in mixtures

> It is only in case the initial conditions are chosen properly that one-component description can be applied

> Green-Kubo formalism is not reliable for mixtures – additional time-modulation must be taken into account

$$\eta/s = \eta/s(T) \rightarrow \eta/s = \eta/s(T) * f(t)$$

Conclusions and Outlook

- Most reliable way to check these conclusions:
  Kinetic transport calculations → BAMPS

See how evolution of an expanding “QGP” with σ_{gg}, σ_{gq}, σ_{qq} $\sim 1/T^2$ can be reproduced by one-component calculations.

How the cross section (i.e. η/s) must be chosen in one-component case? Can any hint of the time-depence of η be seen?

Mixture in BAMPS → (isochronous) freeze-out → flow observables
vs
One-component fluid in BAMPS → (isochronous) freeze-out → flow observables

Work in progress