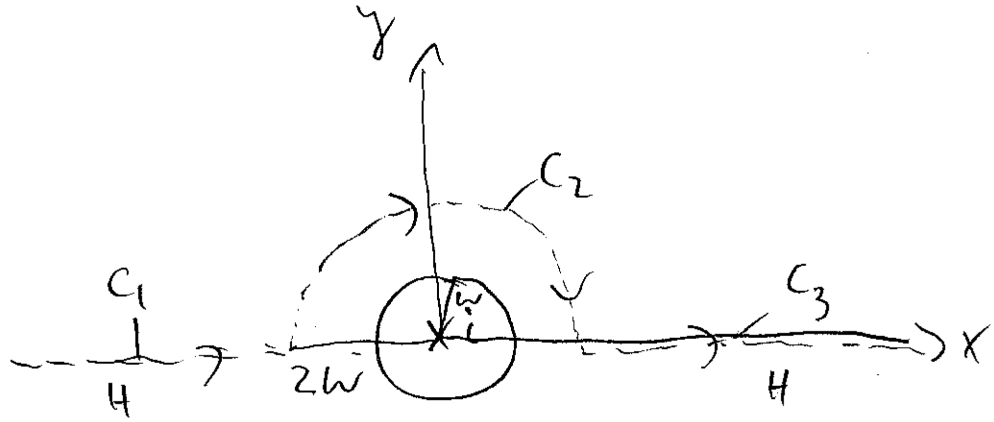
Exam III (make up)



(a) $\vec{B} = -\frac{\mu_0 i}{2\pi\rho}\,\vec{i}_\varphi$ for $\rho > w$

(from Ampère's Law)

$= -\frac{\mu_0 i \rho}{2\pi w^2}\,\vec{i}_\varphi$ for $\rho < w$

$C_1 : \vec{r} = x\,\vec{i}_x$ for $x \in (-2w-H, -2w)$

$= x\,\vec{i}_\rho(\varphi=0)$

$\Rightarrow d\vec{r}\,\vec{B} = 0 \Rightarrow$ no contribution

$C_3 : \vec{r} = x\,\vec{i}_x$ for $x \in (2w, 2w+H)$ same $\Rightarrow$ no contribution

$C_2 : \vec{r} = 2w\,\vec{i}_\rho(\varphi) \quad \varphi : \pi \to 0$

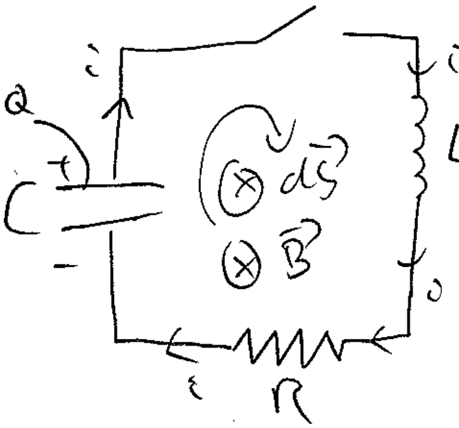
$d\vec{r} = 2w\,\vec{i}_\varphi(\varphi)$

$$\int_{C_2} d\vec{r}\,\vec{B} = -\frac{\mu_0 i}{2\pi(\cancel{2w})}(-)\int_0^\pi d\varphi\,\cancel{2w} = \frac{\mu_0 i}{2}$$

(b) again C_1 and C_3 don't contribute

C_2: $\displaystyle\int_{C_2} d\vec{r}\,\vec{B} = \frac{\mu_0 i \,\frac{\cancel{\pi}}{2}\,\frac{\cancel{w}}{2}}{2\cancel{\pi}\cancel{w}} = \frac{\mu_0 i}{8}.$

(2)



(a) Faraday's law

$$\oint d\vec{r}\,\vec{E} = -\frac{Q}{C} + Ri = -L\frac{di}{dt}$$

$$i = -\frac{dQ}{dt}$$

$$\Rightarrow -\frac{Q}{C} - R\dot{Q} = +L\ddot{Q}$$

$$\Rightarrow L\ddot{Q} + R\dot{Q} + \frac{Q}{C} = 0.$$

(b) For $R=0$,

$$L\ddot{Q} + \frac{Q}{C} = 0 \Rightarrow \ddot{Q} = -\frac{Q}{LC}$$

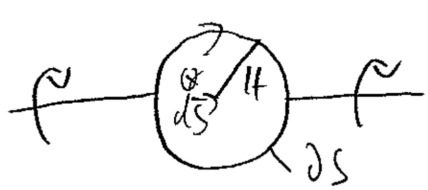
$$Q(t) = A\cos(\omega t) + B\sin(\omega t)$$

$$i(t) = -\dot{Q}(t) = A\omega\sin(\omega t) - B\omega\cos(\omega t)$$

$$\left.\begin{aligned} Q(0) &= A = Q_0 \\ i(0) &= -B\omega = 0 \end{aligned}\right\} \Rightarrow Q(t) = Q_0\cos(\omega t).$$

(3)

$\otimes\ \vec{B} = -B_0\,\vec{i}_z$



$$\varphi = \frac{\alpha}{2}t^2$$

$$\Phi_B = \int_S d\vec{S}\cdot\vec{B} = \pi a^2 B_0 \cos\left(\frac{\alpha}{2}t^2\right)$$

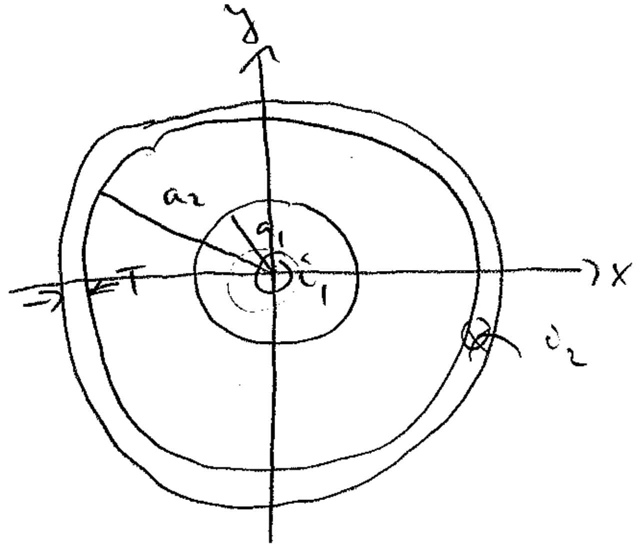
Faraday's Law

$$\oint d\vec{r}\,\vec{E} = Ri = -\frac{d\Phi_B}{dt} = \pi H^2 B_0 \alpha t \sin\left(\frac{\alpha}{2}t^2\right)$$

$$R = \frac{\rho\, 2\pi H}{TW} \Rightarrow i = \frac{\cancel{\pi} H^{\cancel{2}} B_0 \alpha t \sin\left(\frac{\alpha}{2}t^2\right)}{\frac{\rho\, 2\cancel{\pi}\cancel{H}}{TW}}$$

$$i = \frac{TW H B_0 \alpha t \sin\left(\frac{\alpha}{2}t^2\right)}{2\rho}$$

(4)



Magnetic field

Ampère's law with circles around the center (ccw)

$$\vec{B} = B_\varphi \vec{i}_\varphi$$

$$\oint d\vec{r}\,\vec{B} = 2\pi\rho B_\varphi = \mu_0 i_{inside}$$

$$B_\varphi = \frac{\mu_0}{2\pi\rho} \cdot \begin{cases} \frac{i_1\rho^2}{a_1^2} & \text{for } 0 \le \rho < a_1 \\ i_1 & \text{for } a_1 \le \rho < a_2 \\ i_1 - i_2\frac{\rho^2 - a_2^2}{(a_2+T)^2 - a_2^2} & \text{for } a_2 < \rho < a_2 + T \\ i_1 - i_2 & \text{for } \rho > a_2 + T \end{cases}$$

(a) $x > a_2 + T$

$$B_\varphi = \frac{\mu_0}{2\pi x}(i_1 - i_2) \overset{!}{=} 0 \Rightarrow i_1 = i_2$$

(b) $\cancel{B_\varphi =} \frac{\mu_0}{2\pi x} i_1 \overset{!}{=} 0 \Rightarrow i_1 = 0$, no restriction on i_2

(c) $x = a_2 + \frac{T}{2}$

$$B_\varphi = \frac{\mu_0}{2\pi x}\left[i_1 - i_2 \frac{(a_2 + \frac{T}{2})^2 - a_2^2}{(a_2+T)^2 - a_2^2}\right] \overset{!}{=} 0$$

$$\Rightarrow i_1 = i_2 \frac{(a_2 + \frac{T}{2})^2 - a_2^2}{(a_2+T)^2 - a_2^2}$$