Renormalisation of self-consistent resummation schemes[G]

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For the description of physical systems with strong interactions generally perturbative methods are insufficient. Rather on the basis of effective field theories non-perturbative methods such as partial resummation schemes have to be applied. There are a couple central questions connected with such methods which concern symmetries and conservation laws as well as thermodynamic consistency and detailed balance. While the question of conserving approximations was addressed by Baym's Φ-functional method [1, 2, 3], in field theoretical descriptions a further complication arises, namely that of renormalization. Loop integrals generally diverge and renormalization concepts were developed in perturbation theory. In the context of resummation schemes the question arises under which conditions they are still renormalizable with temperature and density independent counter terms. In the past only a few specific examples were investigated.

Our purpose was to analyze self consistent partial Dyson resummation schemes defined by a set of basic generating self energy diagrams with dressed propagators. In terms of perturbative diagrams this leads to an infinite iterative insertion of all basic diagrams. The sum of all these perturbative diagrams defines the self consistent self energy which determines the dressed propagator, cf. (1). All diagram subpieces with the topology of a single loop which are connected to the rest of the diagram at most via two vertices, are divergent and have to be renormalized. These structures, however, appear in a nested way such that first the most inner ones have to be renormalized through counter-terms given by the reduced diagrams where the divergent sub-pieces are contracted to a point. The so obtained reduced diagrams themselves are to be subjected to the same procedure. This iterative process is formalized as the BPHZ-renormalization scheme. For the self consistent scheme under consideration the key issue is to find a compact iteration scheme that generates all the required counter terms at once.

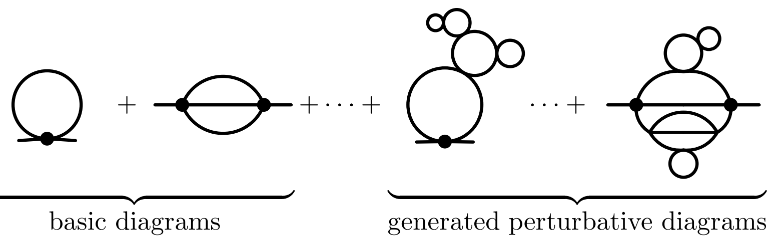
For an initial study we choose a simple scalar field theory model, the ϕ^4-model. For the Hartree approximation given by the tadpole self-energy diagram the subtraction scheme can be formulated as a gap equation. As a new part we included a genuine two-point contribution, namely the sunset diagram. The latter gives rise to an imaginary part in the self-energy, i.e., a finite width for the particles in the medium. In terms of perturbative diagrams the self consistent scheme then leads to all kinds of "super-daisy", "super-sunset" diagrams and all possible mixtures of them, cf. (1).

It could be shown that also this approximation can be renormalized in a BPHZ-type procedure (for details see [4]). The considerations show that one first has to solve the problem in the vacuum where subtracted dispersion relations can be used to renormalize the self-energy. In the same way one obtains the renormalized vertex functions which are needed to replace the corresponding divergent vacuum sub-diagrams in the finite temperature case. This means that for the renormalization of the finite temperature case one needs only *local temperature independent vacuum counter-terms* in perfect analogy to the well known theorem for perturbative finite-temperature quantum field theory.

We argue that the Φ-functional properties of the self energies indeed enforces the consistency of the counter-terms and symmetry factors for the explicit as well as the hidden nested and overlapping divergences since the divergent sub-diagrams are given by higher derivatives of the Φ-functional with respect to the dressed Green's function.

This substitution of divergent sub-diagrams together with direct subtractions of counter-terms on the level of the integrands provides a scheme where one has to deal only with convergent integrals without any explicite beforehand regularization which opened the possibility to calculate the self–energy and thermodynamic quantities such as the entropy in full self–consistency.

The results are important, e.g., for the description of hadrons in dense matter, e.g. [5] or for an effective description of gauge fields such as QCD in cases where the damping width of the particles is of considerable importance. Supplementary to the renormalization question functional methods have been developed on the basis of the Φ-functional concept, which permit to investigate and cure possible violations of symmetries and conservation laws at the level of higher order correlation functions [4]. This is of particular importance for gauge theories[5].

$\Sigma =$  (1)

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