Problems – Chapter VI

(1) $C = \frac{A\varepsilon_0}{d} \Rightarrow A = \frac{C\cdot d}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\, 4\pi C d$

With $C = 1F$ and $d = 1\,cm = 10^{-2}\,m$: $A = 1.13\cdot 10^{9}\,m^2$

(2) $C = \frac{A\varepsilon_0}{d}$; $\varepsilon_0 \simeq 8.84\cdot 10^{-12}\,\frac{F}{m}$

$C = 2.65\cdot 10^{-10}\,F = 265\,pF$

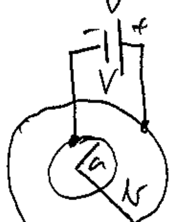
$Q = CV = 3.18\cdot 10^{-9}\,C$ on the plate connected to the + terminal ($-3.18\cdot 10^{-9}\,C$ on the other plate)

After the battery is disconnected these charges stay on the plates. Moving the plates together gives

$$C' = \frac{A\varepsilon_0}{d'} = C\,\frac{d}{d'} = 3C = 796\,pF$$

The voltage is

$$V' = \frac{Q}{C'} = \frac{Q}{C}\,\frac{C}{C'} = \frac{V}{3} = 4V$$

(3) 

The field between the plates is a Coulomb field (see last week's problems and exercises)

$$\Rightarrow V(r) = -\frac{Q}{4\pi\varepsilon_0}\,\frac{1}{r}$$

The voltage then is

$$V = V(b) - V(a) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$ (>0 as it should be)

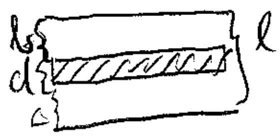
Then

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}} = \frac{4\pi\varepsilon_0\, ab}{b-a}$$

(iv) $C_{\text{single sphere}} = 4\pi\varepsilon_0 a$

For a single plate this doesn't work, because we made the assumption that $d \ll \sqrt{A}$. It would be very complicated to solve the boundary-value problem for a finite single plate!

15)



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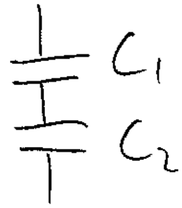


$$C_{total} = \frac{C_1 C_2}{C_1 + C_2} \quad \text{(capacities in series)}$$

and $C_1 = \frac{\varepsilon_0 A}{b}$; $C_2 = \frac{\varepsilon A}{c} = \frac{\varepsilon_0 A}{l-b-d}$

$$C_{total} = \frac{(\varepsilon_0 A)^2}{b(l-b-d)} \; \frac{1}{\varepsilon_0 A\left(\frac{1}{b} + \frac{1}{l-b-d}\right)}$$

$$= \frac{\varepsilon_0 A}{l-b-d+b} = \frac{\varepsilon_0 A}{l-d}$$

It doesn't matter, where we insert the plate!

Exercises – Chapt. VI

(1) $C = \frac{Q}{V} = \frac{0.1\,C}{100\,V} = 10^{-3}\,F\,(= 1\,mF)$

$C = \frac{\varepsilon_0 A}{d} \Rightarrow A = \frac{C d}{\varepsilon_0} = \frac{4\pi\, C d}{4\pi\,\varepsilon_0} = 1.13 \cdot 10^5\, m^2$

(2) (a+b) $C_{total} = \frac{C_1 C_2}{C_1 + C_2}$

Set $C_1 + C_2 = C_3 = \text{const.}$ Then we have

$C_{total} = \frac{C_1 (C_3 - C_1)}{C_3}$

Look for extreme:

$\frac{dC_{total}}{dC_1} = \frac{1}{C_3}\left(C_3 - 2C_1\right) \overset{!}{=} 0 \Rightarrow C_1 = \frac{C_3}{2} = C_2$

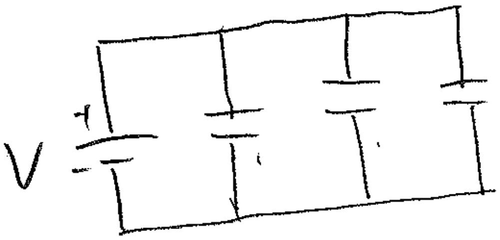
$\frac{d^2 C_{total}}{dC_1^2} = -2\frac{C_1}{C_3} < 0 \Rightarrow$ maximum

minimum for $C_1 = 0$ or $C_2 = 0$

(c) $C_{tot} = C_1 + C_2$

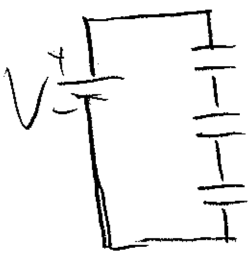
$\Rightarrow$ it's always the same, since $C_{tot} = C_1 + C_2$ is fixed.

(3) Find all combinations



$$C_{tot} = C_1 + C_2 + C_3 = 14\,\mu F$$

order doesn't play any role

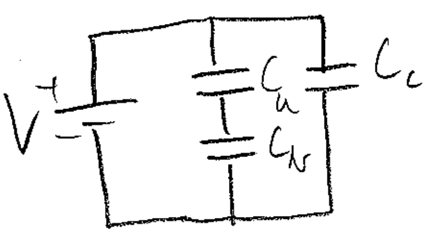


$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)\frac{1}{\mu F}$$

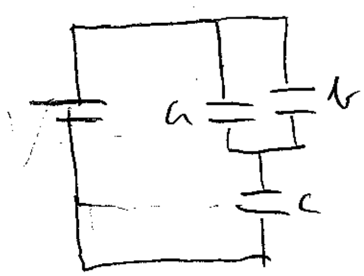
$$= \frac{7}{8}\,\frac{1}{\mu F} \Rightarrow C_{tot} = \frac{8}{7}\,\mu F$$

order doesn't play a role



$$C_{tot} = \frac{C_a C_b}{C_a + C_b} + C_c$$

a	b	c	total
1	2	3	$\frac{28}{3}\,\mu F$
1	3	2	$\frac{28}{5}\,\mu F$
2	3	1	$\frac{14}{3}\,\mu F$



$$\frac{1}{C_{tot}} = \frac{1}{C_a + C_b} + \frac{1}{C_c}$$

a	b	c	total
1	2	3	$24/7\,\mu F$
1	3	2	$20/7\,\mu F$
2	3	1	$12/7\,\mu F$

(4) A series of two spherical capacitors with

$$C_1 = 4\pi\varepsilon_0 \frac{ab}{b-a} \quad \text{and} \quad C_2 = 4\pi\varepsilon_0 \frac{c(b+T)}{c-b-T}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b} + \frac{1}{b+T} - \frac{1}{c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{b(b+T)c - a(b+T)c + abc - ab(b+T)}{ab(b+T)c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{b^2c + bcT - \cancel{abc} - acT + \cancel{abc} - ab^2 - abT}{ab(b+T)c}\right)$$

$$C = \frac{4\pi\varepsilon_0\, abc(b+T)}{b^2c + bcT - acT - ab^2 - abT}$$

(5)(a) $Q_1 = VC_1$, $Q_2 = VC_2 \Rightarrow Q = Q_1 + Q_2 = V(C_1 + C_2)$

(b) Then (suppose $C_1 > C_2$) Then on the total capacity C

$$C = \frac{C_1 C_2}{C_1 + C_2}$$

the charge $Q_1 - Q_2 = Q = (C_1 - C_2)V$

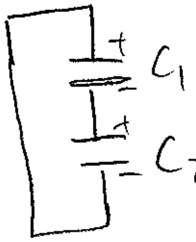
The new voltage is

$$V' = \frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = V\left(1 - \frac{C_2}{C_1} + \frac{C_1}{C_2} - 1\right)$$

$$= V\left(\frac{C_1}{C_2} - \frac{C_2}{C_1}\right)$$

5.67 (4)

$C_1 > C_2$



Total charge on the system is

$Q = (C_1 - C_2)V$

The total voltage difference is 0

$$\Rightarrow \frac{Q_1}{C_1} - \frac{Q_2}{C_2} = 0 \Rightarrow \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_2 = \frac{C_2}{C_1} Q_1$$

but $Q_1 + Q_2 = Q \Rightarrow Q_1 + \frac{C_2}{C_1} Q_1 = Q$

$$\Rightarrow Q_1 = \frac{Q}{1 + \frac{C_2}{C_1}} = \frac{Q C_1}{C_1 + C_2} = \frac{C_1}{C_1 + C_2}(C_1 - C_2)V$$

$$\Rightarrow Q_2 = \frac{Q C_2}{C_1 + C_2} = \frac{C_2}{C_1 + C_2}(C_1 - C_2)V$$