Contents

A quantum field theoretical renormalizable model for the $\pi\rho$-system

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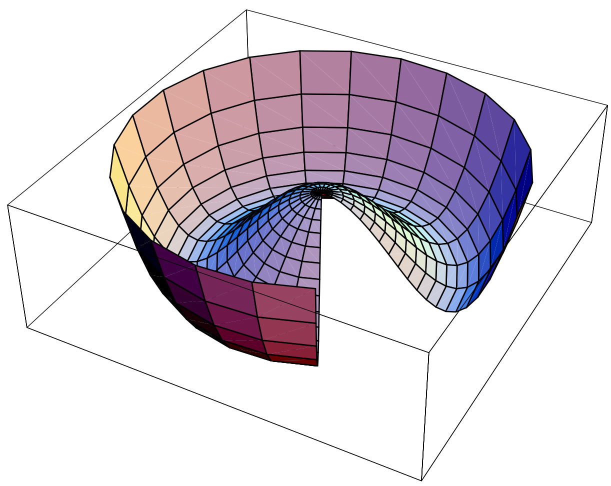
The Model

The ρ-mesons

- Renormalizable model for massive ρ-mesons $\Rightarrow$ Higgs-Kibble-formalism for Gauge theories
- Start with a $SU(2)$ duplett with gauged symmetry group

$$\mathscr{L}_1 = -\frac{1}{2}\operatorname{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}(D_\mu\Phi)^\dagger D^\mu\Phi - V(\Phi)$$

- Mexican hat potential $V(\Phi) = -\frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2$



- Physical gauge (around the stable vacuum):
  - ρ-fields become massive $m_\rho^2 = g^2\mu^2/(4\lambda)$
  - Three Φ-degrees of freedom become ρ degrees of freedom
  - One Φ-degree of freedom gives a massive "Higgs-particle"

The Model

The Pions

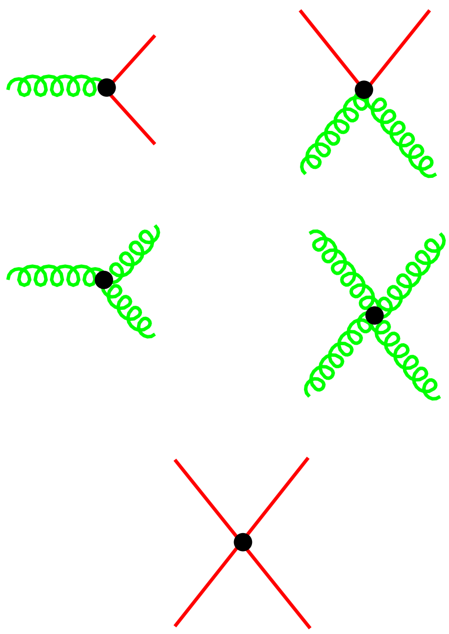
- Introduce Pions as adjoint representation, i.e., SO(3)-triplett

$$\mathscr{L}_2 = \frac{1}{2}(D_\mu \vec{\pi}) \cdot (D^\mu \vec{\pi}) - \frac{\lambda_2}{8}(\vec{\pi}^2)^2 - \frac{\lambda_3}{4}\vec{\pi}^2 \Phi^\dagger \Phi$$

- Consistency condition:

$$m_\pi^2 = \frac{2m_\rho^2}{g}\lambda_3$$

Unitary Gauge - Physical Vertices I



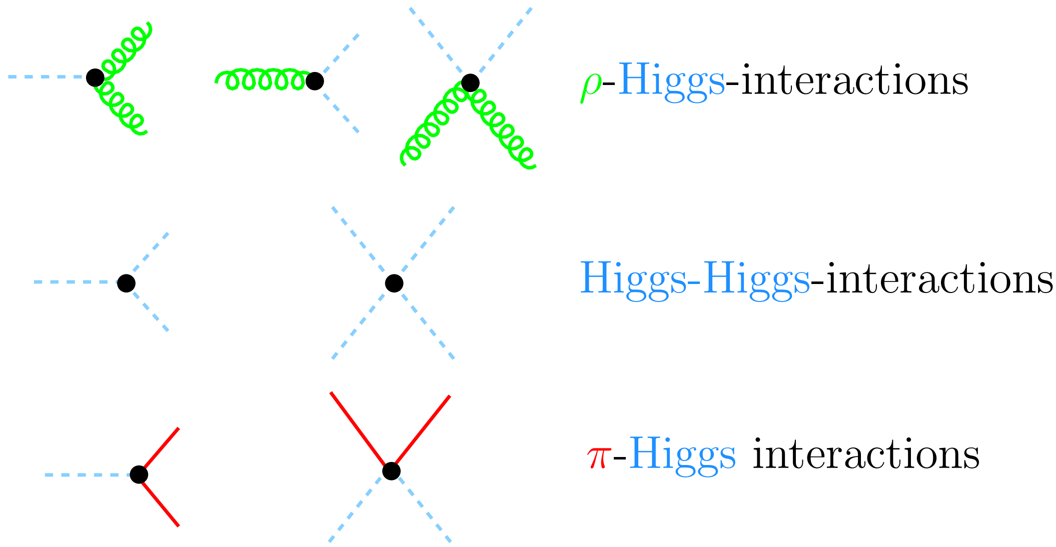
$\pi\rho$-interactions

$\rho\rho$-interactions

π self interactions

The Model

Unitary Gauge - Physical Vertices II



Remarks about Quantization

- Unitary gauge contains only physical dof. $\Rightarrow$ manifestly unitary
- To get renormalizable gauge $\Rightarrow$ Introducing R_ξ-gauges ('t Hooft)
- R_ξ-gauge: manifestly renormalizble
- R_ξ-gauge: Faddeev-Popov-ghosts
- BRST-invariance $\Rightarrow$ S-Matrix gauge invariant
- R_ξ-gauge has unitary gauge as limit $\Rightarrow$ Renormalized theory also unitary

The Model

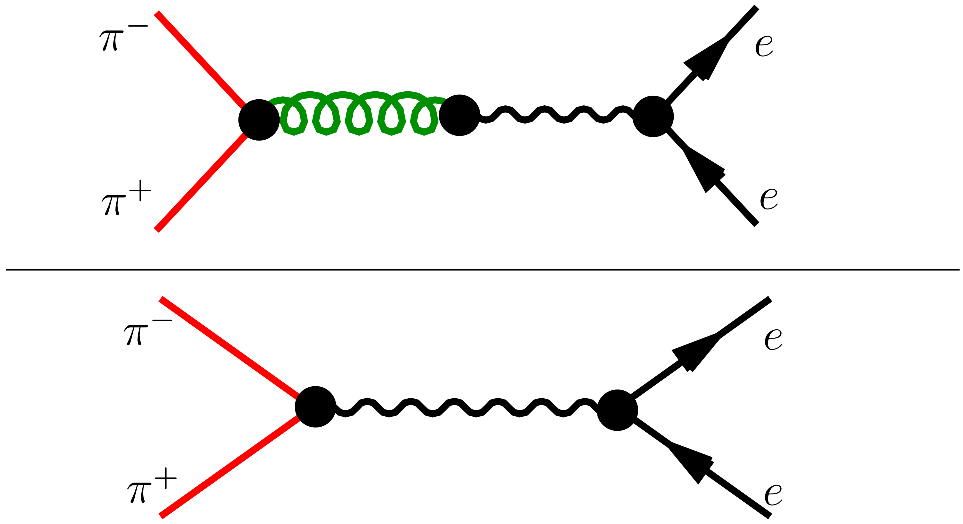
The Photon

- Extending the gauge group to $U(1) \times SU(2)$
- $U(1)$ unbroken $\Rightarrow$ One of the four gauge bosons remains massless $\Rightarrow$ photon
- Equations of Motion $\Rightarrow$ Pions couple to photons only through ρ $\Rightarrow$ Vector-Meson-Dominance

The Form Factor

- Electromagnetic Form Factor of the Pion:

$F(k^2) =$



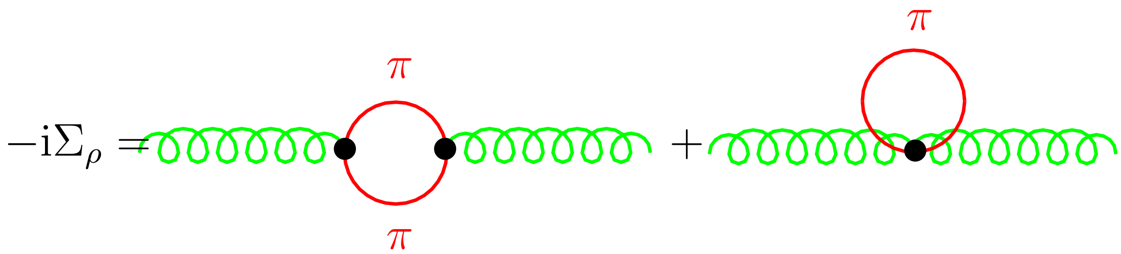
- Feynman rules: $\Gamma_{\rho\gamma} = \mathrm{i}\delta^{a3} M_\rho^2 e/g \Rightarrow$

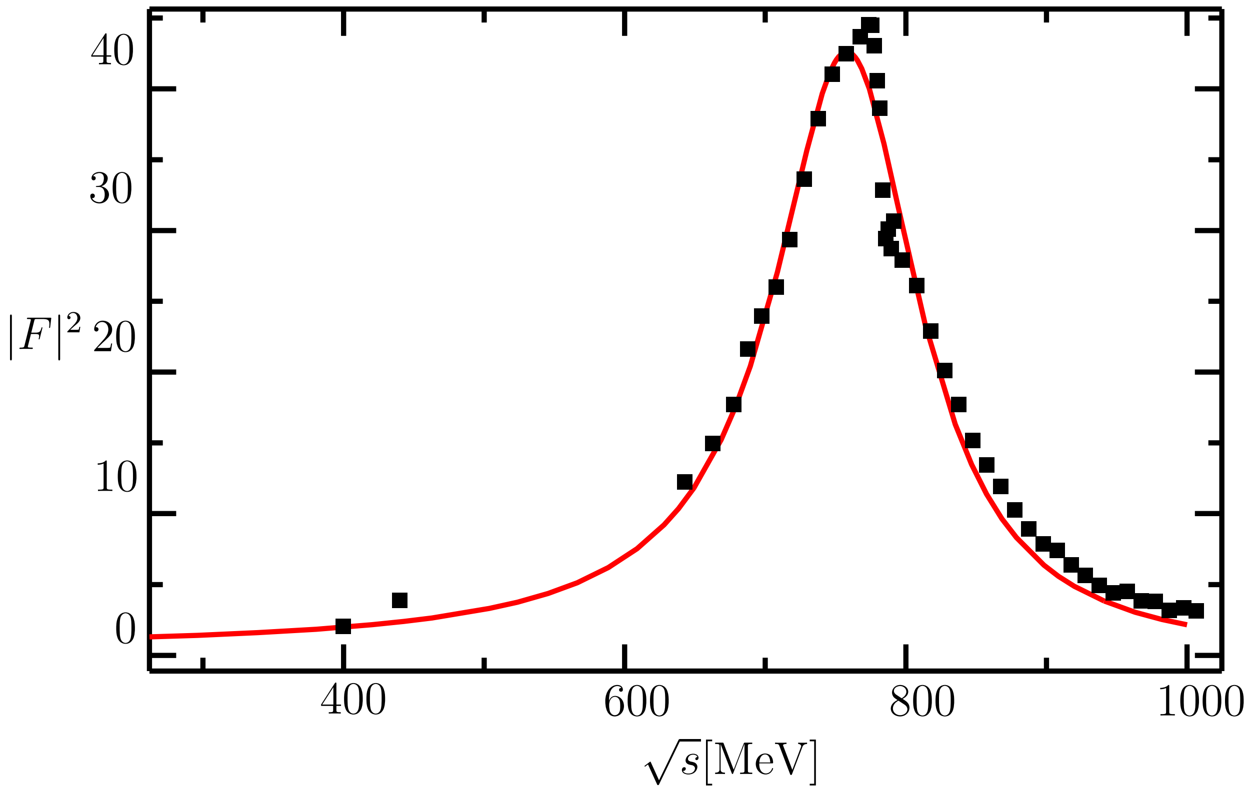
$$|F(s)|^2 = \frac{m_\rho^4}{[s - m_\rho^2 - \mathrm{Re}\,\Pi_\rho(s)]^2 + [\mathrm{Im}\,\Pi_\rho(s)]^2}$$

Fit of the parameters

Form factor and Phase Shift

- Using dimensional regularization and renormalization of the one-loop-self-energy diagrams





Data: Amendolia et al. Phys. Lett. **138B** (1984) 454
Barkov et al. Nucl. Phys. **B256** (1985) 365

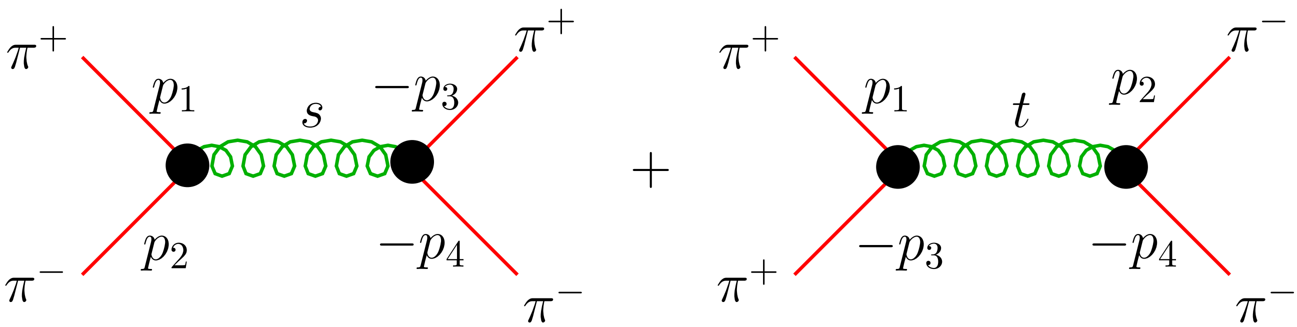
Fit of the parameters

Total $\pi^+\pi^-$ elastic cross-section

- Four π-vertex

$$\Gamma^{abcd}(p_1,\ldots,p_4) = \begin{cases} & A(s,t,u)\delta_{ab}\delta_{cd} + \\ + & A(t,s,u)\delta_{ac}\delta_{bd} + \\ + & A(u,t,s)\delta_{ad}\delta_{bc} \end{cases}$$

- With the invariants $s=(p_1+p_s)^2$, $t=(p_1-p_3)^2$ and $u=(p_1-p_4)^4$



- Feynman rules $\Rightarrow$ invariant transition amplitude:

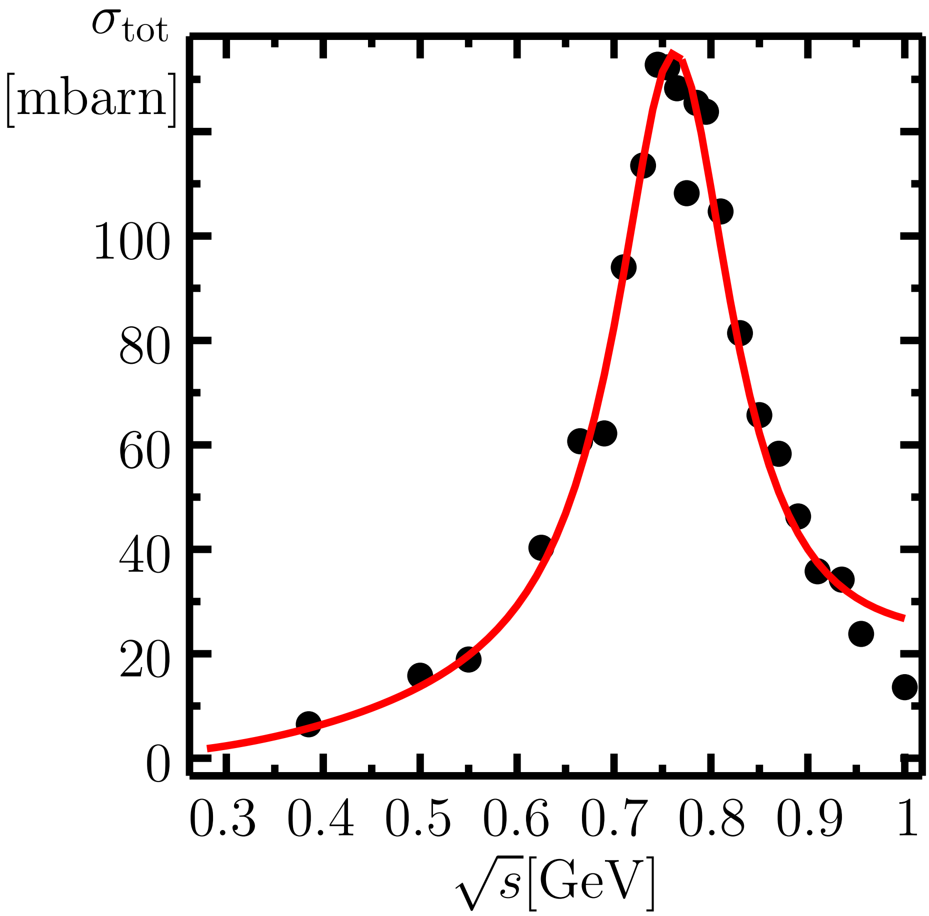
$$M_{fi}(s,t) = A(s,t,u) + A(t,s,u)|_{u=4m_\pi^2-s-t}$$

- Total cross section:

$$\sigma_{\text{tot}} = \frac{1}{64\pi}\frac{1}{s(s-4m_\pi)}\int_{4m_\pi^2-s}^{0}|M_{fi}(s,t)|^2$$

Fit of the parameters

- With the parameters from the fitting to phase-shift and form-factor:



- Data from: Forgatt, Petersen, Nucl. Phys. **B129** (1977) 89

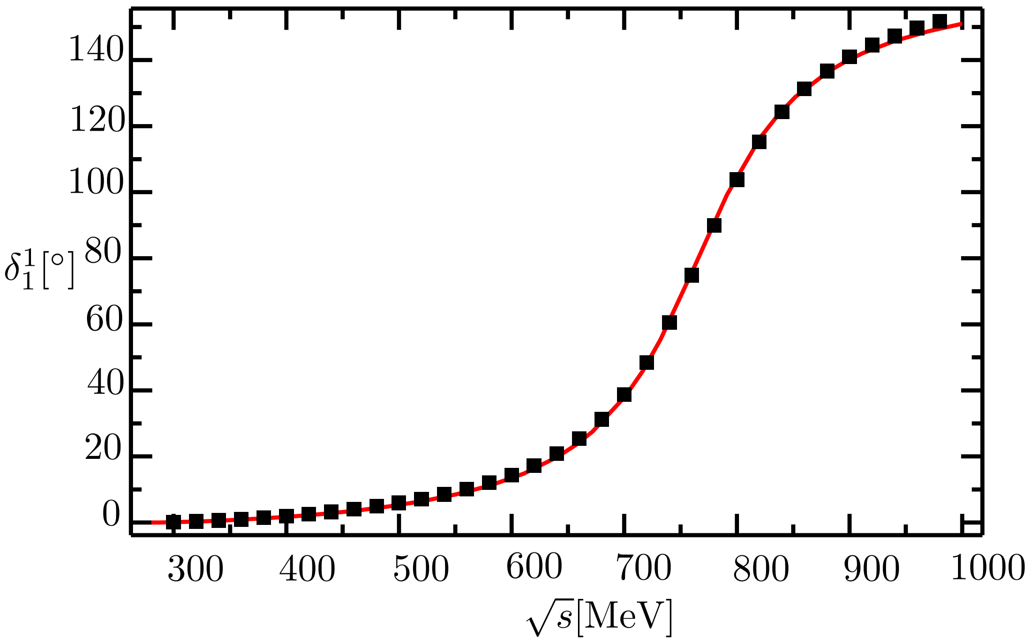
Fit of the parameters

Phaseshift in $t = 1$, $l = 1$-channel

- Projection to isospin $I = 1$:
  $M^{I=1} = A(s,u,t) - A(s,t,u)$
- From ρ-exchange ($s = E^2_{\mathrm{CM}}$, θ scattering angle in CM):

$$M^{I=1}(s,\theta) = 2g^2 \frac{(s - 4m_\pi^2)\cos\theta}{s - m_\rho^2 - \Pi_\rho(s)}$$

- Projection to angular momentum $l = 1$:
  $t_1^1(s) = \frac{1}{64\pi}\int_{-1}^{1} \mathrm{d}(\cos\theta)\cos\theta M^{I=1}(s,\theta)$
- Parametrization with phase shift
  $\delta_1^1(s) = \arccos\left[\frac{\mathrm{Re}\, G_\rho(s)}{|G_\rho(s)|}\right]$



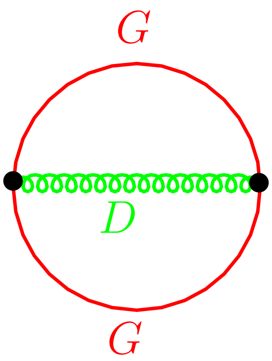
Data: Frogatt, Petersen, Nucl. Phys. **B129** (1977) 89

Selfconsistent approximations

Generating functional

- $\Phi[G, D]$: sum over all 2PI closed diagrams with at least two loops

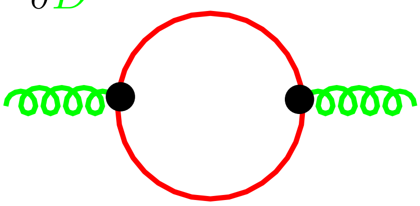
$$i\Phi[G, D] =$$



$$+ \cdots$$

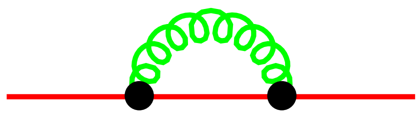
- Variation with respect to Green's functions $\Rightarrow$ self energies fulfilling Dyson's equations

$$\frac{\delta i\Phi}{\delta D} = -i\Pi_\rho =$$



$$\Pi_\rho = D_0^{-1} - D^{-1}$$

$$\frac{\delta i\Phi}{\delta G} = -i\Sigma_\pi =$$



$$\Sigma_\pi = G_0^{-1} - G^{-1}$$

- Sum up to a certain loop order $\Rightarrow$ Selfconsistent effective approximation
- Respects all conservation laws basing on global symmetries
- In thermal field theory: Thermodynamically consistent approximation

Renormalization

Renormalizing the selfconsistent approximation

- Can be seen as resummation of all self energy insertions $\Rightarrow$ Infinities to all orders
- Renormalizable theory $\Rightarrow$ finite by renormalizing parameters already present in Lagrangian
- Physical renormalization conditions

$$\Sigma_\pi(m_\pi^2) = \partial_s \Sigma_\pi(m_\pi^2) = 0, \; \Pi_\rho(0) = \partial_s \Pi_\rho(0) = 0$$

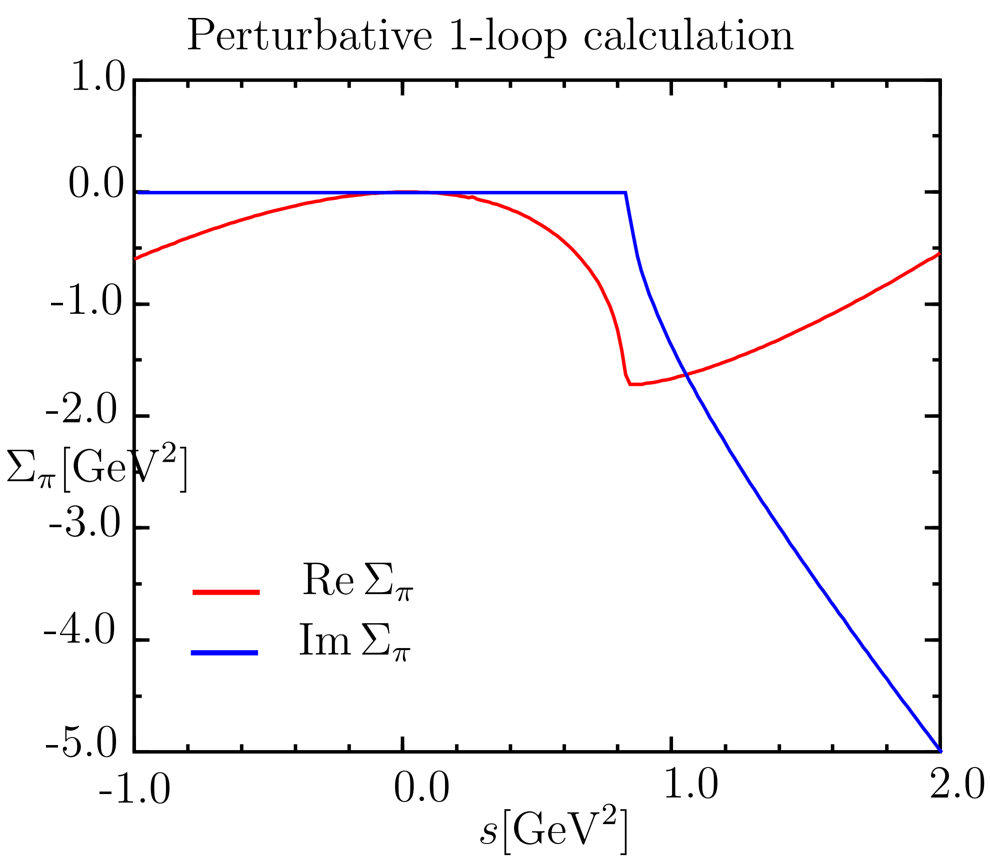
- Analytical properties of Green's functions

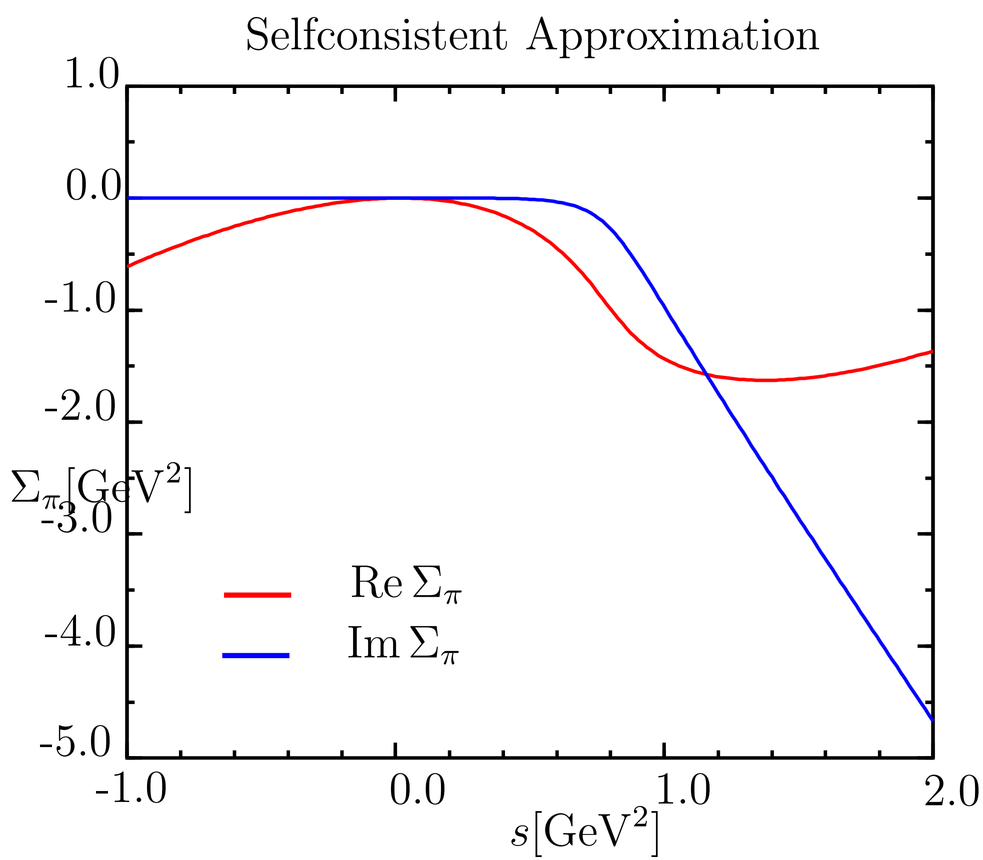
$$G(s) = \frac{1}{\pi} \int_0^\infty dm^2 \Delta(m^2, s) A(m^2) \text{ with } A(s) = -\operatorname{Im} G(s)$$

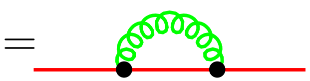
- $\Delta(m^2, s)$: Feynman-propagator $\Rightarrow$ integral kernels $\Rightarrow$ can be renormalized using standard techniques
- self consistent finite set of coupled integral equations solvable numerically by iteration
- Tadpole in vacuum absorbed into mass renormalization

Results in vacuum

The π-Self-Energy



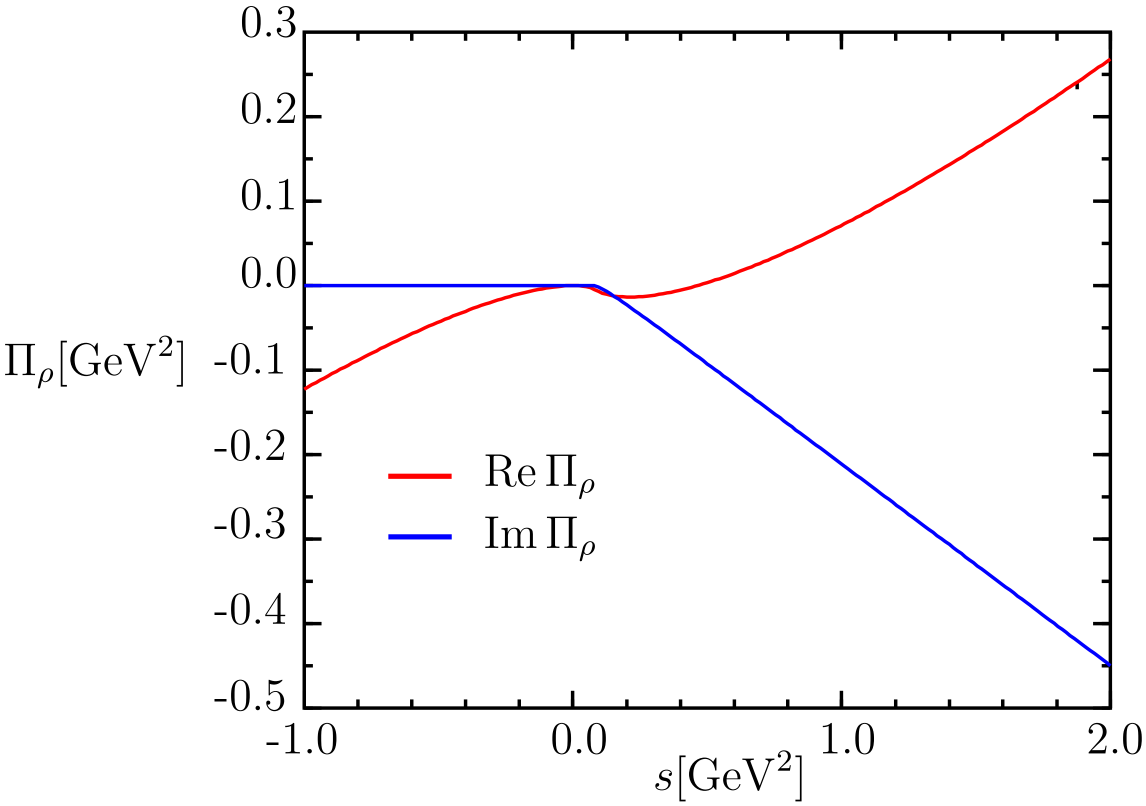


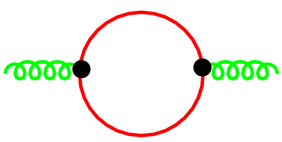
$-i\Sigma_\pi =$ 

Result in vacuum

The ρ-Self-Energy

Perturbative 1-loop and selfconsistent calculation



$-\mathrm{i}\Pi_\rho =$ 

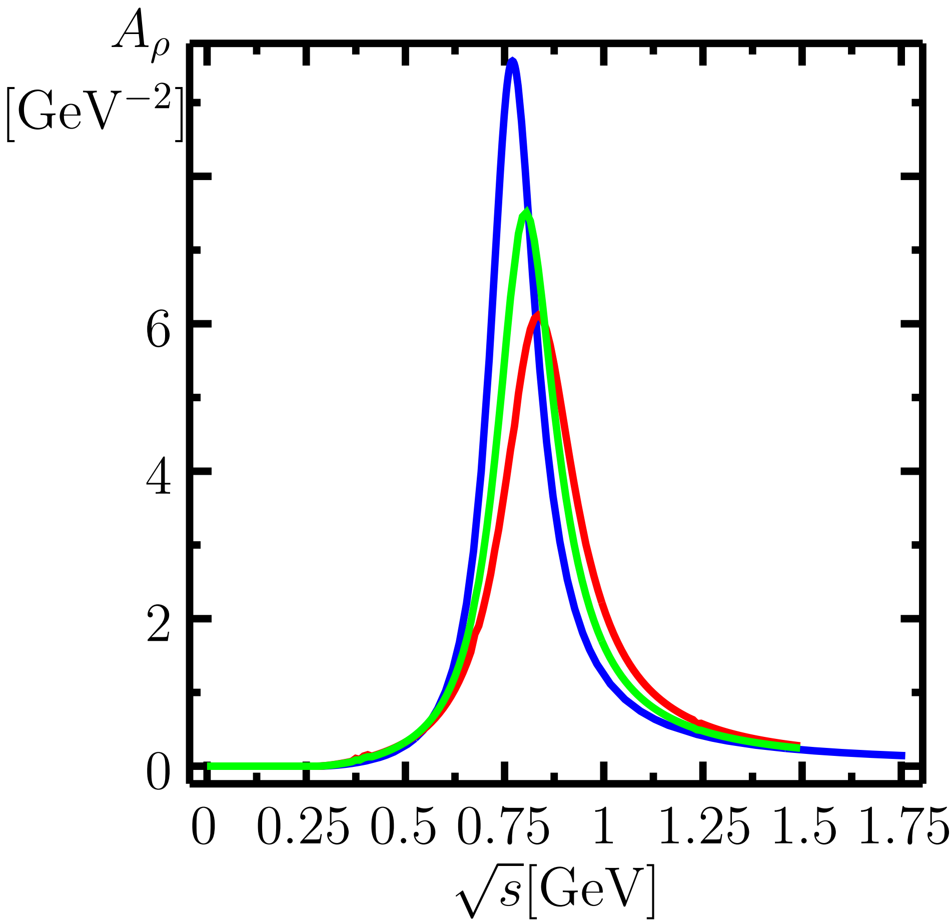
Finite Temperature

Perturbative results at finite temperature

- Imaginary or real-time Formalism $\Rightarrow$ Retarded Green's Function
- Spectral function

$$A_\rho(p_0, |\vec{p}|) = -\,\mathrm{Im}\, G_\rho^{\mathrm{Ret}}$$

- $T = 0$, $T = 150\mathrm{MeV}$, $T = 200\mathrm{MeV}$



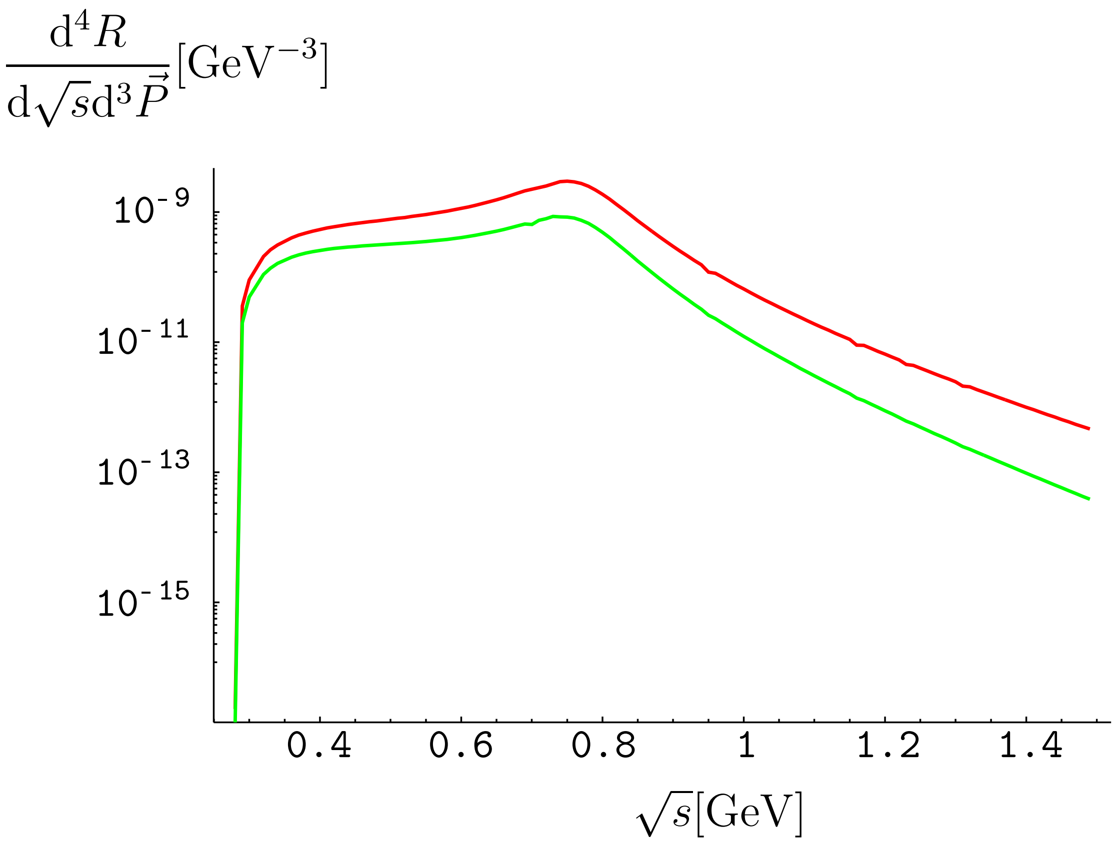
Dilepton Rate

The Dilepton Rate

- Kadanoff-Baym-Equations: Exact result for strong coupling:

$$\left.\frac{\mathrm{d}^4 R}{\mathrm{d}\sqrt{s}\,\mathrm{d}P^3}\right|_{\vec{P}=0} = \frac{2\alpha^2}{(2\pi)^3}\frac{m_\rho^2}{g^2}\frac{1}{s}A_\rho(\sqrt{s},0)f_B(\sqrt{s})$$

- Dilepton Production Rate



- $T = 150\text{MeV}$, 200MeV

Outlook

Work to do

- Exploit non-abelian part of the ρ-interaction
- Selfconsistent approximation for $T, \mu > 0 \Rightarrow$
  Need to include tadpole contributions $\Rightarrow$
  Renormalization of the vertex
- Gauge invariance?