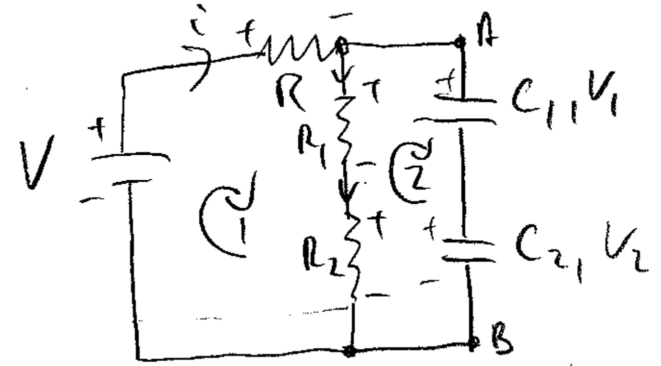
Practice exam -2

1) (a) First I draw it a bit different:



First I calculate the circuit without capacitors. The only unknown is the current i, but that's easily calculated:

$$1: \; V - iR - iR_1 - iR_2 = 0$$

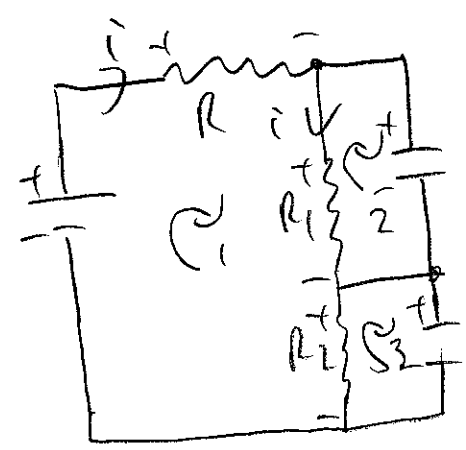
$$\Rightarrow (R + R_1 + R_2) i = V \Rightarrow i = \frac{V}{R + R_1 + R_2}$$

$$Q_1 = Q_2 = \frac{(R_1 + R_2) i}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{(R_1 + R_2) C_1 C_2}{C_1 + C_2} \; \frac{R_1 + R_2}{R + R_1 + R_2} V$$

because the voltage between points A and B is $(R_1 + R_2) i$, and the capacity of the capacitors in series is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

(b)



1: gives the same i as before

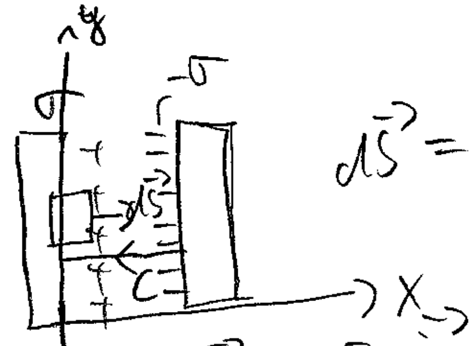
$$2: \; R_1 i - V_1 = 0 \Rightarrow V_1 = R_1 i$$

$$Q_1 = C_1 V_1 = C_1 \frac{R_1}{R + R_1 + R_2} V$$

$$3: \; R_2 i - V_2 = 0 \Rightarrow V_2 = R_2 i$$

$$Q_2 = C_2 V_2 = C_2 \frac{R_2}{R + R_1 + R_2} V$$

(2)



$d\vec{S} = dS\,\vec{i}_x$

(a) Ansatz: $\vec{E} = E(x)\vec{i}_x$ Gauss's law:

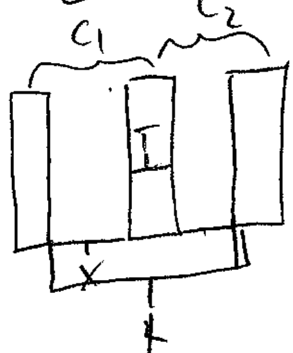
$$\Rightarrow E(x)\,dS = \frac{\sigma\,dS}{\varepsilon_0} \Rightarrow E(x) = \frac{\sigma}{\varepsilon_0} = \text{const for } 0 \le x < d$$

For $x = d \Rightarrow E(d) = 0 \Rightarrow$ there must be a charge spreading area of $-\sigma$ on the right plate (induced by σ on the left plate)

(b) $$U = -\int_C d\vec{r}\,\vec{E}(\vec{r}) = \frac{\sigma d}{\varepsilon_0}$$

$$Q = CU = \sigma A = \frac{\varepsilon_0 U}{d} A \Rightarrow C = \frac{Q}{U} = \frac{\varepsilon_0 A}{d}$$

(c)



Two capacitors in series with

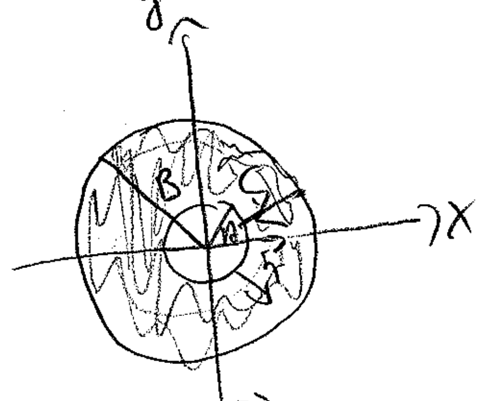
$$C_1 = \frac{\varepsilon_0 A}{x}\ ;\ C_2 = \frac{\varepsilon_0 A}{L-x-T}$$

$$U = \frac{Q}{C} = \sigma A\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \sigma A\left(\frac{x}{\varepsilon_0 A} + \frac{L-x-T}{\varepsilon_0 A}\right)$$

$$U = \frac{\sigma}{\varepsilon_0}(L-T)$$

(3)

(3)(a) Assume the current flows radially:



$$\vec{j}(\vec{r}) = j(r)\,\vec{i}_r$$

Due to charge conservation,

$$\oint_{\partial V} d\vec{S}\,\vec{j} = 0$$

Use for V the hollow ball with radius r for the outward radius, A for the inner ball. Then

$$\int_{S_r} dS\, j(r) - \int_{S_A} j(A)\, dS = 0$$

or $\quad 4\pi r^2 j(r) = 4\pi A^2 j(A)$,

but that's the total current:

$$4\pi r^2 j(r) = i \Rightarrow$$

$$\vec{j} = \frac{i}{4\pi r^2}\vec{i}_r$$

(b) $\vec{E} = \rho\vec{j} = \dfrac{i\rho}{4\pi r^2}\vec{i}_r$; $C: \vec{r} = r\vec{i}_r$; $d\vec{r} = dr\,\vec{i}_r$

$$\Delta V = -\int_C d\vec{r}\cdot\vec{E} = -\frac{i\rho}{4\pi}\int_B^A dr\,\frac{1}{r^2} = -\frac{i\rho}{4\pi}\left[\frac{1}{A} - \frac{1}{B}\right]$$

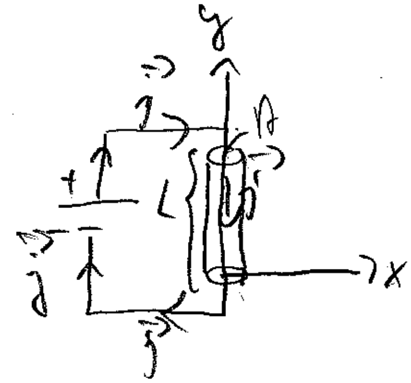
$$\left(= V(A) - V(B) > 0\right)$$

(c) It's 0, because per unit time through each shell passes the same amount of charge. The charge doesn't change compared to the situation, when no current flows.

"Without words":

$$\int_{\partial V} d\vec{S}\cdot\vec{E} = 0 \quad (\text{for all volumes } V)$$

(4) (a)



$$R = \frac{\rho L}{A}\ ; \quad i = \frac{V}{R} = \frac{VA}{\rho L} \quad \text{down}$$

(b) $\vec{j} = -\frac{i}{A}\vec{i}_y = -\frac{V}{\rho L}\vec{i}_y$

(c) the same in magnitude because i is the same everywhere. Directions given in figure.

(d) The surface charge on A is given by the jump of the component of $\vec{E}$:

$$\sigma = \left[E_y(L+0^+) - E_y(L-0^+)\right]\varepsilon_0$$

Now: $\vec{E} = \rho\,\vec{j}$ For the ideal wires $\rho = 0$

$$\Rightarrow \quad E_y(L+0^+) = 0$$

$$\text{and} \quad E_y(L-0^+) = -\rho\frac{V}{\rho L} = -\frac{V}{L}$$

$$\Rightarrow \quad \sigma = \frac{\varepsilon_0 V}{L}$$

Total charge on surface: $Q = \sigma A = \frac{\varepsilon_0 VA}{L}$ //