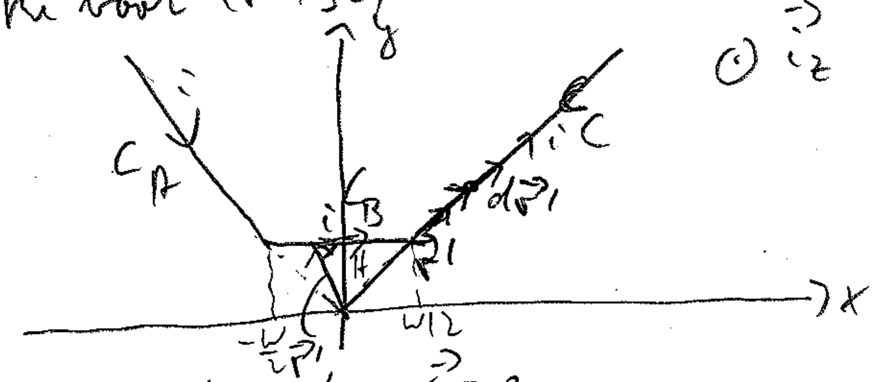
Alternative solution for Problem 2

Which the book (p. 192)



Biot-Savart law for $\vec{r}=0$

$$\vec{B} = -\frac{\mu_0 i}{4\pi}\int_{\mathcal{C}} d\vec{r}' \times \frac{\vec{r}'}{|\vec{r}'|^3}$$

Along parts A and C we have

$$d\vec{r}' \times \vec{r}' = 0$$

$\Rightarrow$ only part along C_B contributes

$d\vec{B} = -\frac{\mu_0 i}{4\pi}\frac{d\vec{r}' \times \vec{r}'}{|\vec{r}'|^3}$ is always $\parallel -\vec{i}_z$, because

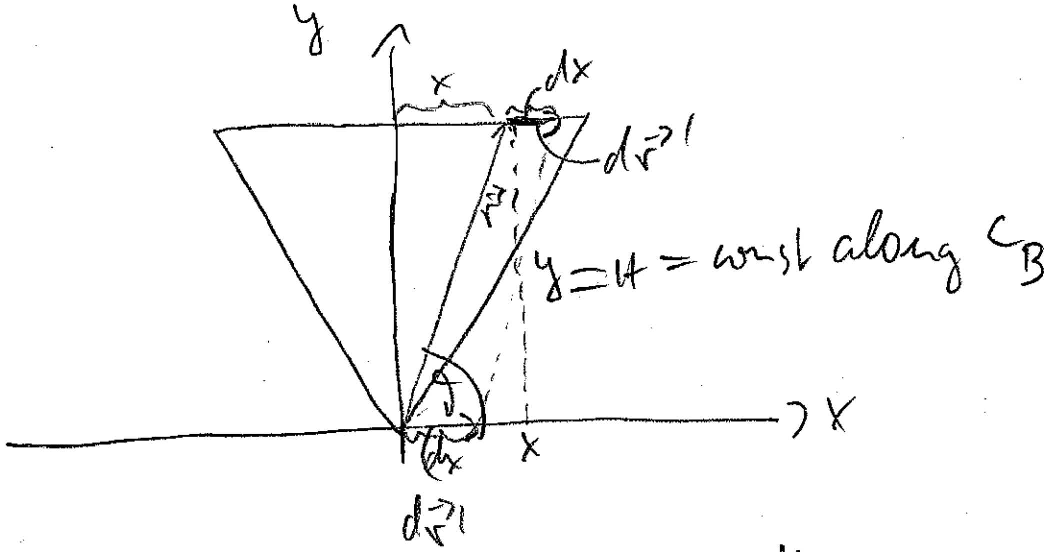
$\vec{r}' = x\vec{i}_x + H\vec{i}_y$; $d\vec{r}' = dx\,\vec{i}_x$

$d\vec{r}' \times \vec{r}' = H\,dx\,\vec{i}_z$

$\Rightarrow$ I can sum up $|d\vec{B}|$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi}\frac{r'\,dx\,\sin\vartheta}{r'^3}$$

$$= \frac{\mu_0 i}{4\pi r'^2}\,dx\,\sin\vartheta$$



$$x = H \tan\vartheta \Rightarrow \frac{dx}{d\vartheta} = \frac{H}{\cos^2\vartheta}$$

$$\Rightarrow dx = \frac{H}{\cos^2\vartheta}\, d\vartheta$$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi} \frac{H \sin\vartheta}{r'^2 \cos^2\vartheta}\, d\vartheta$$

$$r'^2 = H^2 + x^2 = H^2(1+\tan^2\vartheta)$$

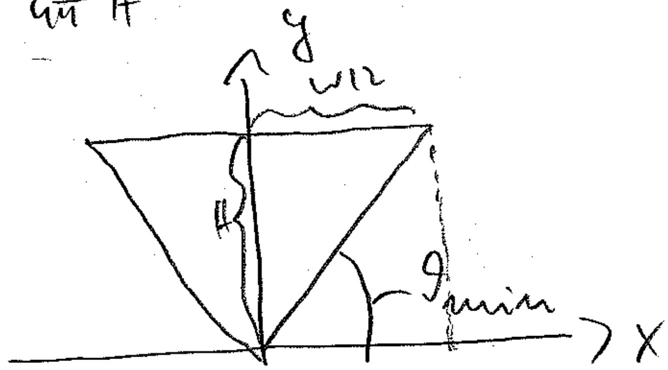
$$= H^2\left(1 + \frac{\sin^2\vartheta}{\cos^2\vartheta}\right) = H^2 \frac{\sin^2\vartheta + \cos^2\vartheta}{\cos^2\vartheta}$$

$$r'^2 = \frac{H^2}{\cos^2\vartheta}$$ (note the typo in the book!)

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi} \frac{\cancel{H}\sin\vartheta}{\frac{H^{\cancel{2}}}{\cancel{\cos^2\vartheta}}\cancel{\cos^2\vartheta}} = \frac{\mu_0 i}{4\pi H} \sin\vartheta$$

$$|\vec{B}| = -\int_{\vartheta_{min}}^{\vartheta_{max}} d\vartheta \, \frac{\mu_0 i}{4\pi H} \sin\vartheta$$ (– from integration direction!)

$$= -\frac{\mu_0 i}{4\pi H}\left[\cos\vartheta_{max} - \cos\vartheta_{min}\right]$$



$$\cos\vartheta_{min} = -\cos\vartheta_{max} = \frac{W/2}{\sqrt{(W/2)^2 + H^2}}$$

$$\Rightarrow |\vec{B}| = \frac{\mu_0 i}{4\pi H}\, \cancel{2}\, \frac{W/\cancel{2}}{\sqrt{(W/2)^2 + H^2}}$$

$$= \frac{\mu_0 i W}{4\pi H\sqrt{(W/2)^2 + H^2}} = \frac{\mu_0 i W}{2\pi H\sqrt{W^2 + 4H^2}}$$

$$\vec{B} = -\frac{\mu_0 i W}{2\pi H\sqrt{W^2 + 4H^2}}\, \mathbf{i}_z$$