Heavy-quark transport coefficients
Sensitivity to hadronization description

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1. Bulk-medium evolution

2. Heavy-quark interactions in the sQGP

3. Non-perturbative HQ interactions
   - Resonance model for HQ-q Scattering
   - T-matrix approach with lQCD potentials

4. Hadronization and HQs at FAIR
Bulk-medium evolution (e.g., LHC)

- transport (UrQMD) + hydro (SHASTA) hybrid model
- nuclei: Woods-Saxon profile; binary interactions
- string excitation, fragmentation (PYTHIA)
- after $t_{\text{start}} = 0.5 \text{ fm}/c$: particle distributions, momenta, energy density from UrQMD → via Gaussian smearing to initial conditions for hydro
- during hydro evolution: HG EoS or chiral DE-EoS
- switch back to transport: Cooper-Frye freezeout at constant $\tau$-hypersurfaces for $\epsilon \lesssim 5\epsilon_0 \simeq 730 \text{ MeV/fm}^3$

Bulk-medium evolution

- Pb-Pb collisions $\sqrt{s_{NN}} = 2.76$ TeV
- particle multiplicities at mid rapidity $|\eta| < 0.5$
Bulk-medium evolution

- Pb-Pb collisions $\sqrt{s_{NN}} = 2.76$ TeV
- $p_T$ distribution of charged hadrons

![Graph showing $p_T$ distribution of charged hadrons](image)
**Bulk-medium evolution**

- **Pb-Pb collisions** $\sqrt{s_{NN}} = 2.76$ TeV
- $v_2$ of charged hadrons

![Graph showing $v_2$ of charged hadrons as a function of $p_T$]
Bulk-medium evolution

- Pb-Pb collisions $\sqrt{s_{NN}} = 2.76$ TeV
- $\epsilon_2$ and $\epsilon_3$ distributions

$\epsilon_n = \sqrt{\langle r^n \cos(n\phi) \rangle^2 + \langle r^n \sin(n\phi) \rangle^2}$

Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[
\begin{align*}
\mathrm{d} \vec{x} &= \frac{\vec{p}}{E_p} \mathrm{d}t, \\
\mathrm{d} \vec{p} &= -A \vec{p} \mathrm{d}t + \sqrt{2} \mathrm{d}t \left[ \sqrt{B_0} P_\perp + \sqrt{B_1} P_\parallel \right] \vec{w}
\end{align*}
\]

- \( \vec{w} \): normal-distributed random variable
- \( A \): friction (drag) coefficient
- \( B_{0,1} \): diffusion coefficients
- Einstein dissipation-fluctuation relation \( B_1 = E_p T A \).
- flow via Lorentz boosts between “heat-bath frame” and “lab frame”
- \( A \) and \( B_0 \) from microscopic models for \( qQ \), \( gQ \) scattering
- **medium**: UrQMD → hydro → UrQMD vs. UrQMD/coarse-graining

Free Lagrangian: Particle Content

- Chiral symmetry $SU_V(2) \otimes SU_A(2)$ in light-quark sector of QCD

\[ \mathcal{L}_D^{(0)} = \sum_{i=1}^{2} \left[ (\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i \right] + \text{massive (pseudo-)vectors } D^* \]

- $\Phi_i$: two doublets: pseudo-scalar $\sim (\frac{D_0}{D_-})$ and scalar

- $\Phi_i^*$: two doublets: vector $\sim (\frac{D_0^*}{D_{-*}})$ and pseudo-vector

\[ \mathcal{L}_q^c = \bar{q} i \gamma \Phi q + \bar{c} (i \gamma - m_c) c \]

- $q$: light-quark doublet $\sim (u_d)$

- $c$: singlet
Interactions determined by chiral symmetry

For transversality of vector mesons:

heavy-quark effective theory vertices

\[ \mathcal{L}_{\text{int}} = -G_S \left( \bar{q} \frac{1 + \psi}{2} \Phi_1 c_v + \bar{q} \frac{1 + \psi}{2} i\gamma^5 \Phi_2 c_v + h.c. \right) \]

\[ -G_V \left( \bar{q} \frac{1 + \psi}{2} \gamma^\mu \Phi^*_1 c_v + \bar{q} \frac{1 + \psi}{2} i\gamma^\mu \gamma^5 \Phi^*_2 c_v + h.c. \right) \]

- \( v \): four velocity of heavy quark
- in HQET: spin symmetry \( \Rightarrow G_S = G_V \)
Non-perturbative interactions: Resonance Scattering

- General idea: Survival of $D$- and $B$-meson like resonances above $T_c$
- Model based on chiral symmetry (light quarks) HQ-effective theory
- Elastic heavy-light-(anti-)quark scattering

- $D$- and $B$-meson like resonances in sQGP

- Parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV

- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked $\leftrightarrow$ resonance isotropic
- resonance scattering more effective for friction and diffusion
Transport coefficients: pQCD vs. resonance scattering

- three-momentum dependence

\[
\begin{align*}
\text{T}=200 \text{ MeV} & \quad \text{γ} [1/\text{fm}] \\
0 & \quad 0.5 & \quad 1 & \quad 1.5 \\
0.05 & \quad 0.1 & \quad 0.15 & \quad 0.2 \\
\text{pQCD: } \alpha_s = 0.3 & \quad \text{pQCD: } \alpha_s = 0.4 & \quad \text{pQCD: } \alpha_s = 0.5 \\
\text{resonances } \Gamma = 0.3 \text{ GeV} & \quad \text{resonances } \Gamma = 0.4 \text{ GeV} & \quad \text{resonances } \Gamma = 0.5 \text{ GeV}
\end{align*}
\]

- resonance contributions factor \(~ 2 \ldots 3\) higher than pQCD!
Temperature dependence

- Resonances: $\Gamma = 0.4$ GeV
- pQCD: $\alpha_s = 0.4$
- Total

Graphs show the dependence of $\gamma$ and $D$ on temperature $T$ in GeV. The graphs compare $\gamma$ and $D$ for pQCD and resonances at various temperatures.
**T-matrix**

- Brueckner many-body approach for elastic $Qq, Q\bar{q}$ scattering

\[
T = V + \sum \Sigma_{\text{glu}} + VT
\]

- $V$: static $q\bar{q}$ potential from lattice QCD ($F$ and $U$)
- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lipmann-Schwinger
- $S$- and $P$ waves
- Relation to invariant matrix elements

\[
\sum |M(s)|^2 \propto \sum q d_a \left( |T_{a,l=0}(s)|^2 + 3 |T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}} \right)
\]

[HvH, M. Mannarelli, V. Greco, R. Rapp, Phys. Rev. Lett. 100, 192301 (2008)]
Static heavy-quark potentials from lattice QCD

- color-singlet free energy from lattice $\rightarrow$ internal energy
  
  $$U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T},$$
  
  $$V_1(r, T) = U_1(r, T) - U_1(r \rightarrow \infty, T)$$

- Casimir scaling of Coulomb part for other color channels; confining part color blind \cite{Riek:2010}. 

  $$V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1$$
• **resonance formation** at lower temperatures $T \approx T_c$
• melting of resonances at higher $T$
• model-independent assessment of elastic $Qq, Q\bar{q}$ scattering!
Transport coefficients

- T-matrix resonance-scattering coefficients: decrease with $T$
- from non-pert. interactions reach $A_{\text{non-pert}} \approx 1/(7 \text{ fm}/c) \approx 4A_{\text{pQCD}}$
- results for free-energy potential, $F$ considerably smaller

\[ P_{\text{coa}} = \exp \left\{ \left( (\Delta p^0)^2 - \sum_{i=1}^{3} (\Delta p^i)^2 - (\Delta_m)^2 \right) \sigma^2 \right\}, \quad \sigma^2 = \frac{8}{3} r_D^2 \]

Petersen fragmentation \((z: \text{momentum fraction } c \to D)\)

\[ D(z) = \frac{H}{z[1-(1/z)-\epsilon_p/(1-z)]^2}. \]
Hadronic transport coefficients

- based on unitarized chiral HQ model

- coupled channel T-matrix approach with pseudo-Goldstone mesons ($\pi$, $\eta$, $K$) and baryons ($N$, $\bar{N}$, $\Delta$, $\bar{\Delta}$)

- D-meson scattering cross sections used for HQ drag and diffusion coefficients
Hadronic transport coefficients

- drag and diffusion coefficients for D from mesonic interactions
Hadronic transport coefficients

- Drag and diffusion coefficients for D from baryonic interactions

![Graph](image-url)
• drag and diffusion coefficients for D at finite $\mu_B$
D-mesons at FAIR

- $E_{\text{kin}} = 25 AGeV$
- bulk-medium evolution: UrQMD-hydro hybrid vs. UrQMD coarse graining
- QQP HQ transport coefficients from resonance model
- influence of hadronization description: sensitivity to hadronization temperature
- coalescence vs. fragmentation (dependence on $r_D$, $\epsilon_P$)

![Graphs showing $p_T$ distribution for D-mesons at different energies under UrQMD/hybrid and UrQMD/hybrid, b=3fm conditions.](image)

- strong rise in $R_{AA}$ due to energy constraint in pp ($p_T \lesssim 2.5 $GeV)

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![Graph showing D-meson yield vs. p_T for different masses and interaction zones.](image)

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![Graph 1: $V_2$ vs. $p_T$ for different D-meson masses and hadronization temperatures.](image1)

- influence of longer hadronic phase negligible

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- large influence on hadronization procedure