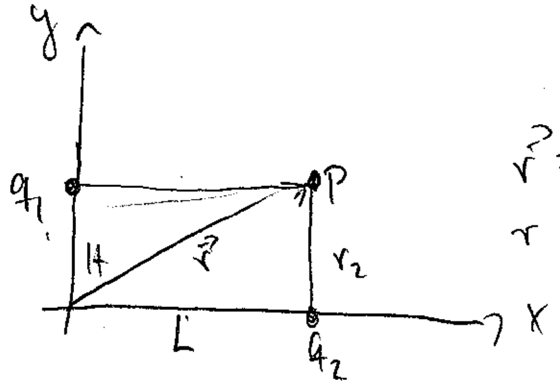
Problems Chapter III

(1) $V(r) = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}$; $q = 3\cdot 10^{-6}\,C$

$V(20\,cm) - V(10\,cm) =$

(2)



$\vec{r} = x\vec{i}_x + y\vec{i}_y$

r

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) \quad \text{(for a general point } P: \vec{r}\text{)}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{\sqrt{x^2+(y-H)^2}} + \frac{q_2}{\sqrt{(x-L)^2+y^2}}\right)$$

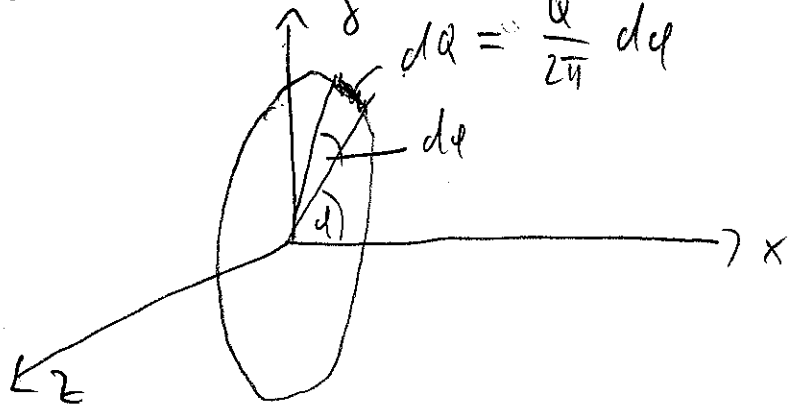
$$W = -[V(\vec{r}_2) - V(\vec{r}_1)]\,q_0$$

$$= -\frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{L} + \frac{q_2}{H} - \frac{q_1}{H} - \frac{q_2}{L}\right)$$

$$W = -\frac{q_0(q_1 - q_2)}{4\pi\varepsilon_0}\left(\frac{1}{L} - \frac{1}{H}\right)$$

(3) $$V = \int_0^{2\pi} d\varphi \frac{Q}{8\pi^2\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}} = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}}$$

$dQ = \frac{Q}{2\pi}\,d\varphi$



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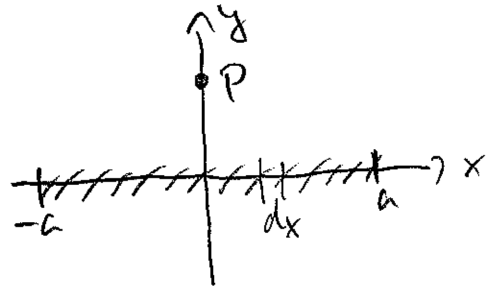


$$dQ = \frac{Q}{2a}\,dx$$

$$V(y) = \int_{-a}^{a} dx\,\frac{Q}{8\pi\varepsilon_0 a}\,\frac{1}{\sqrt{x^2+y^2}} = \int_{0}^{a} dx\,\frac{Q}{4\pi\varepsilon_0 a}\,\frac{1}{\sqrt{x^2+y^2}}$$

Substitution: $x = y\sinh u$

$$dx = du\; y\cosh u$$

$$\Rightarrow \int dx\,\frac{1}{\sqrt{x^2+y^2}} = \int du\,\frac{y\cosh u}{\sqrt{y^2(1+\sinh^2 u)}}$$

$$= u(y),\quad u = \operatorname{arsinh}\frac{x}{|y|}$$

$$V(y) = \frac{Q}{4\pi\varepsilon_0 a}\operatorname{arsinh}\left(\frac{a}{|y|}\right)$$

Since $\operatorname{arsinh}(\alpha) = \ln\left(\alpha + \sqrt{1+\alpha^2}\right)$

$$V(y) = \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{a}{|y|} + \sqrt{1+\frac{a^2}{y^2}}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0 a}\ln\left(\frac{a+\sqrt{y^2+a^2}}{|y|}\right)$$

Check with book

$$\frac{\sqrt{y^2+a^2}+a}{\sqrt{y^2+a^2}-a} = \frac{(\sqrt{y^2+a^2}+a)^2}{y^2}$$

$$\Rightarrow \ln\left(\frac{\sqrt{y^2+a^2}+a}{\sqrt{y^2+a^2}-a}\right) = 2\ln\frac{a+\sqrt{y^2+a^2}}{|y|} \qquad (i)$$

(5) We look for the potential of the force

$$\vec{F} = \frac{A}{r^4}\hat{r} = \frac{A}{r^4}\vec{i}_r \quad ; \quad A = \alpha q_1 q_2$$

In spherical coordinates we have

$$\text{grad } u = \frac{\partial u}{\partial r}\vec{i}_r + \frac{\vec{i}_\varphi}{r\sin\vartheta}\frac{\partial u}{\partial\varphi} + \frac{1}{r}\frac{\partial u}{\partial\vartheta}\vec{i}_\vartheta$$

Since $\vec{F}$ is a central force, we have

$$u = u(r)$$

and

$$\frac{du}{dr} = -F_r = -\frac{A}{r^4} = -Ar^{+4}$$

$$\Rightarrow u = \frac{-A}{-4+1}r^{-4+1} = \frac{A}{3r^3}$$

The "electric potential" would be

$$V = \frac{\alpha q_1}{3r^3}$$

(for a source charge q_1)

(c) We have

$$\frac{m}{2} v_x^2 + U(x) = \text{const.}$$

Now I want

$$\frac{m}{2} v_0^2 + U(x_0) = \frac{m}{2} v_1^2 + U(x \to \infty)$$

$$\Rightarrow \frac{m}{2} v_0^2 = \frac{m}{2} v_1^2 + U(x \to \infty) - U(x_0)$$

Choosing

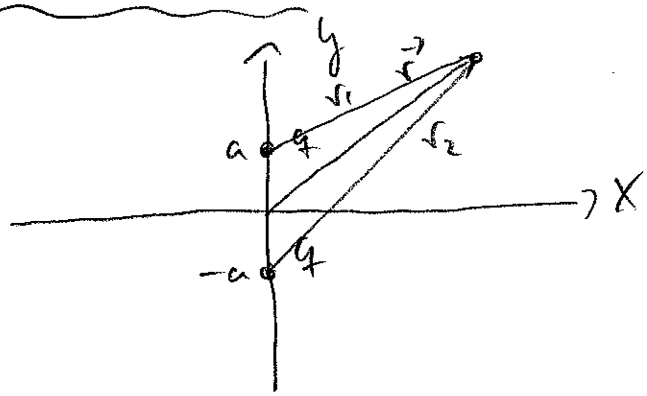
$$U(x) = -\frac{Zq^2}{4\pi\varepsilon_0 x} \Rightarrow U(x \to \infty) = 0$$

The minimal v_0 follows if $v_1 = 0$

$$\Rightarrow \quad v_0 = \sqrt{-\frac{2U(x_0)}{m}} = \sqrt{\frac{2Zq^2}{4\pi\varepsilon_0 m x}} = 20.37 \cdot 10^7 \frac{m}{s}$$

Exercises Chapter III

(1)



(a) $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1}+\frac{1}{r_2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+(y-a)^2}}+\frac{1}{\sqrt{x^2+(y+a)^2}}\right)$

(b) q doesn't exist in nature
However if it would, we had

$U = -eV$ and

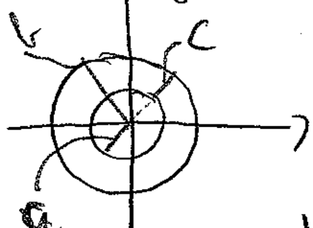
$W = -[U(\vec{r}_2) - U(\vec{r}_1)]$

with $\vec{r}_2 = b\,\vec{i}_x$, $\vec{r}_1 = 0$

$\Rightarrow W = \frac{eq}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{a^2+b^2}}+\frac{1}{\sqrt{a^2+b^2}}-\frac{2}{a}\right)$

$= \frac{2eq}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{a^2+b^2}}-\frac{1}{a}\right) = -2.59\cdot10^{-27}\,J$

(2)

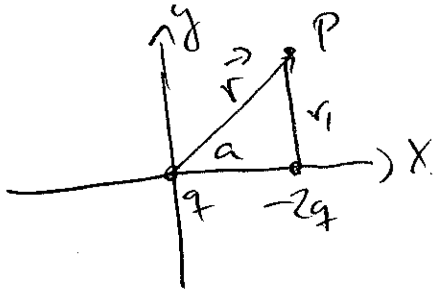


$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{i}_r$

Integrate along arbitrary radius

$V = -\int_C d\vec{r}\,\vec{E} = -\int_a^b dr\,\frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left|\frac{b}{a}\right|$

(3)



(a) $V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{2}{\sqrt{(x-a)^2+y^2}}\right)$

$V_\infty = 0$

$\Rightarrow$ Look for at $y=0$ for x_0 such that $V(x,0) = 0$

$$\Rightarrow \quad \frac{1}{|x_0|} - \frac{2}{|x_0-a|} = 0$$

$$\Leftrightarrow \quad |x_0| = \frac{|x_0-a|}{2}$$

$$\text{or} \quad x_0^2 = \frac{(x_0-a)^2}{4} = \frac{x_0^2 - 2ax_0 + a^2}{4}$$

$$3x_0^2 + 2ax_0 - a^2 = 0$$

$$x_{0_{1,2}} = -\frac{a}{3} \pm \sqrt{\frac{a^2}{9} + \frac{a^2}{3}}$$

$$= -\frac{a}{3} \pm \frac{2}{3}a = -a \text{ and } \frac{a}{3}$$

(b) Electric field

$$\vec{E} = -\operatorname{grad} V = \frac{q}{4\pi\varepsilon_0}\left(\frac{\vec{r}}{r^3} - \frac{2\left[(x-a)\vec{i}_x + y\vec{i}_y\right]}{\left[(x-a)^2+y^2\right]^{3/2}}\right]$$

For $y=0$:

$$\vec{E}(x) = \frac{q}{4\pi\varepsilon_0}\vec{i}_x\left(\frac{1}{x|x|} - \frac{2}{(x-a)|x-a|}\right)$$

$$\frac{1}{x|x|} - \frac{2}{(x-a)|x-a|} = 0 \Rightarrow (x-a)|x-a| = 2x|x|$$

Cases:

(A) $x > a > 0$

$$(x-a)^2 = 2x^2$$

$$x-a = \sqrt{2}\,x$$

$$\Rightarrow x = \frac{-a}{\sqrt{2}-1} = -a(1+\sqrt{2}) \approx -a \cdot 2.414 < 0$$

$$\text{?}$$

(B) $0 \leq x \leq a$

$$\Rightarrow -(x-a)^2 = 2x^2$$

no solutions

(γ) $x < 0$

$$-2x^2 = -(x-a)^2$$

$$\Rightarrow 2x^2 = (x-a)^2$$

Same equation as before, but now $x < 0$

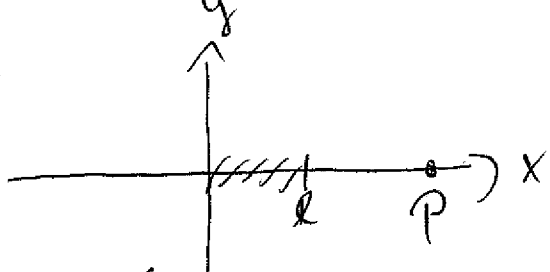
$$2x^2 = (|x|+a)^2$$

$$\Rightarrow \sqrt{2}\,|x| = |x| + a \Rightarrow x = -a(1+\sqrt{2}) = -2.414a$$

That's the only solution.

(c) What should the equipotential lines have to do with vanishing fields (those are the stationary points of the potential!) 6

(4)



$$V(x) = \int_0^l dx' \, \frac{Q}{4\pi\varepsilon_0 l} \, \frac{1}{x-x'} \qquad (x > L)$$

$$V(x) = -\frac{Q}{4\pi\varepsilon_0 l} \ln\left(\frac{x-l}{x}\right)$$

(5) $$\frac{m}{2} v_1^2 + U_1 = \frac{m}{2} v_2^2 + U_2$$

$v_1 = 0$:

$$v_2 = \sqrt{\frac{2(U_1 - U_2)}{m}} = \sqrt{\frac{2(-e)(V_1 - V_2)}{m}}$$

$$= \sqrt{\frac{2eV}{m}} = 1.325 \cdot 10^{-6} \frac{m}{s}$$

(6) $$V_{\text{Oxygen}} = \frac{8e}{4\pi\varepsilon_0} \frac{1}{r} \; ; \; U_{el} = -\frac{8e^2}{4\pi\varepsilon_0 r}$$

$$\frac{m}{2} v_\infty^2 + U_\infty = \frac{m}{2} v_r^2 + U_{el,r} = 0$$

$$\Rightarrow v_r^2 = \sqrt{\frac{2|U_{el,r}|}{m}} = \sqrt{\frac{1}{4\pi\varepsilon_0} \frac{16e^2}{r}} = 2.84 \cdot 10^{+6} \frac{m}{s}$$