Heavy-Quark Diffusion in the QGP in Heavy-Ion Collisions

Hendrik van Hees

Justus-Liebig Universität Gießen

January 14, 2011



Institut für Theoretische Physik

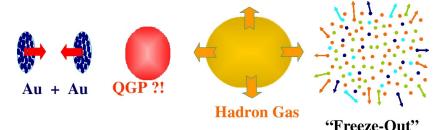


Outline

- Heavy-ion phenomenology
 - Hydrodynamical collective flow
 - Thermal models for chemical freezeout
 - Jet quenching
 - ullet Contituent-quark-number scaling of v_2
- Heavy-quark transport in the sQGP
 - Open heavy-flavor observables in heavy-ion collisions
 - Transport equations
 - The Fokker-Planck equation
 - Realization as Langevin process
 - Langevin simulation for heavy-ion collisions
- 3 In-medium interactions of heavy quarks I
 - Elastic pQCD heavy-quark scattering
 - Non-perturbative interactions: effective resonance model
- Mon-photonic electrons at RHIC
 - Estimate on transport properties of the sQGP
- Summary and Outlook
- $lue{o}$ Backup: Static heavy-quark potentials from lattice QCD + Brückner T-matrix

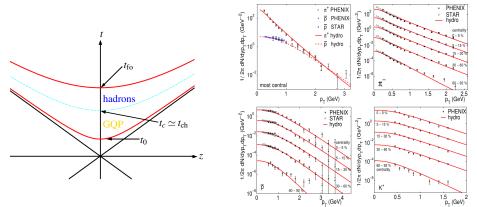
Heavy-ion collisions

- collisions of relativistic (heavy) nuclei
- many collisions of partons inside nucleons
- creation of many particles ⇒ hot and dense fireball
- formation of (thermalized) QGP?
- how to learn about properties of QGP?



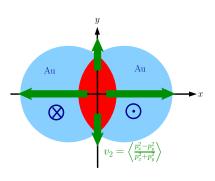
Hydrodynamical radial flow of the bulk

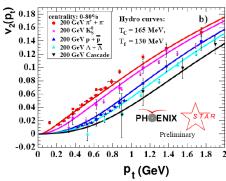
- ideal fluid in local thermal equilibrium \Rightarrow low viscosity/(entropy density), η/s
- needs strong interactions
- hydrodynamical model for ultra-relativistic heavy-ion collisions
 - after short formation time $(t_0 \lesssim 1 \; {\rm fm}/c)$
 - QGP in local thermal equilibrium \rightarrow hadronization at $T_c \simeq 160-190~{
 m MeV}$
 - chemical freeze-out: (inelastic collisions cease) $T_{\rm ch} \simeq 160-175~{
 m MeV}$
 - thermal freeze-out: (also elastic scatterings cease)



Hydrodynamical elliptic flow of the bulk

- particle spectra compatible with collective flow of a (nearly) ideal fluid \Rightarrow small η/s
- medium in local thermal equilibrium



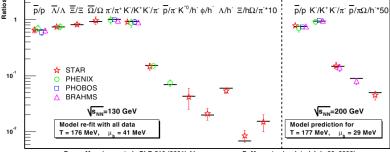


Thermal Models for Chemical Freezeout

- particle abundancies compatible with thermalized hadron-resonance gas
- grand-canonical ensemble
 - ullet fix mean energy \Rightarrow temperature $T_{\sf ch}$ (expect $T_c \simeq T_{\sf ch}$)
 - fix mean conserved "charges" \Rightarrow chemical potentials μ_b , μ_s , μ_q .

$$n_i = \frac{g_i}{(2\pi)^3} 4\pi \int_0^\infty dp \frac{p^2}{\exp\left(\frac{\sqrt{p^2 + m_i^2} - \mu_i}{T_{\text{ch}}}\right) \pm 1}$$

$$\mu_i = \mu_b B_i + \mu_s S_i + \mu_q Q_i$$



Braun-Munzinger et al., PLB 518 (2001) 41

[A. Andronic, P. Braun-Munzinger, Lect. Notes Phys. 652, 3567 (2004); arXiv:hep-ph/0402291

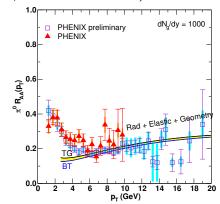
D. Magestro (updated July 22, 2002)

Jet Quenching

comparison to proton-proton collisions: nuclear-modification factor

$$R_{AA} = \frac{\mathrm{d}N_{AA}/\mathrm{d}p_t}{N_{\mathrm{coll}}\mathrm{d}N_{\mathrm{pp}}/\mathrm{d}p_t}$$

- $R_{AA} < 1$ for large p_t : jets absorbed by medium
- density $> \rho_{\rm crit}$ (comparison to lattice QCD)

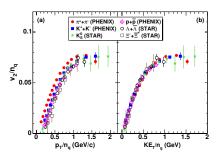


Constituent-quark-number scaling of v_2

ullet elliptic flow, v_2 scales with number of constituent quarks

$$v_2^{(\text{had})}(p_T^{(\text{had})}) = n_q v_2^{(q)}(p_T^{(\text{had})}/n_q)$$

ullet suggests coalescence of quarks at T_c



- possible microscopic mechanism hadron-resonance formation at $T_c \Rightarrow$ resonance-recombination model [Ravagli, HvH, Rapp, PRC 79, 064902 (2009)]
- other hint to quark coalescence:
 enhanced baryon/meson ratio compared to pp collisions

Heavy quarks in the sQGP





HQ rescattering in QGP: Langevin simulation drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP



Hadronization to D,B mesons via quark coalescence + fragmentation



 $\mbox{semileptonic decay} \Rightarrow \\ \mbox{"non-photonic" } \mbox{electron observables}$

The relativistic Boltzmann equation

- describe heavy-quark scattering in the QGP by (semi-)classical transport equation
- $f_Q(t, \vec{x}, \vec{p})$: phase-space distribution of heavy quarks
- equation of motion for HQ-fluid cell at time t at (\vec{p}, \vec{x}) :

$$\mathrm{d}f_{\mathbf{Q}} = \mathrm{d}t \left(\frac{\partial}{\partial t} + \vec{v} \frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right) f_{\mathbf{Q}}$$

- change of phase-space distribution with time (non-equilibrium)
- drift of HQ-fluid cell with velocity $\vec{v}=\vec{p}/E_{\vec{p}},~E_{\vec{p}}=\sqrt{m_Q^2+\vec{p}^2}$
- ullet change of momentum with mean-field force, $ec{F}$
- change must be due to collisions with surrounding medium

$$\frac{\mathrm{d}}{\mathrm{d}t}f_{\mathbf{Q}} = C[f_{\mathbf{Q}}] = \int \mathrm{d}^3\vec{k} [\underbrace{w(\vec{p} + \vec{k}, \vec{k})f_{\mathbf{Q}}(t, \vec{x}, \vec{p} + \vec{k})}_{\mathrm{gain}} - \underbrace{w(\vec{p}, \vec{k})f_{\mathbf{Q}}(t, \vec{x}, \vec{p})}_{\mathrm{loss}}]$$

• $w(\vec{p}, \vec{k})$: transition rate for collision of a heavy quark with momentum, \vec{p} with a heat-bath particle with momentum transfer, \vec{k}

Transition rates

- relation to cross sections of microscopic scattering processes
- e.g., elastic scattering of heavy quark with light quarks

$$w(\vec{p}, \vec{k}) = \gamma_q \int \frac{\mathrm{d}^3 \vec{q}}{(2\pi)^3} f_q(\vec{q}) v_{\mathrm{rel}}(\vec{p}, \vec{q} \to \vec{p} - \vec{k}, \vec{q} + \vec{k}) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

- $\gamma_q = 2 \times 3 = 6$: spin-color-degeneracy factor
- $v_{\rm rel} := \sqrt{(p \cdot q)^2 (m_Q m_q)^2}/(E_Q E_q)$; covariant relative velocity
- in terms of invariant matrix element

$$\begin{split} C[f_{\mathbf{Q}}] = & \frac{1}{2E_{\mathbf{Q}}} \int \frac{\mathrm{d}^{3}\vec{q}}{(2\pi)^{3}2E_{q}} \int \frac{\mathrm{d}^{3}\vec{p}'}{(2\pi)^{3}2E'_{p}} \int \frac{\mathrm{d}^{3}\vec{q}'}{(2\pi)^{3}2E'_{q}} \\ & \times \frac{1}{\gamma_{Q}} \sum_{c,s} \left| \mathcal{M}_{(\vec{p}',\vec{q}') \leftarrow (\vec{p},\vec{q})} \right|^{2} \\ & \times (2\pi)^{4} \delta^{(4)}(p+q-p'-q') [f_{\mathbf{Q}}(\vec{p}')f_{q}(\vec{q}') - f_{\mathbf{Q}}(\vec{p})f_{q}(\vec{q})] \end{split}$$

- \vec{p} , \vec{q} (\vec{p}' , \vec{q}') initial (final) momenta of heavy and light quark
- momentum transfer: $\vec{k} = \vec{q}' \vec{q} = \vec{p} \vec{p}'$
- sum over all ("relevant") scattering processes

The Fokker-Planck Equation

- heavy quarks ↔ light quarks/gluons: momentum transfers small
- $w(\vec{p} + \vec{k}, \vec{k})$: peaked around $\vec{k} = 0$
- ullet expansion of collision term around $ec{k}=0$

$$w(\vec{p} + \vec{k}, \vec{k}) f_{\mathbf{Q}}(\vec{p} + \vec{k}) \simeq w(\vec{p}, \vec{k}) f_{\mathbf{Q}}(\vec{p}) + \vec{k} \cdot \frac{\partial}{\partial \vec{p}} [w(\vec{p}, \vec{k}) f_{\mathbf{Q}}(\vec{p})]$$
$$+ \frac{1}{2} k_i k_j \frac{\partial^2}{\partial \vec{p}_i \vec{p}_k} [w(\vec{p}, \vec{k}) f_{\mathbf{Q}}(\vec{p})]$$

collision term

$$C[f_{\mathbf{Q}}] = \int \mathrm{d}^3\vec{k} \left[k_i \frac{\partial}{\partial p_i} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} \right] [w(\vec{p}, \vec{k}) f_{\mathbf{Q}}(\vec{p})].$$

The Fokker-Planck Equation

Boltzmann equation ⇒ simplifies to Fokker-Planck equation

$$\begin{split} \partial_t f_{\mathbf{Q}}(t, \vec{x}, \vec{p}) + \frac{\vec{p}}{E_{\vec{p}}} \cdot \frac{\partial}{\partial \vec{x}} f_{\mathbf{Q}}(t, \vec{x}, \vec{p}) &= \frac{\partial}{\partial p_i} \Bigg\{ A_i(\vec{p}) f_{\mathbf{Q}}(t, \vec{x}, \vec{p}) \\ &+ \frac{\partial}{\partial p_j} [B_{ij}(\vec{p}) f_{\mathbf{Q}}(t, \vec{p})] \Bigg\} \end{split}$$

with drag and diffusion coefficients

$$A_{i}(\vec{p}) = \int d^{3}\vec{k} \ k_{i}w(\vec{p}, \vec{k}), \quad B_{ij}(\vec{p}) = \frac{1}{2} \int d^{3}\vec{k} \ k_{i}k_{j}w(\vec{p}, \vec{k})$$

- · equilibrated light quarks and gluons: coefficients in heat-bath frame
- matter homogeneous and isotropic

$$\begin{split} A_{i}(\vec{p}) &= A(p)p_{i}, \quad B_{ij}(\vec{p}) = B_{0}(p)P_{ij}^{\perp} + B_{1}(p)P_{ij}^{\parallel} \\ \text{with} \quad P_{ij}^{\parallel}(\vec{p}) &= \frac{p_{i}p_{j}}{\vec{p}^{2}}, \quad P_{ij}^{\perp}(\vec{p}) = \delta_{ij} - \frac{p_{i}p_{j}}{\vec{p}^{2}} \end{split}$$

Meaning of the Coefficients

- Simplified equation for momentum distribution, $F_{Q}(t, \vec{p})$
- Integrate Fokker-Planck equation over whole spatial volume:

$$\begin{split} F_{\mathbf{Q}}(t,\vec{p}) &= \int_{V} \mathrm{d}^{3}\vec{x} f_{\mathbf{Q}}(t,\vec{x},\vec{p}), \\ \int_{V} \mathrm{d}^{3}\vec{x} \operatorname{div}_{\vec{x}} \left[\frac{\vec{p}}{E_{\vec{p}}} f(t,\vec{x},\vec{p}) \right] &= \int_{\partial V} \mathrm{d}\vec{S} \cdot \left[\frac{\vec{p}}{E_{\vec{p}}} f(t,\vec{x},\vec{p}) \right] = 0 \ \Rightarrow \\ \frac{\partial}{\partial t} F_{\mathbf{Q}}(t,\vec{p}) &= \frac{\partial}{\partial p_{i}} \left\{ A_{i}(\vec{p}) F_{\mathbf{Q}}(t,\vec{p}) + \frac{\partial}{\partial p_{j}} [B_{ij}(\vec{p}) F_{\mathbf{Q}}(t,\vec{p})] \right\} \end{split}$$

• most simple case in non-relativistic limit $A(\vec{p}) = A = \text{const}$, $B_0(\vec{p}) = B_1(\vec{p}) = B = \text{const}$

$$F_{Q}(t, \vec{p}) = \left\{ \frac{A}{2\pi D} [1 - \exp(-2\gamma t)] \right\}^{-3/2} \times \exp\left[-\frac{A}{2B} \frac{[\vec{p} - \vec{p}_0 \exp(-At)]^2}{1 - \exp(-2\gamma t)} \right]$$

Meaning of the Coefficients

• solution: Gaussian with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-At), \quad \Delta \vec{p}^2(t) = \langle \vec{p}^2 \rangle - \langle \vec{p} \rangle^2 = \frac{3B}{A} [1 - \exp(-2At)].$$

- A: friction/drag coefficient \Rightarrow dissipation
- 1/A: relaxation time to reach equilibrium
- B: momentum-diffusion coefficient
- measures size of momentum fluctuations (result of random uncorrelated collisions of heavy quarks with medim)
- \Rightarrow effective description of collisions: white-noise-random force
- equilibrium limit $(t \to \infty)$

$$F_{\mathbf{Q}}(t, \vec{p}) \underset{t \to \infty}{\cong} \left(\frac{2\pi B}{A}\right)^{3/2} \exp\left(-\frac{A\vec{p}^2}{2B}\right)$$

has to be Maxwell-Boltzmann distribution ⇒

$$B = m_{\mathcal{O}}AT$$

- \bullet T: given temperature of the QGP
- Einstein's dissipation-flucutation relation (1905)

Realization as Langevin process

- Langevin process: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

$$\mathrm{d}\vec{x} = \frac{\vec{p}}{E_p} \mathrm{d}t,$$

$$\mathrm{d}\vec{p} = -A \vec{p} \mathrm{d}t + \hat{C}\vec{w} \sqrt{\mathrm{d}t}$$

• $\vec{w}(t)$: Gaussian-distributed random variable

$$\langle \vec{w}(t) \rangle = 0, \quad \langle w_j(t)w_k(t') \rangle = \delta(t - t')$$

- $\hat{C} = \hat{C}^t$: covariance matrix of random force
- ullet stochastic process depends on choice of momentum argument of \hat{C}

$$\hat{C} \rightarrow \hat{C}(t, \vec{x}, \vec{p} + \xi d\vec{p}), \quad \xi \in [0, 1]$$

- usual values of ξ
 - $\xi = 0$: pre-point Ito realization
 - $\xi = 1/2$: Stratonovich realization
 - $\xi = 1$: post-point Ito (Hänggi-Klimontovich) realization

Langevin \leftrightarrow Fokker-Planck

heavy-quark phase-space distribution

$$f_{\mathbf{Q}}(t, \vec{x}, \vec{p}) = \left\langle \delta^{(3)}[\vec{x} - \vec{x}'(t)]\delta^{(3)}[\vec{p} - \vec{p}'(t)] \right\rangle \tag{1}$$

• $[\vec{x}'(t), \vec{p}'(t)]$: trajectories according to stochastic Langevin process

$$d\vec{x} = \frac{\vec{p}}{E_p} dt,$$

$$d\vec{p} = -A \vec{p} dt + \hat{C} \vec{w} \sqrt{dt}$$
(2)

perform timestep of Eq. (1) using (2)

$$\begin{split} \frac{\partial f_{\mathbf{Q}}}{\partial t} + \frac{p_{j}}{E} \frac{\partial f_{\mathbf{Q}}}{\partial x_{j}} &= \frac{\partial}{\partial p_{j}} \left[\left(A p_{j} - \xi C_{lk} \frac{\partial C_{jk}}{\partial p_{l}} \right) f_{\mathbf{Q}} \right] + \frac{1}{2} \frac{\partial^{2}}{\partial p_{j} \partial p_{k}} (C_{jl} C_{kl} f_{\mathbf{Q}}) \\ \Rightarrow C_{jk} &= \sqrt{2B_{0}} P_{jk}^{\perp} + \sqrt{2B_{1}} P_{jk}^{\parallel} \end{split}$$

• Form of Fokker-Planck equation ok, but how to chose ξ ?

Langevin ↔ Fokker-Planck

• Choice of ξ : $f_Q \to \text{Maxwell-Boltzmann distribution for } t \to \infty$:

$$f_{\textcolor{red}{Q}}^{\rm eq}(\vec{p}) \propto \exp(-\sqrt{\vec{p}^2 + m_{\textcolor{red}{Q}}^2}/T)$$

- Langevin process with $B_0 = B_1 = D(E) \Rightarrow C_{jk} = \sqrt{2D(E)}\delta_{jk}$
- MB distribution solution of stationary FP equation ⇒

$$A(E)ET - D(E) + (1 - \xi)TD'(E) \stackrel{!}{=} 0$$

- simples choice: $\xi = 1$ (post-point Ito realization)
- then simple Einstein dissipation-fluctuation relation

$$D = TEA$$

- ullet for models for FP coefficients: relation not well satisfied for B_1
- \Rightarrow use $\xi = 1$ and $B_1 = TEA$
- numerical check: Langevin simulation has right equilibrium limit

Langevin simulation for heavy-ion collisions

- need to simulate heavy-quark diffusion in sQGP
- "bulk" (light quarks + gluons) described by thermal fireball model
- flowing medium in local thermal equilibrium
- FP coefficients and Langevin process in restframe of the heat bath
- ullet way out: boost to local heat-bath frame with flow velocity $v(t,ec{x})$
- do time step to "update" momenta
- boost back to "lab frame"
- defines HQ distribution as "freezeout at constant lab time"
- NB: leads to covariant equilibrium distribution

$$dN_{\mathbf{Q}} = \frac{\gamma_{\mathbf{Q}}}{(2\pi)^3} d^3 \vec{x}^{(\mathsf{hb})} \frac{d^3 \vec{p}}{p_0} p \cdot u(x) \exp\left(-\frac{p \cdot u(x)}{T(x)}\right)$$

- $u(t, \vec{x}) = [1, \vec{v}(t, \vec{x})] / \sqrt{1 \vec{v}^2(t, \vec{x})}$: velocity-flow field (4-vector)
- T(x): temperature field (4-scalar)

Fire-ball model

 Elliptic fire-ball parameterization fitted to hydrodynamical flow pattern [Kolb '00]

$$\begin{split} V(t) &= \pi(z_0 + v_z t) a(t) b(t), \quad a,b \text{: semi-axes of ellipse,} \\ v_{a,b} &= v_\infty [1 - \exp(-\alpha t)] \mp \Delta v [1 - \exp(-\beta t)] \end{split}$$

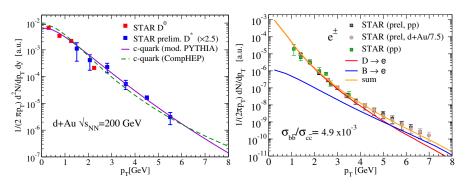
- Isentropic expansion: S = const (fixed from N_{ch})
- QGP Equation of state:

$$s = \frac{S}{V(t)} = \frac{4\pi^2}{90}T^3(16 + 10.5n_f^*), \quad n_f^* = 2.5$$

- obtain $T(t) \Rightarrow A(t,p)$, $B_0(t,p)$ and $B_1 = TEA$
- for semicentral collisions ($b=7~{\rm fm}$): $T_0=340~{\rm MeV}$, QGP lifetime $\simeq 5~{\rm fm}/c$.
- simulate FP equation as relativistic Langevin process

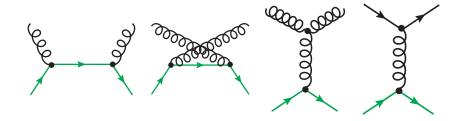
Initial conditions

- ullet need initial p_T -spectra of charm and bottom quarks
 - (modified) PYTHIA to describe exp. D meson spectra, assuming δ -function fragmentation
 - ullet exp. non-photonic single- e^\pm spectra: Fix bottom/charm ratio



Elastic pQCD processes

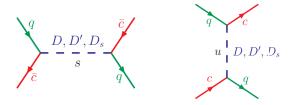
• Lowest-order matrix elements [Combridge 79]



- Debye-screening mass for t-channel gluon exch. $\mu_q = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on "non-photonic" electrons
 [Moore, Teaney PRC 71, volume 71, 064904 (2005)]

Non-perturbative interactions: Resonance Scattering

- ullet General idea: Survival of D- and B-meson like resonances above T_c
- model based on chiral symmetry (light quarks) HQ-effective theory
- elastic heavy-light-(anti-)quark scattering

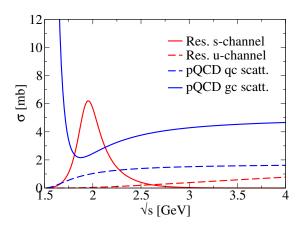


ullet D- and B-meson like resonances in sQGP



- parameters
 - $m_D = 2 \text{ GeV}, \Gamma_D = 0.4 \dots 0.75 \text{ GeV}$
 - $m_B = 5 \text{ GeV}, \Gamma_B = 0.4 \dots 0.75 \text{ GeV}$

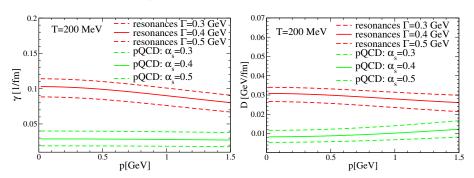
Cross sections



- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion

Transport coefficients: pQCD vs. resonance scattering

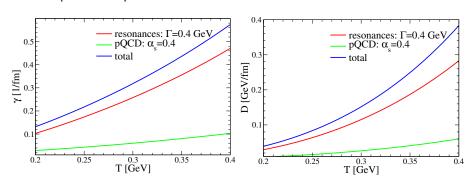
• three-momentum dependence



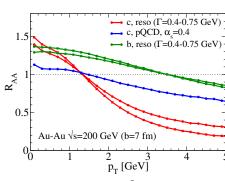
• resonance contributions factor $\sim 2 \dots 3$ higher than pQCD!

Transport coefficients: pQCD vs. resonance scattering

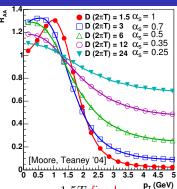
• Temperature dependence



Spectra and elliptic flow for heavy quarks

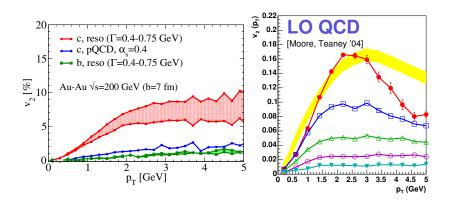


- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances ⇒ c-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro



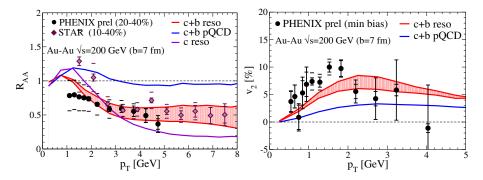
- $\mu_D = 1.5T$ fixed
- spatial diff. coefficient: $D = D_s = \frac{T}{mA}$
- $2\pi TD \simeq \frac{3}{2\alpha^2}$

Spectra and elliptic flow for heavy quarks



Observables: p_T -spectra (R_{AA}) , v_2

- Hadronization: Coalescence with light quarks + fragmentation $\Leftrightarrow c\bar{c}, \ b\bar{b}$ conserved
- single electrons from decay of *D* and *B*-mesons

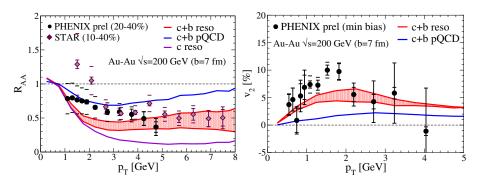


• Without further adjustments: data quite well described

[HvH. V. Greco, R. Rapp. Phys. Rev. C 73, 034913 (2006)]

Observables: p_T -spectra (R_{AA}) , v_2

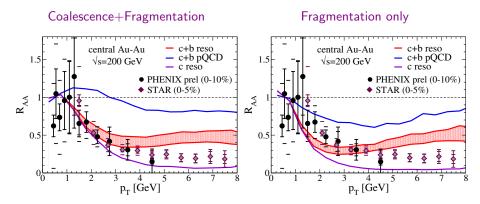
- Hadronization: Fragmentation only
- single electrons from decay of *D* and *B*-mesons



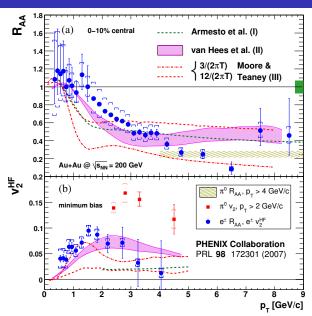
- coalescence brings up both, R_{AA} and v_2
- due to additional momentum kick from light quarks

Observables: p_T -spectra (R_{AA}) , v_2

- Central Collisions
- single electrons from decay of *D* and *B*-mesons



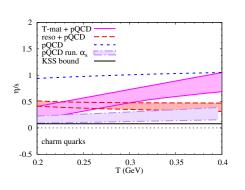
Comparison to newer data

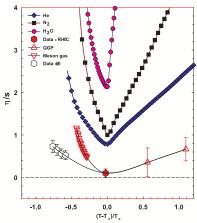


Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck $\Rightarrow D_s = \frac{T}{mA} = \frac{T^2}{D}$
- ullet coupling strength in plasma: viscosity/entropy density, η/s

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \qquad (\text{AdS/CFT}), \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \qquad (\text{wQGP})$$





[Lacey, Taranenko, FRNC2006, 021 (2006)]

Summary

Boltzmann Transport Equations

- can be derived from classical mechanics or quantum-many-body theory
- (semi-)classical statistical description of interacting many-body systems
- equations for single-particle phase-space distribution
- collision term: transition probabilities from microscopic cross sections
- many-body systems \Leftrightarrow microscopic properties of constituents

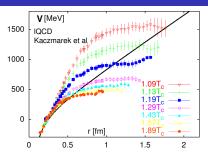
Fokker-Planck Equations

- heavy particles immersed in medium of light particles
- momentum transfer in single collision small ⇒ integro-differential Boltzmann equation ⇒ partial differential equation
- HQ-medium interactions ⇒ friction/drag coefficient + diffusion coefficients
- related by Einstein dissipation-fluctuation relation

Summary

- Langevin Equations
 - stochastic differential equation equivalent to Fokker-Planck equation
 - drag/friction force + random forces = uncorrelated Gaussian noise
 - depends on realization of stochastic process
 - right process ⇒ equilibrium limit = relativistic MB distribution
 - application to flowing sQGP
- Heavy-quark interactions in the sQGP
 - elastic scattering with light quarks and gluons: pQCD + screening
 - resonance scattering with light (anti-)quarks
- Non-photonic single electron observables
 - ullet $R_{AA}(p_T)$ and $v_2(p_T)$ of electrons from D- and B-meson decays
 - ullet Langevin simulation ullet coalescence+fragmentation hadronization ullet semi-leptonic decay
 - pQCD (with realistic α_s) too weak
 - with resonance-scattering interactions good description of data

Microscopic model: Static potentials from lattice QCD



- color-singlet free energy from lattice
- use internal energy

$$U_1(r,T) = F_1(r,T) - T \frac{\partial F_1(r,T)}{\partial T},$$

$$V_1(r,T) = U_1(r,T) - U_1(r \to \infty, T)$$

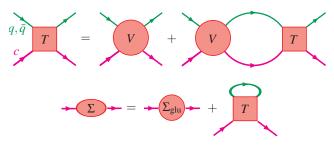
Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

$$V_{\bar{3}} = \frac{1}{2}V_1, \quad V_6 = -\frac{1}{4}V_1, \quad V_8 = -\frac{1}{8}V_1$$

HvH, M. Mannarelli, V. Greco, R. Rapp, PRL f 100, 192301 (2008); HvH, M. Mannarelli, R. Rapp, EJC f 61, 799 (2009)]

T-matrix

ullet Brueckner many-body approach for elastic Qq, $Qar{q}$ scattering

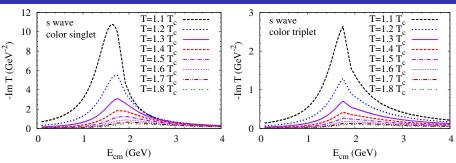


- ullet reduction scheme: 4D Bethe-Salpeter o 3D Lippmann-Schwinger
- S- and P waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

$$\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a \left(|T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos \theta_{\rm cm} \right)$$

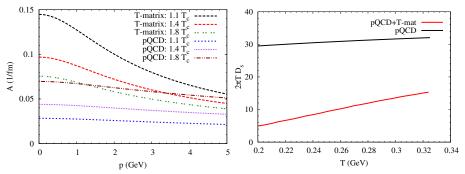
HvH, M. Mannarelli, V. Greco, R. Rapp, PRL 100, 192301 (2008); HvH, M. Mannarelli, R. Rapp, EJC 61, 799 (2009)]

T-matrix



- resonance formation at lower temperatures $T \simeq T_c$
- melting of resonances at higher $T! \Rightarrow \mathsf{sQGP}$
- ullet P wave smaller
- ullet resonances near T_c : natural connection to quark coalescence [Ravagli, Rapp 07; Ravagli, HvH, Rapp 08]
- ullet model-independent assessment of elastic $Qq,\ Qar{q}$ scattering
- problems: uncertainties in extracting potential from IQCD
- ullet in-medium potential U vs. F?

Transport coefficients



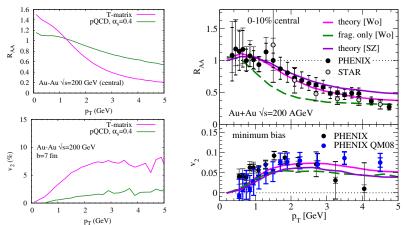
- from non-pert. interactions reach $A_{\rm non-pert} \simeq 1/(7~{\rm fm}/c) \simeq 4 A_{\rm pQCD}$
- A decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

increases with temperature

Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence+fragmentation $\rightarrow D/B \rightarrow e + X$



- coalescence crucial for description of data
- increases both, R_{AA} and $v_2 \Leftrightarrow$ "momentum kick" from light quarks!
- ullet "resonance formation" towards $T_c\Rightarrow$ coalescence natural [Ravagli, Rapp 07]