Makeup exam I

(1) $\vec{F} = (ky + QE - Mg)\,\vec{i}_y$; $ky = \frac{Mg}{2k}$

$$\Rightarrow \quad E = \frac{Mg - ky}{Q} = \frac{Mg}{2Q}$$

(2) set $\vec{r}\,' = \vec{r} - (a\vec{i}_x + b\vec{i}_y)$ then Q in center of new coord. system. Thus

$$\partial_{r'} V(r') = -\frac{\beta Q}{r'^6} \Rightarrow V(r') = \frac{\beta Q}{5r'^5} = \frac{\beta Q}{5\sqrt{(x-a)^2+(y-b)^2+z^2}^{\,5}}$$

$$= V(\vec{r})$$

(3) (a) $\Phi_E = \int_S d\vec{S}\,\vec{E}$

$d\vec{S} = dx\,dy\,\vec{i}_z \Rightarrow \vec{E}\,d\vec{S} = 0 \Rightarrow \Phi_E = 0$

(b) $d\vec{S} = dy\,dz\,\vec{i}_x \Rightarrow d\vec{S}\cdot\vec{E} = \alpha\,dy\,dz =$ const.

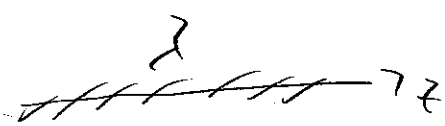
$\Rightarrow \Phi_E = A\alpha = \frac{\pi R^2}{4}\alpha$

(c) $d\vec{S} = dx\,dz\,\vec{i}_y = \beta x^2\,dx\,dz$

rectangle: $x \in (0, L)$; $y \in (0, w)$

$$\Phi_E = \int_{\text{rectangle}} d\vec{S}\cdot\vec{E} = \int_0^L dx \int_0^w dy\,\beta x^2 = \frac{\beta}{3}L^3 w$$

(4) $\vec{E}(\vec{r}) = \int_{z'=-\infty}^{z'=\infty} \frac{dQ(\vec{r}\,')}{4\pi\varepsilon_0}\,\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}$



$dQ = \lambda\,dz'$

(2)

$$|\vec{r}-\vec{r}'| = \sqrt{x^2+y^2+(z-z')^2} = \sqrt{\rho^2+(z-z')^2}$$

$$\boxed{\vec{E} = \int_{-\infty}^{\infty} dz' \, \frac{\lambda}{4\pi\varepsilon_0} \, \frac{\rho\,\vec{i}_\rho + (z-z')\,\vec{i}_z}{\sqrt{\rho + (z-z')^2}^{\,3}}}$$

$E_z = 0$ due to anti symmetry of integrand. Thus we need

$$I = \int_{-\infty}^{\infty} dz' \, \frac{1}{\sqrt{\rho^2+(z-z')^2}^{\,3}} = \int_{-\infty}^{\infty} dz' \, \frac{1}{\sqrt{\rho^2+z'^2}^{\,3}}$$

$z' = \rho \sinh u$

$dz' = \rho \, du \cosh u$

$$I = \int_{-\infty}^{\infty} du \, \rho \cosh u \, \frac{1}{\rho^3 \cosh^3 u}$$

$$= \frac{1}{\rho^2} \int_{-\infty}^{\infty} du \, \frac{1}{\cosh^2 u} = \frac{1}{\rho^2} \tanh u \Big|_{-\infty}^{\infty} = \frac{2}{\rho^2}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \frac{\vec{i}_\rho}{\rho}}$$