Asymptotic description of finite lifetime effects on the photon emission from a quark-gluon plasma

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(Dated: October 18, 2013)

Abstract

Direct photons play an important role as electromagnetic probes from the quark-gluon plasma (QGP) which occurs during ultrarelativistic heavy-ion collisions. In this context, it is of particular interest how the finite lifetime of the QGP affects the resulting photon production. Earlier investigations on this question were accompanied by a divergent contribution from the vacuum polarization and by the remaining contributions not being integrable in the ultraviolet (UV) domain. In this work, we provide a different approach in which we do not consider the photon number density at finite times, but for free asymptotic states obtained by switching the electromagnetic interaction according to the Gell-Mann and Low theorem. This procedure eliminates a possible unphysical contribution from the vacuum polarization and, moreover, renders the photon number density UV integrable. It is emphasized that the consideration of free asymptotic states is, indeed, crucial to obtain such physically reasonable results.

PACS numbers: 05.70.Ln,11.10.Ef,25.75.Cj
Keywords: heavy-ion collision, non-equilibrium quantum field theory, first-order photon production

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I. INTRODUCTION

Direct photons play an important role as electromagnetic probes for the quark-gluon plasma (QGP) which occurs during ultrarelativistic heavy-ion collisions [1–5]. Since photons interact only electromagnetically with the surrounding hadronic medium their mean free path is much larger than the spatial extension of the QGP. For that reason, they leave it almost undisturbed once they have been produced and therefore provide a direct insight into all stages of the collision. In this context, it is of particular interest how non-equilibrium effects such as the finite lifetime of the QGP affect the resulting photon emission.

Earlier investigations on this question [6–8] found that this finite lifetime gives rise to contributions from first-order QED processes, i.e., processes linear in the electromagnetic coupling constant, α_e, which are kinematically forbidden in thermal equilibrium. Moreover, the photon spectrum resulting from these processes flattened into a power-law decay for photon energies $\omega_{\vec{k}} > 1.5$ GeV ($\omega_{\vec{k}} = |\vec{k}|$ with $\vec{k}$ denoting the three-momentum of the emitted photon), which would imply that in this domain the first-order contributions dominate over leading-order thermal contributions. The latter are linear in the electromagnetic coupling constant, α_e, and the strong coupling constant, α_s, in each case and thus of overall second order.

On the other hand, the investigations in [6–8] were accompanied by two serious artifacts. First, the photon number density contained a divergent contribution from the vacuum polarization for a given photon energy, $\omega_{\vec{k}}$. Moreover, the photon number density arising from the remaining contributions scaled as $1/\omega_{\vec{k}}^3$ in the ultraviolet (UV) domain. This implies that the total number density and the total energy density of the emitted photons are logarithmically and linearly divergent, respectively.

Recently, we have followed two other approaches in order to handle these problems in a consistent manner. In the first approach [9], we have pursued a model description in which we have simulated the finite lifetime of the QGP by introducing time dependent quark/antiquark occupation numbers in the photon self-energy. This procedure allows for a consistent renormalization of the divergent contribution from the vacuum polarization. It does, however, not lead to a UV integrable photon number density for the general case.

At first we had suspected that this shortcoming results from a violation of the Ward-Takahashi identities within the model description [9]. For that reason, we have also pursued a second approach [10], where we have modeled the creation of the QGP by a Yukawa-like source term in the QED-Lagrangian coupling the quarks and antiquarks to a purely time dependent, scalar background field. This effectively assigns the quarks and antiquarks a time dependent mass, which is consistent with the Ward-Takahashi identities. We have again restricted ourselves to first-order and thus purely non-equilibrium QED processes. These are kinematically possible in this case since the quarks and antiquarks obtain additional energy by the coupling to the time dependent background field. Similar investigations have been performed in [11–15] on electron-positron pair annihilation into a single photon in the presence of a strong laser field. There the preceding pair creation (and the subsequent annihilation) has been induced by a time dependent electromagnetic background field (see also [16–20]).

Another crucial difference to the approaches in [6–9] has been the consideration of the photon number density not at finite times, but for free asymptotic states employing the standard Gell-Man and Low switching of the interaction Hamiltonian. Through this procedure, the photon number density is not plagued by the aforementioned unphysical contribution from the vacuum polarization anymore and, furthermore, has been rendered UV integrable for suitable mass parameterizations, $m(t)$. In particular, our investigations have shown that the photon number density indeed has to be considered for free asymptotic states in order to obtain such physically reasonable results. In this context, we have seen that a consistent definition of the photon number density is actually only possible for such free asymptotic states, whereas a similar interpretation of the respective expression is usually not justified at finite times, t. Such a conceptual problem also occurs if the electromagnetic interaction is only switched on from $t \to -\infty$ but not off again for $t \to \infty$, which has been suggested in [21] in order to implement initial correlations at some $t = t_0$ developing from an uncorrelated initial state at $t \to -\infty$. Hence, the results from [10] raise the question whether the artifacts encountered in [6–9] result from an inconsistent definition of the 'photon number density' at finite times and whether they are removed if this quantity is considered for free asymptotic states instead.

Accordingly, in this work we revisit the previous approach [9]. This means that we again simulate the time-evolution of the QGP during a heavy-ion collision by introducing strongly time dependent quark/antiquark occupation numbers in the photon self-energy, but we consider photon number density not at finite times, but for free asymptotic states. Hence, we adhere to our principle approach from [10] but consider an alternative description for our time dependent emitting system. We shall demonstrate that in direct analogy to [10], this procedure again eliminates a potential unphysical contribution from the vacuum polarization. Moreover, it leads to a UV integrable photon number density if the time evolution of the quark/antiquark occupation numbers in the photon self-energy is described in a physically reasonable manner, i.e., if it is taken into account that these occupation numbers are populated over a finite interval of time. In this context, we emphasize again that considering the photon number density for free asymptotic states is, indeed, crucial to obtain such physically reasonable results and that the artifacts encountered in [6–8] and still partly in [9] would reappear if this quantity were considered at finite times.

This paper is organized as follows: In section II, we provide a detailed description of our (revised) model approach on first-order photon production from a QGP. In particular, we demonstrate how we simulate the time evolution of the QGP by introducing fastly populating, time dependent quark/antiquark occupation numbers in the photon self-energy and how our asymptotic description eliminates a possible unphysical contribution from the vacuum polarization. After that, we present our numerical investigations in section III. We show that in this present setting, our description also leads to a UV integrable photon number density. There we also provide detailed considerations on the dependence of the photon number density on the time scale, τ, over which the quark/antiquark occupation numbers are assumed to build up. Then we compare our results to leading-order thermal photon emission in section IV. In section V, we again highlight the necessity to consider the photon number density for free asymptotic states before we finish with a summary and an outlook to future investigations in section VI. Technical details are given in appendix A.

II. ASYMPTOTIC PHOTON NUMBER DENSITY

Before we start with our numerical investigations, we provide a more extensive description of our model approach than given in [9]. The starting point is the photon number density for a homogeneous, but non-stationary emitting system of deconfined quarks and antiquarks. At first order in α_e, this quantity is given by

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x \mathrm{d}^3k} = \frac{1}{(2\pi)^3} \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t} \mathrm{d}t_2 \mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k}, t_1, t_2) \mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1 - t_2)} \ . \tag{1}$$

Here $\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k}, t_1, t_2)$ denotes the transverse part of the photon self-energy, i.e.,

$$\mathrm{i}\Pi_{\mathrm{T}}^{<}(\vec{k}, t_1, t_2) = \gamma^{\mu\nu}(\vec{k}) \mathrm{i}\Pi_{\nu\mu}^{<}(\vec{k}, t_1, t_2) \ . \tag{2}$$

$\gamma^{\mu\nu}(\vec{k})$ is the photon tensor reading

$$\gamma^{\mu\nu}(\vec{k}) = \sum_{\lambda=\perp} \epsilon^{\mu,*}(\vec{k},\lambda)\epsilon^{\nu}(\vec{k},\lambda) = \begin{cases} -g^{\mu\nu} - \frac{k^\mu k^\nu}{\omega_{\vec{k}}^2} & , \quad \text{for} \quad \mu,\nu \in \{1,2,3\} \\ 0 & , \quad \text{otherwise} \end{cases} , \tag{3}$$

where the sum runs over all physical (transverse) polarizations. Moreover, we have introduced the four vector $k^\mu = (\omega_{\vec{k}}, \vec{k})$. The photon self-energy, $\mathrm{i}\Pi^<_{\mu\nu}(\vec{k}, t_1, t_2)$, in turn is given by the thermal one-loop approximation

$$\mathrm{i}\Pi^<_{\mu\nu}(\vec{k}, t_1, t_2) = e^2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu S^<_{\mathrm{F}}(\vec{q}, t_1, t_2)\gamma_\nu S^>_{\mathrm{F}}(\vec{p}, t_2, t_1)\right\} , \tag{4}$$

where e denotes the electromagnetic coupling and $\vec{q} = \vec{p} + \vec{k}$. In thermal equilibrium, the fermion propagators entering (4) read

$$S^<_{\mathrm{F}}(\vec{q}, t_1, t_2) = S^<_{\mathrm{Q}}(\vec{q}, t_1, t_2) + S^<_{\mathrm{AQ}}(\vec{q}, t_1, t_2) , \tag{5a}$$

$$S^>_{\mathrm{F}}(\vec{p}, t_1, t_2) = S^>_{\mathrm{Q}}(\vec{p}, t_1, t_2) + S^>_{\mathrm{AQ}}(\vec{p}, t_1, t_2) , \tag{5b}$$

with the quark (Q) and antiquark (AQ) components

$$S^<_{\mathrm{Q}}(\vec{q}, t_1, t_2) = \mathrm{i} n_{\mathrm{F}}(q_0) \frac{\not{q} + m}{2q_0} \cdot \mathrm{e}^{-\mathrm{i}q_0(t_1 - t_2)} , \tag{6a}$$

$$S^<_{\mathrm{AQ}}(\vec{q}, t_1, t_2) = \mathrm{i}\left[1 - n_{\mathrm{F}}(q_0)\right] \frac{\bar{\not{q}} - m}{2q_0} \cdot \mathrm{e}^{\mathrm{i}q_0(t_1 - t_2)} , \tag{6b}$$

$$S^>_{\mathrm{Q}}(\vec{p}, t_1, t_2) = -\mathrm{i}\left[1 - n_{\mathrm{F}}(p_0)\right] \frac{\not{p} + m}{2p_0} \cdot \mathrm{e}^{-\mathrm{i}p_0(t_1 - t_2)} , \tag{6c}$$

$$S^>_{\mathrm{AQ}}(\vec{p}, t_1, t_2) = -\mathrm{i} n_{\mathrm{F}}(p_0) \frac{\bar{\not{p}} - m}{2p_0} \cdot \mathrm{e}^{\mathrm{i}p_0(t_1 - t_2)} . \tag{6d}$$

Here $n_{\mathrm{F}}(E)$ is the Fermi-Dirac distribution function

$$n_{\mathrm{F}}(E) = \frac{1}{1 + \mathrm{e}^{\beta E}} , \tag{7}$$

with $\beta = 1/T$ and T denoting the temperature of the system. Moreover, we have introduced the four-vector notations $p^\mu = (E_{\vec{p}}, \vec{p})$ and $\bar{p}^\mu = (E_{\vec{p}}, -\vec{p})$. Here $E_{\vec{p}} \equiv \sqrt{p^2 + m^2}$ is the free relativistic quark/antiquark energy with p and m describing the absolute value of the three-momentum, $\vec{p}$, and the quark/antiquark mass, respectively.

It follows from (6) that expression (4) contains the contributions from the four first-order QED processes. These processes are (one-body) quark Bremsstrahlung (QBS), (one-body) antiquark Bremsstrahlung (ABS), quark-antiquark pair annihilation into a single photon (ANH), and the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum (PAC). Hence it is convenient to split up (4) accordingly, i.e.,

$$\mathrm{i}\Pi^<_{\mu\nu}(\vec{k}, t_1, t_2) = \mathrm{i}\Pi^{\mathrm{QBS}}_{\mu\nu}(\vec{k}, t_1, t_2) + \mathrm{i}\Pi^{\mathrm{ABS}}_{\mu\nu}(\vec{k}, t_1, t_2) + \mathrm{i}\Pi^{\mathrm{ANH}}_{\mu\nu}(\vec{k}, t_1, t_2) + \mathrm{i}\Pi^{\mathrm{PAC}}_{\mu\nu}(\vec{k}, t_1, t_2) , \tag{8}$$

with the particular contributions given by

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{QBS}}(\vec{k},t_1,t_2) = e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Tr}\left\{\gamma_\mu S_{\mathrm{Q}}^{<}(\vec{q},t_1,t_2)\gamma_\nu S_{\mathrm{Q}}^{>}(\vec{p},t_2,t_1)\right\} , \tag{9a}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{ABS}}(\vec{k},t_1,t_2) = e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Tr}\left\{\gamma_\mu S_{\mathrm{AQ}}^{<}(\vec{q},t_1,t_2)\gamma_\nu S_{\mathrm{AQ}}^{>}(\vec{p},t_2,t_1)\right\} , \tag{9b}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{ANH}}(\vec{k},t_1,t_2) = e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Tr}\left\{\gamma_\mu S_{\mathrm{Q}}^{<}(\vec{q},t_1,t_2)\gamma_\nu S_{\mathrm{AQ}}^{>}(\vec{p},t_2,t_1)\right\} , \tag{9c}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{PAC}}(\vec{k},t_1,t_2) = e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Tr}\left\{\gamma_\mu S_{\mathrm{AQ}}^{<}(\vec{q},t_1,t_2)\gamma_\nu S_{\mathrm{Q}}^{>}(\vec{p},t_2,t_1)\right\} . \tag{9d}$$

It follows from (6a)-(6d) that the contraction with $\gamma^{\mu\nu}(\vec{k})$ yields

$$\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{QBS}}(\vec{k},t_1,t_2) = 2e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1-\frac{px(px+\omega_{\vec{k}})+m^2}{p_0q_0}\right\}n_{\mathrm{F}}(q_0)\left[1-n_{\mathrm{F}}(p_0)\right] \times \mathrm{e}^{-\mathrm{i}(q_0-p_0)(t_1-t_2)} , \tag{10a}$$

$$\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{ABS}}(\vec{k},t_1,t_2) = 2e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1-\frac{px(px+\omega_{\vec{k}})+m^2}{p_0q_0}\right\}n_{\mathrm{F}}(p_0)\left[1-n_{\mathrm{F}}(q_0)\right] \times \mathrm{e}^{\mathrm{i}(q_0-p_0)(t_1-t_2)} , \tag{10b}$$

$$\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{ANH}}(\vec{k},t_1,t_2) = 2e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1+\frac{px(px+\omega_{\vec{k}})+m^2}{p_0q_0}\right\}n_{\mathrm{F}}(q_0)n_{\mathrm{F}}(p_0) \times \mathrm{e}^{-\mathrm{i}(q_0+p_0)(t_1-t_2)} , \tag{10c}$$

$$\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{PAC}}(\vec{k},t_1,t_2) = 2e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1+\frac{px(px+\omega_{\vec{k}})+m^2}{p_0q_0}\right\}\left[1-n_{\mathrm{F}}(q_0)\right]\left[1-n_{\mathrm{F}}(p_0)\right] \times \mathrm{e}^{\mathrm{i}(q_0+p_0)(t_1-t_2)} . \tag{10d}$$

Here p and x denote the absolute value of the loop momentum, $\vec{p}$, and the cosine of the angle between $\vec{p}$ and $\vec{k}$, respectively, i.e., $\vec{p}\cdot\vec{k} = p\omega_{\vec{k}}x$. By making the substitutions $\vec{p}\to\vec{p}-\vec{k}$ and $x\to -x$ in (10b), it follows that this expression agrees with (10a) for all values of t_1 and t_2. It is hence convenient to take these two contributions together as one single contribution describing (one-body) quark/antiquark Bremsstrahlung (BST), i.e.,

$$\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{BST}}(\vec{k},t_1,t_2) = 4e^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{1-\frac{px(px+\omega_{\vec{k}})+m^2}{p_0q_0}\right\}n_{\mathrm{F}}(q_0)\left[1-n_{\mathrm{F}}(p_0)\right] \times \mathrm{e}^{-\mathrm{i}(q_0-p_0)(t_1-t_2)} . \tag{11}$$

Accordingly, the photon number density (1) can be decomposed as

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{BST}} = \frac{1}{(2\pi)^3}\int_{-\infty}^{t}\mathrm{d}t_1\int_{-\infty}^{t}\mathrm{d}t_2\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{BST}}(\vec{k},t_1,t_2)\mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1-t_2)} , \tag{12a}$$

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{ANH}} = \frac{1}{(2\pi)^3}\int_{-\infty}^{t}\mathrm{d}t_1\int_{-\infty}^{t}\mathrm{d}t_2\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{ANH}}(\vec{k},t_1,t_2)\mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1-t_2)} , \tag{12b}$$

$$2\omega_{\vec{k}}\frac{\mathrm{d}^6n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{PAC}} = \frac{1}{(2\pi)^3}\int_{-\infty}^{t}\mathrm{d}t_1\int_{-\infty}^{t}\mathrm{d}t_2\mathrm{i}\Pi_{\mathrm{T}}^{\mathrm{PAC}}(\vec{k},t_1,t_2)\mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1-t_2)} . \tag{12c}$$

That (12a)-(12c) correspond to the contribution from the indicated processes can be seen by carrying out the multiplication of the respective expression for the photon self-energy with the factor $e^{i\omega_{\vec{k}}(t_1-t_2)}$. It follows from (11) and (10a)-(10d) that this procedure gives rise to an oscillating behavior in $t_1 - t_2$. The corresponding process can then be deduced from the specific oscillation frequency. Furthermore, when we show in appendix A that each of the contributions (12a)-(12c) can be written as the absolute square of a first-order QED transition amplitude, this interpretation also becomes evident from the underlying spinor structure.

In order to remove the artifacts encountered in [6–8], we have to find an adequate ansatz for the fermion propagators (5). For this purpose, we take into account that the vacuum contribution to (4) occurs for all times, whereas the medium contributions only occur as long as the QGP is actually present. The former aspect is the reason why we have taken the initial time, i.e., the lower bound of the time integrals entering (1), to $-\infty$. The aforementioned time dependence is implemented into the fermion propagators (5) by introducing time dependent occupation numbers

$$n_{\mathrm{F}}(E) \to n_{\mathrm{F}}(E,t) = f(t) n_{\mathrm{F}}(E) \ , \tag{13}$$

and replacing the fermion occupation numbers and the number of holes entering the fermion propagators (5) by their geometric mean from the different points of time, t_1 and t_2, i.e.,

$$n_{\mathrm{F}}(E) \ \to \ \sqrt{n_{\mathrm{F}}(E,t_1) n_{\mathrm{F}}(E,t_2)} \ , \tag{14a}$$

$$1 - n_{\mathrm{F}}(E) \ \to \ \sqrt{[1 - n_{\mathrm{F}}(E,t_1)]\,[1 - n_{\mathrm{F}}(E,t_2)]} \ . \tag{14b}$$

By means of this procedure, the coincidence between (10a) and (10b) is left unchanged. Moreover, the time evolution of the QGP is coupled to the interaction vertices. As we demonstrate in appendix A, this ansatz ensures that (12a)-(12c) can be written as an absolute square and, as a consequence, are positive (semi-)definite. Therefore, each of these contributions and thus the overall photon number density (1) cannot adopt unphysical negative values. Moreover, the absolute-square representation ensures that (12a)-(12c) can be identified with the first-order QED process indicated in each case.

The crucial difference to [9] is that here we do not consider (1) at finite times, but in the limit $t \to \infty$ for free asymptotic states. In analogy to [10], such states are obtained in this limit by introducing an adiabatic switching of the electromagnetic interaction according to the Gell-Mann and Low theorem, i.e.,

$$\hat{H}_{\mathrm{EM}} \to f_\varepsilon(t) \hat{H}_{\mathrm{EM}} \ , \quad \text{with} \quad f_\varepsilon(t) = e^{-\varepsilon|t|} \quad \text{and} \quad \varepsilon > 0 \ . \tag{15}$$

As a result, the time integrals entering (1) are effectively regulated by a factor of $e^{-\varepsilon|t_i|}$ with $i = 1, 2$. At the very end of our calculation, i.e., after taking the limit $t \to \infty$ in expression (1), we take the limit $\varepsilon \to 0$. As in [10], the physical photon number density is thus defined as

$$2\omega_{\vec{k}} \frac{d^6 n_\gamma}{d^3x d^3k} = \lim_{\varepsilon \to 0} \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 f_\varepsilon(t_1) f_\varepsilon(t_2) i\Pi^{<}_{\mathrm{T}}(\vec{k}, t_1, t_2) e^{i\omega_{\vec{k}}(t_1-t_2)} \ . \tag{16}$$

We shall briefly demonstrate that (16) does not contain any unphysical contribution from the vacuum polarization. The latter is extracted from $i\Pi^{<}_{\mathrm{T}}(\vec{k}, t_1, t_2)$ by taking the limit $T \to 0$ (which corresponds to the absence of the medium) and reads

$$i\Pi^{<}_{\mathrm{T},0}(\vec{k}, t_1 - t_2) = 2e^2 \int \frac{d^3p}{(2\pi)^3} \left\{ 1 + \frac{px(px + \omega_{\vec{k}}) + m^2}{p_0 q_0} \right\} e^{i(q_0+p_0)(t_1-t_2)} \ . \tag{17}$$

Upon insertion of (17) into (16), we obtain

$$
\begin{aligned}
\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma}{\mathrm{d}^3x \mathrm{d}^3k}\bigg|_{T\to 0} &= \lim_{\varepsilon\to 0} \frac{e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{1 + \frac{px(px+\omega_{\vec{k}})+m^2}{p_0 q_0}\right\} \cdot \left\{\frac{2\varepsilon}{\varepsilon^2 + \left(q_0+p_0+\omega_{\vec{k}}\right)^2}\right\}^2 \\
&\leq \lim_{\varepsilon\to 0} \frac{4e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{1 + \frac{px(px+\omega_{\vec{k}})+m^2}{p_0 q_0}\right\} \cdot \frac{\varepsilon^2}{\left(q_0+p_0+\omega_{\vec{k}}\right)^4} \\
&= 0 \ , \qquad\qquad (18)
\end{aligned}
$$

where we have taken into account that $q_0 + p_0 + \omega_{\vec{k}} > 0$ in the second step.

III. NUMERICAL INVESTIGATIONS AND RESULTS

In the previous section, we have presented the key features of our earlier model description on finite lifetime effects on the photon emission from a QGP. Now, we turn to our numerical investigations within this model approach. In this context we demonstrate that the consideration of the photon number density for free asymptotic states leads to UV integrable photon spectra if the time evolution of the quark/antiquark occupation numbers is modeled in a physically reasonable manner. For this purpose, we consider different switching functions, $f_i(t)$, for (13). These switching functions are given by

$$f_1(t) = \theta(t) \ , \qquad (19a)$$

$$f_2(t) = \theta(t) - \frac{\mathrm{sign}(t)}{2} \mathrm{e}^{-2|t|/\tau} \ , \qquad (19b)$$

$$f_3(t) = \frac{1}{2}\left[1 + \tanh\frac{2t}{\tau}\right] \ , \qquad (19c)$$

and are depicted in Fig. 1.

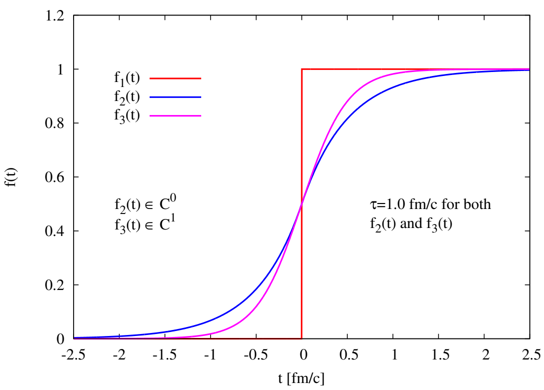

FIG. 1: The time evolution of the QGP is modeled by different switching functions, $f_i(t)$.

$f_1(t)$ describes an instantaneous formation at $t = 0$, whereas $f_2(t)$ and $f_3(t)$ describe a formation over a finite interval, τ, in each case. Another difference between the latter two switching functions is that $f_2(t)$ is continuously differentiable once, whereas $f_3(t)$ is continuously differentiable infinitely many times. As in [9], the photon self-energy, $\mathrm{i}\Pi^{<}_{\mathrm{T}}(\vec{k}, t_1, t_2)$, is summed over the two light-quark flavors, up and down, such that $\sum_f e_f^2/e^2 = 5/9$, and the three colors. In order to avoid possible infrared and/or anticollinear singularities the quark/antiquark masses have been left finite, $m_{\mathrm{u}} = m_{\mathrm{d}} = 0.01$ GeV.

Fig. 2 compares the asymptotic photon spectra for the different switching functions, $f_i(t)$. For $f_2(t)$ and $f_3(t)$ a switching time of $\tau = 1.0$ fm/c has been chosen.

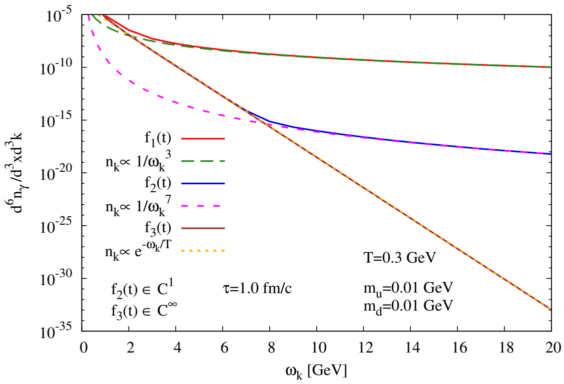

FIG. 2: The scaling behavior of the photon number density in the UV domain is highly sensitive to the choice of $f(t)$. In particular, it is rendered UV integrable if the QGP is assumed to be created over a finite interval of time, τ.

For all three parameterizations, the loop integrals entering (10c)-(10d) and (11) are rendered finite by the Fermi-Dirac distribution function (7). In particular, this is also the case for (10d) since the contribution from the vacuum polarization characterized by the term proportional to 1 is removed under the successive limits $t \to \infty$ and $\varepsilon \to 0$, which also follows from Eqs. (16)-(18). For $f(t) = f_1(t)$ representing an instantaneous formation at $t = 0$, the photon number density scales as $1/\omega_{\vec{k}}^3$ for large photon momenta, which means that the total number density and the total energy density of the emitted photons are logarithmically and linearly divergent, respectively.

In contrast to [9], however, this artifact is now fully removed if we turn form an instantaneous formation to a formation over a finite interval of time, τ, representing a physically more reasonable scenario. For $f_2(t)$, which is continuously differentiable once, the photon number density is suppressed to $\propto 1/\omega_{\vec{k}}^7$, which means that the total photon number density and the total energy density are both UV finite. Moreover, if we turn from $f_2(t)$ to $f_3(t)$, which is continuously differentiable infinitely many times and hence represents the most physical scenario, the photon number density is suppressed even further to an exponential decay in $\omega_{\vec{k}}$.

One remarkable feature in this context is that the slope of the photon spectrum, i.e., the energy scale over which the photon number density decreases by a factor of 1/e for large $\omega_{\vec{k}}$, coincides with $\beta = 1/T$ for $\tau = 1.0$ fm/c. This suggests that the photon spectrum starts looking thermal with τ increasing from 0 (where it coincides with the one for $f_1(t)$) if the quark/antiquark occupation numbers are switched on according to $f_3(t)$. A comparison of the photon spectra for different switching times, which is provided in Fig. 3, supports this.

Nevertheless, in this context the exact dependence of the photon number density on the switching time, τ, is counterintuitive for $f_3(t)$: If the quark/antiquark occupation numbers are switched on according to $f_2(t)$, the suppression of the photon number density in the UV domain with respect to the instantaneous case is the stronger the larger τ is chosen, i.e., the more slowly the formation of the QGP is assumed to take place. Furthermore, $f_2(t)$ reproduces the photon spectrum for the instantaneous case in the limit $\tau \to 0$, as it must be. The latter is also the case if the quark/antiquark occupation numbers are switched on by means of $f_3(t)$. In the limit $\tau \to \infty$, however, the photon number density seems to converge against some finite value and, as a conse-

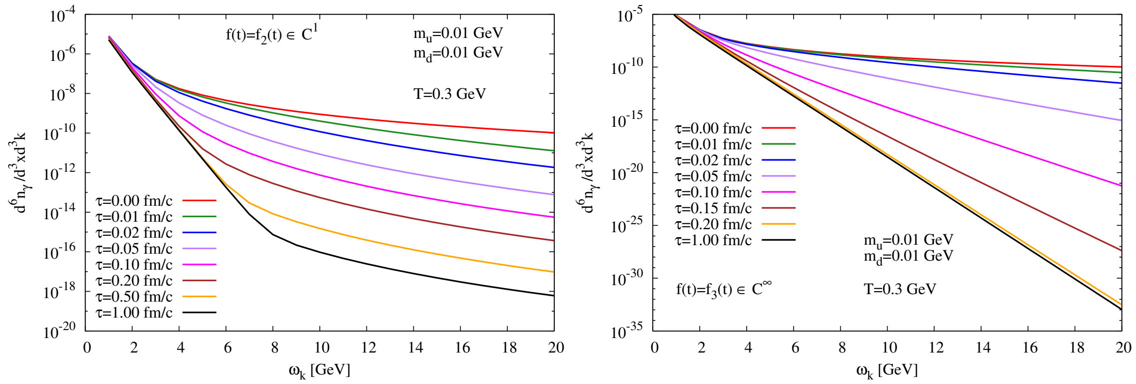

FIG. 3: For both $f_2(t)$ (left panel) and $f_3(t)$ (right panel) the photon spectrum for $f_1(t)$ is reproduced in the limit $\tau \to 0$. For $f_2(t)$, the suppression of the photon number density with respect to the instantaneous case is the stronger the larger τ is chosen. To the contrary, this quantity seems to converge against some finite value with increasing τ for $f_3(t)$ with the slope of the photon spectrum then given by β.

quence, to become independent of τ. To the contrary, one would expect intuitively that in this limit said quantity disappears. Then one effectively has a static plasma such that first-order QED processes become kinematically impossible.

In the following, we demonstrate that the latter is indeed the case. For this purpose, we first consider the photon spectra for each of the processes contributing to (16) separately. Fig. 4 shows the photon spectra arising from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon.

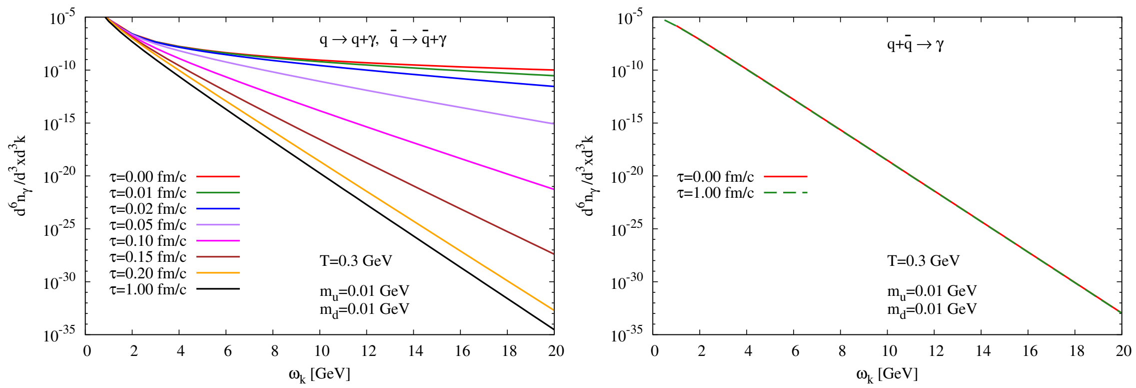

FIG. 4: Dependence of the contributions from quark/antiquark Bremsstrahlung (left panel) and from quark-antiquark pair annihilation into a single photon (right panel) on the switching time, τ, for $f_3(t)$. The contribution from quark/antiquark Bremsstrahlung seems to saturate in the limit $\tau \to \infty$ with the slope of the spectrum then given by β. Furthermore, the photon spectrum arising from quark-antiquark pair annihilation into a single photon seems to be entirely independent of τ and exhibits the same slope.

We see that the inverse slope of the photon spectrum arising from quark/antiquark Bremsstrahlung seems to converge against β with increasing τ, and that the photon number density appears to converge to a finite value in the limit $\tau \to \infty$ for a given photon energy, $\omega_{\vec{k}}$. Furthermore, the photon spectrum arising from quark-antiquark pair annihilation into a single photon seems to be independent of τ with its slope also given by β. To the contrary, for the contribution from the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum one can infer from Fig. 5 that its suppression with respect to the instantaneous case is the stronger the larger τ is chosen and that it accordingly disappears in the limit $\tau \to \infty$.

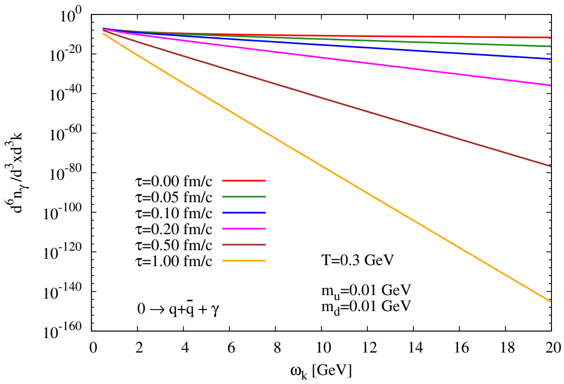

FIG. 5: For the contribution arising from the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum, it is evident that its suppression with respect to the instantaneous case is the stronger the more slowly (τ increasing) the formation of the QGP is assumed to take place and that it eventually disappears in the limit $\tau \to \infty$.

This implies that the apparent saturation of the overall photon number density in the limit $\tau \to \infty$ results from the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon. As one expects intuitively, however, these contributions (and hence the overall photon number density) do not saturate but also vanish in the above limit. In order to see this, one has to consider them for switching times that exceed the expected (from the phenomenological point of view) formation time of the QGP of $\tau_{\mathrm{QGP}} \simeq 1.0$ fm/c [22] by several orders of magnitude. This can be inferred from Fig. 6. For the contribution from the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum, to the contrary, the expected disappearance in the limit $\tau \to \infty$ already becomes visible for switching times being of the same order of magnitude as the expected formation time of the QGP.

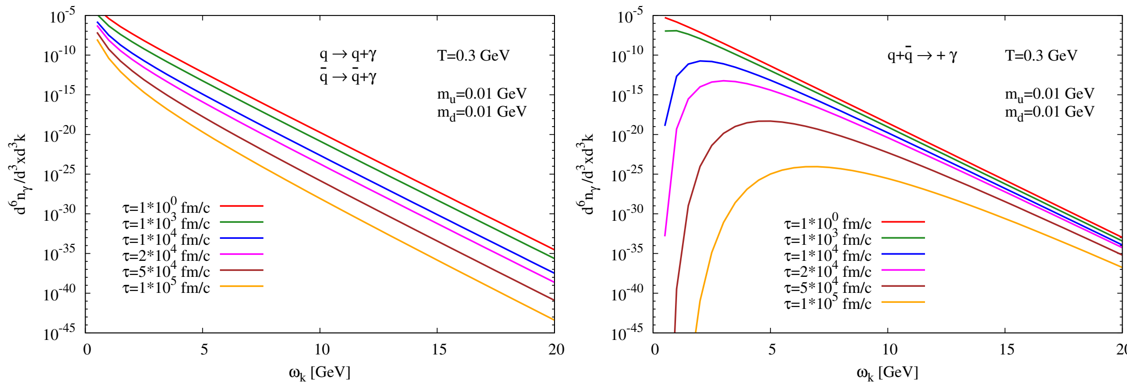

FIG. 6: If the photon spectra arising from quark/antiquark Bremsstrahlung (left panel) and quark-antiquark pair annihilation into a single photon are considered for switching times exceeding the expected formation time of the QGP by several orders of magnitude, one sees that both contributions also vanish in the limit $\tau \to \infty$.

We shall give an explanation for how such a different dependence on τ comes about for the individual contributions to (16). For this purpose, we take into account that each of them is given by a loop integral over the different loop-momentum modes contributing to the respective underlying first-order QED process. Each of these modes is characterized by a specific formation time. For the individual first-order QED processes, these formation times read

$$\tau_{\mathrm{BST}}(\vec{p},\vec{k}) = \frac{2\pi}{\left|q_0 - p_0 - \omega_{\vec{k}}\right|} , \tag{20a}$$

$$\tau_{\mathrm{ANH}}(\vec{p},\vec{k}) = \frac{2\pi}{q_0 + p_0 - \omega_{\vec{k}}} , \tag{20b}$$

$$\tau_{\mathrm{PAC}}(\vec{p},\vec{k}) = \frac{2\pi}{q_0 + p_0 + \omega_{\vec{k}}} , \tag{20c}$$

with the denominators denoting the required virtuality, i.e. the 'offshellness', of the respectively considered process. In equation (20a) we have taken into account that the frequency $q_0 - p_0 - \omega_{\vec{k}}$ is negative definite.

For a specific photon-emission mode that contributes to a particular process to be suppressed with respect to the instantaneous case, the switching time, τ, has to be chosen significantly larger than the formation time of the considered mode. The reason is that then the QGP appears to be static for this mode by which the associated process becomes effectively kinematically impossible. When considering the contribution to the photon number density from this particular process, this implies that τ has to be chosen significantly larger than the formation times of all contributing emission modes such that the disappearance of respective contribution in the limit $\tau \to \infty$ becomes evident.

On the other hand, for the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon the formation times of the collinear ($x = 1$) and the anticollinear modes ($x = -1$) in the domain $p \leq \omega_{\vec{k}}$, respectively, exhibit formation times exceeding the expected formation time of the QGP by several orders of magnitude, which can be read from Table I. As a consequence, the switching time has to be chosen significantly larger than these formation times and hence by several orders of magnitude larger than the expected formation time of the QGP such that it becomes visible that the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon vanish in the limit $\tau \to \infty$. For the sake of clarity, we would like to stress again that x denotes the cosine of the angle between the photon momentum, $\vec{k}$, and the fermion-loop momentum, $\vec{p}$, i.e., $\vec{p}\cdot\vec{k} = p\omega_{\vec{k}}x$, such that the collinear and the anticollinear photon-emission modes are characterized by $x = 1$ and $x = -1$, respectively.

This can be seen by restricting the integration range over d^3p such that the collinear (quark/antiquark Bremsstrahlung) and the anti-collinear modes for $p \leq \omega_{\vec{k}}$ (quark-antiquark pair annihilation into a single photon) are excluded. In this case, the respective contribution decreases much faster with increasing τ and, depending on the exact restriction of the integration range, it becomes visible that both of them disappear for large τ already for values around 1 fm/c. For the contribution from quark/antiquark Bremsstrahlung, this can be seen in Fig. 7, where the upper bound of the integration over $\mathrm{d}x$ is varied. If we choose $x_{\mathrm{MAX}} = 0.9$ such that the collinear modes are excluded, the contribution from quark/antiquark Bremsstrahlung decreases much faster with increasing τ. In particular, it becomes evident that it disappears in the limit $\tau \to \infty$ even if τ is of the order of 1 fm/c, which coincides with the expected formation time of the QGP. On the other hand, if x_{MAX} is increased gradually back to 1 the collinear modes are successively re-included such that the decrease of the Bremsstrahlung contribution with increasing τ is delayed accordingly.

Analogously, the contribution from quark-antiquark pair annihilation into a single photon decreases considerably faster with increasing τ if either the anticollinear modes or the modes for which $p \leq \omega_{\vec{k}}$ are excluded. This is shown in Fig. 8, where the lower bound of the integrations over

	p [GeV]	$\tau_{\rm BST}(\vec{p},\vec{k})$ [fm/c] x=1.0	$\tau_{\rm BST}(\vec{p},\vec{k})$ [fm/c] x=0.9	p [GeV]	$\tau_{\rm ANH}(\vec{p},\vec{k})$ [fm/c] x=-1.0	$\tau_{\rm ANH}(\vec{p},\vec{k})$ [fm/c] x=-0.9
$\omega_{\vec{k}} = 5.0$ GeV	2.0	$3.52 \cdot 10^5$	$8.70 \cdot 10^0$	1.0	$2.01 \cdot 10^4$	$1.02 \cdot 10^1$
	4.0	$9.05 \cdot 10^5$	$5.58 \cdot 10^0$	2.0	$3.02 \cdot 10^4$	$3.97 \cdot 10^0$
	6.0	$1.66 \cdot 10^6$	$4.55 \cdot 10^0$	3.0	$3.02 \cdot 10^4$	$1.95 \cdot 10^0$
	8.0	$2.61 \cdot 10^6$	$4.04 \cdot 10^0$	4.0	$2.01 \cdot 10^4$	$1.02 \cdot 10^0$
	10.0	$3.77 \cdot 10^6$	$3.73 \cdot 10^0$	5.0	$1.26 \cdot 10^0$	$5.62 \cdot 10^{-1}$
	12.0	$5.13 \cdot 10^6$	$3.52 \cdot 10^0$	6.0	$6.28 \cdot 10^{-1}$	$3.45 \cdot 10^{-1}$

TABLE I: Formation times of the collinear modes for the process of quark/antiquark Bremsstrahlung (left part) and of the anticollinear modes for the process of quark-antiquark pair annihilation into a single photon (right part) for $\omega_{\vec{k}} = 5.0$ GeV and $m_{\rm u} = m_{\rm d} = 0.01$ GeV. One can see that the formation times of the collinear modes and of the anticollinear modes in the domain $p \leq \omega_{\vec{k}}$ exceed the expected formation time of the QGP by several orders of magnitude. In this context, it is particularly remarkable that the formation times in turn decrease by several orders of magnitude if they are considered for modes outside these domains, i.e., if one decreases x from 1.0 to 0.9 for the contribution from quark/antiquark Bremsstrahlung or if one either increases p from some $p \leq \omega_{\vec{k}}$ to 6.0 GeV or x from -1.0 to -0.9 for the contribution from quark-antiquark pair annihilation into a single photon. In each case, the formation time is of the same or even in a smaller order of magnitude than the expected formation time of the QGP.

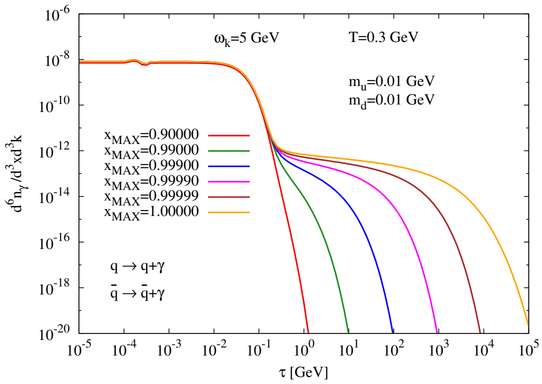

FIG. 7: Dependence of the contribution arising from quark/antiquark Bremsstrahlung on the switching time, τ, for different upper bounds, $x_{\rm MAX}$, for the integration over dx. If the collinear modes are excluded, this contribution decreases much faster with increasing τ. As it must be, the actual decreasing behavior is reproduced if $x_{\rm MAX}$ is increased back to 1.

dp and dx are varied from 5.2 GeV down to 0 GeV (left panel) and from -0.9 down to -1.0 (right panel), respectively. If we choose either $p_{\rm MIN} = 5.2$ GeV or $x_{\rm MIN} = -0.9$ the anticollinear modes in the domain $p \leq \omega_{\vec{k}}$ are excluded, and the contribution from quark-antiquark pair annihilation into a single photon decreases much faster with increasing τ than it does for a full integration over d^3p. As a consequence, it becomes evident that this contribution disappears in the limit $\tau \to \infty$ already if τ is chosen around 10 fm/c. If $p_{\rm MIN}$ and $x_{\rm MIN}$ are gradually decreased back to 0.0 GeV and -1.0, respectively, the anticollinear modes from the range $p \leq \omega_{\vec{k}}$ are re-included and the decrease of the of the pair-annihilation contribution is effectively delayed.

This shows that the apparent saturation of the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon for τ being varied from $0-1$ fm/c results from the large formation times of the collinear and anticollinear modes in the range $p \leq \omega_{\vec{k}}$, respectively. To the contrary, for the spontaneous creation of a quark-antiquark

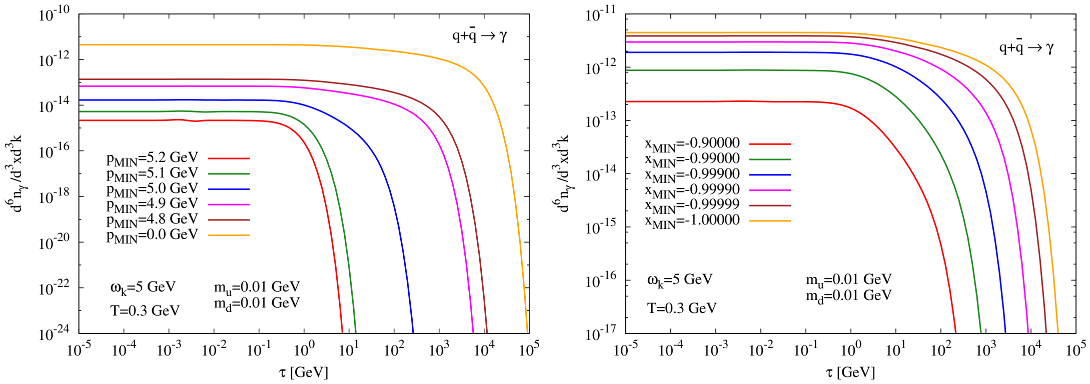

FIG. 8: Dependence of the contribution arising from quark-antiquark pair annihilation into a single photon on the switching time, τ, with different lower bounds, $p_{\rm MIN}$, for integration over dp (left panel) and different lower bounds, $x_{\rm MIN}$, for the integration over dx (right panel). If the anticollinear modes at $p \leq \omega_{\vec{k}}$ are excluded this contribution also decreases much faster with increasing τ. As expected, the actual decreasing behavior is reproduced if we decrease $p_{\rm MIN}$ back to 1.0 GeV and $x_{\rm MIN}$ back to 1.0, respectively.

pair together with a photon out of the vacuum the formation times of all contributing modes are bounded by

$$\tau_{\rm PAC}(\vec{p},\vec{k}) \leq \frac{2\pi}{2m_{\rm u,d}+\omega_{\vec{k}}} \ , \tag{21}$$

for a specific photon energy, $\omega_{\vec{k}}$, such that the contribution from this process decreases much faster with increasing τ. Accordingly, its vanishing in the limit $\tau \to \infty$ manifests itself already for switching times of the same order of magnitude as the formation time of the QGP.

We have seen that the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon decrease much faster with increasing τ if the collinear and the anticollinear modes at $p \leq \omega_{\vec{k}}$ are excluded from the integration over d^3p in each case. Accordingly, said modes lead to an enhancement of the respective contribution to the overall photon number density by several orders of magnitude for the physically motivated choice of $\tau \simeq \tau_{\rm QGP} \simeq 1.0$ fm/c. Such an enhancement, which eventually might turn into a (anti-) collinear divergence $m_{\rm u,d} \to 0$, requires an HTL-resummation of the quark/antiquark propagators. This effectively assigns the quarks and antiquarks a thermal mass. The full quark/antiquark mass hence reads

$$m_{\rm u,d}^{\rm full,2} = m_{\rm u,d}^{\rm bare,2} + m^2(T) \ , \tag{22}$$

where we have chosen $m_{\rm u,d}^{\rm bare} = 0.01$ GeV, and the thermal component, $m(T)$, given by

$$m^2(T) = \frac{4\pi\alpha_s}{3}\left(N_{\rm c}+\frac{N_{\rm f}}{2}\right)T^2 \ . \tag{23}$$

Here $N_{\rm c}$ and $N_{\rm f}$ denote the number of colors and flavors, respectively. If we consider three colors and the two light-quark flavors, up and down, expression (23) turns into

$$m^2(T) = \frac{16\pi\alpha_s}{3}T^2 \ . \tag{24}$$

For a temperature of $T = 0.3$ GeV and $\alpha_s \approx 0.3$, the thermal component of the quark/antiquark mass is of the order of several hundred MeV and hence significantly larger than the bare component. This in turn implies that if the thermal component of (22) is taken into account the

		$\tau_{\rm BST}(\vec{p},\vec{k})$ [fm/c]			$\tau_{\rm ANH}(\vec{p},\vec{k})$ [fm/c]	
	p [GeV]	$m_{\rm u,d} = m_{\rm u,d}^{\rm bare}$	$m_{\rm u,d} = m_{\rm u,d}^{\rm full}$	p [GeV]	$m_{\rm u,d} = m_{\rm u,d}^{\rm bare}$	$m_{\rm u,d} = m_{\rm u,d}^{\rm full}$
$\omega_{\vec{k}} = 5.0$ GeV	2.0	$3.52 \cdot 10^5$	$1.61 \cdot 10^1$	1.0	$2.01 \cdot 10^4$	$6.81 \cdot 10^0$
	4.0	$9.05 \cdot 10^5$	$4.05 \cdot 10^1$	2.0	$3.02 \cdot 10^4$	$4.81 \cdot 10^0$
	6.0	$1.66 \cdot 10^6$	$7.38 \cdot 10^1$	3.0	$3.02 \cdot 10^4$	$5.60 \cdot 10^{-1}$
	8.0	$2.61 \cdot 10^6$	$1.16 \cdot 10^2$	4.0	$2.01 \cdot 10^4$	$2.06 \cdot 10^{-1}$
	10.0	$3.77 \cdot 10^6$	$1.67 \cdot 10^2$	5.0	$1.26 \cdot 10^0$	$1.25 \cdot 10^{-1}$
	12.0	$5.13 \cdot 10^6$	$2.27 \cdot 10^2$	6.0	$6.28 \cdot 10^{-1}$	$8.94 \cdot 10^{-2}$

TABLE II: If the thermal component of the quark/antiquark mass is taken into account, the formation times of the collinear modes and the anticollinear modes at $p \leq \omega_{\vec{k}}$ contributing to the processes of quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon, respectively, are significantly smaller compared to the case where only the bare component is considered. For the thermal component of the quark/antiquark mass, we have chosen $T = 0.3$ GeV and $\alpha_s \approx 0.3$, which implies that $m(T) \approx 0.67$ GeV.

actual formation times of the collinear modes and the anticollinear modes at $p \leq \omega_{\vec{k}}$ contributing to the processes quark/antiquark Bremsstrahlung and quark/antiquark pair annihilation into a single photon, respectively, are significantly smaller compared to the case in which only the bare component is considered. This is shown in Table II.

One hence expects that the contributions from quark/antiquark Bremsstrahlung and quark/antiquark pair annihilation into a single photon then accordingly decrease considerably faster with increasing τ. As a consequence, the disappearance of these contributions for $\tau \to \infty$ should become evident even if τ is chosen from the same order of magnitude as the expected formation time of the QGP. One can infer from Fig. 9 that this is indeed the case.

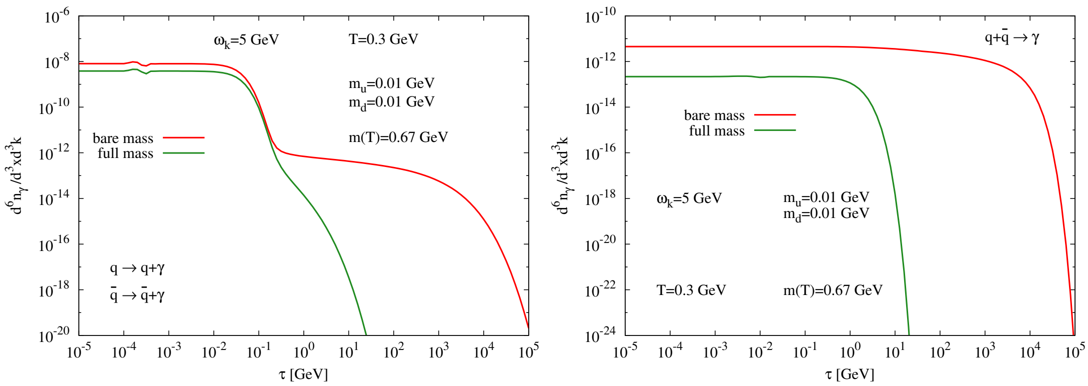

FIG. 9: Dependence of the contributions from quark/antiquark Bremsstrahlung (left panel) and from quark-antiquark pair annihilation into a single photon (right panel) on the switching time, τ, for the bare and the full quark/antiquark masses. If the thermal component of the mass is included, both contributions decrease much faster with increasing τ compared to the case where only the bare masses are considered.

In this work, we have only presented results on the scenario in which the quark/antiquark occupation numbers are switched on and maintained, but not the scenario in which they are switched back off after a certain period of time, $\tau_{\rm L}$, to take into account the finite lifetime of the QGP during a heavy-ion collision. The reason is that for the latter scenario the principle sensitivity of the photon number density on the switching function, $f(t)$, and the switching time, τ, is as in the one presented here. Firstly, the photon number density again scales as $1/\omega_{\vec{k}}^3$ in the UV domain if the quark/antiquark occupation numbers are switched on and off instantaneously, with this artifact being removed if both switchings take place over a finite interval of time, τ, instead. In particular, the photon spectrum again starts looking thermal with increasing τ if we consider a switching function being continuously differentiable infinitely many times. For such a switching function, the photon number density again seems to converge against some finite value for $\tau \to \infty$, where this apparent saturation can again be traced back to the large formation times of the collinear modes and the anticollinear modes at $p \leq \omega_{\vec{k}}$ for the processes of quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon, respectively.

When comparing the exact dependence of the asymptotic photon spectra within our (revisited) model description to those from [10], the fact that the slope of the overall photon spectrum converges against the inverse temperature, β, (provided that the quark/antiquark occupation numbers are switched on according to $f_3(t)$) might seem unphysical at first. In [10], we have investigated the photon emission arising from a change of the quark/antiquark mass. We have seen that the slope of the resulting photon spectrum increases with the transition time of the quark/antiquark mass if the time evolution of the latter is modeled by a function being continuously differentiable infinitely many times. Hence, one might expect a similar dependence of the photon spectra on the switching time, τ, within our revisited model description if the quark/antiquark occupation numbers are switched according to $f_3(t)$ since this function is also continuously differentiable infinitely many times.

Here it is important to point out, however, that within our (revisited) model description, we always switch on the same distribution function for the quarks and antiquarks for all switching functions, $f(t)$, and, in particular, for all considered switching times, τ. To the contrary, in [10] we pursue a first-principle approach in which the quark/antiquark occupation numbers are determined by solving the Dirac equation with a time dependent mass. This has the direct consequence that the quark/antiquark occupation numbers decrease exponentially with increasing momentum, p, and that the slope of the respective spectrum increases with the transition time (provided that the mass function is continuously differentiable infinitely many times). This in turn manifests itself in form of a very similar sensitivity of the asymptotic photon spectrum on this time. To the contrary, such a specific dependence does not occur within our model description since by construction the latter features quark/antiquark occupation numbers which solely depend on the temperature and are hence independent of τ.

IV. COMPARISON TO LEADING-ORDER THERMAL PHOTON PRODUCTION

The investigations from [6–8] indicated that non-equilibrium photon production arising from first-order QED processes possibly dominates over leading-order thermal photon emission in the UV domain. On the other hand, these investigations came along with the mentioned artifacts, which in turn questions the explanatory power of the comparison performed therein. Since the artifacts from [6–8] have been resolved to a satisfactory extend within this work, we again perform a comparison to leading-order thermal photon production in order to get a more significant picture. Here we note again that the contributions from first-order QED processes to photon production vanish in a static thermal equilibrium such that there the first non-trivial contribution starts at two-loop order. Since a loop expansion does not coincide with a coupling-constant expansion, resummations of so-called ladder diagrams are necessary in order to obtain the thermal rate at second order in the perturbative coupling constants, i.e., at linear order in α_e and at linear order in α_s [23].

Within the scope of our investigations on chiral photon production [10], we have already made a rather rudimentary comparison to leading-order thermal photon emission by simply integrating the rate from [23] over the assumed lifetime of the chirally restored phase at constant temperature. This comparison indicated that (first-order) non-equilibrium photon production is subdominant compared to leading-order thermal production for photon energies $\omega_{\vec{k}} \gtrsim 1.0$ GeV.

Since the actual question from [6–8] on the role of finite lifetime effects on direct photon emission from a QGP is readdressed within this work, we now perform a more detailed comparison. In this context we take into account that the QGP as it occurs in a heavy-ion collision is not a static medium but instead expands and cools down over a finite interval of time before it hadronizes finally. To begin with, the time dependence of the temperature effectively leads to a time dependent photon-production rate, i.e.,

$$\frac{\mathrm{d}^7 n_\gamma}{\mathrm{d}^4 x \mathrm{d}^3 k} = \frac{\mathrm{d}^7 n_\gamma(T(t))}{\mathrm{d}^4 x \mathrm{d}^3 k} \equiv \frac{\mathrm{d}^7 n_\gamma(t)}{\mathrm{d}^4 x \mathrm{d}^3 k} \ . \tag{25}$$

In order to obtain the overall photon number accessible to experiment, one has to convolute (25) with the time dependent volume, $V_{\mathrm{QGP}}(t)$, of the expanding QGP from the initial time, t_0, at which the QGP has thermalized until the time t_{had}, at which the full hadronic phase is reached. This leads to

$$\left.\frac{\mathrm{d}^3 n_\gamma}{\mathrm{d}^3 k}\right|_{\mathrm{eq.}} = \int_{t_0}^{t_{\mathrm{had}}} \mathrm{d}t V_{\mathrm{QGP}}(t) \frac{\mathrm{d}^7 n_\gamma(t)}{\mathrm{d}^4 x \mathrm{d}^3 k} \ . \tag{26}$$

For the time evolution of the volume and the temperature of the QGP, we consider the same fireball model that has been used in [24] for $0-20$ % central Au+Au collisions at 200 AGeV.

When calculating the overall photon number arising from the first-order non-equilibrium contributions, we multiply our asymptotic photon number density directly with the initial volume of the QGP, i.e.,

$$\left.\frac{\mathrm{d}^3 n_\gamma}{\mathrm{d}^3 k}\right|_{\mathrm{non\text{-}eq.}} = V_{\mathrm{QGP}}(t_0) \frac{\mathrm{d}^6 n_\gamma}{\mathrm{d}^3 x \mathrm{d}^3 k} \ . \tag{27}$$

The reason is that the first-order non-equilibrium photon production occurs during the formation of the QGP, which we model by the switching-on of the quark/antiquark occupation numbers. The adiabatic switching-off of the electromagnetic interaction then removes the artificial contributions occurring at finite times. As a consequence, the asymptotic photon number density has also to be computed for the initial temperature, T_0.

For our numerical analysis, we chose the same values for the parameters of the fireball model as done in [24]. In particular, we assume an initial volume of the QGP of $V_{\mathrm{QGP}}(t_0) = 73.76$ fm^3 which from the underlying equation of state leads to an initial temperature of $T_0 = 0.36$ GeV. For the photon numbers emerging from first-order non-equilibrium emission process, we consider a switching time of $\tau = 1.0$ fm/c. The latter are considered both for the bare and the full quark/antiquark mass. As a consequence, the thermal mass is taken with respect to the initial temperature, which accordingly to (23) leads to $m(T_0) = 0.81$ GeV.

Fig. 10 compares the photon spectra for first-order non-equilibrium production to those for leading-order thermal production. If we only take into account the bare component of the quark/antiquark mass, the non-equilibrium photon emission exceeds the thermal emission by one one order of magnitude for photon energies $\omega_{\vec{k}} \gtrsim 1.0$ GeV. At first sight this seems to support the qualitative picture from [6–8]. Here it is important to point out, however, that the photon spectrum for the full quark/antiquark mass is the more realistic one since the included thermal component effectively provides the required HTL-resummation of the (anti-)collinear photon-emission modes. In this case we see that in contrast to [6–8], the photon numbers arising from first-order non-equilibrium processes are clearly below those arising from leading-order thermal photon production for $\omega_{\vec{k}} = 1-5$ GeV.

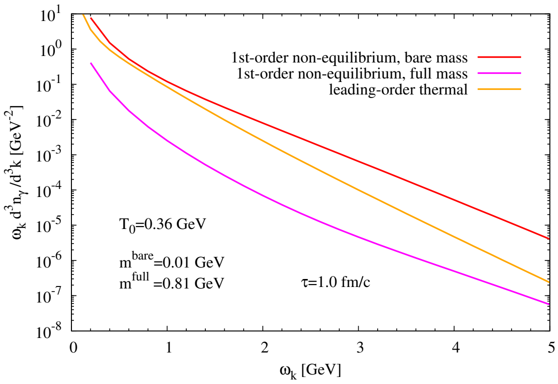

FIG. 10: Comparison of first-order non-equilibrium photon production to leading-order thermal production. If one takes into account the full quark/antiquark mass, the former is subdominant for $\omega_{\vec{k}} = 1 - 5$ GeV.

On the other hand, the photon spectrum emerging from leading-order thermal contributions features a steeper decay that the one from the first-order non-equilibrium contributions since it incorporates the entire time evolution of the temperature of the QGP and not only its initial temperature, T_0. This in turn indicates that non-equilibrium photon production becomes dominant somewhere above $\omega_{\vec{k}} = 5$ GeV.

This does, however, effectively not change the principle idea that direct photon production from the QGP phase during a heavy-ion collision can be addressed by integrating the leading-order thermal rate on a hydrodynamic background (quasi-static calculation) and that a full dynamic treatment is not crucial quantitatively. Comprehensive comparisons of the contributions from the different sources of direct photon emission to the overall photon spectra measured in RHIC and LHC experiments [25–27] have shown that medium contributions from the hadronic phase dominate in the infrared (IR) domain, whereas the photon emission arising from initial nucleon-nucleon scatterings and jet-medium interactions outshine the medium contributions both from the QGP and the hadronic phase in the UV domain. To the contrary, a dominance of the medium contribution from the QGP phase could only possibly be observed at intermediate photon energies with the exact range increasing with the collision energy. On the other hand, our investigations have shown that for these intermediate energies, leading-order thermal photon production clearly dominates over the first-order non-equilibrium one.

The principal reason why the contributions form initial nucleon-nucleon scatterings and jet-medium interactions dominate over the pure medium contributions from the QGP and the subsequent hadronic phase in the UV domain is that the photon spectra from the former two sources flatten into a power-law decay, whereas those from the latter feature an exponential decay. Such an exponential decay is also observed for the photon spectra arising from first-order non-equilibrium production from the QGP. This implies that even though this photon production starts to dominate over the leading-order thermal one at photon energies $\omega_{\vec{k}} \gtrsim 5$ GeV such that a quasi-static description strictly speaking becomes invalid in this domain, this does not effectively matter since the medium contributions from the QGP are outshone by the contributions from initial nucleon-nucleon scatterings and jet-medium interactions in any case.

V. REMARKS ON THE IMPORTANCE OF FREE ASYMPTOTIC STATES

We would like to stress again that the exact sequence of limits, i.e., taking *first* $t \to \infty$ and *then* $\varepsilon \to 0$, is crucial to eliminate a possible unphysical contribution from the vacuum polarization and, in general, to obtain a UV integrable photon number density from the medium contributions to $\mathrm{i}\Pi^{<}_{\mathrm{T}}(\vec{k}, t_1, t_2)$. If one interchanges both limits, i.e., if one first takes $\varepsilon \to 0$ at some finite time, t, it can be shown [10] that the contribution from the vacuum polarization does not vanish, but instead turns into

$$\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma}{\mathrm{d}^3x \mathrm{d}^3k}\bigg|_{T\to 0} = \frac{e^2}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3} \left\{ 1 + \frac{px(px + \omega_{\vec{k}}) + m^2}{p_0 q_0} \right\} \frac{1}{\left(q_0 + p_0 + \omega_{\vec{k}}\right)^2} \,. \tag{28}$$

Since the integration measure, d^3p, contributes an additional factor of p^2 to the integrand, the loop integral is linearly divergent for a given photon energy, $\omega_{\vec{k}}$. Furthermore, since (28) is time independent, it persists under the subsequent limit $t \to \infty$.

On the other hand, since (28) is time independent and hence already present before any medium contributions to (1) can appear, one might still argue that it can be identified with the virtual cloud of the vacuum and accordingly needs to be subtracted since it is unobservable. The reason for this time independence, which suggests such an identification, is that in contrast to [6–8], our description takes into account that the vacuum contribution to the photon self-energy always occurs, whereas for the medium contributions this is only the case as long as the QGP is actually present. After subtracting the divergent vacuum contribution and taking the subsequent limit $t \to \infty$, however, one in general still encounters the problem that the photon number density arising from the remaining medium contributions to the photon self-energy is not integrable in the UV domain. This is shown in Fig. 11.

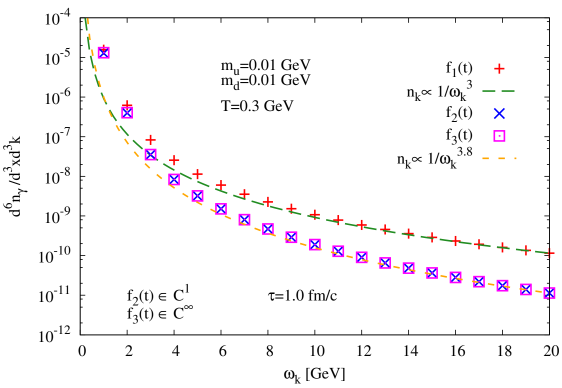

FIG. 11: If both limits are interchanged, the UV scaling behavior of the photon number density solely changes from $\propto 1/\omega_{\vec{k}}^3$ to $\propto 1/\omega_{\vec{k}}^{3.8}$ when turning from an instantaneous switching to a switching over a finite interval of time, τ.

As for the correct sequence of limits the photon number density scales $\propto 1/\omega_{\vec{k}}^3$ for $f_1(t)$. If we turn from $f_1(t)$ to $f_2(t)$ or $f_3(t)$, we see, however, that the photon number density is suppressed to only a slightly steeper decay $\propto 1/\omega_{\vec{k}}^{3.8}$. For that reason, only the total number density of the radiated photons is UV finite, whereas their total energy density remains UV divergent. In particular, the thus obtained photon number density exceeds the value for the correct sequence of limits by several orders of magnitude, which is displayed in Fig. 12.

Here it is important to point out once more that the interpretation of (1) as a photon number density is only justified in the limit $t \to \pm\infty$ for finite ε since only then the electromagnetic field

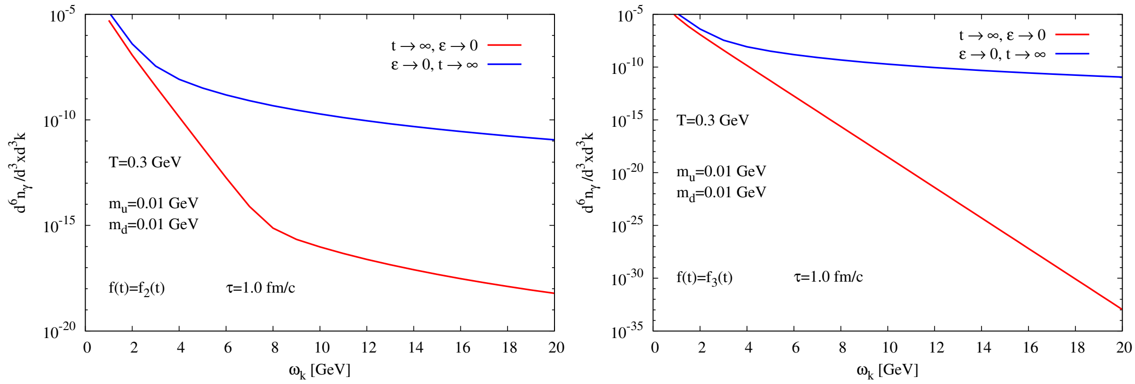

FIG. 12: Comparison of the photon number density for the correct and the interchanged sequence of limits for $f(t) = f_2(t)$ (left panel) and $f(t) = f_3(t)$ (right panel). For the interchanged sequence of limits the photon number density exceeds the value for the correct sequence of limits by several orders of magnitude in the UV domain in both cases.

is asymptotically non-interacting. At finite times, higher order (Fock) contributions to the photon number density still persist due to the remote interactions. This implies that taking first $\varepsilon \to 0$ at some finite time, t, is not correct, as then we would have an *interacting* electromagnetic field such that the interpretation of (1) as a photon number density is not justified. Moreover, such an interpretation remains doubtful even in the limit $t \to \infty$. Since we would have taken $\varepsilon \to 0$ before the electromagnetic field would not evolve into a non-interacting one. The same conceptual problem occurs when only using an adiabatic switching-on of the electromagnetic interaction for $t \to -\infty$ but no adiabatic switching-off for $t \to \infty$. Such a procedure has been suggested in [21] to describe initial correlations at some $t = t_0$ evolving from an uncorrelated initial state at $t \to -\infty$.

VI. SUMMARY, CONCLUSIONS AND OUTLOOK

In this work, we have investigated the role of finite lifetime effects on the photon emission from a rapidly created quark-gluon plasma (QGP) during a heavy-ion collision. We have essentially revisited our earlier model description [9], in which we simulate the time evolution of the QGP by time dependent quark/antiquark occupation numbers in the photon self-energy. In contrast to [9], we have not considered the photon number density at finite times, but for free asymptotic states, as the former is ill-defined. In analogy to [10], we have seen that this procedure does eliminate a possible unphysical contribution from the vacuum polarization and, moreover, leads to a UV integrable photon number density. This result confirms the conjecture that the artifacts encountered in [6–9] arise from an inconsistent definition of the photon number density at finite times. Consequently, our investigations again support the corresponding concern raised in [28, 29] towards [6–8].

When switching the quark/antiquark occupation numbers by an analytic function, which represents the physically most reasonable scenario, we have seen that the photon number density apparently converges to a finite value for large τ if the latter is chosen of the same order of magnitude as the (phenomenologically) expected formation time of the QGP, which amounts to $\tau_{\text{QGP}} \simeq 1.0$ fm/c. In order to see that the photon number density actually vanishes in the limit $\tau \to \infty$, the switching time has to be chosen larger than τ_{QGP} by several orders of magnitude.

We have shown that this apparent saturation results from the contributions describing quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon. In contrast to the spontaneous creation of a quark-antiquark pair together with a photon out of the vacuum, both of these processes feature contributions from individual photon-emission modes for which the formation times exceed $\tau_{\rm QGP}$ by several orders of magnitude. In particular, these modes are the collinear ones for the process of quarks/antiquark Bremsstrahlung and the anticollinear ones at $p \leq \omega_{\vec{k}}$ (with p denoting the absolute value of the loop momentum) for the process of quark-antiquark pair annihilation into a single photon. On the other hand, the switching time, τ, has to be chosen significantly larger than the formation time of *all* modes contributing to a specific process such that the disappearance of the contribution from this process (and hence of the overall photon number density) in the limit $\tau \to \infty$ becomes evident. This can be seen by excluding said modes from the integration range over d^3p. In this case, the contributions from quark/antiquark Bremsstrahlung and quark-antiquark pair annihilation into a single photon decrease much faster with increasing τ. In particular, it becomes evident that these contributions vanish for $\tau \to \infty$ even if τ is chosen to be of the same order of magnitude as $\tau_{\rm QGP}$.

On the other hand, said (anti-)collinear photon-emission modes lead to a significant (by several orders of magnitude) enhancement of the respective contributions to the overall photon number density for $\tau \simeq \tau_{\rm QGP}$. Strictly speaking, such an enhancement requires an HTL resummation of the quark/antiquark propagators, by which the quarks and antiquarks are effectively assigned a thermal mass. This thermal component of the quark/antiquark mass is by $1-2$ orders of magnitude larger than the bare component. If it is taken into account, the formation times of the aforementioned (anti-)collinear modes hence decrease by several orders of magnitude. As a consequence, then the contributions from the processes of quark/antiquark Bremsstrahlung and quark/antiquark pair annihilation into a single photon decrease much faster with increasing τ.

Finally, we have compared our results to leading-order thermal photon production yields. We have seen that if one takes into account the full thermal mass of the quarks and antiquarks the photon numbers arising from leading-order thermal photon emission clearly outshine those from first-order non-equilibrium photon emission for photon energies of $\omega_{\vec{k}} = 1-5$ GeV. On the other hand, our investigations indicate that first-order photon production in turn dominates for $\omega_{\vec{k}} \gtrsim 5$ GeV. This does, however, not affect the quantitative accuracy of the recipe to address direct photon emission from the QGP phase by thermal calculations since both the thermal and the non-equilibrium contributions from this phase are outshone by direct photon emission arising from initial nucleon-nucleon scatterings and jet-medium interaction in that domain.

In summary, we have seen that our approach, which considers the photon number density for free asymptotic states, leads to physically reasonable results (no vacuum contribution, UV integrability) for this quantity. This is the case even though our ansatz for the time evolution of the QGP during a heavy-ion collision formally violates the Ward-Takahashi identities for the photon self-energy. The principal reason is that we (strictly speaking) make *ad hoc* assumptions on the two-time dependence of the latter quantity by introducing time dependent quark/antiquark occupation numbers. On the other hand, it has been pointed out in [30] that the conservation of QED gauge invariance and hence of the Ward-Takahashi identities remains challenging even if one tries to calculate the photon self-energy in a self-consistent framework such as the 2PI approach, where such assumptions are absent. A similar problem usually occurs when trying to calculate direct photon production within a transport framework: It has been shown in [31] that the Thomas-Reiche-Kuhn sum rules, which are a direct consequence of gauge invariance of QED, impose restrictions on the actual applicability of the transport approaches on photon production from non-equilibrated hot hadronic matter presented in [32–36].

For our future investigations, however, the actual role of the Ward-Takahashi identities still requires further consideration. We have seen that even though they are formally violated within our model approach, this approach nevertheless leads to physically reasonable results for the (asymptotic) photon number density. In first sight, this seems to disprove our earlier conjecture that the artifacts encountered in [6–8] and still partly in [9] result from a violation of the Ward-Takahashi identities. Here one has to keep in mind, however, that these identities can be violated in two different ways:

- Firstly, they can be violated directly by making *ad hoc* assumptions on the two-time dependence of the photon-self energy. This has been the case in [9] by introducing time-dependent occupation numbers in the one-loop thermal photon self-energy, which on its own fulfills the Ward-Takahashi identities.

- On the other hand, they can also be violated indirectly by considering the 'photon number density' at finite times and using an inadequate definition of this quantity. This has been the case in [6–8]. The reason is that the definition of the photon number density considered therein would only allow for an accordant interpretation if the electromagnetic interaction was switched off at the point of time, t, at which said quantity is considered. By means of such a switching, however, an effective violation of the Ward-Takahashi identities, which otherwise would be fulfilled, reoccurs. In this context it is important to point out that an adequate definition of a transient particle number density is generally impossible altogether for fundamental reasons except in some special settings [37–40].

In particular, within the scope of our model description such an indirect violation would occur in addition to the direct one if we considered the photon number density at finite times. Consequently, it is of particular interest whether possibly only this indirect violation leads to artificial results.

Acknowledgments

F. M. gratefully acknowledges financial support by the Helmholtz Research School for Quark Matter Studies (H-QM) and by the Helmholtz Graduate School for Hadron and Ion Research (HGS-HIRe for FAIR). This work has been supported (financially) by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse and by the German Federal Ministry of Education and Research (BMBF Förderkennzeichen 05P12RFCTB). The authors thank E. Fraga, S. Leupold, B. Schenke, J. Knoll, P. Danielewicz, and B. Müller for fruitful discussions.

Appendix A: Representation of the photon number density as an absolute square

In this appendix, we show that each of the contributions (12a)-(12c) can be written as the absolute square of a first-order QED transition amplitude and thus is positive (semi-)definite. For this purpose, we first undo the contraction of the individual contributions to the photon self-energy with $\gamma^{\mu\nu}(\vec{k})$. Then (12a)-(12c) turn into

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{BST}} = \frac{\gamma^{\mu\nu}(\vec{k})}{(2\pi)^3} \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t} \mathrm{d}t_2 \mathrm{i}\Pi^{\mathrm{BST}}_{\nu\mu}(\vec{k}, t_1, t_2) \mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1 - t_2)} \,, \tag{A1a}$$

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{ANH}} = \frac{\gamma^{\mu\nu}(\vec{k})}{(2\pi)^3} \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t} \mathrm{d}t_2 \mathrm{i}\Pi^{\mathrm{ANH}}_{\nu\mu}(\vec{k}, t_1, t_2) \mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1 - t_2)} \,, \tag{A1b}$$

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x\mathrm{d}^3k}\bigg|_{\mathrm{PAC}} = \frac{\gamma^{\mu\nu}(\vec{k})}{(2\pi)^3} \int_{-\infty}^{t} \mathrm{d}t_1 \int_{-\infty}^{t} \mathrm{d}t_2 \mathrm{i}\Pi^{\mathrm{PAC}}_{\nu\mu}(\vec{k}, t_1, t_2) \mathrm{e}^{\mathrm{i}\omega_{\vec{k}}(t_1 - t_2)} \,. \tag{A1c}$$

It follows from (6a)-(6d) that the contributions to the uncontracted photon self-energy read

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{BST}}(\vec{k},t_1,t_2) = 2e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{q}+m}{2q_0}\gamma_\nu \frac{\not{p}+m}{2p_0}\right\} n_{\mathrm{F}}(q_0)\left[1-n_{\mathrm{F}}(p_0)\right] \times \mathrm{e}^{-\mathrm{i}(q_0-p_0)(t_1-t_2)} , \tag{A2a}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{ANH}}(\vec{k},t_1,t_2) = e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{q}+m}{2q_0}\gamma_\nu \frac{\not{\bar{p}}-m}{2p_0}\right\} n_{\mathrm{F}}(q_0) n_{\mathrm{F}}(p_0) \times \mathrm{e}^{-\mathrm{i}(q_0+p_0)(t_1-t_2)} , \tag{A2b}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{PAC}}(\vec{k},t_1,t_2) = e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{\bar{q}}-m}{2q_0}\gamma_\nu \frac{\not{p}+m}{2p_0}\right\} \left[1-n_{\mathrm{F}}(q_0)\right]\left[1-n_{\mathrm{F}}(p_0)\right] \times \mathrm{e}^{\mathrm{i}(q_0+p_0)(t_1-t_2)} . \tag{A2c}$$

When incorporating the time evolution of the QGP into (A2a)-(A2c) according to (13) and (14a)-(14b), these expressions turn into

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{BST}}(\vec{k},t_1,t_2) = 2e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{q}+m}{2q_0}\gamma_\nu \frac{\not{p}+m}{2p_0}\right\} f_{\mathrm{BST}}(q_0,p_0,t_1) f_{\mathrm{BST}}(q_0,p_0,t_2) \times \mathrm{e}^{-\mathrm{i}(q_0-p_0)(t_1-t_2)} , \tag{A3a}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{ANH}}(\vec{k},t_1,t_2) = e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{q}+m}{2q_0}\gamma_\nu \frac{\not{\bar{p}}-m}{2p_0}\right\} f_{\mathrm{ANH}}(q_0,p_0,t_1) f_{\mathrm{ANH}}(q_0,p_0,t_2) \times \mathrm{e}^{-\mathrm{i}(q_0+p_0)(t_1-t_2)} , \tag{A3b}$$

$$\mathrm{i}\Pi_{\mu\nu}^{\mathrm{PAC}}(\vec{k},t_1,t_2) = e^2 \int \frac{\mathrm{d}^3p}{(2\pi)^3} \mathrm{Tr}\left\{\gamma_\mu \frac{\not{\bar{q}}-m}{2q_0}\gamma_\nu \frac{\not{p}+m}{2p_0}\right\} f_{\mathrm{PAC}}(q_0,p_0,t_1) f_{\mathrm{PAC}}(q_0,p_0,t_2) \times \mathrm{e}^{\mathrm{i}(q_0+p_0)(t_1-t_2)} . \tag{A3c}$$

In order to keep the notation short, we have introduced

$$f_{\mathrm{BST}}(q_0,p_0,t) = \sqrt{n_{\mathrm{F}}(q_0,t)\left[1-n_{\mathrm{F}}(p_0,t)\right]} , \tag{A4a}$$

$$f_{\mathrm{ANH}}(q_0,p_0,t) = \sqrt{n_{\mathrm{F}}(q_0,t) n_{\mathrm{F}}(p_0,t)} , \tag{A4b}$$

$$f_{\mathrm{PAC}}(q_0,p_0,t) = \sqrt{\left[1-n_{\mathrm{F}}(q_0,t)\right]\left[1-n_{\mathrm{F}}(p_0,t)\right]} . \tag{A4c}$$

As the next step, we take into account that

$$\sum_s u(\vec{p},s)\bar{u}(\vec{p},s) = \frac{\not{p}+m}{2p_0} , \tag{A5a}$$

$$\sum_s v(\vec{p},s)\bar{v}(\vec{p},s) = \frac{\not{\bar{p}}-m}{2p_0} . \tag{A5b}$$

With the help of these relations, (A3a)-(A3c) can be further rewritten as

$$\mathrm{i}\Pi^{\mathrm{BST}}_{\mu\nu}(\vec{k},t_1,t_2) = 2e^2 \sum_{r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\bar{u}(\vec{p},r)\gamma_\mu u(\vec{q},s)\right] \cdot \left[\bar{u}(\vec{q},s)\gamma_\nu u(\vec{p},r)\right] \times f_{\mathrm{BST}}(q_0,p_0,t_1) f_{\mathrm{BST}}(q_0,p_0,t_2) \mathrm{e}^{-\mathrm{i}(q_0-p_0)(t_1-t_2)} \, , \tag{A6a}$$

$$\mathrm{i}\Pi^{\mathrm{ANH}}_{\mu\nu}(\vec{k},t_1,t_2) = e^2 \sum_{r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\bar{v}(\vec{p},r)\gamma_\mu u(\vec{q},s)\right] \cdot \left[\bar{u}(\vec{q},s)\gamma_\nu v(\vec{p},r)\right] \times f_{\mathrm{ANH}}(q_0,p_0,t_1) f_{\mathrm{ANH}}(q_0,p_0,t_2) \mathrm{e}^{-\mathrm{i}(q_0+p_0)(t_1-t_2)} \, , \tag{A6b}$$

$$\mathrm{i}\Pi^{\mathrm{PAC}}_{\mu\nu}(\vec{k},t_1,t_2) = e^2 \sum_{r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left[\bar{u}(\vec{p},r)\gamma_\mu v(\vec{q},s)\right] \cdot \left[\bar{v}(\vec{q},s)\gamma_\nu u(\vec{p},r)\right] \times f_{\mathrm{PAC}}(q_0,p_0,t_1) f_{\mathrm{PAC}}(q_0,p_0,t_2) \mathrm{e}^{\mathrm{i}(q_0+p_0)(t_1-t_2)} \, . \tag{A6c}$$

If we now insert (A6a)-(A6c) into (A1a)-(A1b) and make use of relation (3) we can finally rewrite the individual contributions to the photon number density as

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x \mathrm{d}^3k}\bigg|_{\mathrm{BST}} = \frac{2e^2}{(2\pi)^3} \sum_{\lambda,r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \bigg| \epsilon^{\mu,*}(\vec{k},\lambda) \bar{u}(\vec{p},r)\gamma_\mu u(\vec{q},s) \times \int_{-\infty}^{t} \mathrm{d}u \; f_{\mathrm{BST}}(q_0,p_0,u) \mathrm{e}^{-\mathrm{i}(q_0-p_0-\omega_{\vec{k}})u} \bigg|^2 \, , \tag{A7a}$$

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x \mathrm{d}^3k}\bigg|_{\mathrm{ANH}} = \frac{e^2}{(2\pi)^3} \sum_{\lambda,r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \bigg| \epsilon^{\mu,*}(\vec{k},\lambda) \bar{v}(\vec{p},r)\gamma_\mu u(\vec{q},s) \times \int_{-\infty}^{t} \mathrm{d}u \; f_{\mathrm{ANH}}(q_0,p_0,u) \mathrm{e}^{-\mathrm{i}(q_0+p_0-\omega_{\vec{k}})u} \bigg|^2 \, , \tag{A7b}$$

$$2\omega_{\vec{k}} \frac{\mathrm{d}^6 n_\gamma(t)}{\mathrm{d}^3x \mathrm{d}^3k}\bigg|_{\mathrm{PAC}} = \frac{e^2}{(2\pi)^3} \sum_{\lambda,r,s} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \bigg| \epsilon^{\mu,*}(\vec{k},\lambda) \bar{u}(\vec{p},r)\gamma_\mu v(\vec{q},s) \times \int_{-\infty}^{t} \mathrm{d}u \; f_{\mathrm{PAC}}(q_0,p_0,u) \mathrm{e}^{\mathrm{i}(q_0+p_0+\omega_{\vec{k}})u} \bigg|^2 \, . \tag{A7c}$$

This completes the proof that (12a)-(12c) can be expressed as absolute squares. Furthermore, taking a closer look at the underlying spinor structures shows that (A7a)-(A7c) can be interpreted as the corresponding first-order QED process.

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