Charmonium production in a classical Langevin model

Hendrik van Hees

Goethe-Universität Frankfurt

September 03, 2024

in collaboration with
Naomi Oei, Juan Torres-Rincon, and Carsten Greiner

Outline

Motivation

Langevin equation for $Q\overline{Q}$-pairs

Box calculations for quarkonium formation

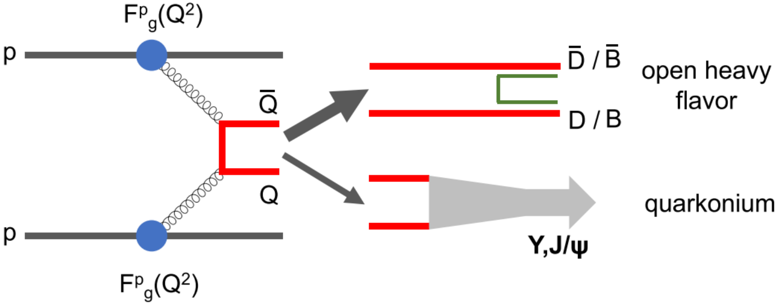
First simulation for heavy-ion collisions

Conclusions and outlook

References

Motivation

- heavy quarks and antiquarks/quarkonia produced in hard initial conditions



- interactions with hot and dense medium during entire evolution of collision
- $m_Q \gg \Lambda_{\text{QCD}}$, $m_Q \gg T_{\text{med}} \Rightarrow$ only partial equilibrium with bulk

⇒ handle on transport properties(?)

- both "drag" of HQs with medium and quarkonium melting ⟷ regeneration
- kinetic process rather than "naive coalescence"

Langevin equation for Q$\overline{\text{Q}}$-pairs

- drag and diffusion of single Q's and $\overline{Q}$'s with bulk medium
- binding Q$\overline{\text{Q}}$ potential ⇒ formation and destruction of bound states

$$\mathrm{d}\vec{x}_{\mathrm{Q}} = \frac{\vec{p}_{\mathrm{Q}}}{E_{\mathrm{Q}}} \mathrm{d}t, \quad \mathrm{d}\vec{p}_{\mathrm{Q}} = -\gamma \, \mathrm{d}t \, \vec{p}_{\mathrm{Q}} - \mathrm{d}t \, \vec{\nabla}_{\mathrm{Q}} V(|\vec{x}_{\mathrm{Q}} - \vec{x}_{\overline{\mathrm{Q}}}|) + \sqrt{2D \, \mathrm{d}t} \, \vec{\rho}(t)$$

$$\mathrm{d}\vec{x}_{\overline{\mathrm{Q}}} = \frac{\vec{p}_{\overline{\mathrm{Q}}}}{E_{\overline{\mathrm{Q}}}} \mathrm{d}t, \quad \mathrm{d}\vec{p}_{\overline{\mathrm{Q}}} = -\gamma \, \mathrm{d}t \, \vec{p}_{\overline{\mathrm{Q}}} - \mathrm{d}t \, \vec{\nabla}_{\overline{\mathrm{Q}}} V(|\vec{x}_{\mathrm{Q}} - \vec{x}_{\overline{\mathrm{Q}}}|) + \sqrt{2D \, \mathrm{d}t} \, \vec{\rho}(t)$$

- analogous for more than one Q$\overline{\text{Q}}$ pair
- γ: drag coefficient, $D = E T \gamma$ diffusion coefficient, $\vec{\rho}$ uncorrelated white noise

HQ potential

- use HQ model in an Abelian plasma by Blaizot et al [BDFG16]
- non-relativistic HQs in plasma of relativistic particles
- influence functional for HQs, $m_{\mathrm{Q}} \rightarrow \infty \Rightarrow$ complex potential

$$\mathcal{V}(r) = -\alpha_{\mathrm{s}} m_{\mathrm{D}} - \alpha_{\mathrm{s}} \frac{\exp(-m_{\mathrm{D}} r)}{r} - \mathrm{i}\alpha_{\mathrm{s}} T \phi(m_{\mathrm{D}} r), \quad r = |\vec{x}_{\mathrm{Q}} - \vec{x}_{\overline{\mathrm{Q}}}|$$

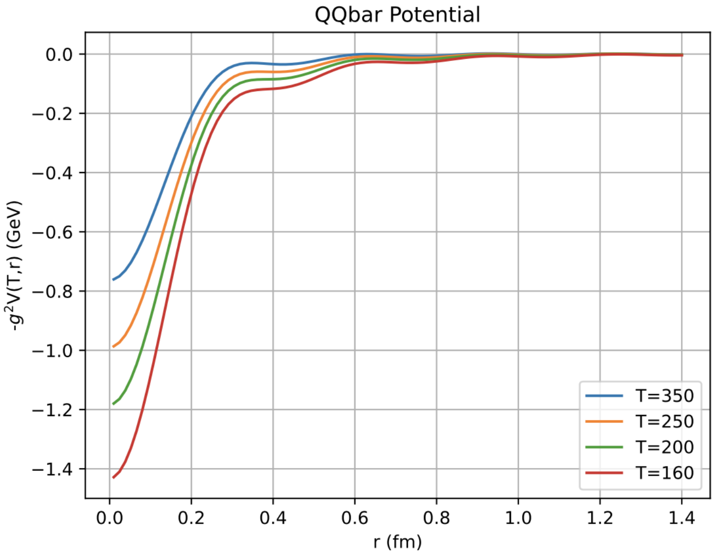
- interaction potential: real part (screened Coulomb potential) with

$$\alpha_{\mathrm{s}} = \frac{\alpha_{\mathrm{s}}(T_{\mathrm{c}})}{1 + C \ln(T/T_{\mathrm{c}})}, \quad m_{\mathrm{c}} = 1.8\,\mathrm{GeV}, \quad T_{\mathrm{c}} = 160\,\mathrm{MeV},$$

$$\alpha_{\mathrm{s}}(T_{\mathrm{c}}) = 0.7, \quad C = 0.76, \quad m_{\mathrm{D}}^2 = 16\pi\alpha_{\mathrm{s}} T^2/3$$

- with momentum cut-off $\Lambda = 4\,\mathrm{GeV}$

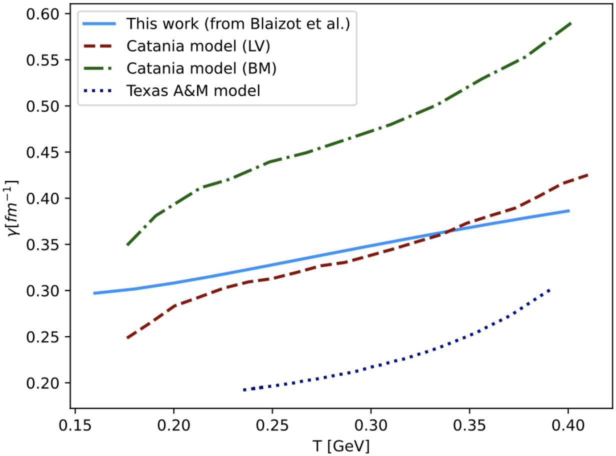
HQ potential



Drag coefficient

- taken **from same model** by Blaizot et al

$$\gamma = \frac{m_{\mathrm{D}}^2}{24\pi m_{\mathrm{c}}}\left[\ln\left(1+\frac{\Lambda^2}{m_{\mathrm{D}}^2}\right)-\frac{\Lambda^2/m_{\mathrm{D}}^2}{1+\Lambda^2/m_{\mathrm{D}}^2}\right]$$

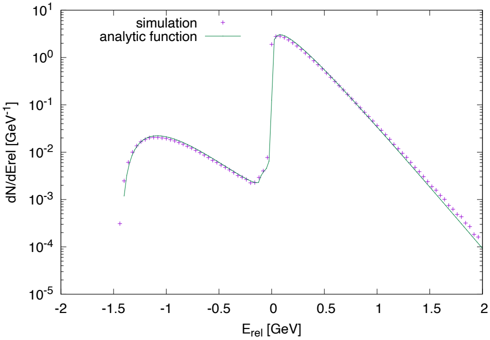


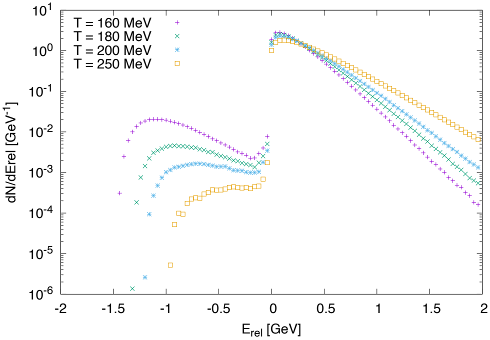
- TAMU: [HR12]
- Catania: [SDM+17]

Energy distribution in equilibrium

- classical distribution for $\mathrm{Q}\overline{Q}$ pair
- bound states: $E_{\mathrm{rel}} < 0$

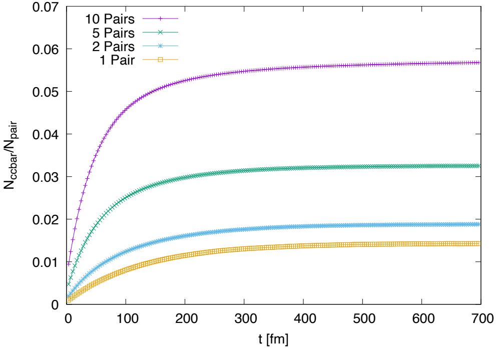
$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{rel}}} = C \int_0^R \mathrm{d}r\, r^2 \sqrt{E_{\mathrm{rel}} - V(r)} \exp(-E_{\mathrm{rel}}/T)$$



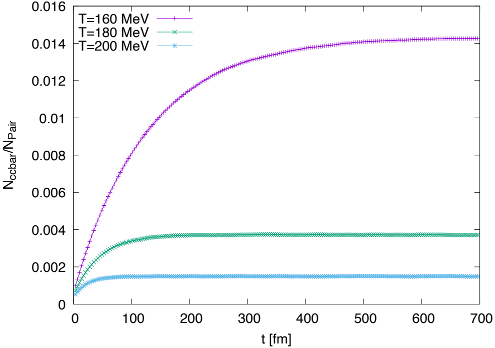


Bound-state formation

- $T = 160\,\mathrm{MeV}$



- $N_{\mathrm{pairs}} = 1$

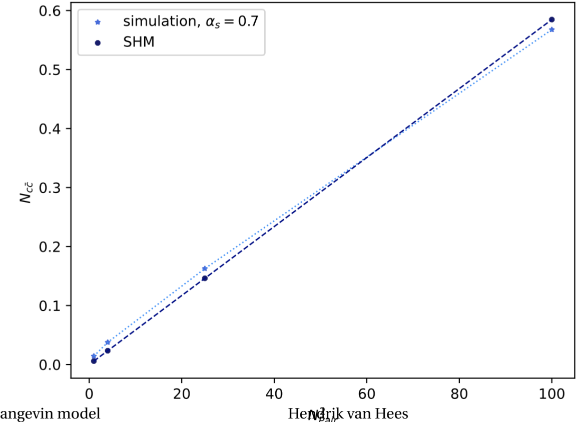


Comparison to statistical hadronization model

- charmonium multiplicity in grand-canonical ensemble

$$N_{\text{charmonium}} = V \sum_i \lambda_{\text{c}}^2 g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp(-m_i/T), \quad i \in \{\eta_c, \text{J}/\psi, \psi', \chi_c\}, \quad \lambda_{\text{c}} = \exp(\mu_{\text{c}}/T)$$

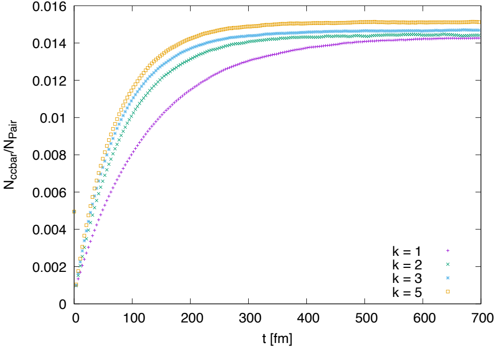
- $V = L_{\text{Box}}^3$, $L_{\text{box}} = 10\,\text{fm}$

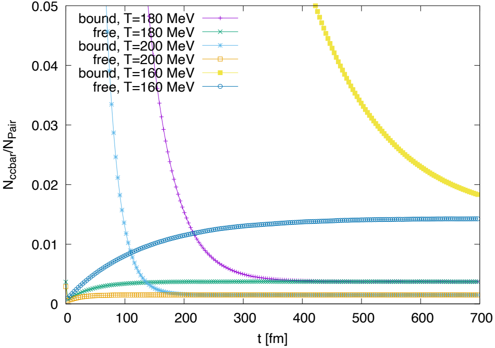


Relaxation time(s)

- charm-quark equilibration time $\tau_{\text{eq}} = 1/\gamma \simeq 3.3\,\text{fm}/c$
- relaxation times for quarkonium number much longer $\tau_{\text{equil}} \simeq 127\,\text{fm}/c$
- c-$\bar{\text{c}}$ must come close (within range of potential $\sim 0.6\,\text{fm}$)

- influence of drag coefficient: $\gamma \rightarrow k\gamma$ (potential kept)
- initial state: free c$\bar{\text{c}}$ pairs vs. all in bound states





First simulation for heavy-ion collisions

- fireball elliptic cylinder

$$\frac{x^2}{b^2(\tau)} + \frac{y^2}{a^2(\tau)} \leq 1$$

- volume

$$V(\tau) = \pi a(\tau) b(\tau)(z_0 + c\tau)$$

- long and short axes

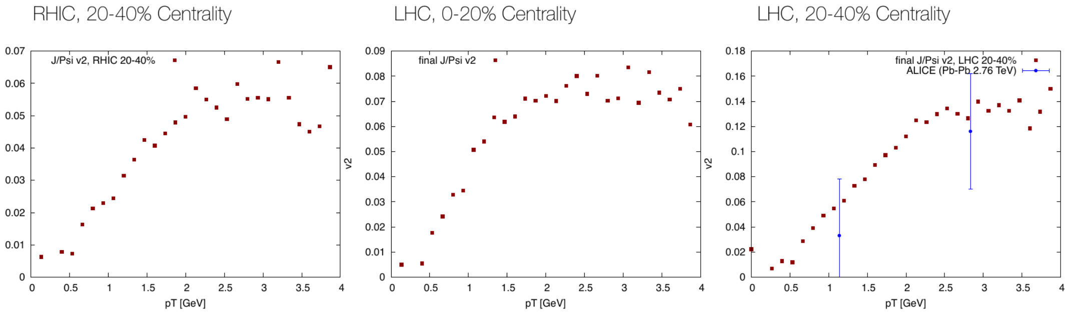
$$a(\tau) = a_0 + \frac{1}{a_a}\left(\sqrt{1 + a_a^2 \tau^2} - 1\right), \quad b(\tau) = b_0 + \frac{1}{a_b}\left(\sqrt{1 + a_b^2 \tau^2} - 1\right)$$

- a_a and a_b chosen to fit p_{T} spectra and v_2 of light hadrons
- 3D and finite rapidity: boost-invariant Bjorken flow

$$\vec{v} = (\tau / t\, v_b(\tau) \cos \nu r / r_{\mathrm{B}}, \tau / t\, v_a(\tau) \sin \nu r / r_{\mathrm{B}}, \tanh \eta)$$

- initial HQ momentum distribution from PYTHIA
- initial spatial distribution according to Glauber model

Charm-quark v_2


RHIC, 20-40% Centrality

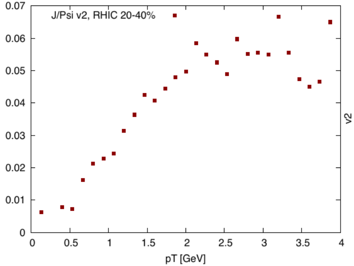
LHC, 0-20% Centrality

LHC, 20-40% Centrality

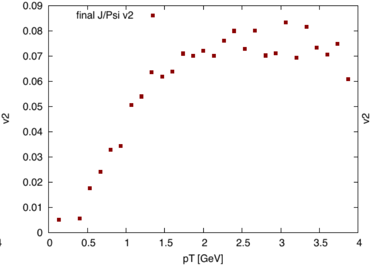
- data: [A^{+}13b]

Charmonium v_2

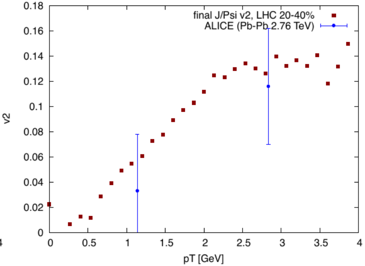
RHIC, 20-40% Centrality



LHC, 0-20% Centrality



LHC, 20-40% Centrality



- data: [A+13a]

Conclusions and outlook

- Conclusions
  - Box simulations: correct equilibrium limit (detailed balance) in agreement with SHM
  - bound-state formation as dynamical/kinetic process, including dissociation ⟷ regeneration
  - in fireball: v_2 of charm quarks and charmonia
- Outlook
  - Nuclear modification factors
  - using PHYTHIA: initialize with primordial charmonium
  - use formalism for bottom quarks and bottomonia
- (Big) Open questions
  - in-medium bound-state formation within many-body non-equilibrium QFT (so far only quantum-mechanical toy model [NRB+24] or Lindblad approach)
  - how to understand hadronization/confinement in dynamical models?

Bibliography I

[A^{+}13a] E. Abbas, et al., J/Ψ Elliptic Flow in Pb-Pb Collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV, Phys. Rev. Lett. **111** (2013) 162301. URL `https://doi.org/10.1103/PhysRevLett.111.162301`

[A^{+}13b] B. Abelev, et al., D meson elliptic flow in non-central Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$TeV, Phys. Rev. Lett. **111** (2013) 102301. URL `https://doi.org/10.1103/PhysRevLett.111.102301`

[BDFG16] J.-P. Blaizot, D. De Boni, P. Faccioli, G. Garberoglio, Heavy quark bound states in a quark-gluon plasma: Dissociation and recombination, Nucl. Phys. A **946** (2016) 49. URL `https://doi.org/10.1016/j.nuclphysa.2015.10.011`

[HR12] K. Huggins, R. Rapp, A T-Matrix Calculation for in-Medium Heavy-Quark Gluon Scattering, Nucl. Phys. A **896** (2012) 24. URL `https://doi.org/10.1016/j.nuclphysa.2012.09.008`

Bibliography II

[NRB+24] T. Neidig, J. Rais, M. Bleicher, H. van Hees, C. Greiner, Open quantum systems with Kadanoff-Baym equations, Phys. Lett. B **851** (2024) 138589.

[SDM+17] F. Scardina, S. K. Das, V. Minissale, S. Plumari, V. Greco, Estimating the charm quark diffusion coefficient and thermalization time from D meson spectra at energies available at the BNL Relativistic Heavy Ion Collider and the CERN Large Hadron Collider, Phys. Rev. C **96** (2017) 044905.
URL `https://doi.org/10.1103/PhysRevC.96.044905`