Tracing the QCD pressure

André Peshier

w/ G Jackson

University of Cape Town

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Goethe University Frankfurt

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161201 –
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Why this talk may be relevant – or not ;)

- idea relevant for many lattice-QCD calculations at $T>0$
- understand QCD where we think it is simple
- revisit first non-perturbative coefficient
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- here: only quenched limit of QCD (no quarks)
- hard work done by others, we only interpret their results
- only 1% effect ... for quenched QCD
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![Graph showing pressure ($p/p_0$) versus temperature ($T/T_c$)]
Methods

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \ldots \text{ where } F^a_{\mu\nu} = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g f_{abc} A^\mu_b A^\nu_c \]
Methods

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu \nu}^a F_{a}^{\mu \nu} + \ldots \text{ where } F_{a}^{\mu \nu} = \partial_{\mu} A^{\nu}_a - \partial_{\nu} A^{\mu}_a - g f_{a b c} A^{\mu}_b A^{\nu}_c \]

Perturbation theory

\[ p = p_{\text{SB}} \left[ 1 + c_2 \sqrt{\alpha^2} + \ldots \right] \]
Methods

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\[ p = p_{\text{SB}} \left[ 1 + c_2 \sqrt{\alpha^2} + \ldots \right] \]
Perturbation theory **CHALLENGES**

\[
\frac{p}{p_{SB}} = 1 + c_2 \alpha^{2/2} + c_3 \alpha^{3/2} + (c_4 + \tilde{c}_4 \ln \alpha) \alpha^{4/2} + c_5 \alpha^{5/2} + (c_6 + \tilde{c}_6 \ln \alpha) \alpha^{6/2} + \ldots
\]

[Shuryak 1978]

[Kajantie et al 2003]
Perturbation theory **CHALLENGES**

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[Shuryak 1978]

[Kajantie et al 2003]
Perturbation theory \textbf{CHALLENGES}

\[
\frac{p}{\rho_{SB}} = 1 + c_2 \alpha^{2/2} + c_3 \alpha^{3/2} + (c_4 + \tilde{c}_4 \ln \alpha) \alpha^{4/2} + c_5 \alpha^{5/2} + (c_6 + \tilde{c}_6 \ln \alpha) \alpha^{6/2} + \ldots
\]

[Shuryak 1978]
[\textit{\ldots}]
[\textit{\ldots}]

[Shuryak 1978]
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Perturbation theory “diverges”

**toy model:** scalar “QFT” in \(d=0\) dimensions

- “Lagrangian” \(L = \frac{1}{2} x^2 + \lambda x^4\) \(\rightarrow\) “partition fnc” \(Z(\lambda) = \int dx \exp(-L(\lambda))\)

- perturbative expansion \(Z(\lambda)/Z_0 = 1 - 3\lambda + \frac{1}{2} 105\lambda^2 + \ldots\)
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lower order better for larger coupling
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![Graph showing perturbation theory divergence](image)

**cut $\lambda$ plane: convergence radius = 0**

**lower order better for larger coupling**
Lattice QCD CHALLENGES

\[ \frac{I(T)}{T^4} \]

- \( N_t = 5 \)
- \( N_t = 6 \)
- \( N_t = 7 \)
- \( N_t = 8 \)
- cont. limit
- Boyd et al.

[Borsanyi et al, 2012]
finite-size artefacts in particular around $T_c$: correlations

[Borsanyi et al, 2012]
Lattice QCD

integral method: pressure from interaction measure

\[ \frac{p(T)}{T^4} = \sigma + \int_{T_0}^T \frac{dT'}{T'} \frac{\mathcal{I}(T')}{T'^4} \]

where \( \sigma = \frac{p(T_0)}{T_0^4} \)

\[ \mathcal{I} = e - 3p \]

where \( e = sT - p \)

with \( s = \partial p / \partial T \)
Lattice QCD

**integral method**: pressure from interaction measure

\[
\frac{p(T)}{T^4} = \sigma + \int_{T_0}^{T} \frac{dT'}{T'} \frac{\mathcal{I}(T')}{T'^4} \quad \text{where} \quad \sigma = \frac{p(T_0)}{T_0^4}
\]

\[\mathcal{I} = e - 3p\]

where \(e = sT - p\)

with \(s = \partial p / \partial T\)

---

The region around \(T_c\) is highlighted with a yellow shade, indicating it "contributes most" to the integral. The graph shows the ratio \(\mathcal{I}/T^4\) and \(\mathcal{I}T_c/T^5\) as functions of \(T/T_c\).
Scrutinize existing results [Borsanyi et al, 2012]

\[
\frac{T}{T_c}^4
\]

\[
\begin{array}{c}
10^{-1} \\
10^{-2} \\
1 \\
10 \\
10^2 \\
10^3
\end{array}
\]

compare to pQCD $\Lambda = 0.73 T_c$ known

$\rightarrow$ fit: $c_6 = -72 \pm 3$
Scrutinize existing results [Borsanyi et al, 2012]

\[ \frac{T}{T_c} \]

\[ \frac{T}{T_c} \]

\[ \frac{\rho(T)}{\rho_{SB}} \]

\[ \frac{\rho(T)}{\rho_{SB}} \]

compare to pQCD \( \Lambda = 0.73 T_c \) known

\[ c_6 = -72 \pm 3 \]
Scrutinize existing results [Borsanyi et al, 2012]

- pQCD should be “better” at large $T$...

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  $\rightarrow$ fit: $c_6 = -72 \pm 3$
Scrutinize existing results [Borsanyi et al, 2012]

\[ T/T_c \]

\[ 1 - p/p_0 \]

\[ \Lambda = 0.73 T_c \] known

\[ c_6 = -72 \pm 3 \]
Scrutinize existing results [Borsanyi et al, 2012]

\[ \frac{T}{T_c} \]

\[ \frac{1 - p/p_0}{p/T_{c}} \]

\[ c_6 = -72 \pm 3 \]

pQCD should be “better” at large \( T \) ...
Theory & models

In practice, we always use models/approximations. Validity check is crucial in QFT.

Adjusting parameters = renormalization.

Theory

\[\text{phenomena, observables}\]

\[\text{ADJUST params}\]

\[\text{VERIFY}\]

\[\text{PREDICT}\]
• in practice, we **always use models/approximations**
  • **validity check** is crucial
in practice, we always use models/approximations
  • validity check is crucial

in QFT
  • adjusting parameters = renormalization
Our approach

- **thermodynamic renormalization**: match perturbative results to lattice data at sufficiently large “renormalization temperature” to specify model parameter(s)

  - use interaction measure (being the actual lattice “observable”)

\[
\mathcal{I}_{\text{model}} = T^5 \frac{\partial \left( \frac{p_{\text{model}}}{T^4} \right)}{\partial T}
\]
Our approach

- **thermodynamic renormalization**: match perturbative results to lattice data at sufficiently large “renormalization temperature” to specify model parameter(s)

use interaction measure (being the actual lattice “observable”)

\[ I_{\text{model}} = T^5 \frac{\partial \left( \frac{p_{\text{model}}}{T^4} \right)}{\partial T} \]

- **check range of applicability** of adjusted model by **comparison with lattice data** (not by vague arguments “coupling small”)
Our approach

- **thermodynamic renormalization**: match perturbative results to lattice data at sufficiently large “renormalization temperature” to specify model parameter(s)
  
  use interaction measure (being the actual lattice “observable”)

\[ I_{\text{model}} = T^5 \frac{\partial (p_{\text{model}}/T^4)}{\partial T} \]

- **check range of applicability** of adjusted model by comparison with lattice data (not by vague arguments “coupling small”)

- make **predictions for other observables** in applicability range
(n|l) models \((n_f=0)\)

Pressure (= thermodynamic potential) to order \(n\):

\[p_{(n)} = p_0 \left[ 1 + \sum_{m=2}^{n} C_m \alpha^{m/2} \right] \quad \text{where} \quad p_0 = 8 \times 2^{\frac{\pi^2}{90}} T^4\]

\[C_2 = -1.2 \]
\[C_3 = +5.4 \]
\[C_4 = 6.8 \ln \alpha + 16.2 \]
\[C_5 = -45.7 \]
\[C_6 = -36.6 \ln \alpha + c_6 \quad \text{(for} \ \mu = 2\pi T)\]

Running coupling to order \(l\):

\[\alpha(\ell) = \sum_{k=1}^{\ell} a_k(L) L^{-k} \]

\[a_1 = 1.14, \quad a_2 = -0.96 \ln L, \quad a_3 = 0.41 + 0.81 (\ln L - 1) \ln L \]

\[L(T) = \ln \left( \frac{2\pi T}{\lambda T_c} \right)^2 \quad \text{where} \quad \lambda = \Lambda / T_c\]
Running coupling

fairly similar for $l=2$ and $l=3$ (and $l=1$ after rescaling)
(5|2)-model \ldots \textbf{how NOT to: } \rho\text{-scheme}
(5|2)-model … how NOT to: $p$-scheme
(5|2)-model … how **NOT** to: \( p \)-scheme

small applicability range, not to \( T \to \infty \)
\((5|2)\)-model \ldots how NOT to: \(p\)-scheme

\[ \frac{I}{T^4} \]

\[ 1 - \frac{p}{p_0} \]

**systematic discrepancy** for interaction measure as actual lattice “observable”

small applicability range, not to \( T \to \infty \)
(5|2)-model

\[ p_{(5|2)}(T; \lambda) = p_{\text{lattice}}(T) \]
(5|2)-model

\[ \mathcal{I}_{(5|2)}(T; \lambda) = \mathcal{I}_{\text{lattice}}(T) \]

\[ p_{(5|2)}(T; \lambda) = p_{\text{lattice}}(T) \]

\[ \lambda(T) \]

\[ T/T_c \]

matching \( p_{(5|2)} \)

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(5|2)-model

\[ \mathcal{I}_{(5|2)}(T; \lambda) = \mathcal{I}_{\text{lattice}}(T) \]

\[ p_{(5|2)}(T; \lambda) = p_{\text{lattice}}(T) \]
\( \lambda^{*}_{(5|2)} = 0.58 \pm 0.11 \)

\[ \mathcal{I}_{(5|2)}(T; \lambda) = \mathcal{I}_{\text{lattice}}(T) \]

\[ p_{(5|2)}(T; \lambda) = p_{\text{lattice}}(T) \]
(5|2)-model

applicability range: $T > 40 T_c$
(5|2)-model

The applicability range is $T > 40T_c$. 

- lattice
- $T_{(5,2)}$
- $T_{(5,2)} \oplus T(10T_c)$
(5|2)-model

applicability range: $T > 40T_c$
(5|2)-model

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in applicability range: discrepancy = constant shift
(5|2)-model

applicability range: \( T > 40T_c \)

in applicability range:
discrepancy = constant shift

\[
\frac{p(T)}{T^4} = \sigma + \int_{T_0}^{T'} \frac{dT'}{T'} \frac{I(T')}{T'^4}
\]
(5|2) model

breakdown at $T^* \sim 40T_c$ because “coupling too large” …?
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\begin{align*}
\alpha(10T_c) &= 0.10 \\
\alpha(40T_c) &= 0.08 \\
\alpha(400T_c) &= 0.06
\end{align*}
(5|2) model

breakdown at $T^* \sim 40T_c$ because "coupling too large" …?

$$p_{(5|2)}^*(40T_c) = p_0[1 - 0.09 + 0.12 - 0.01 - 0.08] \approx p_0[1 - \frac{1}{2}0.09]$$

$$\alpha(10T_c) = 0.10$$
$$\alpha(40T_c) = 0.08$$
$$\alpha(400T_c) = 0.06$$

similar properties as asymptotic series
(6|3) model

more difficult

• 2 parameters

\[ \lambda_{(6|3)}, \ c_6 \]

• expect smaller applicability range
(6|3) model

more difficult
- 2 parameters
  \( \lambda_{(6|3)}, c_6 \)
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fit over interval \([T_f, T_{\text{max}}]\)
(6|3) model

- More difficult
  - 2 parameters: $\lambda_{(6|3)}, c_6$
  - Expect smaller applicability range

Fit over interval $[T_f, T_{\text{max}}]$
Results

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Graph showing the pressure ratio \( p/p_0 \) as a function of the temperature ratio \( T/T_c \). The graph includes lines for \( p_{(6,3)}^* \) and \( p_{(5,2)}^* \), as well as data points for lQCD OLD and lQCD revised.}
\end{figure}
Results

Matching interaction measure at large $T$

"re-calibration" of pressure at $T > 4T_c$

Slower approach to free limit
Results

matching interaction measure at large $T$

\[ \downarrow \]

\textbf{“re-calibration”} of pressure at $T > 4T_c$

slower approach to free limit

non-perturbative coefficient

\[ c_6 = \mathcal{O}(-40) \]

\[ c_6 = -72 \pm 3 \]

\[ c_6 = -95 \pm 6 \]
Results

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Resumé

The graph illustrates the behavior of $T/T_c$ and $p/p_0$ as a function of $T/T_c$. The initial downward trend in $T/T_c$ towards $1$ is followed by a match with the revised data, indicating a corrected trend.

The inset graph shows $p/p_0$ as a function of $T/T_c$. The data points are labeled as "lattice QCD" and "revised". A note indicates a difference of about 1%.
Resumé [arXiv:1610.08530]

- new approach to deal with lattice artefacts (which have physics reasons):
  combine integral method with perturbative QCD

- 1% modification of pressure, slower approach to asymptotic freedom
  ~ relevant to benchmark improved analytical methods

- improved value of 1st nonperturbative coefficient

- outlook: phenomenological implications for physical case
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