Kinetics of the chiral phase transition

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Outline

1. Linear σ model
2. Semiclassical particle-field dynamics
   - Thermal quench (box calculation)
   - Expanding hot-matter droplet
3. Conclusions and outlook

Motivation

- exploring the QCD phase diagram in heavy-ion collisions
- identify observables for different phase transitions (cross-over at low vs. 1st order at high μ_B)
- critical endpoint of 1st-order phase-transition line?!?
- problem: rapidly expanding and cooling “fireballs” ⇒ observables?
- “grand canonical fluctuations” of conserved “charges”?!?
- model fluctuations from dynamics rather than imposed by hand (Langevin/Fokker-Planck)
- here: novel kinetic model based on particle-field dualism
- Phys. Rev. E **91**, 043302 (2015) (arXiv: 1411.7979 [hep-ph])
  J. Phys. Conf. Ser. **636**, 012007 (2015) (arXiv: 1505.04738 [hep-ph])
  C. Wesp, PhD Thesis, Goethe University Frankfurt (2015)

Quark-meson linear σ model

- quark-meson linear σ model
- chiral $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2) \sim \mathrm{SO}(4)$ symmetry
- spontaneously broken to $\mathrm{SU}_V(2) \sim \mathrm{SO}(3)$
- mesons SO(4): σ (scalar) $\vec{\pi}$ (pseudoscalar)
- constituent quarks: $\mathrm{SU}_L(2) \times \mathrm{SU}_R(2)$

$$\mathscr{L} = \overline{\psi}[\mathrm{i}\partial\!\!\!/ - g(\sigma + \mathrm{i}\vec{\pi} \cdot \vec{\tau}\gamma_5)]\psi - \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi})$$

- meson potential

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 + f_\pi m_\pi^2 \sigma.$$

- explicit breaking of chiral symmetry

Semiclassical particle-field dynamics

- treat σ bosons on the mean-field level + $\sigma \longleftrightarrow \overline{q} q$

$$\Box\sigma + \lambda(\sigma^2 - \nu^2)\sigma - f_\pi m_\pi^2 + g \left\langle \overline{\psi}\psi \right\rangle = \text{“}I(\sigma \longleftrightarrow \overline{q} q)\text{”}$$

- (anti-)quarks via Boltzmann equation

$$\left[\partial_t + \frac{p}{E_q} \cdot \vec{\nabla}_{\vec{x}} - \vec{\nabla}_{\vec{x}} E_\psi(t, \vec{x}, \vec{p}) \cdot \vec{\nabla}_{\vec{p}}\right] f_q(t, \vec{x}, \vec{p}) = C(\overline{q} q \rightarrow \overline{q} q, \sigma \longleftrightarrow \overline{q} q)$$

$$\text{with} \quad E(t, \vec{x}, \vec{p}) = \sqrt{\vec{p}^2 + g^2 \sigma^2(t, \vec{x})}$$

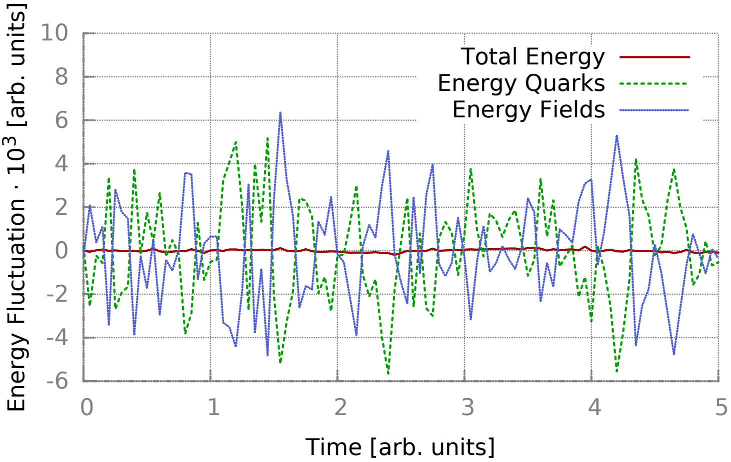
- test-particle ansatz for (anti-)quarks on a spatial grid
- “particle-field dualism” of σ **field** $\longleftrightarrow$ **particle** for “collision terms”

Semiclassical particle-field dynamics

- $\sigma \rightarrow \overline{q} + q$
  - calculate energy and momentum of σ **field** in cell
  - determine local temperature and chemical potential
  - Boltzmann distribution ⇒ σ-**particle** momentum distribution
  - use σ-decay width/rate (matrix element) from QFT in collision terms
  - take out energy and momentum of σ **particle** as a corresponding Gaussian wave packet from the σ **field**
- $\overline{q} + q \rightarrow \sigma$
  - "Monte-Carlo" event according to matrix element from σ model
  - add energy and momentum of σ **particle** as a corresponding Gaussian wave packet to σ **field**
- energy-momentum and baryon-number conservation
- principle of detailed balance fulfilled!

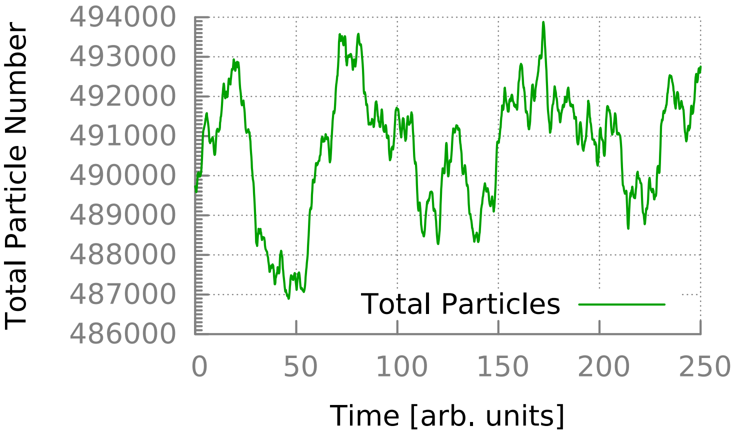
Test: energy conservation (box calculation)

- uncorrelated thermal fluctuations $\Delta E_q / E_q \sim 10^{-3}$ and $\Delta E_\sigma / E_\sigma \sim 10^{-2}$
- $\Delta E_{\text{tot}} / E_{\text{tot}} \lesssim 5 \cdot 10^{-5}$



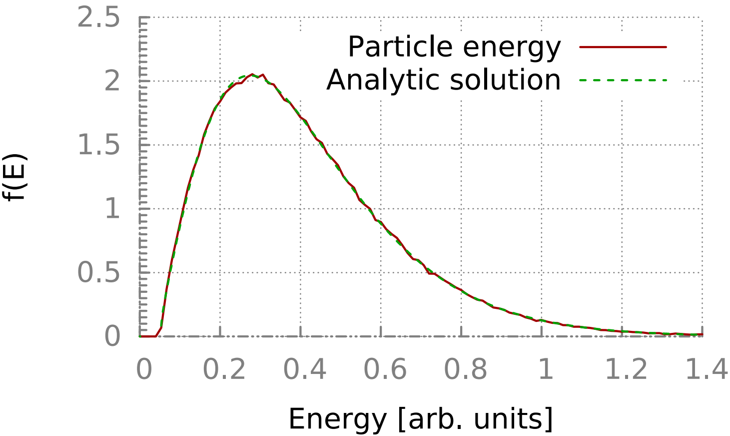
Test: quark-number fluctuations (box calculation)

- total number of (anti-)quarks ($N_q = N_{\overline{q}}$) fluctuates due to $\sigma \longleftrightarrow \overline{q} + q$



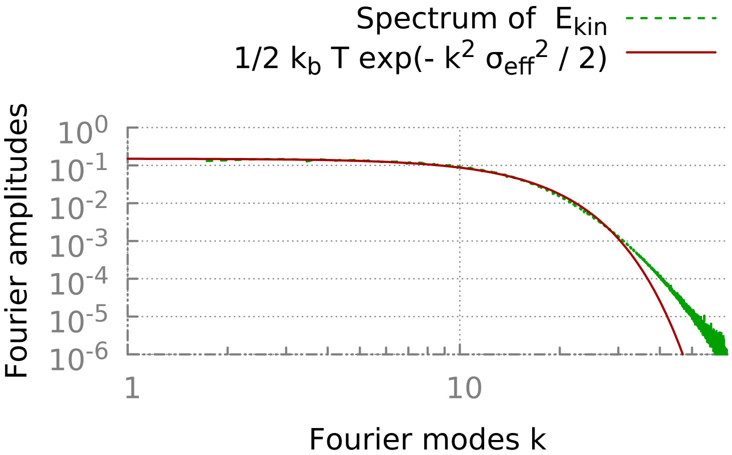
Test: Equilibration limit (box calculation)

- run algorithm in box until q-$\overline{q}$ distribution and σ field stationary
- excellent agreement with relativistic Boltzmann distribution



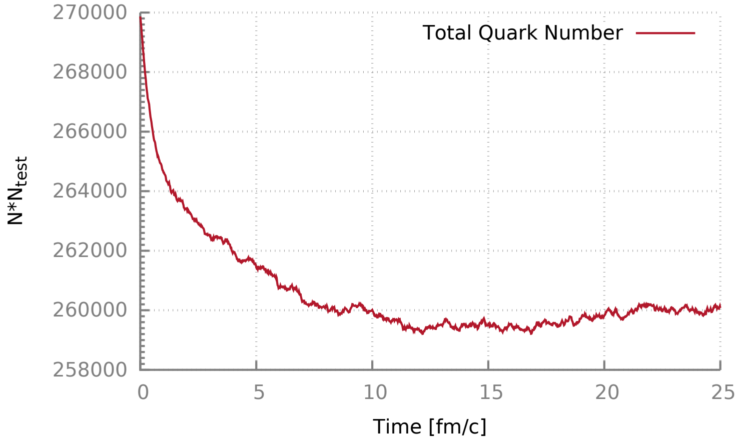
Test: Equilibration limit (box calculation)

- run algorithm in box until q-$\overline{q}$ distribution and σ field stationary
- Fourier spectrum of σ-field energy
- "UV catastrophy" avoided due to finite width σ_{eff} of Gaussian wave packets $\longleftrightarrow$ energy-momentum transfer between particles and fields



Thermal quench (no phase transition)

- start system with $T_\sigma = 180\,\text{MeV}$, $T_{\bar{q}q} = 140\,\text{MeV}$
- system always in chirally restored phase



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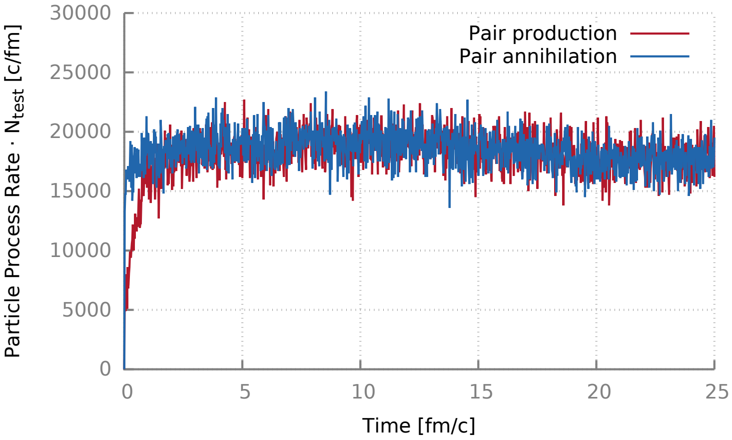

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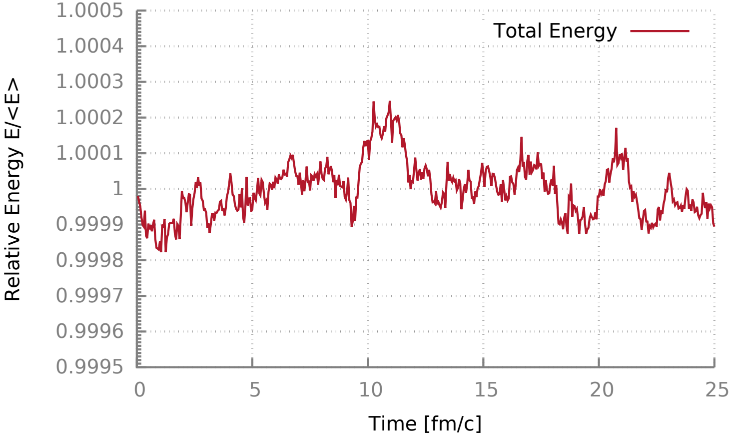

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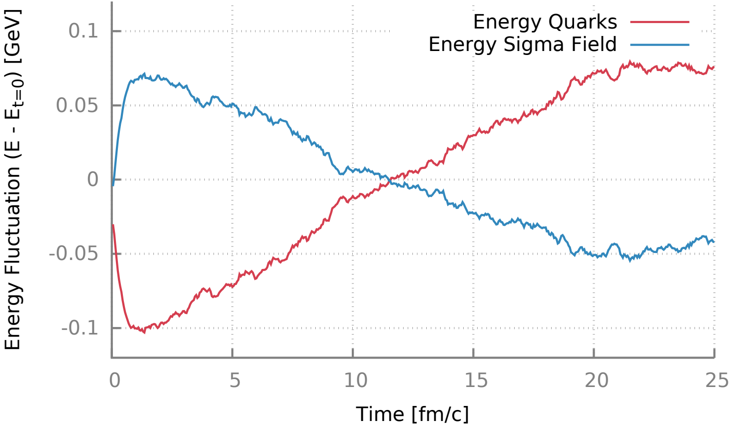
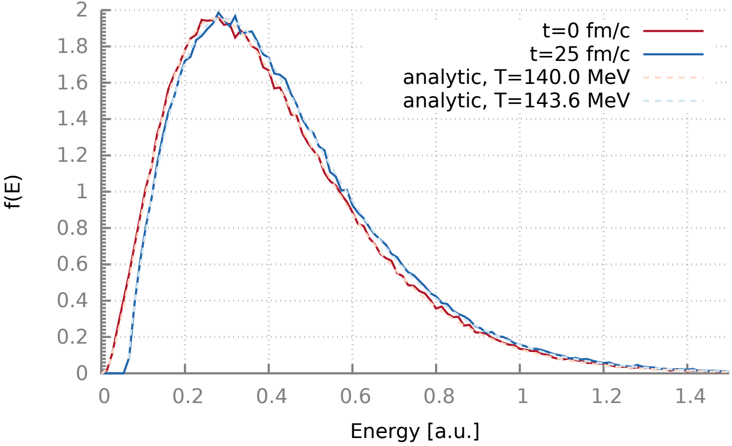


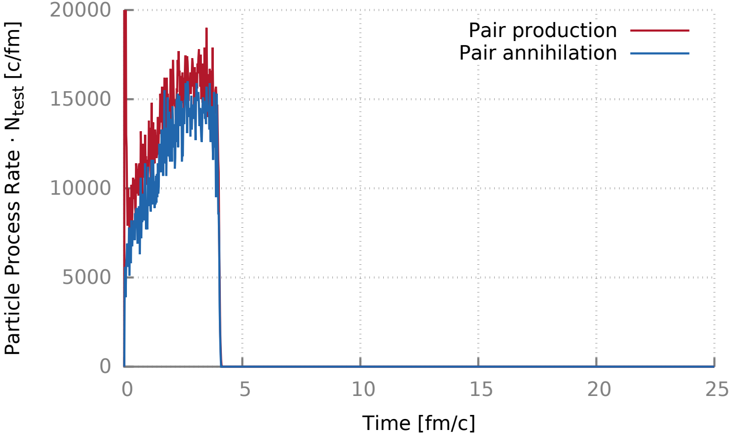
Thermal quench (no phase transition)

- start system with $T_\sigma = 180$ MeV, $T_{\overline{q}q} = 140$ MeV
- system always in chirally restored phase
- system comes to thermal equilibrium ($\sigma \longleftrightarrow \overline{q}q$ always "active")



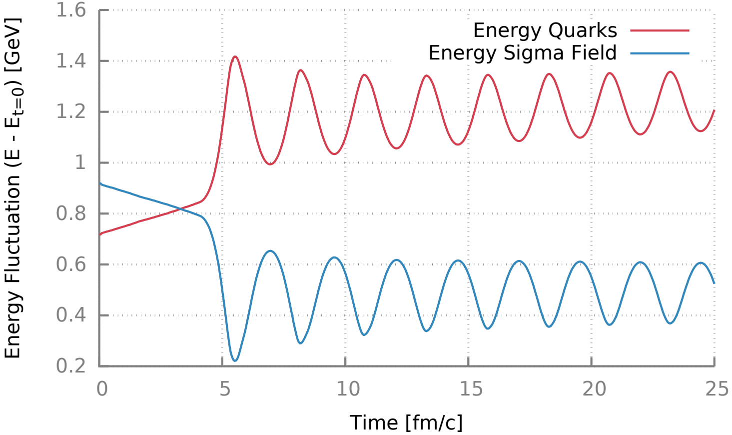
Thermal quench (χ restored $\longrightarrow$ broken phase)

- start system with $T_\sigma = 180$ MeV, $T_{\overline{q}q} = 80$ MeV
- system undergoes transition from chirally restored to broken phase
- system does not come to thermal equilibrium
  ($\sigma \longleftrightarrow \overline{q}q$ becomes impossible because $m_\sigma < 2m_q$)



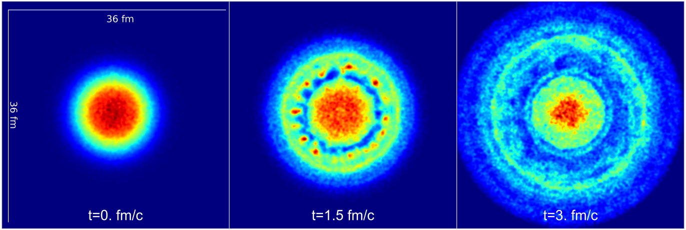
Thermal quench (χ restored $\longrightarrow$ broken phase)

- start system with $T_\sigma = 180$ MeV, $T_{\overline{q}q} = 80$ MeV
- system undergoes transition from chirally restored to broken phase
- ($\sigma \longleftrightarrow \overline{q}q$ becomes impossible because $m_\sigma < 2m_q$)
  after "decoupling" oscillations in σ field $\Leftrightarrow E_{q\overline{q}}$

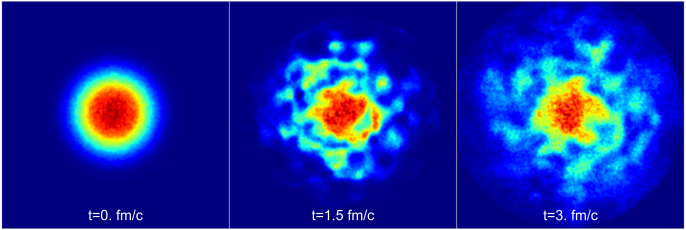


Expanding hot-matter droplet (cross-over at $g = 3.3$)

without “chemical processes”

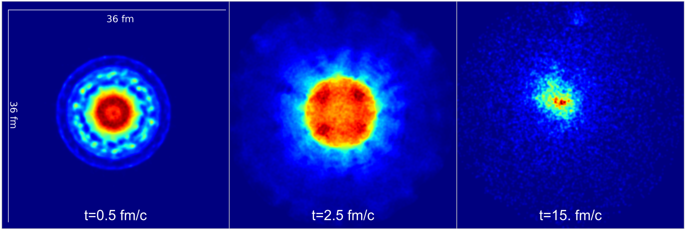


with “chemical processes” ($\sigma \longleftrightarrow \overline{q} + q$)

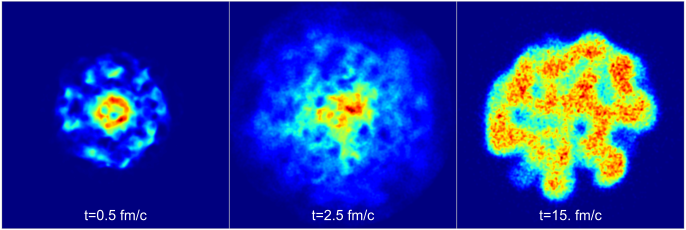


Expanding hot-matter droplet (1^{st}-order PT at $g = 5.5$)

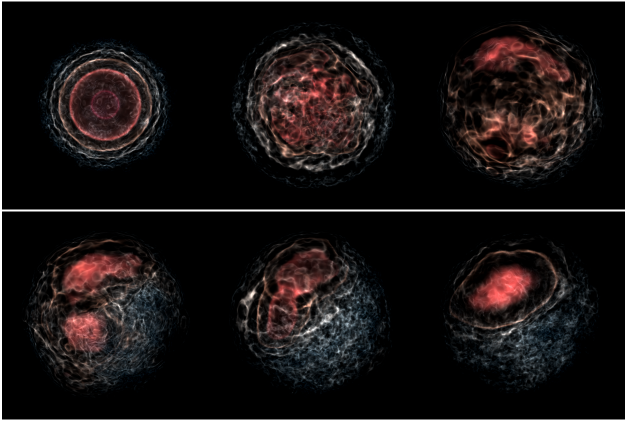
without “chemical processes”



with “chemical processes” ($\sigma \longleftrightarrow \overline{q} + q$)

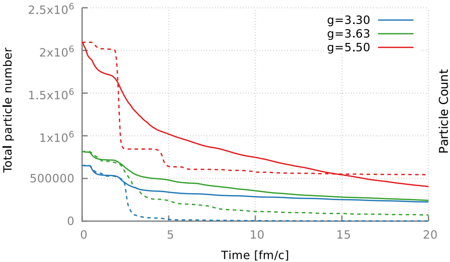


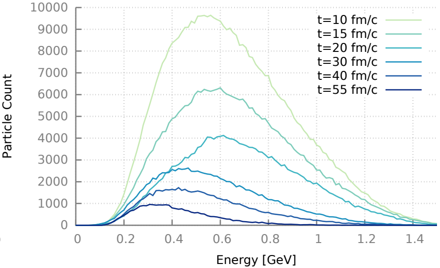
Volumetric view on expanding fireball at $g = 5.5$



- in time steps of $1\,\text{fm}/c$
- $q\overline{q}$ bubbles formed $\Rightarrow$ tend to merge into large bubble
- "cold" particles trapped in local potential wells of σ field
- slowly evaporating

Quark-number distribution in expanding fireball

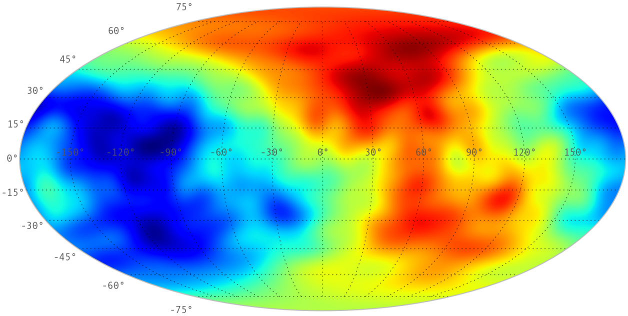




- left: time evolution of total quark number
  - simulations without (dashed) and with (solid) chemical processes $\sigma \longleftrightarrow q\bar{q}$
  - number of quarks decreasing (leaving the fireball)
  - at $g = 5.5$: metastable droplets form ⇒ plateaus (washed out by chemical processes)
- right: energy distribution of quark number vs. time
  - quarks can get trapped in local σ-field potential wells
  - with $g = 5.5$ big metastable quark droplet forms
  - slowly evaporates ⇒ cooling

Angular distribution of quarks

- first attempt at an observable sensitive to nature of phase transition
- angular distribution of quarks
- first attempt at C_ℓ power spectra of angular correlations
- no qualitative difference for different phase transitions
- only overall size of fluctations differs related to g



Conclusions and outlook

- novel scheme to model off-equilibrium kinetics of phase transitions
- based on particle-field duality
- application to linear quark-meson σ model
- obeys conservation laws and detailed balance
- dynamically generated fluctuations (no assumptions as in Langevin!)
- passes box-calculation tests
- thermal quench + expanding fireballs
- qualitative difference between cross-over and $1^{\text{st}}/2^{\text{nd}}$-order scenario?
- to do: how quantifiable?
  - first attempt: angular correlations of quarks
  - no clear signal to distinguish different phase transitions
  - possible observables in heavy-ion collisions
    (e.g., “grand-canonical fluctuations” of baryon number)?