Thermalization of Heavy Quarks in the QGP

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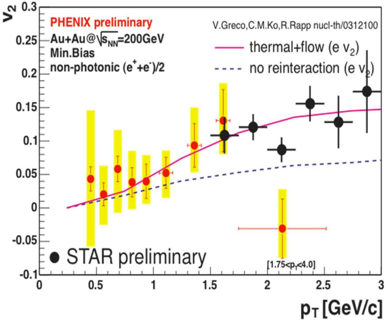
Outline

Motivation

- p_T spectra and v_2 of D mesons
- single-electron v_2 measurements from PHENIX, STAR '04
- coalescence model describes data under assumption of flowing thermalized c quarks



Motivation

- importance of dissociation and regeneration in

$$c + \bar{c} \leftrightarrow J/\psi + X$$

- in-medium spectral properties of charmonia in QGP (Grandchamp, Rapp '02)
- importance of thermalization of heavy quarks

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- importance of thermalization of heavy quarks
- Possible mechanism: Survival of "D-mesonic resonances" above T_c
- suggestive from lattice QCD (Umeda et al '02, Datta et al '03)

Free Lagrangian: Particle Content

Chiral symmetry $\mathrm{SU}_V(2) \otimes \mathrm{SU}_A(2)$ in light-quark sector of QCD

$$\mathscr{L}_D^{(0)} = \sum_{i=1}^{2} [(\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i] + \text{massive (pseudo-)vectors } D^*$$

Φ_i: two doublets: pseudo-scalar $\sim \binom{\overline{D^0}}{D^-}$ and scalar

Φ_i^*: two doublets: vector $\sim \binom{\overline{D^{0*}}}{D^{-*}}$ and pseudo-vector

$$\mathscr{L}_{qc}^{(0)} = \bar{q} \mathrm{i} \not{\partial} q + \bar{c}(\mathrm{i} \not{\partial} - m_c) c$$

q: light-quark doublet $\sim \binom{u}{d}$

c: singlet

Chiral Symmetry

Infinitesimal version:

$$q \longrightarrow (1 + \mathrm{i}\delta\vec{\phi}_V \vec{t} + \mathrm{i}\delta\vec{\phi}_A \vec{t}\gamma_5)q, \quad c \longrightarrow c.$$

Light quarks massless in chiral limit!

$$\Phi_1 \longrightarrow \Phi_1 + \mathrm{i}\delta\vec{\phi}_V \vec{t}\Phi_1 + \mathrm{i}\delta\vec{\phi}_A \vec{t}\Phi_2,$$
$$\Phi_2 \longrightarrow \Phi_2 + \mathrm{i}\delta\vec{\phi}_V \vec{t}\Phi_2 + \mathrm{i}\delta\vec{\phi}_A \vec{t}\Phi_1.$$

Mesons must have chiral partners
In the vacuum: chiral symmetry spontaneously broken
In QGP: chiral symmetry restored

Interactions

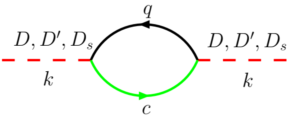
- Interactions determined by chiral symmetry
- Strong interactions also preserve parity
- For transversality of vector mesons: use heavy-quark effective theory vertices

$$
\begin{aligned}
\mathscr{L}_{\text{int}} = & - G_S \left(\bar{q} \frac{1+\not{v}}{2} \Phi_1 c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^5 \Phi_2 c_v + h.c. \right) \\
& - G_V \left(\bar{q} \frac{1+\not{v}}{2} \gamma^\mu \Phi_{1\mu}^* c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^\mu \gamma^5 \Phi_{2\mu}^* c_v + h.c. \right)
\end{aligned}
$$

- v: four momentum of heavy quark in HQET: spin symmetry $\Rightarrow G_S = G_V$

Dressing the D Mesons

Dressing the D mesons with self-energies



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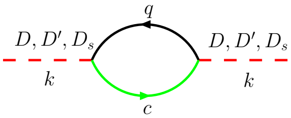

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$$\partial_{k^2}\Pi_D(k^2=0)=0, \quad \Pi_D(k^2=0)=0$$

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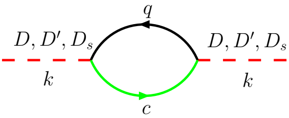

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or dipole form-factor cutoff:

$$F_{\text{dip}} = \left(\frac{2\Lambda^2}{2\Lambda^2 + k_{\text{cm}}^2} \right)^2, \quad k_{\text{cm}} = \frac{s - m_c^2}{2\sqrt{s}}$$

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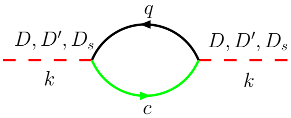

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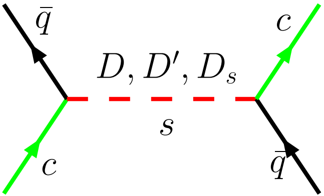
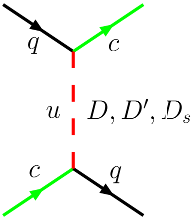
Bare mass and coupling adjusted such that

$$m_D=2\text{ GeV}, \quad \Gamma_D=(0.3\ldots0.8)\text{GeV}$$

Resonance Scattering

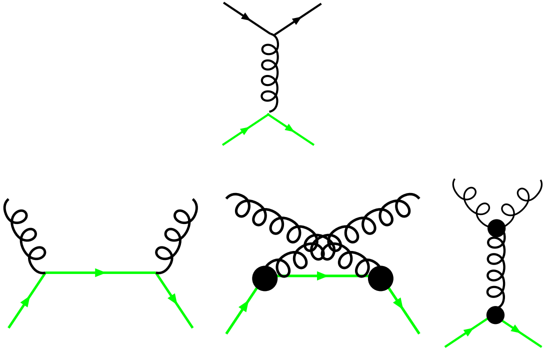
heavy-light-(anti-)quark scattering





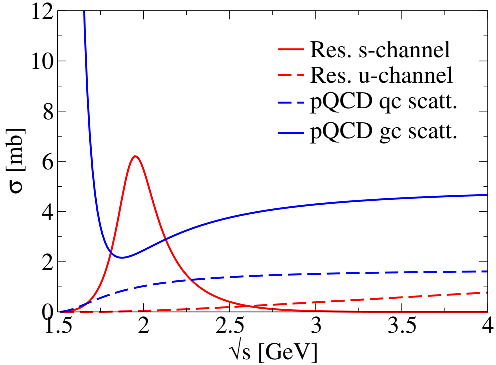
Contributions from pQCD

Lowest-order matrix elements (Combridge '79)



In-medium Debye-screening mass for t-channel gluon exchange:
$\mu_g = gT$, $\alpha_s = 0.3, 0.4, 0.5$

Cross sections



- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion

The Fokker-Planck Equation

heavy particle (c quarks) in a heat bath of light particles (QGP)

$$\frac{\partial f(t,\vec{p})}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(t,\vec{p}) + \frac{\partial}{\partial p_j} B_{ij}(t,\vec{p})\right] f(t,\vec{p})$$

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A_i and B_{ij} given by averages over initial momenta $\vec{q}$ of light particles and summation over final states (Svetitsky '88):

$$\langle X(\vec{p}')\rangle = \frac{1}{\gamma_c}\frac{1}{2E_p}\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q}\int \frac{\mathrm{d}^3\vec{q}'}{(2\pi)^3 2E_{q'}}\int \frac{\mathrm{d}^3\vec{p}'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p+q-p'-q')\hat{f}(\vec{q})X(\vec{p}')$$

Friction and Diffusion Coefficients

For t, $\vec{p}$-independent coefficients:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}} (\vec{p} f) + D \frac{\partial^2}{\partial \vec{p}^2} f$$

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Solution for c quark with given momentum $\vec{p}_0$ at $t = 0$:

$$G(t, \vec{p}|\vec{p}_0) = \left\{ \frac{\gamma}{2\pi D[1 - \exp(-2\gamma t)]} \right\}^{3/2} \exp\left\{ -\frac{\gamma}{2D} \frac{[\vec{p} - \vec{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right\}$$

- $\langle \vec{p} \rangle = \vec{p}_0 \exp(-\gamma t) \Rightarrow \gamma =$ friction coefficient (dissipation)
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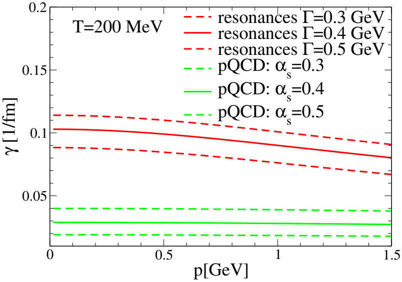
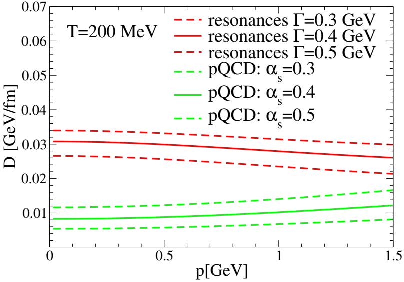
- $t \to \infty$: temperature $T^* = D/\gamma m_c$
- consistency condition $T^* \stackrel{!}{=} T$ (heat bath-temperature)
- fulfilled within $\sim 15\%$ for relevant temperature region

The Coefficients: pQCD vs. resonance scattering





- only weakly p-dependent
- resonance contributions factor $\sim 2 \ldots 3$ higher than pQCD!

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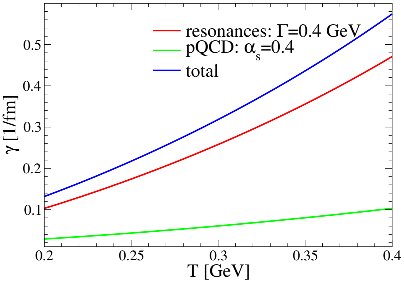

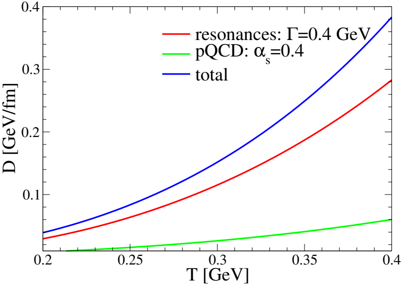

- temperature dependence ⇒ need to treat Fokker-Planck equation with time-dependent coefficients
- Solvable with method of characteristics

Time evolution of the fire ball

- Simple fire-ball parameterization:

$$V(\tau) = \pi(z_0 + v_z \tau)(r_0 + \frac{1}{2} a_\perp \tau^2)^2$$

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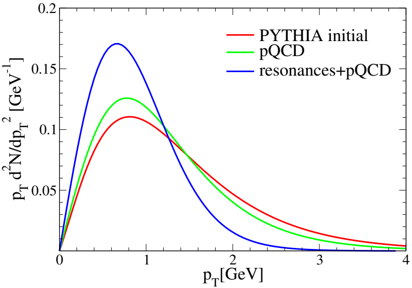
- initial condition from PYTHIA

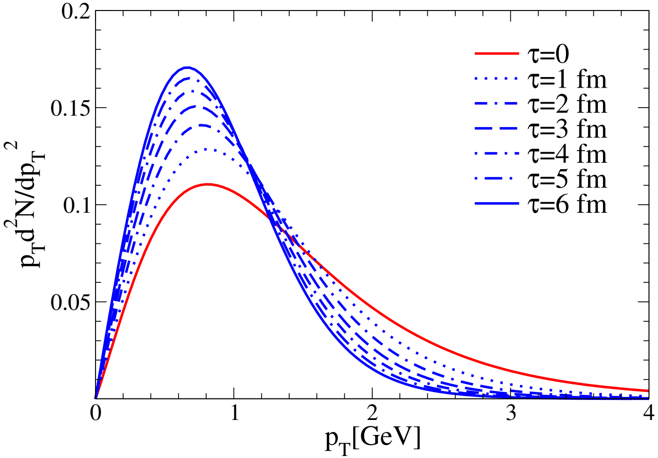
$$\frac{\mathrm{d}^2 N}{\mathrm{d}p_T^2} := f(p_T, t=0) \propto \frac{(p_T + A)^2}{(1 + p_T/B)^\alpha}$$

Evolved p_T spectra



- initially $\sqrt{\langle p_T^2 \rangle} = 1.66$ GeV
- with pQCD: not much change in spectrum
- with resonance contributions: $p_T^{(\max)} \sim 0.66$ GeV
- nearly thermal: $T \sim 290$ MeV

Evolution of p_T spectra



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$$\delta\vec{x} = \frac{\vec{p}}{E}\delta t$$

$$\delta\vec{p} = -\gamma(t, \vec{p}+\delta\vec{p})\vec{p}\delta t + \delta W(t, \vec{p}+\delta\vec{p})$$

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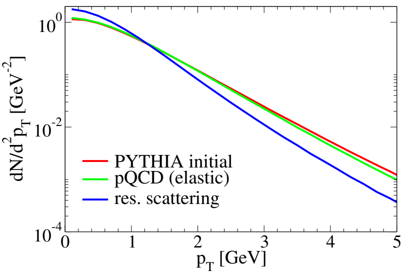
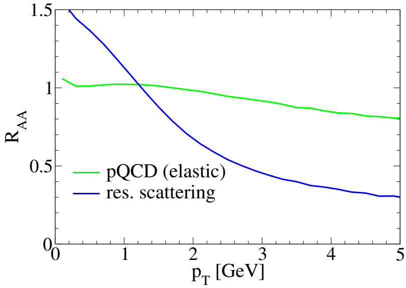
- Hänggi-Klimontovich realization $\Leftrightarrow$ rel. Maxwell distribution as equilibrium limit with unchanged FP coefficients

Observables: p_T-spectra, v_2, (R_{AA})

- use elliptic flow parametrization for fireball, based on hydro [Kolb et al]
- boost "labframe" $\leftrightarrow$ local heat-bath-rest frame (see also [Moore, Teaney 2004])





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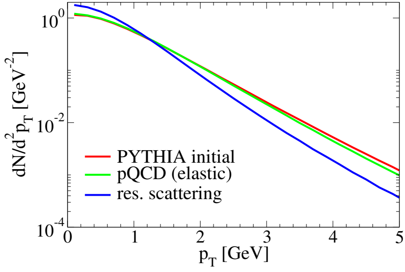

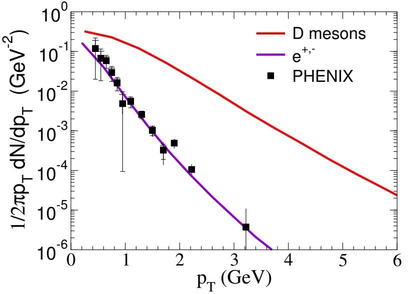

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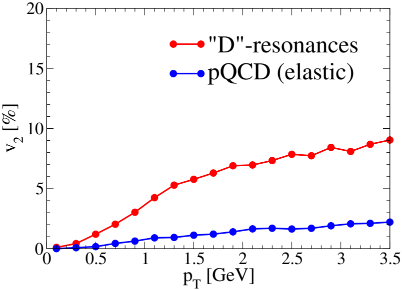

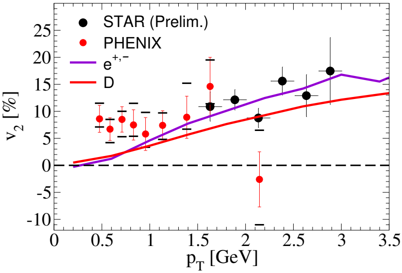

- D-meson v_2 via coalescence
- $e^{\pm}$ from subsequent decay

Conclusions

- Assumption: survival of resonances in the QGP
- possible mechanism for strong interactions beyond T_c
- Equilibration of heavy quarks in QGP
- Observables via Langevin approach and coalescence