

Heavy-Quark Diffusion in the Quark-Gluon Plasma

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1 Heavy-ion phenomenology

- The phase diagram of strongly interacting matter
- Hydrodynamical collective flow
- Constituent-quark-number scaling of v_2

2 Heavy-quark interactions in the sQGP

- Heavy quarks in heavy-ion collisions
- Heavy-quark diffusion: The Langevin Equation
- Elastic pQCD heavy-quark scattering
- Non-perturbative interactions: effective resonance model
- Comparison to RHIC data: “non-photonic electrons”

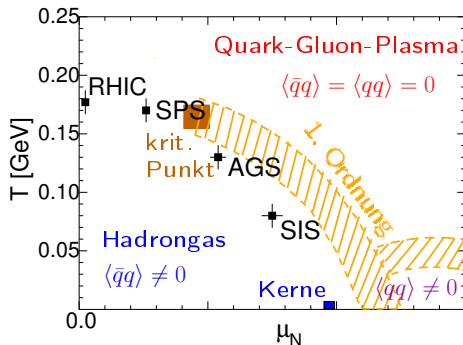
3 Microscopic model for non-perturbative HQ interactions

- Static heavy-quark potentials from lattice QCD
- T-matrix approach
- Estimate on transport properties of the sQGP

4 Summary and Outlook

The phase diagram of strongly interacting matter

- hot and dense matter: **Quarks and Gluons** close to each other
- relativistic collisions \Rightarrow **Deconfinement**
- **Quarks** and **gluons** relevant degrees of freedom \Rightarrow **Quark-Gluon-Plasma**
- still strongly interacting: **fast thermalization!**

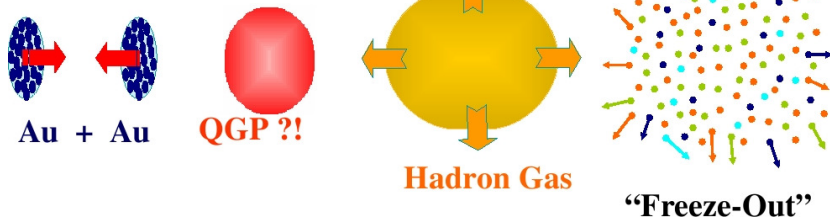


NB: $100 \text{ MeV} \simeq 1.16 \cdot 10^{12} \text{ K}$

$T_{\text{center of sun}} \simeq 1.57 \cdot 10^7 \text{ K} \simeq 1.3 \cdot 10^{-3} \text{ MeV}$

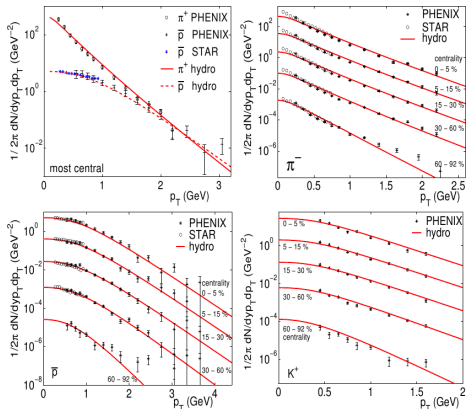
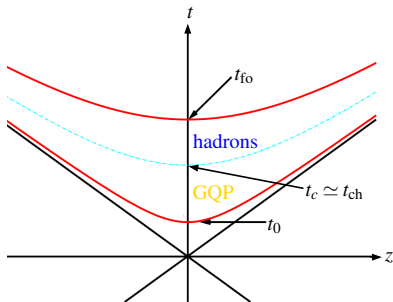
Heavy-ion collisions

- collisions of relativistic (heavy) nuclei
- many collisions of **partons** inside nucleons
- creation of many particles \Rightarrow **hot and dense fireball**
- formation of (thermalized) QGP?
- how to learn about properties of QGP?



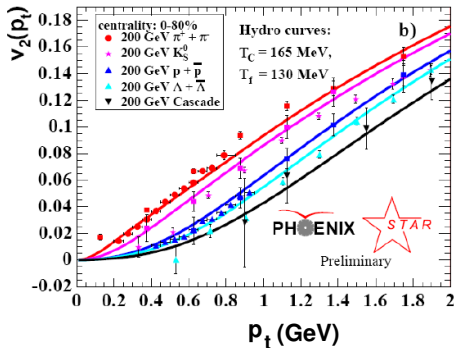
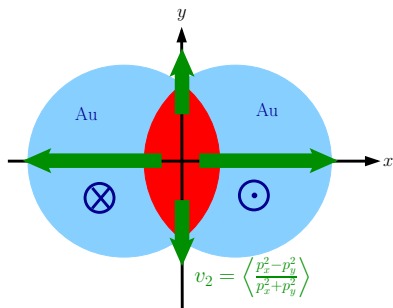
Hydrodynamical radial flow of the bulk

- ideal fluid in **local thermal equilibrium** \Rightarrow low viscosity/(entropy density), η/s
- **needs strong interactions**
- **hydrodynamical model** for ultra-relativistic heavy-ion collisions
 - after short formation time ($t_0 \lesssim 1$ fm/c)
 - **QGP** in **local thermal equilibrium** \rightarrow **hadronization** at $T_c \simeq 160 - 190$ MeV
 - chemical freeze-out: (**inelastic collisions cease**) $T_{ch} \simeq 160 - 175$ MeV
 - thermal freeze-out: (**also elastic scatterings cease**)



Hydrodynamical elliptic flow of the bulk

- particle spectra compatible with collective flow of a (nearly) ideal fluid \Rightarrow small η/s
- medium in local thermal equilibrium

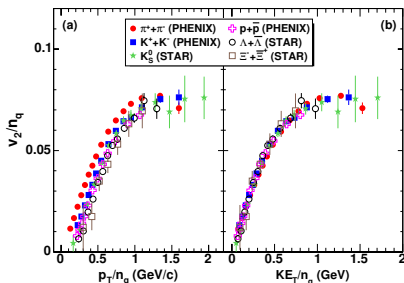


Constituent-quark-number scaling of v_2

- elliptic flow, v_2 scales with **number of constituent quarks**

$$v_2^{(\text{had})}(p_T^{(\text{had})}) = n_q v_2^{(q)}(p_T^{(\text{had})}/n_q)$$

- suggests coalescence of **quarks** at T_c



- possible microscopic mechanism **hadron-resonance formation** at $T_c \Rightarrow$ resonance-recombination model [Ravagli, HvH, Rapp, PRC 79, 064902 (2009)]
- other hint to quark coalescence: enhanced **baryon/meson** ratio compared to **pp** collisions

Heavy quarks in heavy-ion collisions: motivation

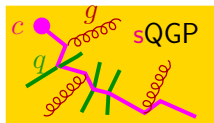
- Fast equilibration of hot and dense matter in heavy-ion collisions: collective flow (nearly ideal hydrodynamics) \Rightarrow sQGP
- Heavy quarks as calibrated probe of QGP properties
 - produced in early hard collisions: well-defined initial conditions
 - not fully equilibrated due to large masses
 - **heavy-quark diffusion** \Rightarrow probes for QGP-transport properties
- Langevin simulation
- drag and diffusion coefficients
 - T -matrix approach with static lattice-QCD **heavy-quark potentials**
 - **resonance formation** close to T_c
 - mechanism for **non-perturbative strong interactions**

Heavy Quarks in Heavy-Ion collisions

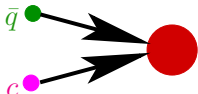


hard production of HQs
described by PDF's + pQCD (PYTHIA)

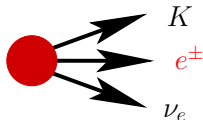
c, b quark



HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP



Hadronization to D, B mesons via
quark coalescence + fragmentation



semileptonic decay \Rightarrow
"non-photonic" electron observables
 $R_{AA}^{e^+e^-}(p_T), v_2^{e^+e^-}(p_T)$

Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

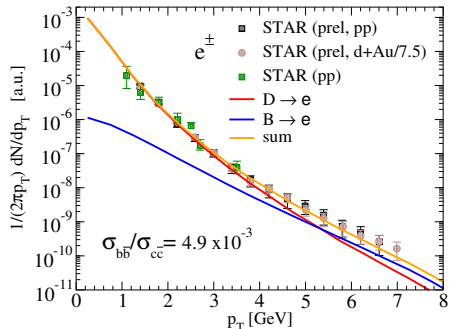
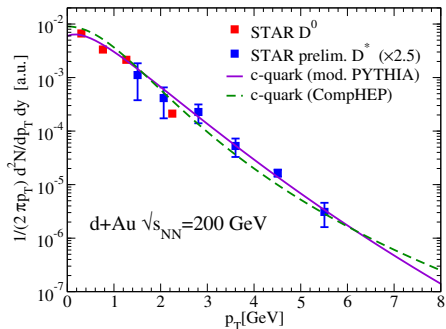
$$d\vec{x} = \frac{\vec{p}}{E_p} dt,$$

$$d\vec{p} = -A\vec{p}dt + \sqrt{2dt}[\sqrt{B_0}P_\perp + \sqrt{B_1}P_\parallel]\vec{w}$$

- \vec{w} : normal-distributed random variable
- A : friction (drag) coefficient
- $B_{0,1}$: diffusion coefficients
- Einstein dissipation-fluctuation relation $B_1 = E_p T A$.
- flow via Lorentz boosts between “heat-bath frame” and “lab frame”
- A and B_0 from microscopic models for qQ , gQ scattering

Bulk evolution and initial conditions

- bulk evolution as elliptic **thermal fireball**
- **isentropic expansion** with **QGP Equation of State**
- initial p_T -spectra of **charm** and **bottom** quarks
 - (modified) PYTHIA to describe exp. **D meson spectra**, assuming **δ -function fragmentation**
 - exp. **non-photonic single- e^\pm spectra**: Fix bottom/charm ratio



Meaning of the Coefficients

- solution for nonrelativistic case: **Gaussian** with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-At), \quad \Delta \vec{p}^2(t) = \langle \vec{p}^2 \rangle - \langle \vec{p} \rangle^2 = \frac{3B}{A} [1 - \exp(-2At)].$$

- A : **friction/drag** coefficient \Rightarrow **dissipation**
- $1/A$: **relaxation time** to reach **equilibrium**
- B : **momentum-diffusion** coefficient
- measures size of **momentum fluctuations**
(result of random **uncorrelated collisions** of **heavy quarks** with **medium**)
- \Rightarrow effective description of collisions: **white-noise-random force**
- **equilibrium limit** ($t \rightarrow \infty$)

$$F_Q(t, \vec{p}) \underset{t \rightarrow \infty}{\cong} \left(\frac{2\pi B}{A} \right)^{3/2} \exp\left(-\frac{A\vec{p}^2}{2B}\right)$$

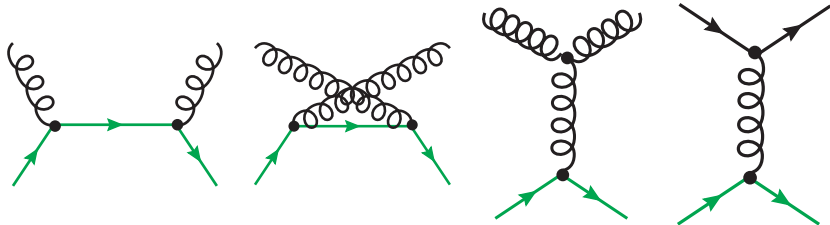
- has to be **Maxwell-Boltzmann distribution** \Rightarrow

$$B = m_Q AT$$

- T : given temperature of the **QGP**
- Einstein's **dissipation-fluctuation** relation (1905)

Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

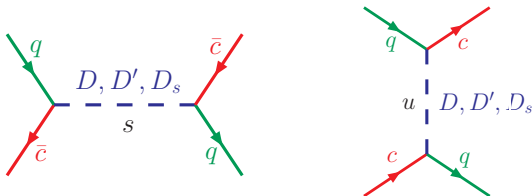


- **Debye-screening mass** for t -channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on “non-photonic” electrons

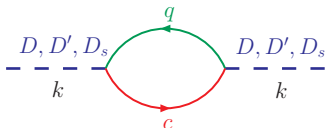
[Moore, Teaney PRC 71, volume 71, 064904 (2005)]

Non-perturbative interactions: Resonance Scattering

- General idea: Survival of D - and B -meson like **resonances** above T_c
- model based on chiral symmetry (light quarks) HQ-effective theory
- **elastic heavy-light-(anti-)quark scattering**



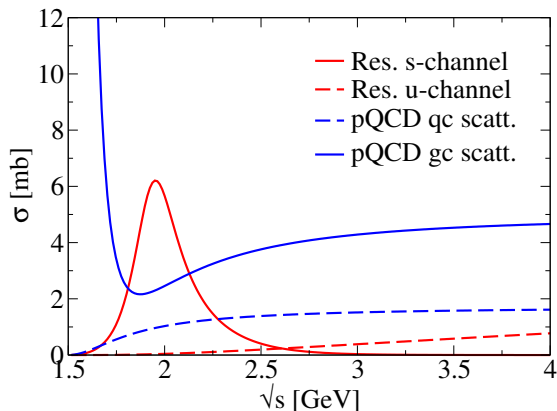
- D - and B -meson like resonances in **s**QGP



- parameters

- $m_D = 2 \text{ GeV}$, $\Gamma_D = 0.4 \dots 0.75 \text{ GeV}$
- $m_B = 5 \text{ GeV}$, $\Gamma_B = 0.4 \dots 0.75 \text{ GeV}$

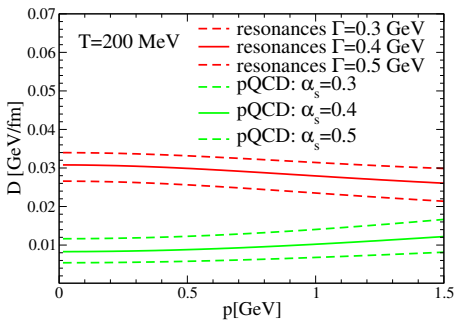
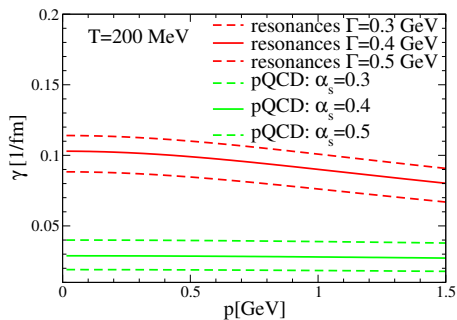
Cross sections



- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked \leftrightarrow resonance isotropic
- resonance scattering more effective for friction and diffusion

Transport coefficients: pQCD vs. resonance scattering

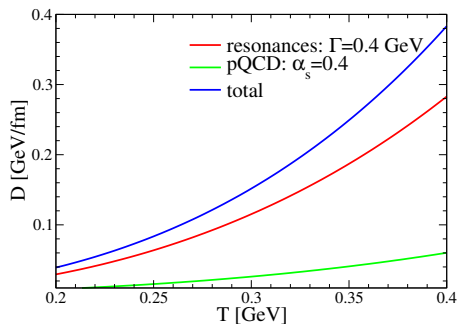
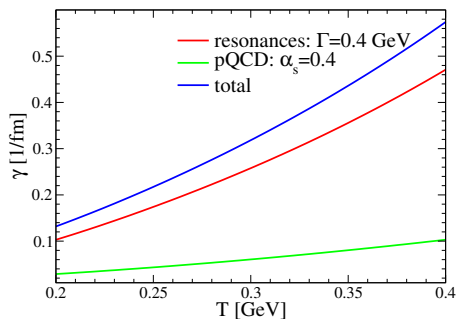
- three-momentum dependence



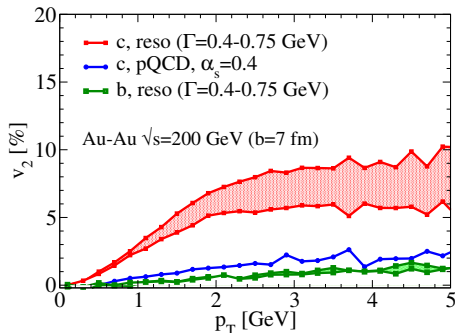
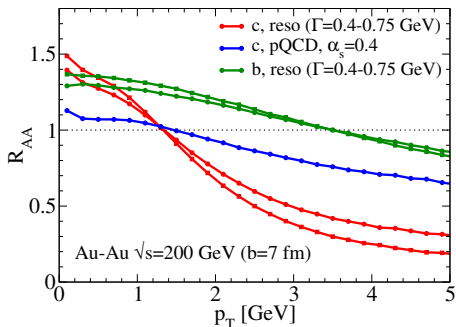
- resonance contributions factor $\sim 2 \dots 3$ higher than pQCD!

Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence



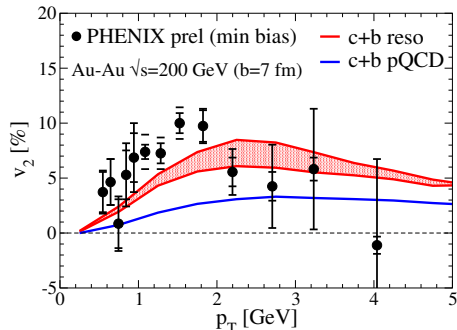
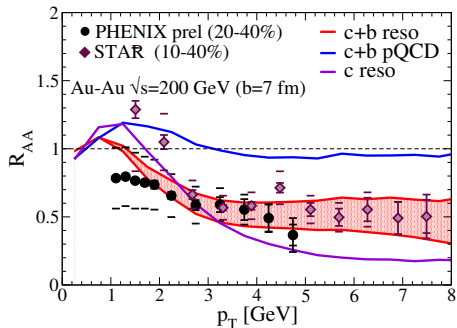
Spectra and elliptic flow for heavy quarks



- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances \Rightarrow c -quark thermalization **without upscaling of cross sections**
- **Fireball parametrization** consistent with hydro

Observables: p_T -spectra (R_{AA}), v_2

- **Hadronization: Coalescence** with light quarks + **fragmentation**
 $\Leftrightarrow c\bar{c}, b\bar{b}$ conserved
- single electrons from decay of D - and B -mesons

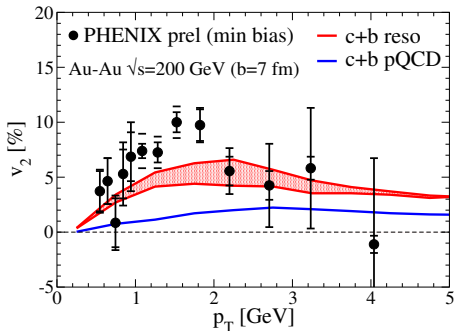
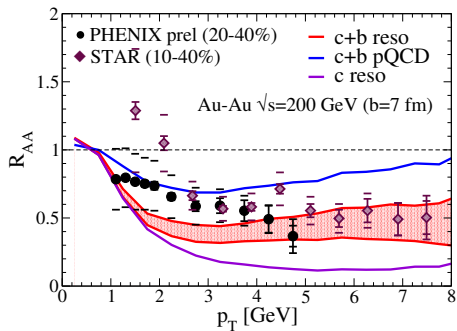


- Without further adjustments: data quite well described

[HvH, V. Greco, R. Rapp, Phys. Rev. C **73**, 034913 (2006)]

Observables: p_T -spectra (R_{AA}), v_2

- Hadronization: Fragmentation only
- single electrons from decay of D - and B -mesons

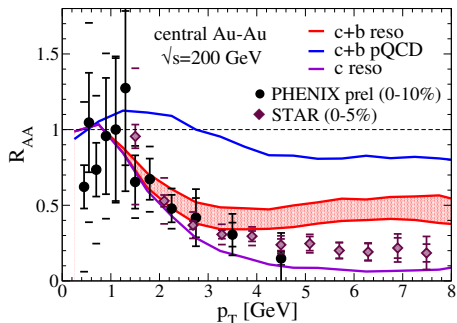


- coalescence brings up both, R_{AA} and v_2
- due to additional momentum kick from light quarks

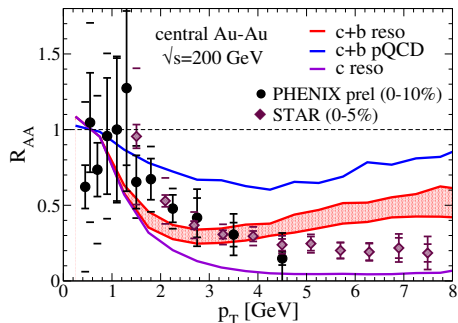
Observables: p_T -spectra (R_{AA}), v_2

- Central Collisions
- single electrons from decay of D - and B -mesons

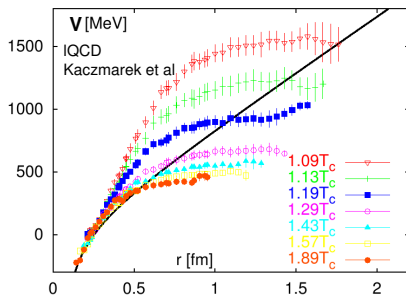
Coalescence+Fragmentation



Fragmentation only



Microscopic model: Static potentials from lattice QCD



- color-singlet free energy from lattice \rightarrow internal energy

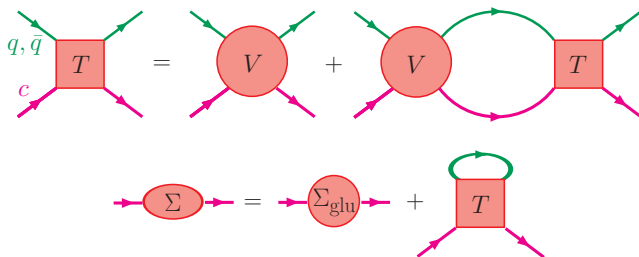
$$U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T},$$

$$V_1(r, T) = U_1(r, T) - U_1(r \rightarrow \infty, T)$$

- Casimir scaling of Coulomb part for other color channels;
confining part color blind [F. Riek, R. Rapp, Phys. Rev. C **82**, 035201 (2010)].

$$V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1$$

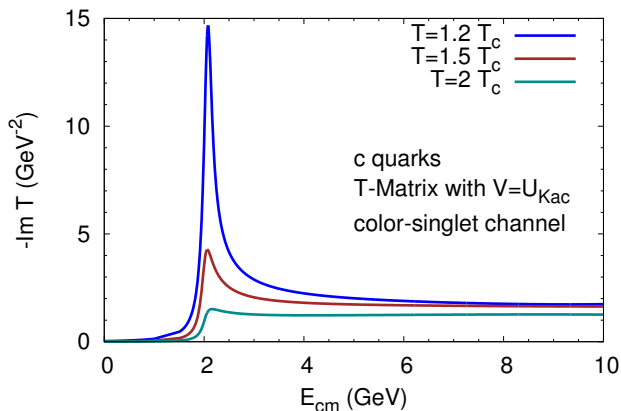
- Brueckner many-body approach for elastic $Qq, Q\bar{q}$ scattering



- reduction scheme: 4D Bethe-Salpeter \rightarrow 3D Lipmann-Schwinger
- S - and P waves
- Relation to invariant **matrix elements**

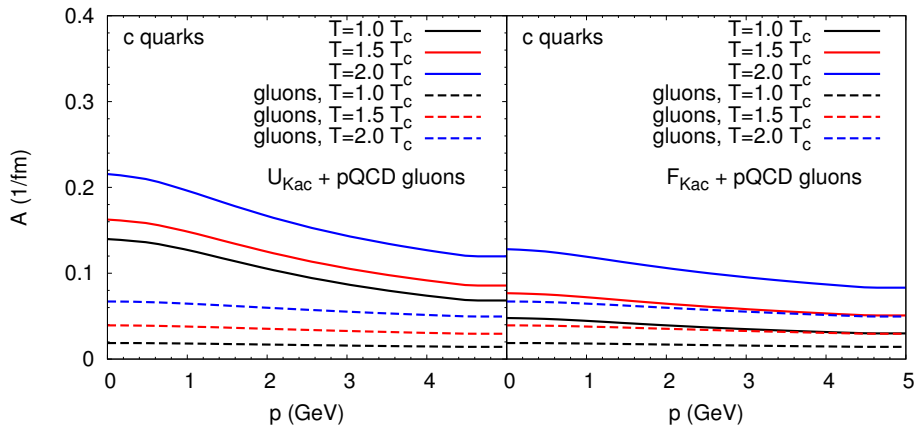
$$\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a (|T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}})$$

T-matrix results



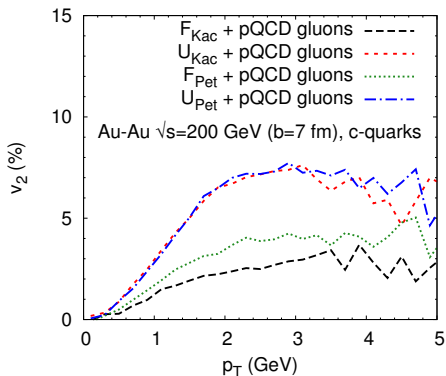
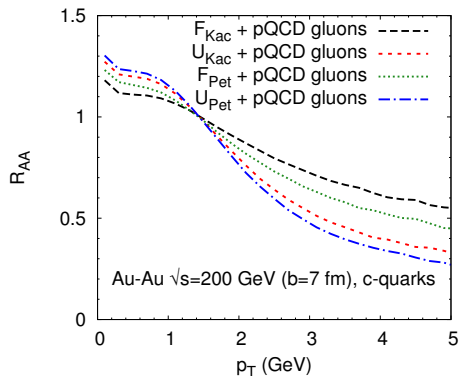
- **resonance formation** at lower temperatures $T \simeq T_c$
- melting of resonances at higher T
- model-independent assessment of elastic Qq , $Q\bar{q}$ scattering!

Transport coefficients

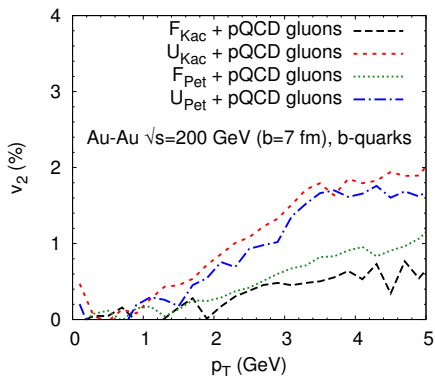
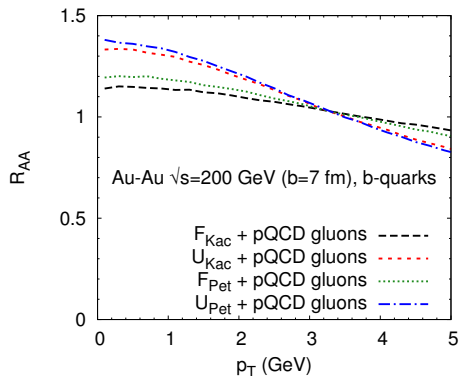


- from **non-pert.** interactions reach $A_{\text{non-pert}} \simeq 1/(7 \text{ fm}/c) \simeq 4A_{\text{pQCD}}$
- results for **free-energy potential**, F considerably smaller

Spectra and elliptic flow for c-quarks

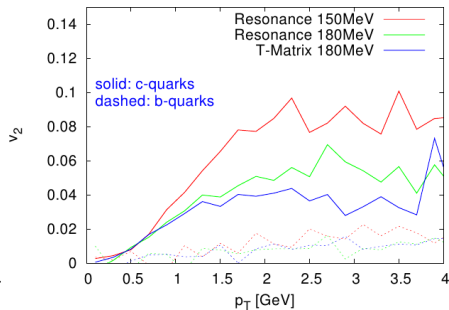
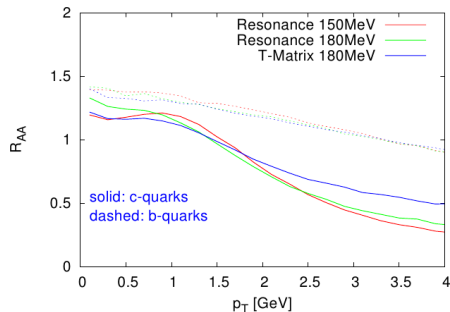


Spectra and elliptic flow for b -quarks



Implementation in UrQMD

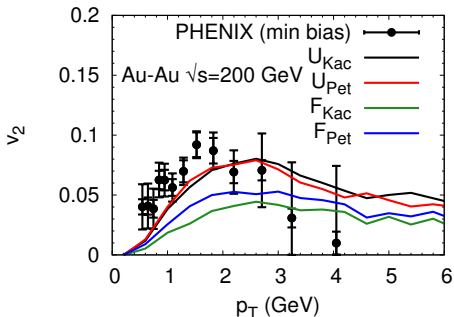
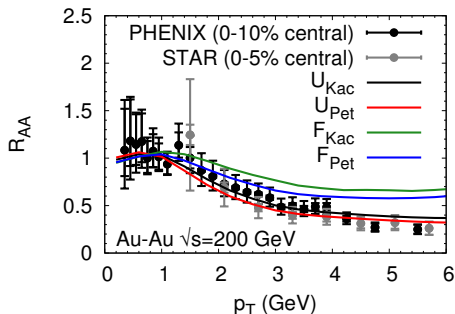
- Langevin simulation easily implemented into any “bulk background”
- UrQMD (hybrid cascade/hydro mode)
 - more realistic **fireball evolution**
 - possibility to study effects of **fluctuations**



[T. Lang, J. Steinheimer, HvH, work in progress]

Non-photonic electrons at RHIC

- here: fireball model!
- quark **coalescence**+**fragmentation** $\rightarrow D/B \rightarrow e + X$



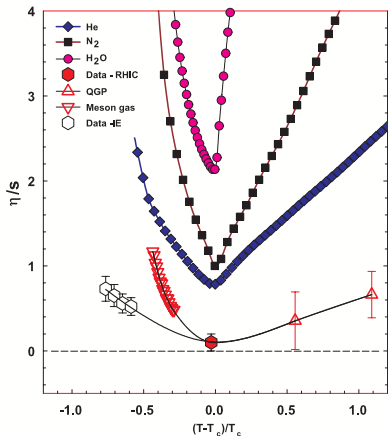
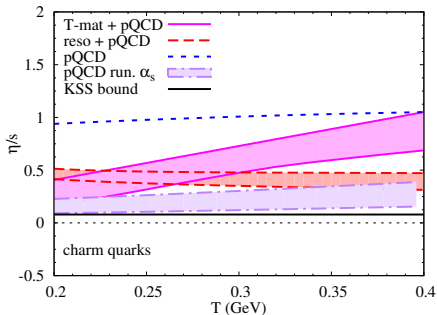
- **coalescence crucial for description of data**
- increases **both**, R_{AA} and $v_2 \Leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” **towards T_c** \Rightarrow **coalescence natural**

[L. Ravagli, R. Rapp, Phys. Lett. B **655**, 126, (2007); L. Ravagli, HvH, R. Rapp, Phys. Rev. C **79**, 064902 (2009)]

Transport properties of the sQGP

- spatial diffusion coefficient: **Fokker-Planck** $\Rightarrow D_s = \frac{T}{m_A} = \frac{T^2}{D}$
- coupling strength in plasma: viscosity/entropy density, η/s

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad (\text{AdS/CFT}), \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad (\text{wQGP})$$



[Lacey, Taranenko, FRNC2006, 021 (2006)]

Summary and Outlook

- Heavy quarks in the sQGP
- non-perturbative interactions
 - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
 - lattice-QCD potentials parameter free
 - resonances melt at higher temperatures
 \Leftrightarrow consistency betw. R_{AA} and v_2 !
- also provides “natural” mechanism for quark coalescence
- resonance-recombination model
- potential approach at finite T : F , V or combination?
- Outlook
 - include inelastic heavy-quark processes (gluo-radiative processes)
 - other heavy-quark observables like charmonium suppression/regeneration
 - implementation of RRM in transport models (BAMPS)
as a consistent hadronization model
 - study QCD phase transition(s)
 - fluctuations; finite μ_B ; cross-over \leftrightarrow 1st order; CEP(!?!)