Renormalization of Conserving Dyson resummation schemes

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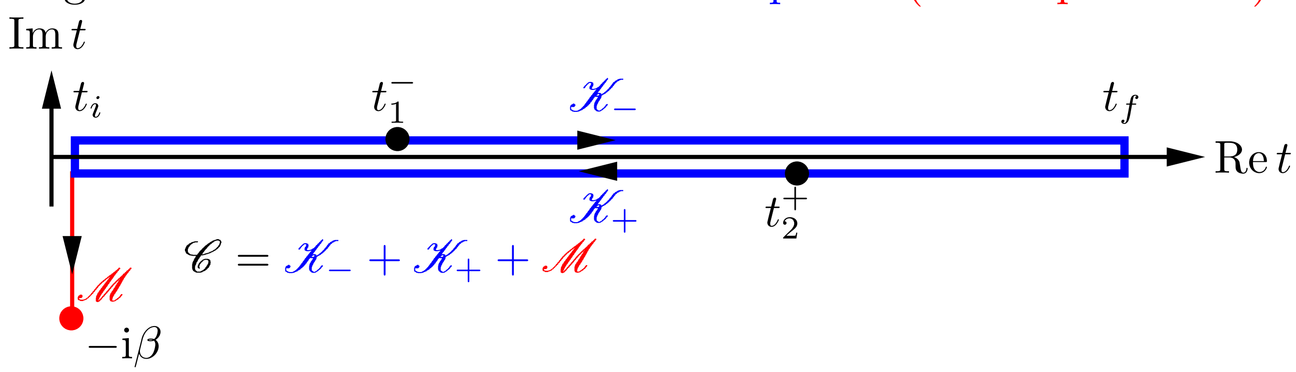
Content

- 2PI-Functionals of quantum field theory
- Renormalization with temperature independent counter terms
- Symmetry properties
- Numerical Results
- Conclusions and Outlook

2PI Formalism

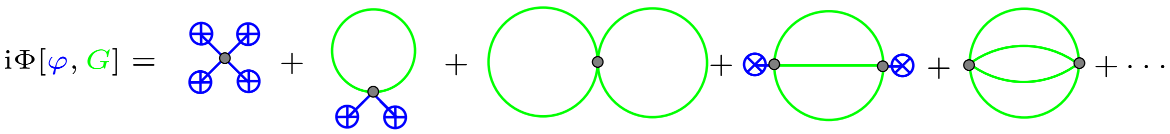
- Diagrams defined for real time path (for equilibrium)



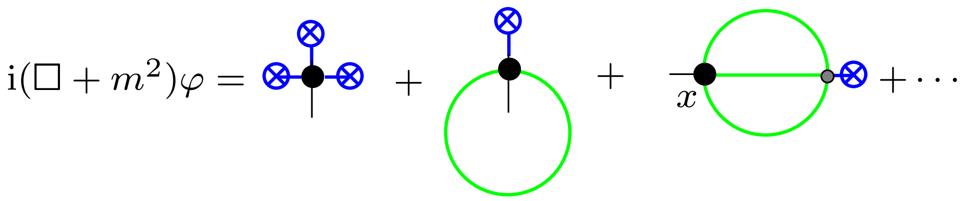
- $O(N)$-theory

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \vec{\phi})(\partial^\mu \vec{\phi}) - \frac{m^2}{2}\vec{\phi}^2 - \frac{\lambda}{4!}(\vec{\phi}^2)^2$$

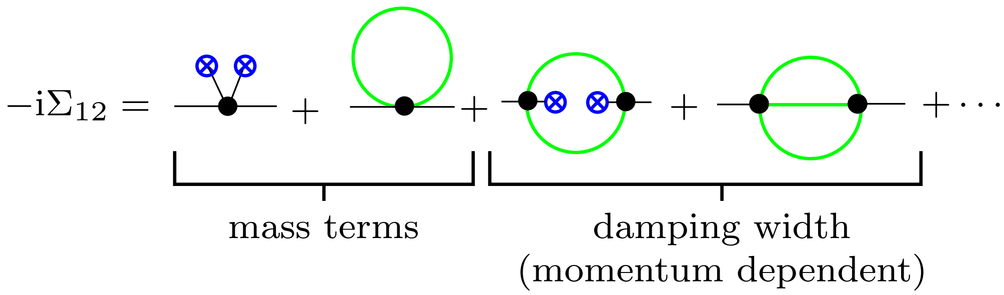
- 2PI Generating Functional



- Mean field equation of motion



- Self-energy



- Dyson-equation:

$$G^{-1} = D^{-1} - \Sigma[\varphi, G]$$

- Closed set of equations of motion for φ and G

Self-consistent Renormalization

First step: Vacuum

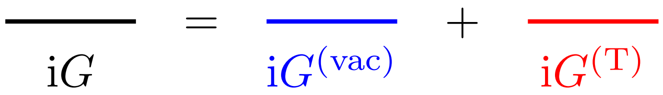
- Power-counting for self-consistent propagators as in perturbation theory: $\delta = 4 - E$
- Usual BPHZ-renormalization for wave function, mass and coupling constant renormalization
- In practice: Use Lehmann-representation and dimensional regularization

✔ Closed self-consistent finite Dyson-equations of motion

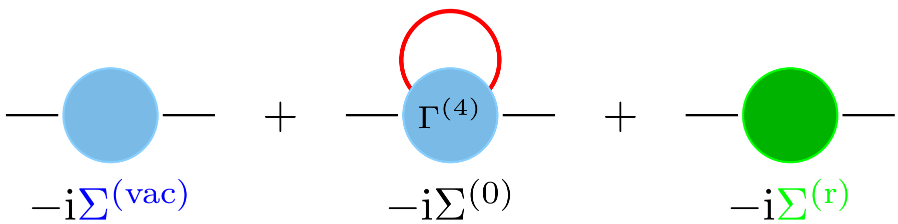
✔ Numerically treatable

Second step: Finite Temperature

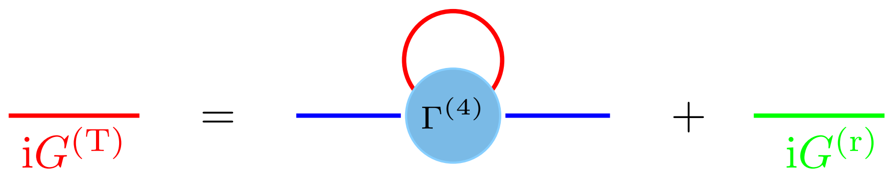
- Split propagator in vacuum and T-dependent part



- Expand self-energy around vacuum part



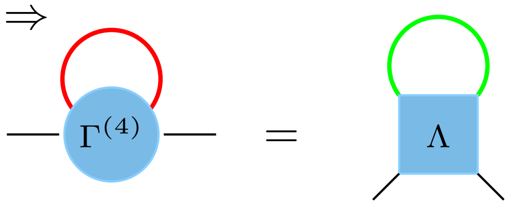
- Need further splitting of propagator



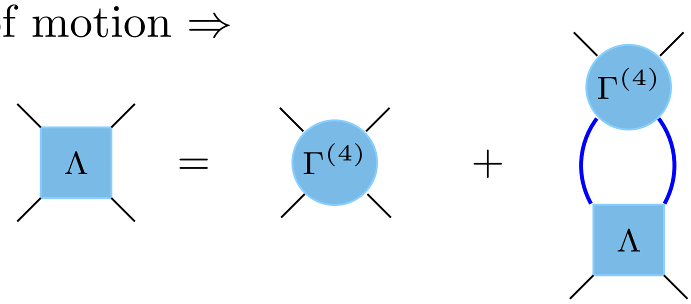
Self-consistent Renormalization

Third step: 4-point vertex renormalization

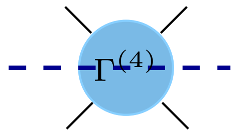
- Σ^0 linear in $G^{(\mathrm{r})}$ $\Rightarrow$



- Equation of motion $\Rightarrow$



☞ s-channel Bethe-Salpeter equation



cuts more than three lines!

$\Rightarrow$ "BPHZ Boxes" in ladder-diagrams do not cut inside $\Gamma^{(4)}$.

$\Rightarrow$ Asymptotics + BPHZ-formalism:

$$\Gamma^{(4)}(l,p) - \Gamma^{(4)}(l,0) \cong O(l^{-\alpha}) \text{ with } \alpha > 0$$

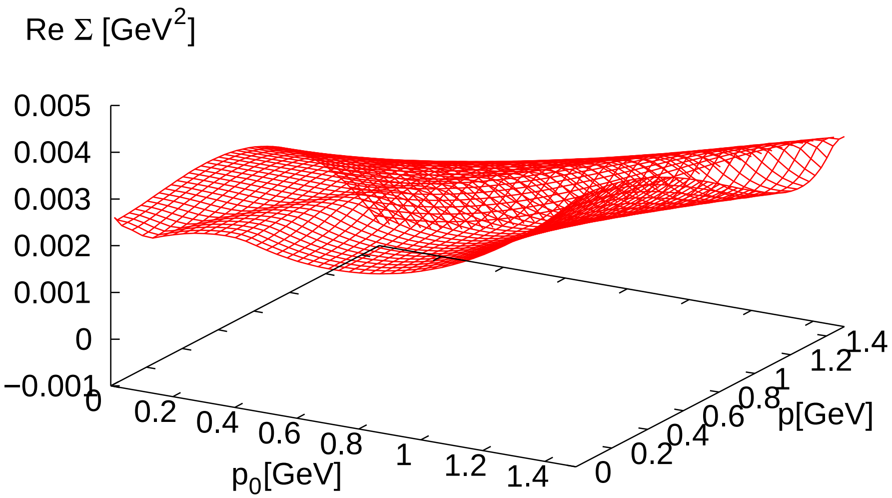
$\Rightarrow$ Renormalized eq. of motion for Λ:

$$\begin{aligned}\Lambda(p,q) =& \Lambda(0,0) + \Gamma^{(4)}(p,q) - \Gamma^{(4)}(0,0) \\ &+ \mathrm{i} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} [\Gamma^{(4)}(p,l) - \Gamma^{(4)}(0,l)][G^{\mathrm{vac}}]^2(l)\Lambda(l,q) \\ &+ \mathrm{i} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \Lambda(0,l)[G^{\mathrm{vac}}]^2(l)[\Gamma^{(4)}(l,q) - \Gamma^{(4)}(l,0)]\end{aligned}$$

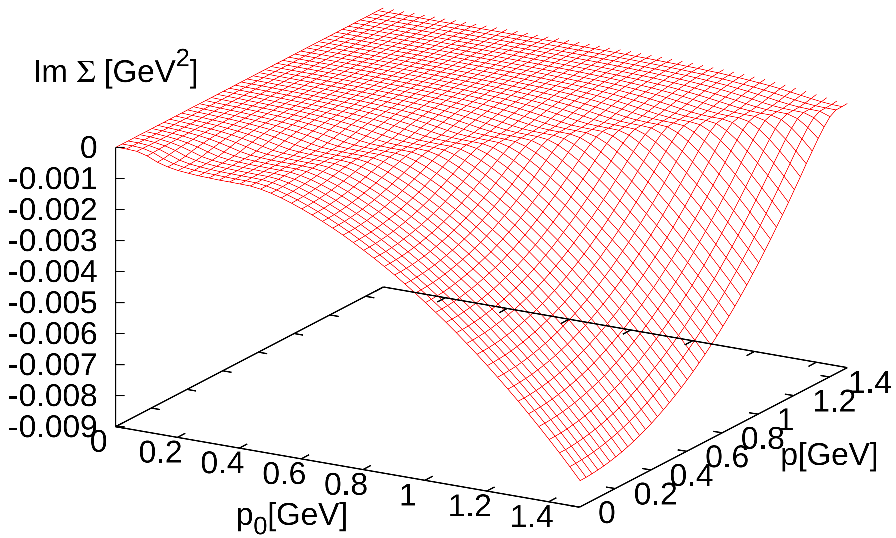
✔ Self-energy finite with vacuum counter terms

Results for "Sunset + Tadpole" at $T > 0$

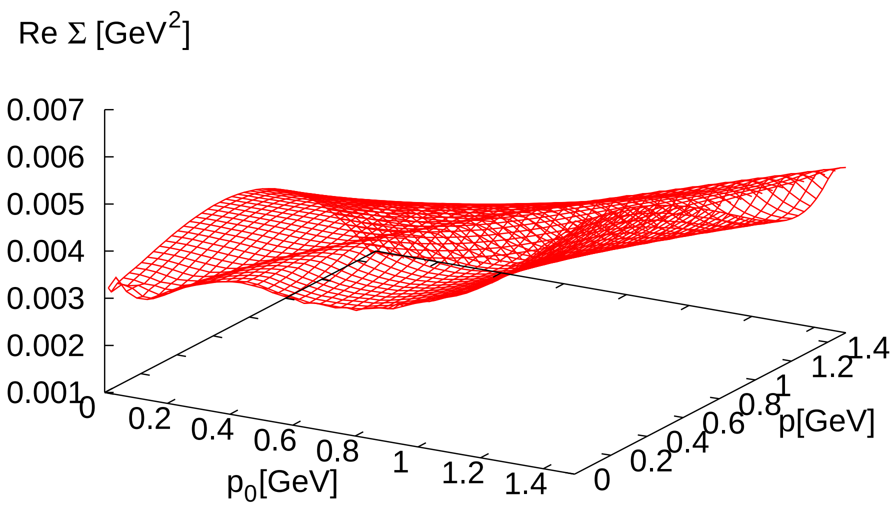
Pert. Re Σ for T=100MeV, λ=20



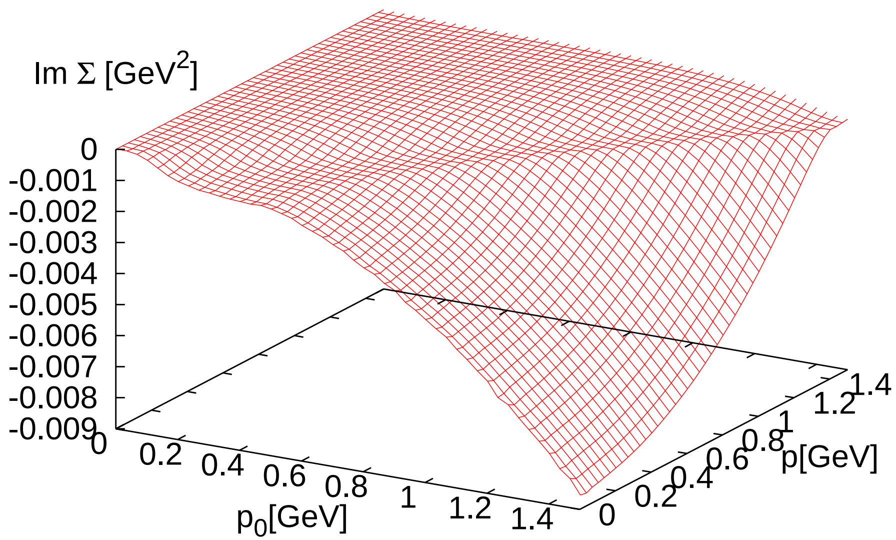
Pert. Im Σ for T=100MeV, λ=20



Re Σ for T=100MeV, λ=20

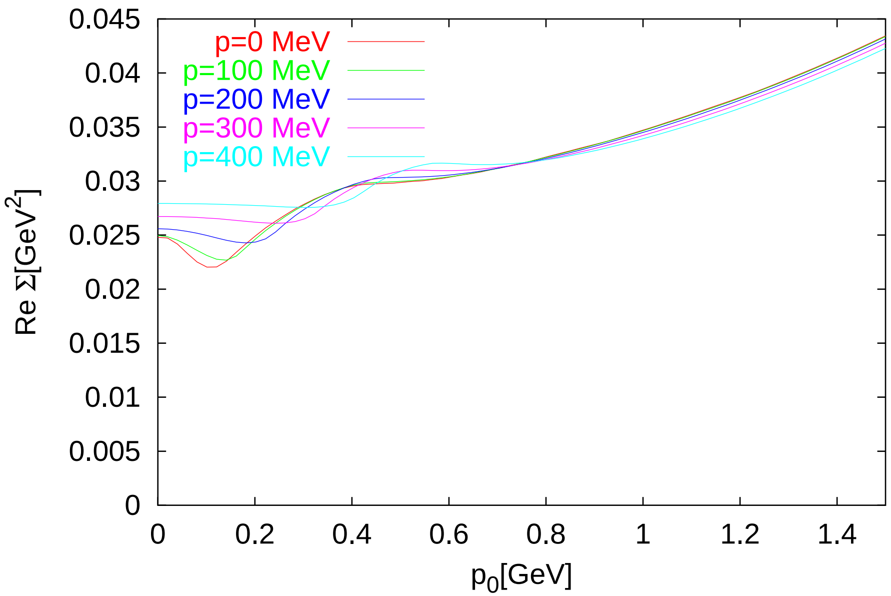


Im Σ for T=100MeV, λ=20

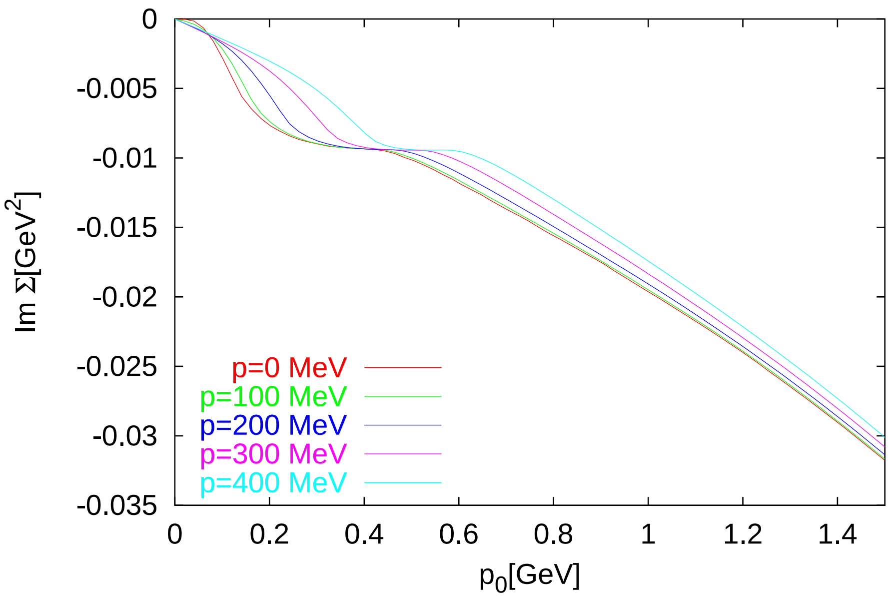


Results for "Sunset + Tadpole" at $T > 0$

Pert. Re Σ for λ=30, T=200 MeV



Pert. Im Σ for λ=30, T=200 MeV



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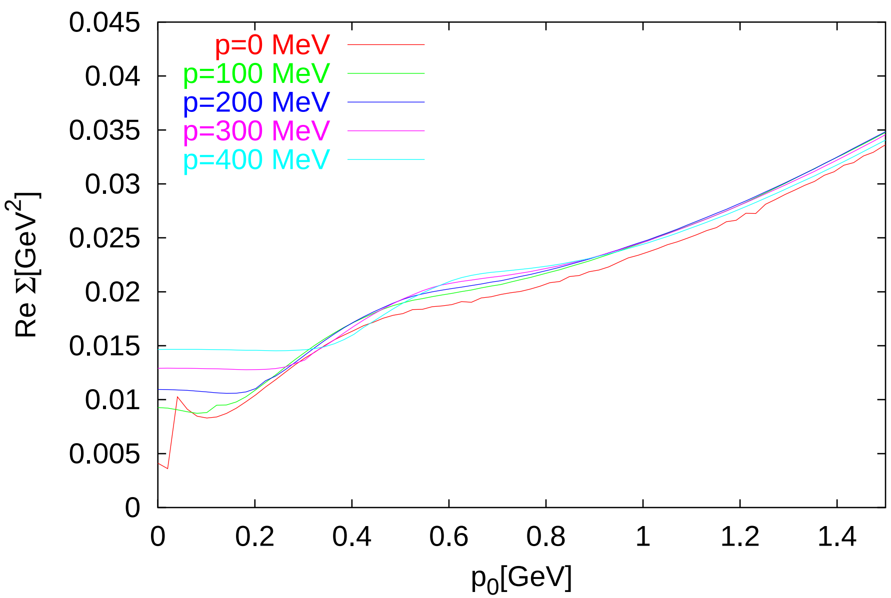

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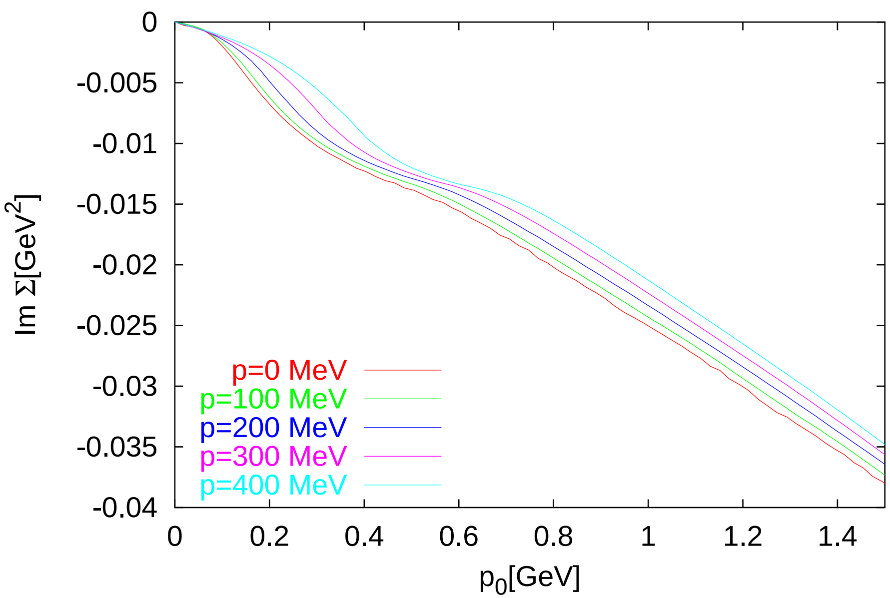


Symmetry properties

- Symmetry: Expectation values of Noether currents exactly conserved
- Approximations are only partial resummations of perturbation series

☞ Crossing symmetry violated

☞ Ward-Takahashi identities for n-point functions violated

- Non-perturbative approximation for effective action:

$$\tilde{\Gamma}[\varphi] = \Gamma[\varphi, \tilde{G}[\varphi]]$$

$$\left.\frac{\delta\Gamma[\varphi, G]}{\delta G}\right|_{G=\tilde{G}[\varphi]} \stackrel{!}{=} 0$$

- Crossing symmetric proper vertex functions

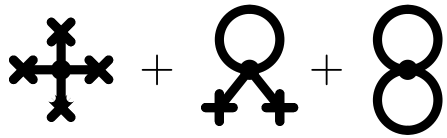
$$\tilde{\Gamma}^{(n)}(x_1, x_2, \ldots, x_n) := \mathrm{i}\frac{\delta\tilde{\Gamma}[\varphi]}{\delta\varphi_1\delta\varphi_2\cdots\delta\varphi_n}$$

  fulfill Ward-Takahashi identities

- Calculation of $\tilde{\Gamma}^{(n)}$: Bethe-Salpeter equation like resummations in terms of self-consistent propagator
- Renormalization in the same way as self-consistent scheme ⇒ Recovers symmetry also for counter terms!

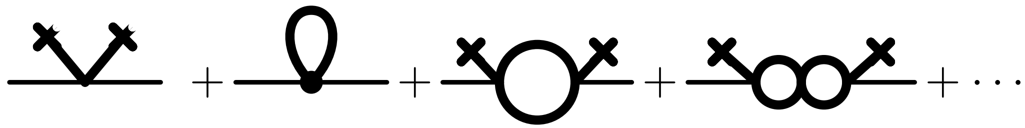
Example: Hartree approximation

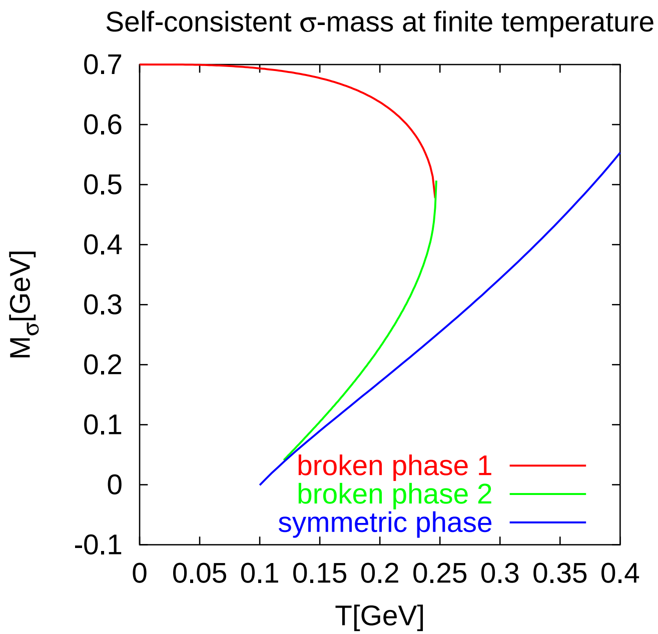
- Hartree approximation:

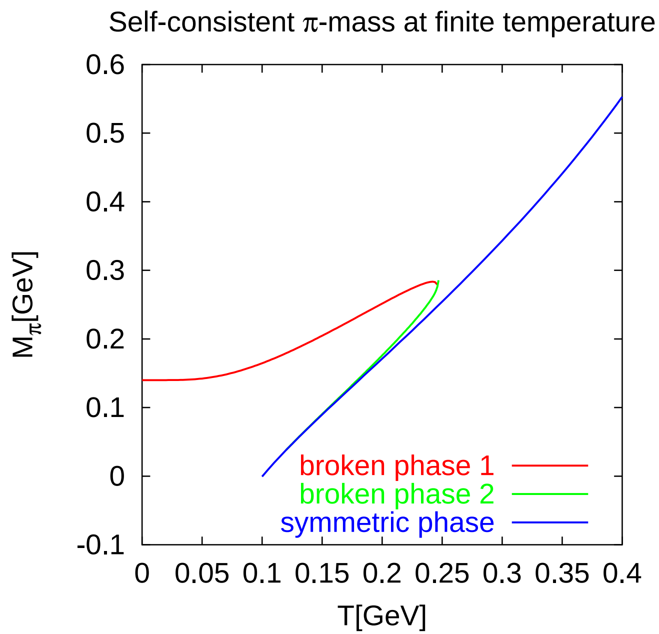
$$i\Phi =$$ 

- 1PI self–energy defined on top of Hartree approximation

☞ Random phase approximation (RPA):

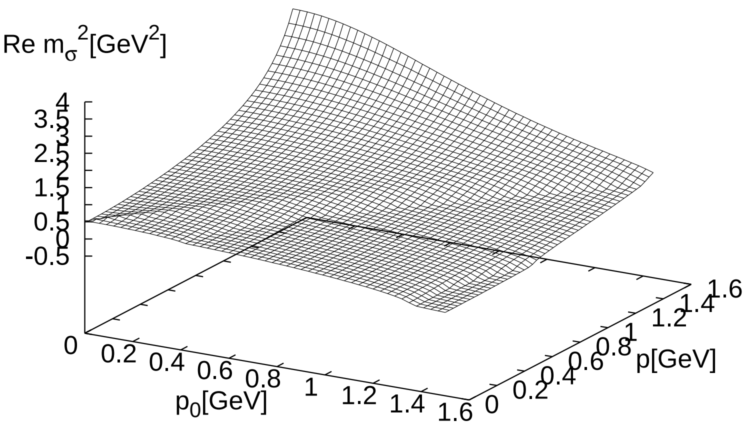
$$-i\tilde{\Sigma} =$$ 





RPA-resummation

External σ-mass at T=150 MeV (stable solution)



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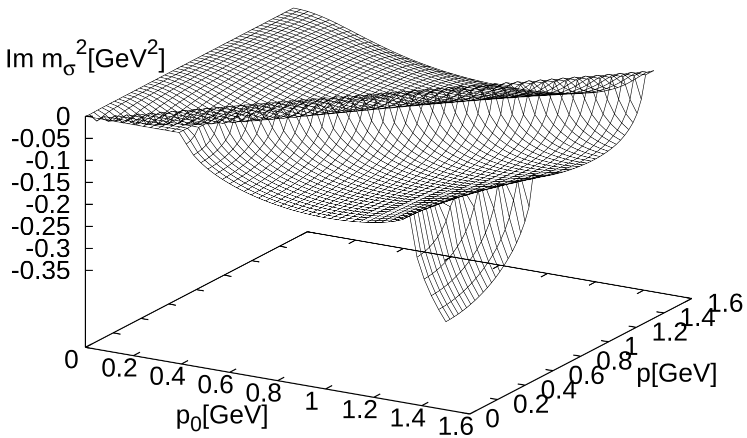
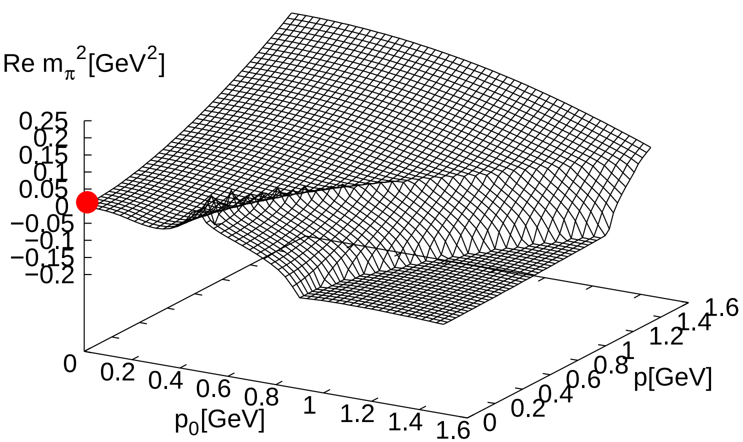
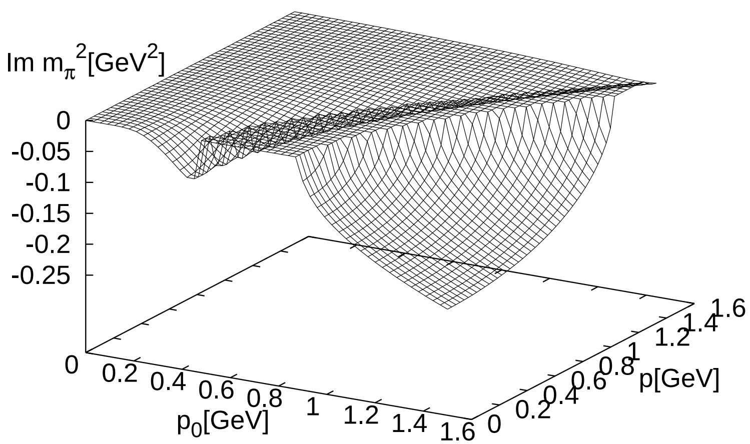


External π−mass at T=150 MeV (stable solution)



External σ-mass at T=150 MeV (stable solution)



Conclusions and Outlook

- ✔ Self–consistent Φ–derivable schemes
- ✔ Renormalization: Phys. Rev. **D65**, 025010 (2002), hep-ph/0107200
- ✔ Numerical treatment: hep-ph/0111193 (Phys. Rev. D, in press)
- ✔ Symmetry properties: hep-ph/0203008

- ✔ "Toolbox" for application to realistic models
- ✔ Perspectives for self–consistent treatment of vector particles: Nucl. Phys. **A683** 369, hep-ph/0002087
- ✘ General gauge theories?
- ✘ QCD e.g. beyond HTL?
- ✔ Transport equations for particles with finite width

`http://theory.gsi.de/~vanhees/index.html`
`http://theory.gsi.de/~knoll/index.html`