From Quantum Field Theory to Semiclassical Transport in Heavy-Ion Collisions

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September 06, 2022

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

ITP Institute for Theoretical Physics

CRC-TR 211 Strong-interaction matter under extreme conditions

HFHF Helmholtz Forschungsakademie Hessen für FAIR

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Nucl. Phys. A **484**, 661 (1988)

FLAVOUR KINETICS IN AN EXPANDING QUARK–GLUON PLASMA

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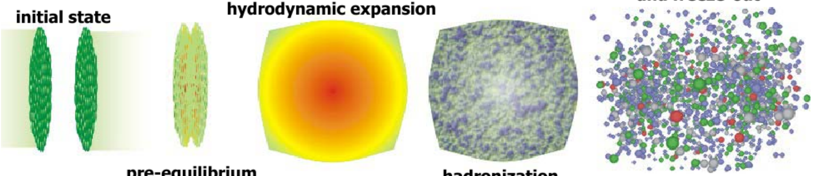
Received 4 March 1988

Abstract: A phenomenological model for the expansion of a quark-gluon plasma is presented and applied to ultra-relativistic nucleus–nucleus collisions. The flavour kinetics is described by rate equations allowing the abundances of the different constituents of the system to be out of equilibrium. The basic constituents are the quarks and gluons in the plasma phase and hadrons in the hadronic phase. Besides ordinary kinetic processes within the two phases (inelastic collisions), we construct a model for the confinement mechanism which is based on the string phenomenology of hadronization. The latter aspect constitutes the novel and essential part of the paper. Abundances of the various hadrons have been calculated for a baryon-poor and a baryon-rich scenario. The results show that during the hadronization process, the original quark content is roughly doubled. The resulting hadron abundances are found to depend on the hadronization mechanism, while the subsequent hadronic reactions do not have a significant effect on the final hadronic abundances. We find a K^-/π^- ratio as much as a factor 6 larger than in $p\bar{p}$ collisions.

Rate equations for particle numbers in heavy-ion collisions



- confinement: at low temperatures/densities quarks and gluons bound in hadrons
- describe transition between hot "partonic" to later "hadronic" stages of fireball evolution
- here: abundances of quarks, gluons, and hadrons

$$A_j \in \{g, u, \bar{u}, d, \bar{d}, s, \bar{s}; \pi, \mathrm{K}, \bar{\mathrm{K}}, \mathrm{N}, \bar{\mathrm{N}}, \mathrm{Y}, \bar{\mathrm{Y}}, \ldots\}$$

- radially exanding fireball with $v_r \approx 0.5c$ in globel thermal and pressure equilibrium
- off-chemical equilibrium: chemical potentials for each particle $\mu_j(t)$

Rate equations for particle numbers in heavy-ion collisions

- reaction formulae for various reactions, i

$$\sum_j l_i(j) A_j \longleftrightarrow \sum_j r_i(j) A_j$$

- rate equations ("detailed balance")

$$\dot{y}_i = \langle \text{forward rate} \rangle \left\{ 1 - \exp[(\mu_r - \mu_l)/T] \right\},$$

$$\dot{N}_j = \sum_i [r_i(j) - l_i(j)] \dot{y}_i$$

- guarantees validity of H-theorem (increasing entropy)
- correct equilibrium limit

Reactions

- QGP: $g+g \longleftrightarrow q\bar{q}, \quad q_1+\bar{q}_1 \longleftrightarrow q_2\bar{q}_2$ (pQCD cross sections)

- QGP⟷hadrons: String fragmentation model

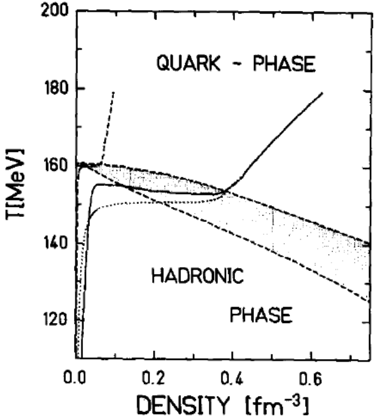
$$\begin{aligned} q_1-\bar{q}_2 &\rightleftharpoons q_1-\bar{q}_3 q_3-\bar{q}_2\,, & q_1q_2-q_3 &\rightleftharpoons q_1q_2-q_4\bar{q}_4-q_3\,,\\ q_1-\bar{q}_2 &\rightleftharpoons (q_1\bar{q}_3)q_3-\bar{q}_2\,, & q_1q_2-q_3 &\rightleftharpoons (q_1q_2q_4)\bar{q}_4-q_3\,,\\ q_1-\bar{q}_2 &\rightleftharpoons q_1-\bar{q}_3(q_3\bar{q}_2)\,, & q_1q_2-q_3 &\rightleftharpoons q_1q_2-q_4(\bar{q}_4q_3)\,,\\ q_1-\bar{q}_2 &\rightleftharpoons (q_1\bar{q}_3)(q_3\bar{q}_2)\,. & q_1q_2-q_3 &\rightleftharpoons (q_1q_2q_4)(\bar{q}_4q_3)\,. \end{aligned}$$

  forward rate $\propto \langle\lambda\rangle$ (length of string) and $\propto \#_{\text{strings}}$; backward rate: detailed balance $\propto \exp[(\mu_r-\mu_l)/T]$

- hadrons (Boltzmann Eq. with empirical cross sections [P. Koch, B. Müller, J. Rafelski, Phys. Rept. **142**, 167])
  $\pi\pi \longleftrightarrow \mathrm{K}\overline{K}, \quad \ldots$

Equations of state

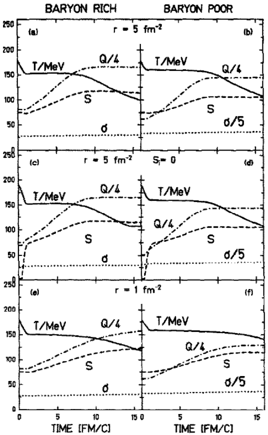
- QGP: bag-model equation of state $p_{\text{QGP}} = p_g + p_q + p_{\bar{q}} - B$, bag constant $B = (235\,\text{Mev})^{1/4}$
- hadronic equation of state: Walecka model; in-medium masses
- Gibbs conditions for chemical potentials, temperature, and pressure $\Rightarrow$ 1^{st}-order phase transition

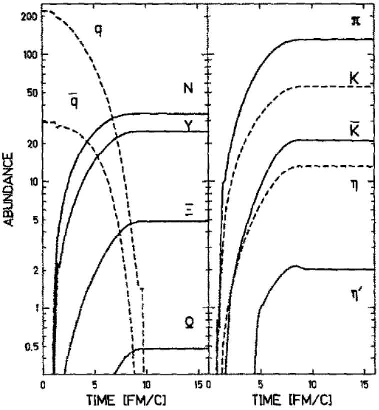


- $T_c \simeq 160\,\text{MeV}$ (!)

Solution of rate equations

- fireball volume: $V(t) = V_0(1 + t/t_0)^n$
- $T(t)$ and $\beta(t) = V_{\text{had}}(t)/V(t)$: energy conservation, pressure equilibrium

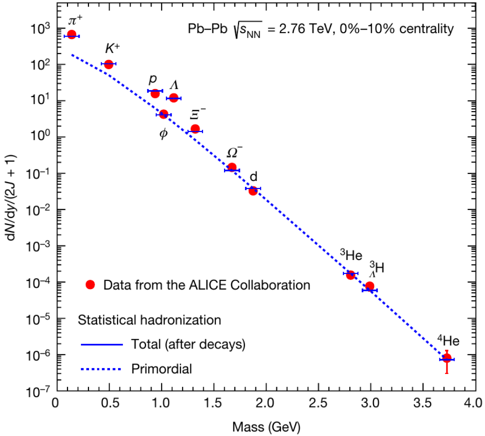




- hadron abundances determined close to phase transition!

Today: Thermal-hadronization model

- from lattice-QCD: at $\mu_B = 0$ cross-over transition, $T_{pc} \simeq 157\,\text{MeV}$
- thermal-hadronization model: hadron abundances determined at chemical freeze-out



- temperature $T_{pc} \simeq 158\,\text{MeV}$ [A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature **561**, 218 (2018)]

Nucl. Phys. A **657**, 413 (1999)

Self-consistent approximations to non-equilibrium many-body theory

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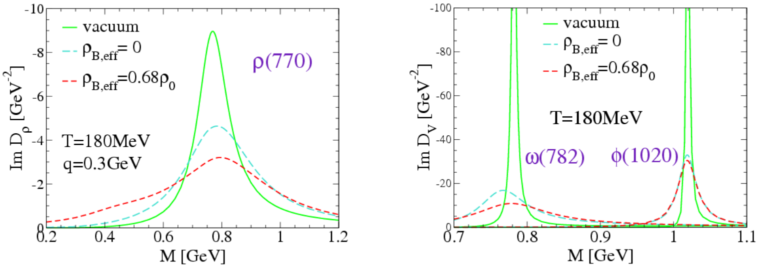
Received 3 March 1999; revised 17 May 1999; accepted 2 June 1999

Abstract

Within the non-equilibrium Green function technique on the real-time contour, the Φ-functional method of Baym is generalized to arbitrary non-equilibrium many-particle systems. The scheme may be closed at any desired order in the number of loops or vertices of the generating functional. It defines effective theories, which provide a closed set of coupled classical-field and Dyson equations, which are self-consistent, conserving and thermodynamically consistent. The approach permits one to include unstable particles and therefore unifies the description of resonances with all other particles, which obtain a mass width by collisions, decays or creation processes in dense matter. The inclusion of classical fields enables the treatment of soft modes and phase instabilities. The method can be taken as a starting point for adequate and consistent quantum improvements of the in-medium rates in transport theories. Properties of resonances are discussed within the dilute density limit in terms of scattering phase shifts.

Motivation

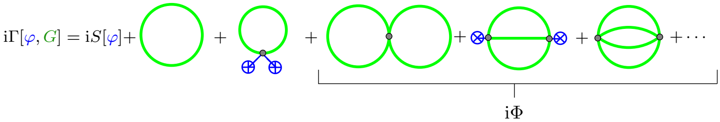
- heavy-collision “fireball” often described with transport models
- uses “cross sections” as in vacuum; $f(x,\vec{p})$ phase-space distribution
- fulfills conservation laws for energy, momentum, particle number,...
- entropy always increasing with time (“H-theorem”) $\Rightarrow$ equilibrium is maximum-entropy state
- equilibrium limit: Fermi-Dirac/Bose-Einstein distributions
- heavy-ion collisions: strongly interacting hot and dense matter
  $\Rightarrow$ particles $\Rightarrow$ “broad resonances” $\Rightarrow$ need “off-shell description”



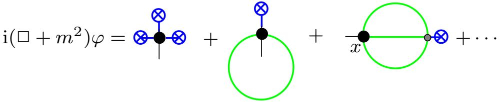
[R. Rapp, J. Wambach, Eur. Phys. J. A **6**, 415 (1999)]

- see also [HvH., K. Knoll, Nucl. Phys. A **683**, 369 (2001)]

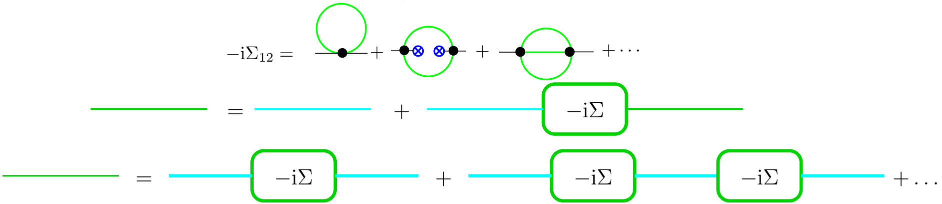
- Feynman diagrams applicable to vacuum, equilibrium, non-equilibrium situations
- generating functional for mean fields and self-energies (lines = full propagators!)



- mean-field equation of motion $\delta\Gamma/\delta\phi(x) = 0$ (take off 1 "sticker")



- self-energy: $\Sigma_{12} \propto \delta\Gamma/\delta G_{21}$ (open one propagator line)



Properties of Φ-derivable approximations

- Φ: all closed two-particle irreducible diagrams (can't disconnect any diagram by cutting 2 lines)
- provides self-consistent equations for mean field and self-energy
- conservation laws from symmetries (energy-momentum conservation, charge conservation,...); Noether theorem
- in equilibrium $\hat{\rho} = \exp(-\hat{H}/T)/\operatorname{Tr}\exp(-\hat{H}/T)$
  - thermodynamic potential: $\Omega(T) = -T\Gamma[\varphi, G]$ with φ, G solutions of equations of motion
  - $\Rightarrow$ Φ-derivable approximations are thermodynamically consistent

Nucl. Phys. A **672**, 313 (2000)

Resonance transport and kinetic entropy

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Received 21 April 1999; revised 20 October 1999; accepted 21 October 1999

Abstract

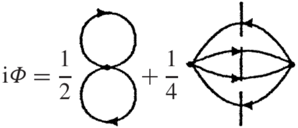
We continue the description of the dynamics of unstable particles within the real-time formulation of nonequilibrium field theory initiated in a previous paper [1]. There we suggest to use Baym's Φ-functional method in order to achieve approximation schemes with 'built in' consistency with respect to conservation laws and thermodynamics even in the case of particles with finite damping width. Starting from Kadanoff–Baym equations we discuss a consistent first order gradient approach to transport which preserves the Φ-derivable properties. The validity conditions for the resulting quantum four-phase-space kinetic theory are discussed under the perspective to treat particles with broad damping widths. This non-equilibrium dynamics naturally includes all those quantum features already inherent in the corresponding equilibrium limit (e.g. Matsubara formalism) at the same level of Φ-derivable approximation. Various collision-term diagrams are discussed including those of higher order which lead to memory effects. As an important novel part we derive a generalized nonequilibrium expression for the *kinetic* entropy flow, which includes contributions from fluctuations and mass-width effects. In special cases an H-theorem is derived implying that the entropy can only increase with time. Memory effects in the kinetic terms provide contributions to the kinetic entropy flow that in the equilibrium limit recover the famous bosonic type $T^3 \ln T$ correction to the specific heat in the case of Fermi liquids like Helium-3.

Collision terms

- Wigner transform: $G(x_1, x_2) \rightarrow G(X, p)$; (extended) phase-space description
- A: spectral function, f phase-space distribution

$$F(X,p) = A(X,p) f(X,p) = \mp \mathrm{i} G^{-+}(X,p), \quad \tilde{F}(X,p) = A(X,p)(1 \mp f(X,p) = \mathrm{i} G^{+-}(X,p)$$

- example from ϕ^4/Fermi-liquid theory



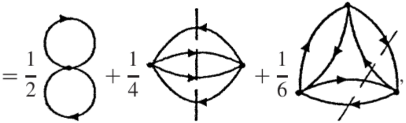
- only two-point part contributes to collision term
- self-energy diagram contains no internal points ⇒ "local self-energy" exact, i.e., all propagators $G^{\pm\mp}$ at same point $X = (x_1 + x_2)/2$
- collision term of usual "Boltzmann-Uehling-Uhlenbeck type"

$$C^{(2)} = d^2 \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{\mathrm{d}^4 p_3}{(2\pi)^4} \left| \; \cdot \; \right|^2$$

$$\times (2\pi)^4 \delta^4(p + p_1 - p_2 - p_3) \left(F_2 F_3 \tilde{F} \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F F_1\right)$$

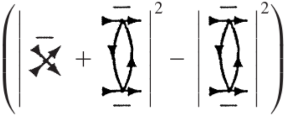
Collision terms (memory effects)

$$\mathrm{i}\Phi = \mathrm{i}\left(\Phi^{(1)} + \Phi^{(2)} + \Phi^{(3)}\right)$$



- local part

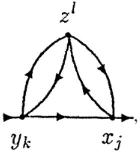
$$C^{(2)} + C^{(3)}_{\mathrm{loc}} = \mathrm{d}^2 \int \frac{\mathrm{d}^4 p_1}{(2\pi)^4} \frac{\mathrm{d}^4 p_2}{(2\pi)^4} \frac{\mathrm{d}^4 p_3}{(2\pi)^4}$$



$$\times (2\pi)^4 \delta^4(p + p_1 - p_2 - p_3)\left(F_2 F_3 \tilde{F} \tilde{F}_1 - \tilde{F}_2 \tilde{F}_3 F F_1\right),$$

- additional non-local/memory correction from diagrams involving internal point in self-energy

$$-\mathrm{i}\Sigma^{(3)}_{jk}(x, y) =$$



Ann. Phys. **293**, 125 (2001)

Exact Conservation Laws of the Gradient Expanded Kadanoff–Baym Equations

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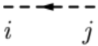
Received February 26, 2001; accepted May 31, 2001

It is shown that the Kadanoff-Baym equations at consistent first-order gradient approximation reveal exact rather than approximate conservation laws related to global symmetries of the system. The conserved currents and energy–momentum tensor coincide with corresponding Noether quantities in the local approximation. These exact conservations are valid, provided a Φ derivable approximation is used to describe the system, and possible memory effects in the collision term are also consistently evaluated up to first-order gradients.

Diagram rules to calculate memory contributions

- "local" diagrams: usual rules with all G's taken at one point X
- 1st-order gradient corrections for $G(X',p)$ with $X' \neq X$

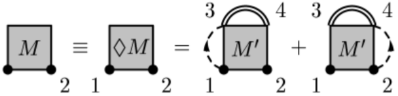
$$= \frac{1}{2}(\partial_i + \partial_j) G(i,j) \rightarrow \partial_X G(X,p),$$



$$= -i(x_i - x_j) \rightarrow -(2\pi)^4 \frac{\partial}{\partial p} \delta(p)$$

- memory contributions to two-point functions

$$\diamond\{M(1,2)\} = \diamond$$

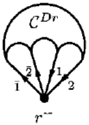


- $M'(12,34) = \mp \frac{\partial M(12)}{\mathrm{i}\delta G(3,4)}$
- apply this to all propagators connecting to a point $\neq x_1, x_2$

Diagrams for one-point functions

- for energy-momentum tensor, currents of conserved charges

$$i\Phi^{D}_{\text{loc}}(X;r^{-}) =$$



$$= \int \frac{d^4 p_1}{(2\pi)^4} \cdots \frac{d^4 \bar{p}_1}{(2\pi)^4} \cdots \mathcal{C}^{Dr}_{k_1 \cdots \bar{k}_1 \cdots}(X; p_1, \ldots, \bar{p}_1, \ldots)$$

$$\times (\mp i) G_1^{-k_1}(X, p_1) \cdots i G_{\bar{1}}^{\bar{k}_1 -}(X, \bar{p}_1) \cdots .$$

- exactly conserved energy-momentum tensor:

$$\Theta^{\mu\nu}_{\text{loc}}(X) = \sum_a \int \frac{d^4 p}{(2\pi)^4} v^{\mu} p^{\nu} (\mp i) G_a^{-+}(X, p) + g^{\mu\nu} \left(\mathcal{E}^{\text{int}}_{\text{loc}}(X) - \mathcal{E}^{\text{pot}}_{\text{loc}}(X) \right)$$

- exactly conserved charge current

$$J^{\mu}(X) = \sum_a \int \frac{d^4 p}{(2\pi)^4} e_a v^{\mu} (\mp i) G_a^{-+}(X, p)$$

Conclusion: Off-shell transport

- starting from quantum field theory
- using the Φ-functional formalism
- $\Rightarrow$ Kadanoff-Baym equations for full Green's functions (and mean fields)
- "particles" $\Rightarrow$ broad spectral functions ("off-shell")
- via systematic gradient expansion for Wigner functions
  $\Rightarrow$ kinetic/transport equations for "off-shell particles"/resonances
    - conservation laws fulfilled for conserved Noether charges of symmetries: charges, energy, momentum
    - definition of off-equilibrium entropy; H-theorem valid at least close to thermal equilibrium
    - thermodynamical consistency: Φ functional defines thermodynamic potential in equilibrium
      $\Rightarrow$ thermodynamic relations fulfilled
    - off-equilibrium entropy $\Rightarrow$ equivalent to thermodynamic entropy in equilibrium

Renormalization in self-consistent approximation schemes at finite temperature: Theory

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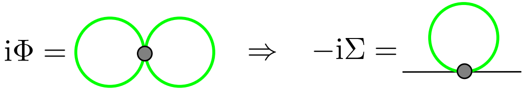
(Received 18 July 2001; published 26 December 2001)

Within finite temperature field theory, we show that truncated nonperturbative self-consistent Dyson resummation schemes can be renormalized with local counterterms defined at the vacuum level. The requirements are that the underlying theory is renormalizable and that the self-consistent scheme follows Baym's Φ-derivable concept. The scheme generates both the renormalized self-consistent equations of motion and the closed equations for the infinite set of counterterms. At the same time the corresponding two-particle irreducible generating functional and the thermodynamical potential can be renormalized, consistent with the equations of motion. This guarantees that the standard Φ-derivable properties such as thermodynamic consistency and exact conservation laws hold also for the renormalized approximation schemes. The proof uses the techniques of Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization to cope with the explicit and the hidden overlapping vacuum divergences.

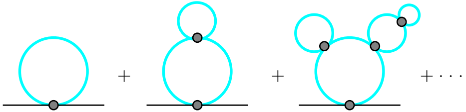
Example: Tadpole in ϕ^4-model



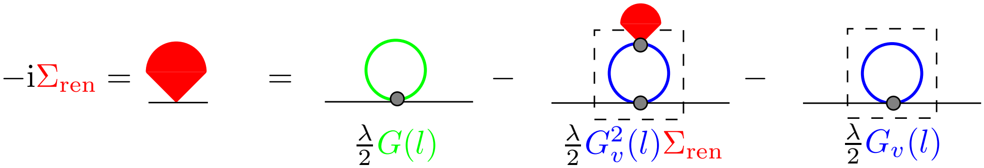
Temperature dependent mass

$$M^2 = m^2 + \Sigma_{\text{ren}}$$

Eq. of motion ⇒ Resummation of “Daisy” and “super-Daisy” diagrams:



- Expand Green’s function in vacuum part and temperature part
- Dyson equation: $G = G_v + G_v \Sigma G_v + \ldots$
- Subtract vacuum divergences and subdivergences only
- Counterterms: Vacuum-mass and coupling-constant counterterm



Renormalization of general approximations

- The same strategy as in the tadpole example
- Renormalize vacuum first
- power counting is the same as for perturbative diagrams
- The temperature part of the self-energy is of power 0
- the asymptotic behavior is governed by the vacuum part alone
- expand Green's function due to Dyson equation

$$G = \underbrace{G_v}_{\delta=-2} + \underbrace{G_v \Sigma_T G_v}_{\delta=-4} + \underbrace{G_r}_{\delta=-6}$$

- coupling constant renormalization more difficult than for tadpole
- can be solved due to the 2PI properties of the Φ-functional!

Phys. Rev. D **65**, 105005 (2001)

Renormalization of self-consistent approximation schemes at finite temperature. II. Applications to the sunset diagram

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(Received 28 November 2001; published 24 April 2002)

The theoretical concepts for the renormalization of self-consistent Dyson resummations, devised in the first paper of this series, are applied to first example cases of ϕ^4 theory. In addition to the tadpole (Hartree) approximation, as a novel part the numerical solutions are presented, which include the sunset self-energy diagram into the self-consistent scheme based on the Φ-derivable approximation or the two-particle irreducible effective action concept.

Renormalization in self-consistent approximation schemes at finite temperature. III. Global symmetries

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(Received 1 March 2002; published 31 July 2002)

We investigate the symmetry properties for Baym's Φ-derivable schemes. We show that in general the solutions of the dynamical equations of motion, derived from approximations of the Φ functional, do not satisfy the Ward-Takahashi identities of the symmetry of the underlying classical action, although the conservation laws for the expectation values of the corresponding Noether currents are satisfied exactly for the approximation. Further we prove that one can define an effective action functional in terms of the self-consistent propagators which is invariant under the operation of the same symmetry group representation as the classical action. The requirements for this theorem to hold true are the same as for perturbative approximations: The symmetry has to be realized linearly on the fields and it must be free of anomalies; i.e., there should exist a symmetry-conserving regularization scheme. In addition, if the theory is renormalizable in Dyson's narrow sense, it can be renormalized with counterterms which do not violate the symmetry.

Conclusion: Renormalization of Φ-derivable approximations

- at finite temperature: Φ-derivable approximations renormalizable with vacuum counter-terms
- parameters (masses, coupling constants) defined in vacuum
- medium modifications follow from dynamics at finite temperature
- thermodynamically consistent
- problem: symmetries violated on the propagator level although conservation laws ("one-point-function level") fulfilled
- symmetries restored for effective 1PI propagators