(1) $f(x) = (x^2)^{-1/3}$; $D = \mathbb{R}/\{0\}$

$f'(x) = 2x\left(-\frac{1}{3}\right)(x^2)^{-4/3} = -\frac{2}{3x}(x^2)^{-1/2}$

$\Rightarrow f_1(x) = x^{-2/3}$; $D = \mathbb{R}_{>0}$ $\Rightarrow f'(x) = -\frac{2}{3}x^{-5/3}$

$f_2(x) = (-x)^{-2/3}$; $D = \mathbb{R}_{<0}$ $\Rightarrow f'(x) = \frac{2}{3}(-x)^{-5/3}$

(b) $f(x) = \frac{x^4-1}{x^2-1} = \frac{\cancel{(x^2-1)}(x^2+1)}{\cancel{x^2-1}} = x^2+1$

$D = \mathbb{R}/\{-1, 1\}$

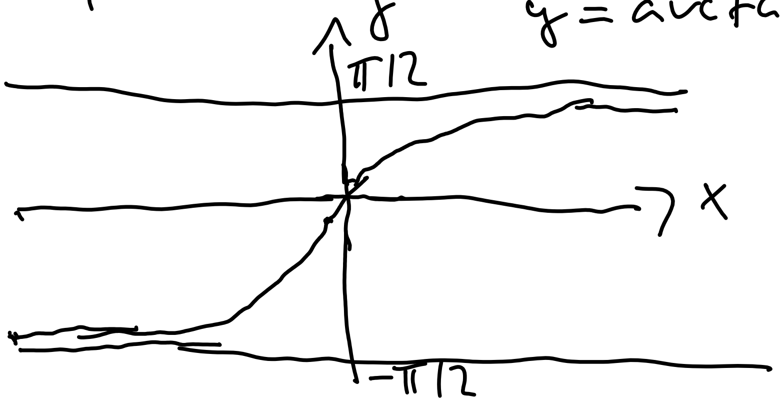
$f'(x) = 2x$

$\tilde{f}(x) = x^2+1$; $D = \mathbb{R}$

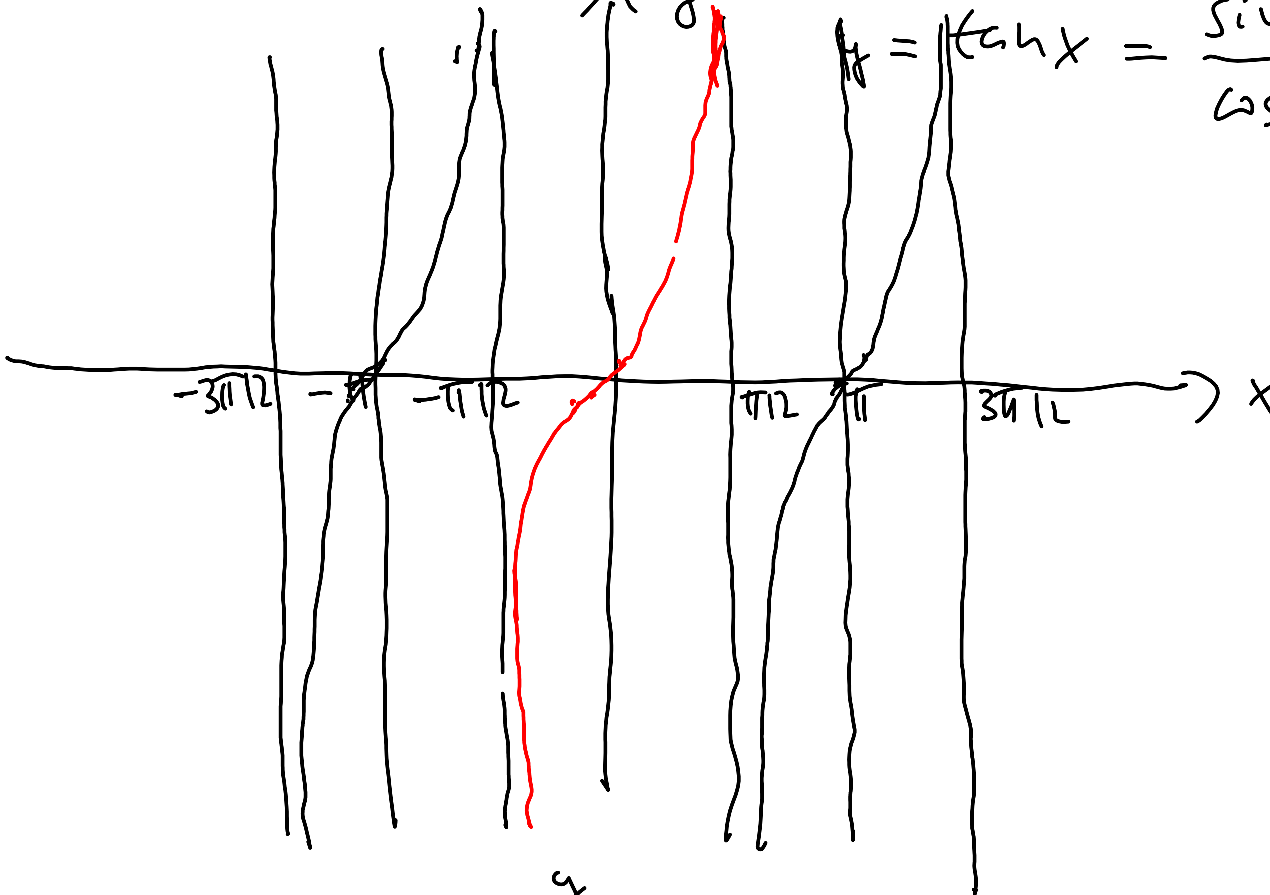
(c) $f(x) = \arctan(x^2)$; $D = \mathbb{R}$

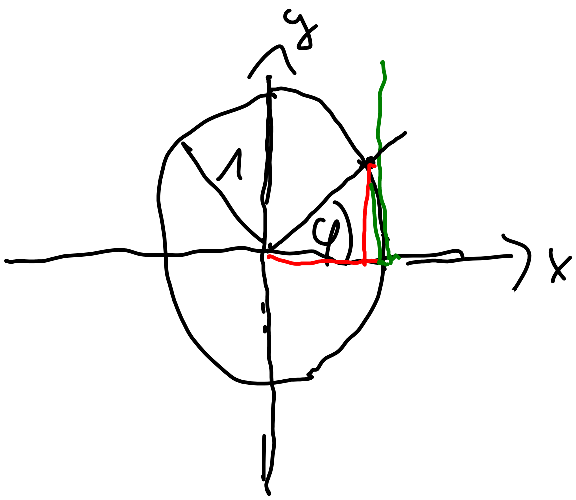
$f'(x) = \frac{2x}{1+x^4}$

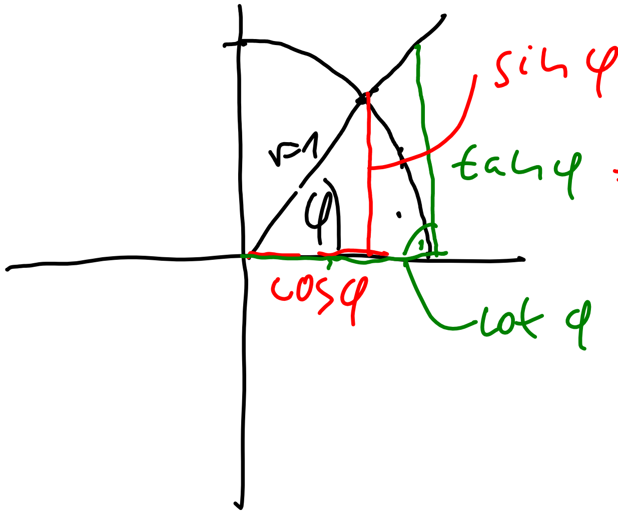
$y = \arctan x$



$$y = \tan x = \frac{\sin x}{\cos x}$$ 2







$$\tan\varphi = \frac{\sin\varphi}{\cos\varphi}$$

$$\cot\varphi = \frac{\cos\varphi}{\sin\varphi} = \frac{1}{\tan\varphi}$$

Aufg. 2

$$\sinh x = \frac{\exp x - \exp(-x)}{2}$$

$$\cosh x = \frac{\exp x + \exp(-x)}{2} \qquad D = \mathbb{R}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{\exp x - \exp(-x)}{\exp x + \exp(-x)}$$

(a) $$\frac{d}{dx}\sinh x = \frac{1}{2}\left[\exp x + \exp(-x)\right] = \cosh x$$

$$\frac{d}{dx}\cosh x = \frac{1}{2}\left[\exp x - \exp(-x)\right] = \sinh x$$

$$\frac{d}{dx}\tanh x = \frac{d}{dx}\frac{\sinh x}{\cosh x} = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

$$= 1 - \left(\frac{\sinh x}{\cosh x}\right)^2 = 1 - \tanh^2 x$$

$$= 1/\cosh^2 x$$

(4)

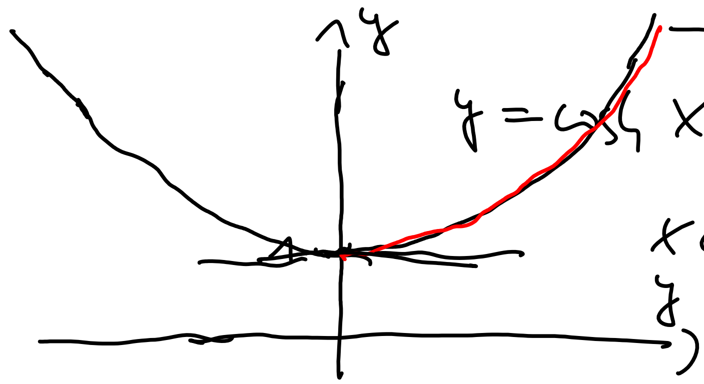
(b) z.z: $\cosh^2 x - \sinh^2 x = 1$

$$\frac{1}{4}\left(\exp x + \exp(-x)\right)^2 - \frac{1}{4}\left(\exp x - \exp(-x)\right)^2$$

$$= \frac{1}{4}\left(\cancel{\exp(2x)} + 2\cdot 1 + \cancel{\exp(-2x)} - \cancel{\exp(2x)} + 2\cdot 1 - \cancel{\exp(-2x)}\right)$$

$= 1$ ✓

(c)



$x \in \mathbb{R}$

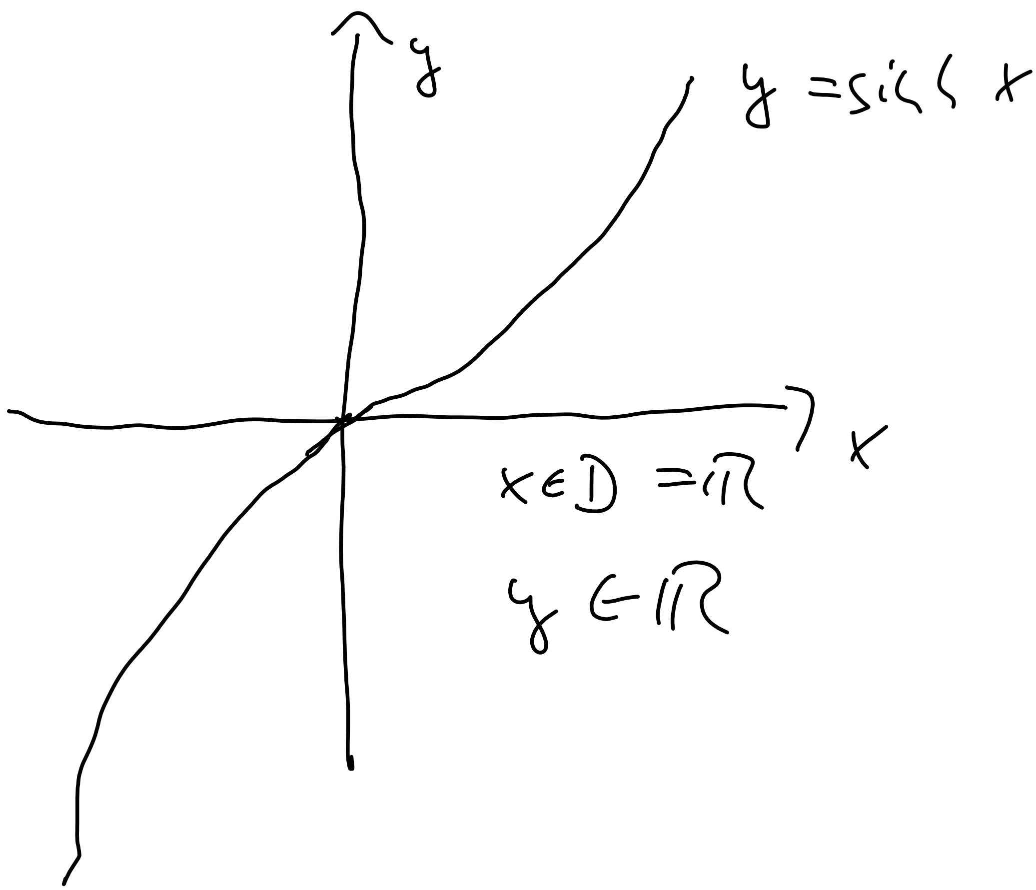
$y \in [1, \infty]$

$$y = \cosh x \Rightarrow x = \operatorname{arcosh} y \geq 1$$

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{\sinh x} \overset{x>0}{=} \frac{1}{\sqrt{\cosh^2 x - 1}}$$

$$= \frac{1}{\sqrt{y^2-1}}$$

$$\boxed{\frac{d}{dx} \operatorname{arcosh} x = \frac{1}{\sqrt{x^2-1}}}$$



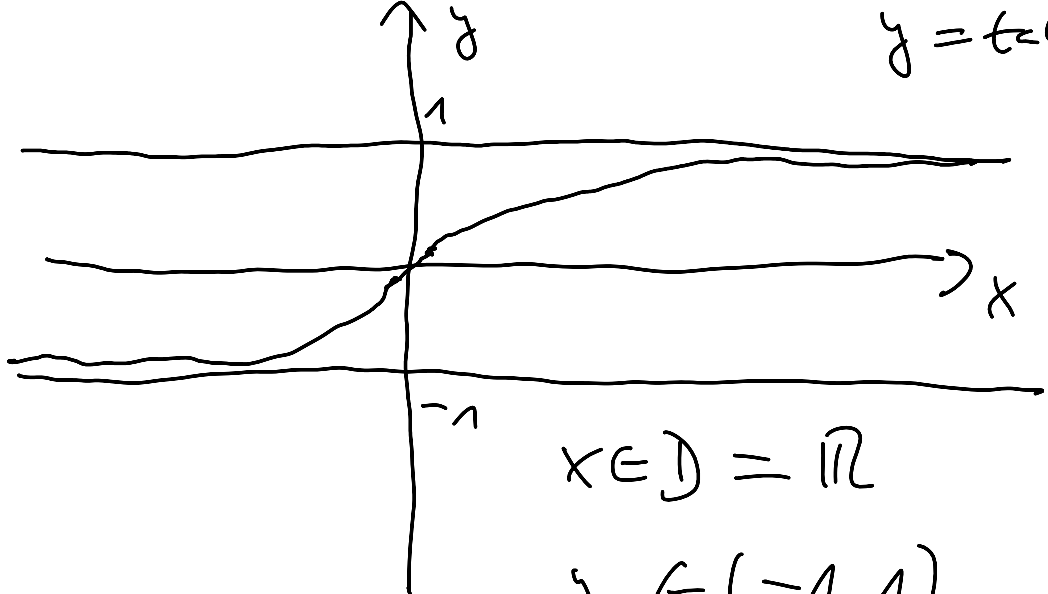
$x = \operatorname{arsinh} y \; ; \; y \in \mathbb{R}$

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{\cosh x} = \frac{1}{\sqrt{1+\sinh^2 x}}$$

$$\frac{d}{dy} \operatorname{arsinh} y = \frac{1}{\sqrt{1+y^2}}$$

$$\boxed{\frac{d}{dx} \operatorname{arsinh} x = \frac{1}{\sqrt{1+x^2}}}$$

$y = \tanh x$ 6



$x \in D = \mathbb{R}$

$y \in (-1, 1)$

$x = \operatorname{artanh} y \quad ; \quad y \in (-1, 1)$

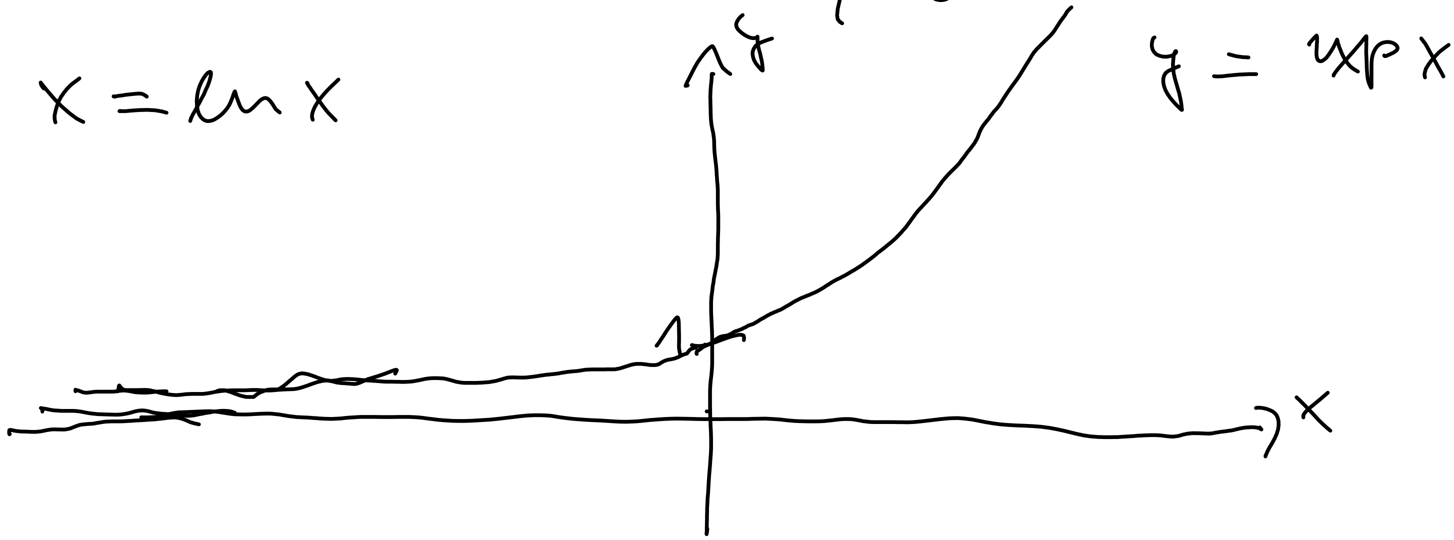
$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}} = \frac{1}{1 + \tanh^2 x} = \frac{1}{1+y^2}$$

$$\frac{d}{dy} \operatorname{artanh} y = \frac{1}{1+y^2}$$

$$\frac{d}{dx} \operatorname{artanh} x = \frac{1}{1+x^2}$$

$y = \exp x \qquad x \in D = \mathbb{R} \; ; \; y > 0 \; ; \; y \in (0, \infty)$

$x = \ln x$



$$y = \sinh x = \frac{1}{2}\Big[\underbrace{\exp x}_{A} - \underbrace{\exp(-x)}_{1/A}\Big]$$

$$y = \frac{1}{2}\left(A - \frac{1}{A}\right) \Big| \cdot 2A$$

$$2Ay = A^2 - 1 \Rightarrow A^2 - 2Ay - 1 = 0$$

$$(A - y)^2 - y^2 - 1 = 0$$

$$(\underbrace{A - y}_{\geq 1})^2 = 1 + y^2$$

$$A - y = \exp x - \frac{1}{2}\left(\exp x - \exp(-x)\right)$$

$$= \frac{1}{2}\left(\exp x + \exp(-x)\right) = \cosh x \geq 1$$

$\hookrightarrow \quad A - y = +\sqrt{1+y^2}$

$$A = \exp x = y + \sqrt{1+y^2} > 0$$

$$x = \ln\left(y + \sqrt{1+y^2}\right) = \operatorname{arsinh} y$$

$$\boxed{\operatorname{arsinh} x = \ln\left(x + \sqrt{1+x^2}\right)}$$

$$\frac{d}{dx} \operatorname{arsinh} x = \frac{1 + \frac{1 \cdot \cancel{2}x}{\cancel{2}\sqrt{1+x^2}}}{x + \sqrt{1+x^2}}$$

$$= \frac{1}{\cancel{x+\sqrt{1+x^2}}} \left(\frac{\cancel{\sqrt{1+x^2}+x}}{\sqrt{1+x^2}} \right)$$

$$= \frac{1}{\sqrt{1+x^2}} \quad \checkmark$$

$$y = \cosh x = \frac{1}{2}\left(\underbrace{\exp x}_{A} + \underbrace{\exp(-x)}_{1/A}\right)$$

$\vdots$

$$x = \operatorname{arcosh} y \geq 0 \quad ; \quad y \geq 1$$

$\vdots$

$$\operatorname{arcosh} y = x = \ln\left(\underbrace{y + \sqrt{y^2-1}}_{\geq 1}\right) \geq 0$$

$$\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\frac{d}{dx}\operatorname{arcosh} x = \frac{1}{x+\sqrt{x^2-1}}\left(1+\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}\right)$$

$$= \frac{1}{\cancel{x+\sqrt{x^2-1}}}\;\frac{\cancel{\sqrt{x^2-1}+x}}{\sqrt{x^2-1}}$$

$$= \frac{1}{\sqrt{x^2-1}} \quad ✓$$

$$y = \tanh x = \frac{\exp x - \exp(-x)}{\exp x + \exp(-x)} \quad ; \; A = \exp x$$

$$y = \frac{A - 1/A}{A + 1/A} = \frac{A^2-1}{A^2+1}$$

$$y(A^2+1) = A^2-1$$

$$(y-1)A^2 + y + 1 = 0 \qquad ; \; y \in (-1,1)$$

$$A^2 = -\frac{y+1}{y-1} = \frac{1+y}{1-y} \quad ; \; A > 0$$

$$\Rightarrow \exp x = A = +\sqrt{\frac{1+y}{1-y}} \Rightarrow x = \ln\left(\sqrt{\frac{1+y}{1-y}}\right)$$

$$\operatorname{artanh} y = x = \ln\left[\left(\frac{1+y}{1-y}\right)^{1/2}\right] = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\frac{d}{dx}\operatorname{artanh} x = \frac{1}{\cancel{2}}\left[\frac{\cancel{1-x}}{1+x}\left(\frac{\overbrace{1\cdot(1-x)+(1+x)}^{\cancel{2}}}{(1-x)^{\cancel{2}}}\right)\right]$$

$$\boxed{x\in(-1,1)} \qquad = \frac{1}{(1-x)(1+x)} = \frac{1}{1-x^2} \checkmark$$