Measuring and interpreting anisotropic flow
Outline

- Intro: what is flow
- Why should you care?
- How do we measure it?
- What have we learned so far?
- Where do we go from here?
Heavy-ion collisions

adapted from Chun Shen

Initial energy
density

Hadronization

QGP phase

Hadron
gas phase

Kinetic
freeze-out

final detected
particle distributions

τ ~ 0 fm/c

viscous hydro

collision evolution

τ ~ 1 fm/c

hadronic cascade

τ ~ 10 fm/c

free streaming

τ ~ 10^{15} fm/c
What is flow?

Anisotropic Flow: anisotropies in the azimuthal distribution of particles in momentum space.
Why does it flow?

It is commonly interpreted as the result of the hydrodynamic behaviour of strongly-interacting QCD matter:

- strongly-interacting non-spherical system
  ⇒ anisotropic pressure ⇒ anisotropic flow

Spatial anisotropies of the initial system are due to:

- event-by-event fluctuations
- impact parameter
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  -\rightarrow anisotropic pressure -\rightarrow anisotropic flow

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Spatial anisotropies

Hydro

Momentum anisotropies
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from B. Schenke
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in HI collisions, strong elliptical anisotropy, depending on centrality / impact parameter.
How do we quantify it?

Flow is quantified in terms of Fourier coefficients:

\[ \frac{dN}{d\varphi} \propto 1 + \sum_{n=1}^{+\infty} 2v_n \cos(n(\varphi - \Psi_{RP})) \]

**Diagram:**
- **V1:** directed flow
- **V2:** elliptic flow
- **V3:** triangular flow
Symmetry planes

There’s not only the reaction plane:

Fluctuating initial conditions

Each harmonic ($v_n$) develops along its corresponding symmetry plane ($\psi_n \neq \psi_{RP}$)
a First Look

Impact parameter dominated fluctuations

fluctuations dominated
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a Realistic Medium

Whatever information you want to extract about the QGP from experimental data requires a realistic modelling of a Heavy-Ion collision:

- Energy loss: jets, heavy flavour
- Charmonia
- Strangeness production
- Photons / dileptons

Most of what we know so far has been inferred from soft hadron observables: $p_T$ spectra, flow
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Flow can be measured with a variety of techniques:

- 2-particle correlation function ($\Delta \eta, \Delta \phi$)
- Scalar Product / Event Plane methods
- Multi-particle cumulants
How do we measure it?

Flow can be measured with a variety of techniques:

- 2-particle correlation function \((\Delta \eta, \Delta \varphi)\)
- Flow
- Jets (near-side and away-side)

\[
S(\Delta \varphi, \Delta \eta) = \frac{1}{N_{\text{trig}}} \frac{d^2N}{d\Delta \varphi d\Delta \eta}
\]

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"the Double Ridge"

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- Scalar Product / Event Plane methods

correlate tracks with an event plane ($\psi_{EP}$) reconstructed with an independent detector:

$$v_n\{EP\} = \frac{1}{R} \langle \cos(n(\phi - \psi_{EP})) \rangle$$

N.B. conceptually, it’s again a 2-particle correlation
How do we measure it?

Flow can be measured with a variety of techniques:

- 2-particle correlation function ($\Delta \eta, \Delta \phi$)
- Scalar Product / Event Plane methods

can be used to correlate reconstructed tracks with event planes from forward detectors (scintillators/calorimeters)
How do we measure it?

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- 2-particle correlation function \((\Delta \eta, \Delta \varphi)\)
- Scalar Product / Event Plane methods
- Multi-particle cumulants
  - Provide additional information on flow fluctuations
  - Analytically suppress background
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Multi-particle cumulants

Possible to measure different cumulants of the underlying flow:

2-particle: $\langle\langle 2 \rangle\rangle = \langle\langle e^{in(\varphi_1 - \varphi_2)} \rangle\rangle = \langle v_n^2 \rangle$

4-particle: $\langle\langle 4 \rangle\rangle = \langle\langle e^{in(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle\rangle = \langle v_n^4 \rangle$

by definition:

$\nu_n\{2\} = \sqrt{\langle v_n^2 \rangle}$

$\nu_n\{4\} = \frac{4}{\sqrt{2}}\sqrt{\langle v_n^2 \rangle^2 - \langle v_n^4 \rangle}$

if $\nu_n$ is constant

$\nu_n\{4\} \neq \nu_n\{2\} \neq \langle \nu_n \rangle$ if $\nu_n$ fluctuates
Multi-particle cumulants

Possible to measure different cumulants of the underlying flow:

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4-particle: \[ \langle\langle 4 \rangle\rangle = \langle\langle e^{i n (\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle\rangle = \langle v_n^4 \rangle \]

\[ v_n\{2\} = \sqrt{\langle v_n^2 \rangle} \]

\[ v_n\{4\} = 4 \sqrt{2 \langle v_n^2 \rangle^2 - \langle v_n^4 \rangle} \]

by definition:

\[ v_n\{4\} = v_n\{2\} = \langle v_n \rangle \quad \text{if } v_n \text{ is constant} \]

\[ v_n\{4\} \neq v_n\{2\} \neq \langle v_n \rangle \quad \text{if } v_n \text{ fluctuates} \]
Multi-particle cumulants

Up to order $\sigma^2$:

$$v_2\{2\} = \langle v_2 \rangle + \frac{1}{2} \frac{\sigma^2}{\langle v_2 \rangle}$$

$$v_2\{4\} = v_2\{6\} = \ldots v_2\{n\} = \langle v_2 \rangle - \frac{1}{2} \frac{\sigma^2}{\langle v_2 \rangle}$$

$v_2\{2\} \neq v_2\{4\}$

flow fluctuations!

ALICE, QM '11
the Flow Hypothesis

2- and multi-particle correlations are based on one simple assumption (a.k.a. the flow hypothesis)

\[
\langle e^{i n (\varphi_1 - \varphi_2)} \rangle = \langle e^{i n (\varphi_1 - \Psi_n - (\varphi_2 - \Psi_n))} \rangle = \\
= \langle e^{i n (\varphi_1 - \Psi_n)} \rangle \langle e^{i n (\varphi_2 - \Psi_n)} \rangle = \langle \nu_n^2 \rangle
\]

* Correlations among produced particles are induced only by correlation of each particle with the event planes.
Non-flow

\[ v_2 > 0, \ v_2 \{2\} > 0 \quad v_2 = 0, \ v_2 \{2\} = 0 \quad v_2 = 0, \ v_2 \{2\} > 0 \]

short-range correlations (jets, resonances) unrelated to the reaction plane enter into multi-particle correlations:

\[ v_2 \{2\} = \sqrt{\langle \langle e^{i2(\varphi_1 - \varphi_2)} \rangle \rangle} = \sqrt{\langle v_n^2 + \delta_2 \rangle} \]

e.g. for two-body decays: \[ \delta_2 \propto 1/M \]
Non-flow

but are suppressed in higher order cumulants:

\[
v_2\{4\} = \sqrt[4]{2} \left( \langle e^{i2(\varphi_1-\varphi_2)} \rangle^2 - \langle e^{i2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \rangle \right)
\]

\[
= \sqrt[4]{2} \left( \langle v_2^2 + \delta_2 \rangle^2 - \langle v_2^4 + 4v_2^2\delta_2 + 2\delta_2^2 \rangle \right)
\]

\[
= \sqrt[4]{\langle v_2^4 - \delta_4 \rangle}
\]

\[
\delta_4 \propto 1/M^3
\]

and/or by imposing a large gap in rapidity ($\Delta\eta > 1$):

the advantage of using forward detectors!
Non-flow

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\[
\begin{align*}
v_2\{4\} &= 4\sqrt{2} \left\langle \langle e^{i2(\varphi_1-\varphi_2)} \rangle \right\rangle^2 - \left\langle \langle e^{i2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)} \rangle \right\rangle \\
&= 4\sqrt{2} \left\langle v_2^2 + \delta_2 \right\rangle^2 - \left\langle v_2^4 + \delta_4 + 4v_2^2\delta_2 + 2\delta_2^2 \right\rangle \\
&= 4\sqrt{\left\langle v_2^4 - \delta_4 \right\rangle}
\end{align*}
\]

\[\delta_4 \propto 1/M^3\]

and/or by imposing a large gap in rapidity (\(\Delta\eta>1\)):

the advantage of using forward detectors!
Flow tutti-frutti

Many different observables measurable (and measured) around flow:

- Centrality dependence
- $p_T$ and $\eta$ dependence
- Identified particles, resonances
- Flow fluctuations (also event-by-event)
- Event planes: $p_T$ and $\eta$ dependence
- Correlations between harmonics
- Event-Shape-Engineering
Flow fluctuations

Possible to reconstruct the complete $v_n$ pdf:

\begin{align*}
P(\varepsilon) &= 2\alpha\varepsilon(1 - \varepsilon^2)^{\alpha - 1} \\
\end{align*}

L. Yan, J. Y. Ollitrault PRL 112, 082301 (2014)
Correlations between harmonics

Correlations between flow harmonics (a.k.a. symmetric cumulants):

\[ SC(m, n) = \frac{\langle \langle e^{i(m\phi_1 + n\phi_2 - m\phi_3 - n\phi_4)} \rangle \rangle - \langle \langle e^{im(\phi_1 - \phi_2)} \rangle \rangle \langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle}{\langle \langle e^{im(\phi_1 - \phi_2)} \rangle \rangle \langle \langle e^{in(\phi_1 - \phi_2)} \rangle \rangle} \]

\[ \approx \frac{\langle v^2_m v^2_n \rangle - \langle v^2_m \rangle \langle v^2_n \rangle}{\langle v^2_m \rangle \langle v^2_n \rangle} \cos^2(c_n \Psi_n - c_m \Psi_m) \]
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\approx \frac{\langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle}{\langle v_m^2 \rangle \langle v_n^2 \rangle} \cos^2 (c_n \Psi_n - c_m \Psi_m)
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correlation between flow fluctuations
Correlations between harmonics

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Correlations between flow harmonics (a.k.a. symmetric cumulants):

\[ v_2 \text{ and } v_4: \text{ correlated} \]

\[ v_2 \text{ and } v_3: \text{ anticorrelated} \]

ALICE, PRL 117, 182301 (2016)
Correlations between harmonics (a.k.a. symmetric cumulants): Show great potential to decouple different model parameters!
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The paradigm

The sizeable values of flow coefficients, up to high harmonics, have been successfully explained by:

fluctuating initial conditions + hydro-like collective expansion

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Implications:

- yes, we create a strongly coupled system
- it quickly expands before hadronizing
- doing so, it behaves like a fluid with very low viscosity
  - initial spatial anisotropies translate into momentum ones
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which initial conditions? which collectivity?
which Initial Conditions?

from J. Noronha-Hostler at Hot Quarks 2016
which Initial Conditions?

The observed $v_n$ always come from an interplay of initial and final state effects: *not straightforward to decouple them!"
which Initial Conditions?

The observed $v_n$ always come from an interplay of initial and final state effects: not straightforward to decouple them!

e.g. $< v_n >$: lumpy IC + high viscosity $\approx$ smooth IC + low viscosity

X. Zhu et al., arXiv:1608.05305

B. Schenke et al., PRL 108 (2012)
which Initial Conditions?

... but looking at the full flow pdf does favour one: IP-Glasma

C. Gale et al., arXiv:1210.5144
What about the longitudinal structure? (default: boost invariance)

- Required to describe forward-backward asymmetric phenomena (directed flow, twist/torque/event plane decorrelations…)
- More important at lower energies!

3D-Glauber

3D-Glasma

the twist

which Collectivity?

Do the final momentum correlations come only from the hydro-like evolution of the system? Where does the “collectivity” come from?

- Initial state momentum correlations? (CGC)
- Hadronic rescattering
which Collectivity?

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flow + mass ordering

CGC + Lund

B. Schenke et al, PRL 117, 162301 (2016)
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Schlichting, Tribedy arXiv:1611.00329
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![Graph showing hydro and hydro + hadronic cascade comparison](attachment:image.png)

ALICE, JHEP 06 (2015)
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a Convoluted Problem

- Flow is a key observable for characterising the collective properties and the evolution of the medium
- However, it develops during different phases (initial state, QGP, hadronic phase): *highly convoluted problem!*
- The problem: how to decouple these?
  - New observables (e.g. symmetric cumulants)
  - New approaches from theory
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New Observables

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ALICE, PRL 117, 182301 (2016)
New Observables

Correlations between flow harmonics (a.k.a. symmetric cumulants):

SC(3,2): set by initial conditions

ALICE, PRL 117, 182301 (2016)
New Observables

Correlations between flow harmonics \((a.k.a.\ symmetric\ cumulants)\): 

\[ SC(4,2):\ \text{set by hydro phase} \]

\[ \text{ALICE, PRL 117, 182301 (2016)} \]
New Approaches

Applying Bayesian parameter estimation to relativistic heavy-ion collisions: simultaneous characterization of the initial state and quark-gluon plasma medium


Using Bayesian methods to perform multi-parameter model-to-data comparison:

\[ P(x_* | X, Y, y_{\text{exp}}) \propto P(X, Y, y_{\text{exp}} | x_*) P(x_*) \]

model parameter \quad model \quad measured obs. \quad computed obs.
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- posterior
- likelihood
- prior
- model parameter
- model
- computed obs.
- measured obs.
New Approaches

simultaneous parameter optimisation of initial state and hydro phase:

<table>
<thead>
<tr>
<th>Initial condition</th>
<th>QGP medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm 120. / 129.</td>
<td>η/s min 0.08</td>
</tr>
<tr>
<td>p 0.0</td>
<td>η/s slope 0.85 / 0.75 GeV⁻¹</td>
</tr>
<tr>
<td>k 1.5 / 1.6</td>
<td>ζ/s norm 1.25 / 1.10</td>
</tr>
<tr>
<td>w 0.43 / 0.49 fm</td>
<td>T_{\text{switch}} 0.148 GeV</td>
</tr>
</tbody>
</table>
one Final Plea

We need more and more synergy between experimentalists and theoreticians: *fast, efficient, frequent exchange of ideas.*

If we want things to move forward, don’t be afraid and go open source:

- Github
- Rivet
I WANT YOU TO GO OPEN SOURCE
NEAREST SOFTWARE REPOSITORY
THANKS FOR THE ATTENTION!
Pre-Equilibrium Dynamics

Qualitatively complete picture of equilibration mechanism at weak coupling

Colliding nuclei → Glasma flux tubes → Over-occupied plasma → Min-jets + soft bath → Equilibrium

Strong fields at 1-2 fm/c

Eff. kinetic theory

Hydro

Classical-statistical lattice gauge theory

Quasi particles

from Soeren Schlichting at “Exploring the QCD Phase Diagram through Energy Scans” 2016
Twist and Shake

\[ r_n(\eta_a, \eta_b) = \frac{\langle \cos[n(\phi(-\eta_a) - \phi(\eta_b))] \rangle}{\langle \cos[n(\phi(\eta_a) - \phi(\eta_b))] \rangle} \]

CMS PbPb \( \sqrt{s_{NN}} = 2.76 \text{ TeV} \)