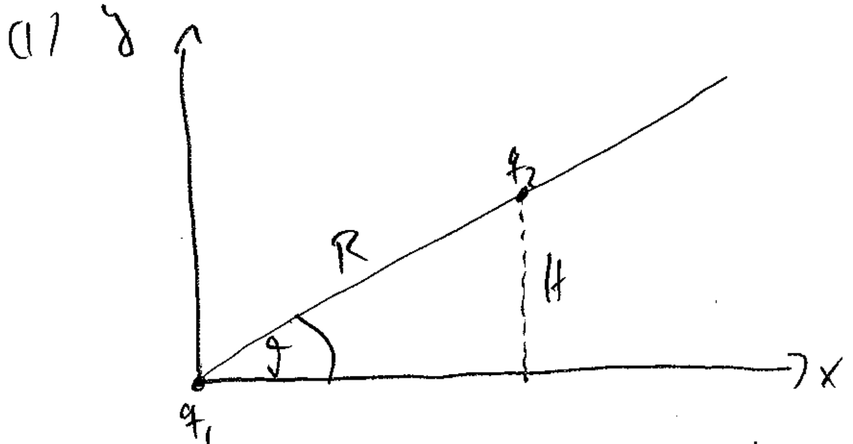
Solutions to exam 1

(1)



The forces on q_2 are the electric force exerted by charge q_1 and the component of $m\vec{g}$ in direction of the plane

$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}\left(\cos\vartheta\,\vec{i}_x + \sin\vartheta\,\vec{i}_y\right) + \vec{F}_{G\parallel}$$

$$\vec{F}_{G\parallel} = -mg\,\vec{i}_y\cdot\left(\cos\vartheta\,\vec{i}_x + \sin\vartheta\,\vec{i}_y\right)\left[\cos\vartheta\,\vec{i}_x + \sin\vartheta\,\vec{i}_y\right]$$

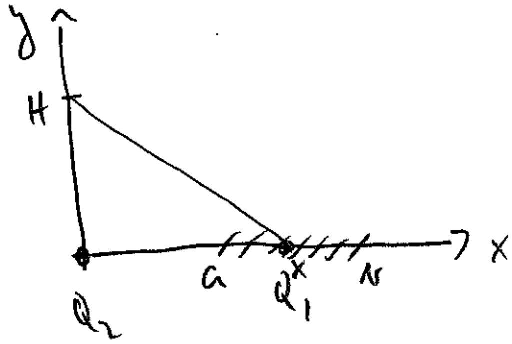
$$= -mg\sin\vartheta\left[\cos\vartheta\,\vec{i}_x + \sin\vartheta\,\vec{i}_y\right]$$

$$\vec{F} = 0 \Leftrightarrow \frac{q_1 q_2}{4\pi\varepsilon_0 R^2} = mg\sin\vartheta \Rightarrow R^2 = \frac{q_1 q_2}{4\pi\varepsilon_0 mg\sin\vartheta}$$

$$H = R\sin\vartheta$$

$$\Rightarrow \boxed{H = \sqrt{\frac{q_1 q_2 \sin\vartheta}{4\pi\varepsilon_0 mg}}}$$

(2)



$$\vec{E}_1 = \frac{Q_2}{4\pi\varepsilon_0 H^2}\vec{i}_y$$

$$\vec{E}_2 = \frac{Q_1}{4\pi\varepsilon_0 (b-a)} \int_a^b dx \, \frac{-x\,\vec{i}_x + H\vec{i}_y}{(x^2+H^2)^{3/2}}$$

$$= \frac{Q_1}{4\pi\varepsilon_0 (b-a)} \left[\vec{i}_x \left(+ \frac{1}{\sqrt{x^2+H^2}} \right) + \frac{H x}{H^2\sqrt{x^2+H^2}} \right]_a^b$$

$$= \frac{Q_1}{4\pi\varepsilon_0 (b-a)} \left[-\vec{i}_x \left(\frac{1}{\sqrt{a^2+H^2}} - \frac{1}{\sqrt{b^2+H^2}} \right) + \frac{\vec{i}_y}{H} \left(\frac{b}{\sqrt{b^2+H^2}} - \frac{a}{\sqrt{a^2+H^2}} \right) \right]$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

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(3) $\vec{E} = \alpha x^2 \vec{i}_x + \beta y^2 \vec{i}_y$

$$E_x = \alpha x^2 = -\frac{\partial V}{\partial x} \Rightarrow V = -\frac{\alpha}{3}x^3 + V_1(y)$$

$$E_y = \beta y^2 = -\frac{\partial V}{\partial y} = -\frac{\partial V_1}{\partial y} \Rightarrow V_1 = -\frac{\beta}{3}y^3$$

$$V = -\frac{\alpha}{3}x^3 - \frac{\beta}{3}y^3 \Rightarrow \vec{E} \text{ conservativo}$$

$$V(0,c) - V(c,0) = -\frac{\beta}{3}c^3 + \frac{\alpha}{3}c^3 = \frac{\alpha-\beta}{3}c^3$$

(4) $\vec{E} = b x^2 \vec{i}_x + c x \vec{i}_z$

(a) $d\vec{S}_{\text{Stück}} = \vec{i}_x \, dy \, dz$ mit $y \in (0,a)$; $z \in (0,a)$; $x = a$

$$\Rightarrow \Phi_{\text{Stück}} = \int_0^a dy \int_0^a dz \; E_x$$

$$= \int_0^a dy \int_0^a dz \; a^2 b = \underline{\underline{a^4 b}}$$

$d\vec{S}_{\text{dot}} = \vec{i}_z \, dx \, dy$; $x \in (0,a)$; $y \in (0,a)$; $z = a$

$$\Rightarrow \Phi_{\text{dot}} = \int_0^a dx \int_0^a dy \; c x = \underline{\underline{\frac{a^3 c}{2}}}$$

(b) $\Phi = \int \vec{E} \, d\vec{S}$

$d\vec{S} = R^2 \vec{i}_R \, d\Omega$; $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \vec{i}_r$

$$\Phi = \int_{\Omega_0} d\Omega \; \frac{q}{4\pi\varepsilon_0} = \frac{q}{4\pi\varepsilon_0} \Omega_0$$