## Study of Strangeness -1 and -2 hypernuclei

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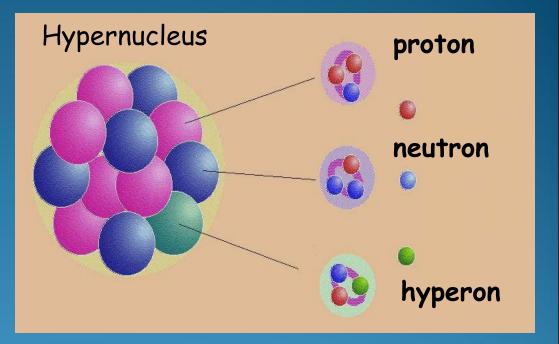


## • Introduction

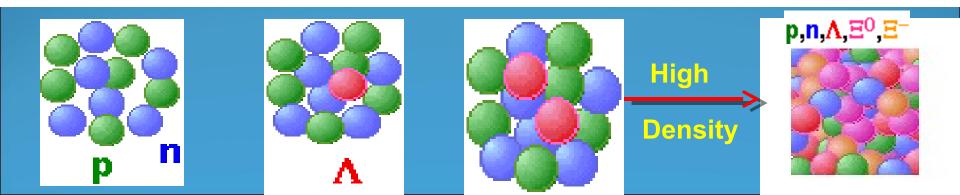
- A review and comparison of production reactions
- Brief sketch of the theoretical model
- Results, cross sections, spectroscopy
- Conclusions

# Nuclei with Strangeness (Hypernuclei)

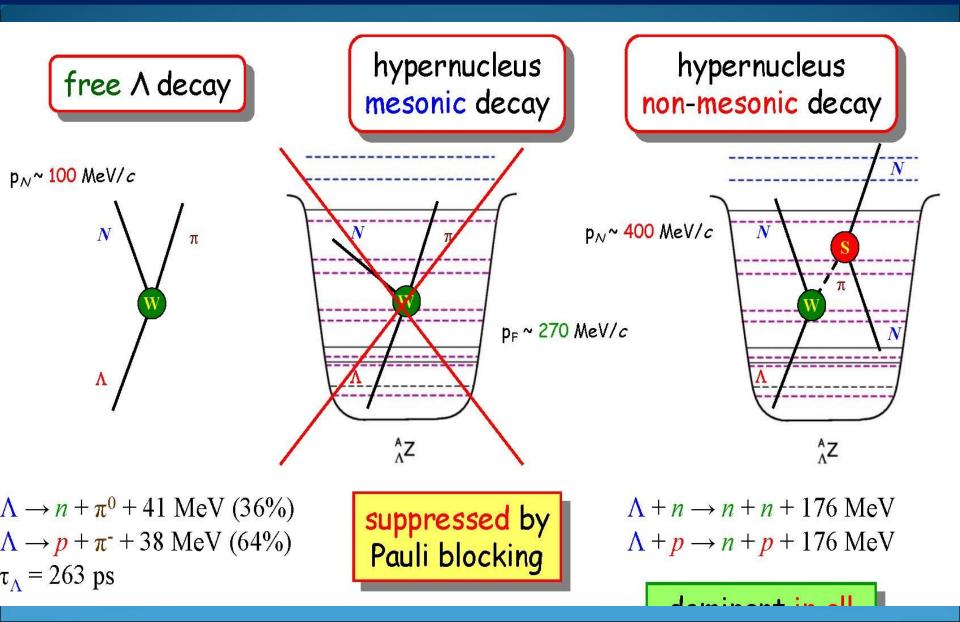
Hypernuclei are nuclear systems where at least one nucleon in one of its orbits is replaced by A hyperon (e.g.  $\Lambda$ ).

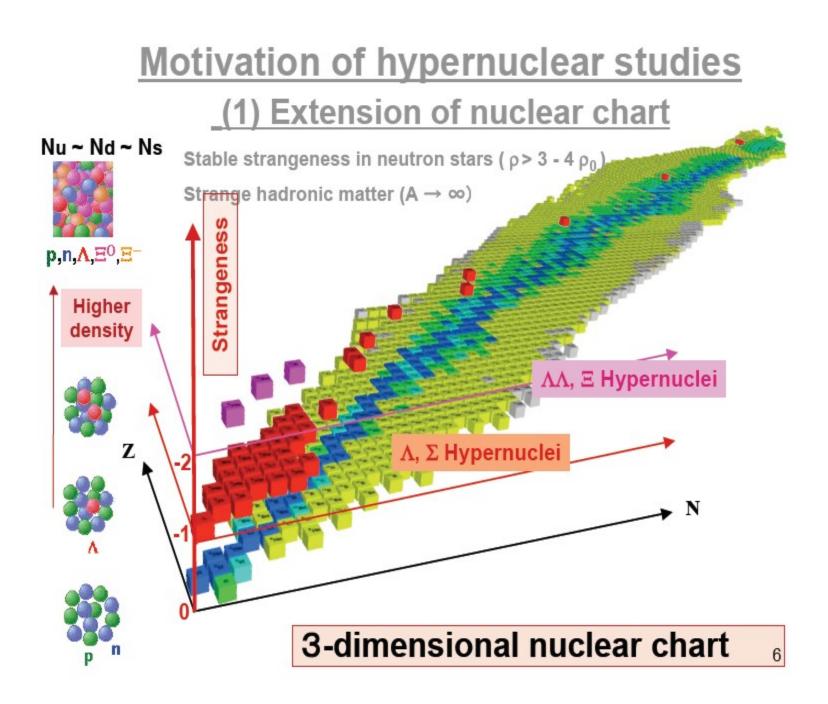


 ${}^{A}{}_{\Lambda}Z$  is a bound state of Z protons (A-Z-1) neutrons and a  $\Lambda$  hyperon

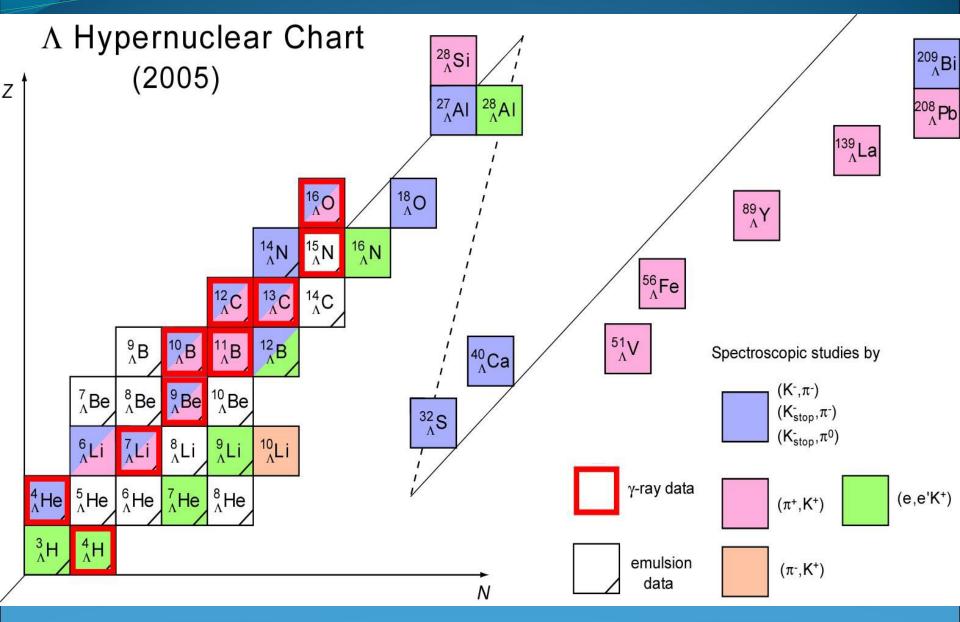


# $\Lambda$ hyperon can stay in contact with nucleons inside a Nucleus





## PRESENT STATUS OF A HYPERNUCLEI



# Why are Hypernuclei interesting!

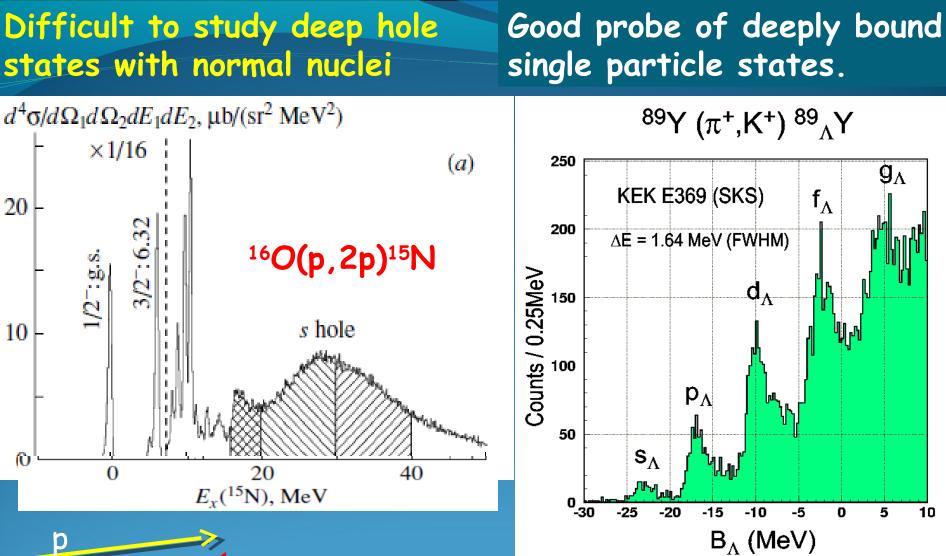
New type of nuclear matter, new symmetries, New selection rules. First kind of flavored nuclei.

Hyperons are free from Pauli principle restrictions

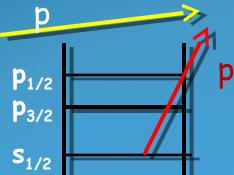
**Can occupy quantum states already filled up with nucleons** 

This makes a hyperon embedded in the nucleus a unique tool for exploring the nuclear structure.

Good probe for deeply bound single particle states.



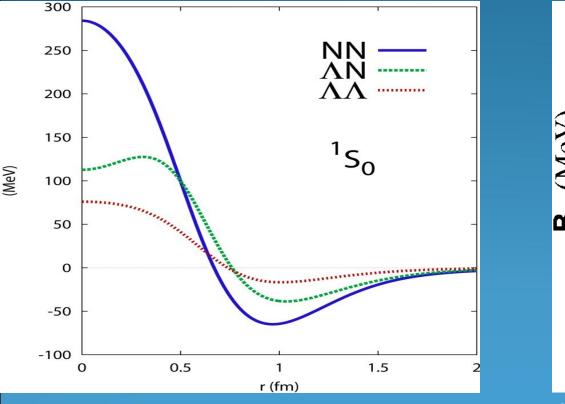
#### Hotchi et al., PRC 64 (2001) 044302

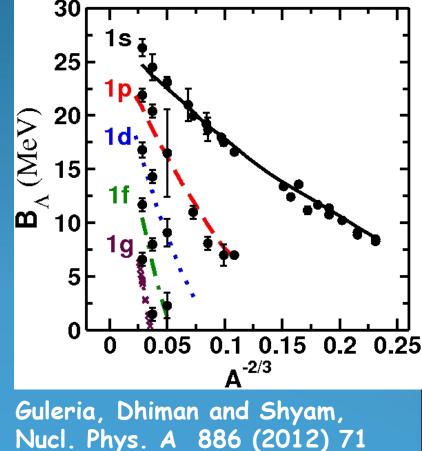


#### S=-1 Hypernuclei: Laboratory for hyperon-Nucleon Interaction

The nuclear structure and the many body nuclear dynamics is extended to new non conventional symmetries.

The Skyrme type  $\Lambda N$  interaction from the known BE of  $\Lambda$  hypernuclei.





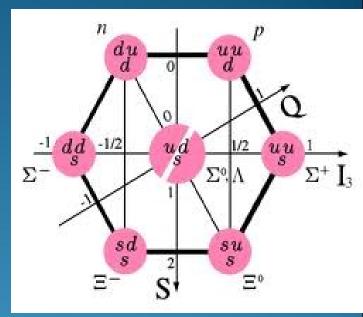
Several other models, DDRH, QMC

## New Physics from the study of S=-2 systems

## Baryon octet

Detailed understanding of the quark aspect of the baryon-baryon forces in the SU(3) space, information on the  $\Xi N$  and  $\Xi \Xi$ channels are essential.

**Spin 1/2** 



 $\Xi$  Bound or quasi-bound states in nuclei,  $\Xi$  hypernuclei

•Search for H particle (uu dd ss) S = -2 system

R. L. Jaffe PRL 38 (1977) 195, bound by about 80 MeV (*N* threshold = 2.231GeV)

Unique object in multi-quark (>3) spectroscopy.

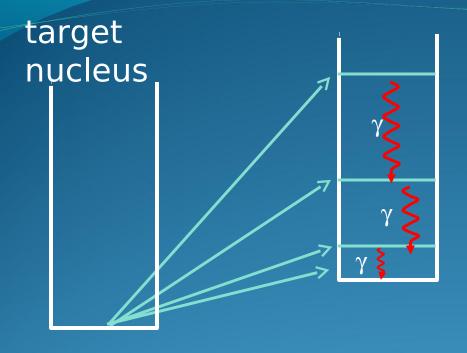
## STRANGENESS IN NEUTRON STARS

Density in the interior of the neutron star ~ few times of  $\rho_0 \longrightarrow$ Hyperon emerge, has profound impact on their structure.

hypernuclei  $\leftarrow \Lambda$ -B Interaction  $\rightarrow$  Neutron Stars quark-hybrid star traditional neutron star N+e N+e+n n,p,e,u hyperon neutron star with S=-2 S=-1 pion condensate superfluid u,d, vy uj crust Fe absolutely stable 10<sup>6</sup> g/cm<sup>3</sup> strange quark -11 matter 10<sup>11</sup> a/cm<sup>3</sup> 10<sup>14</sup> g/cm<sup>3</sup> aito, HYPO6 uds S = -0strange star nucleon star R ~ 10 km M~1.4 M

•Formation of hyperon stars depends on the nature of the YY interaction, compact star, more dense than the normal neutron star

## Hypernuclear production (S=-1)



#### **Strangeness production**

 $(\pi^+, K^+), (\pi^-, K^0)$ BNL,KEK

**Strangeness exchange** 

 $(K^{-}, \pi^{-}), (K^{-}, \pi^{0})$ BNL,KEK

#### **Strangeness deposition**

$$e^+ + e^- \longrightarrow \Phi \longrightarrow K^+ + K^-$$

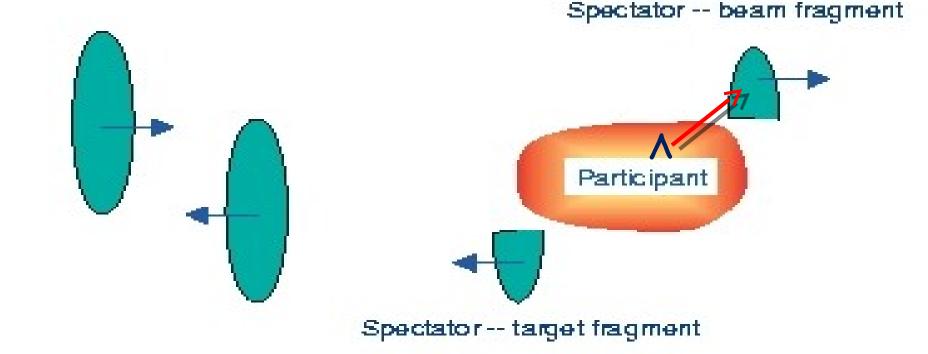
 $K^{-}_{stopped} +^{A} Z \longrightarrow^{A} Z + \pi^{-}$ 

**Electromagnetic production** 

(e,e'K<sup>+</sup>), (γ,K<sup>+</sup>) Jlab, Mainz

#### FINUDA

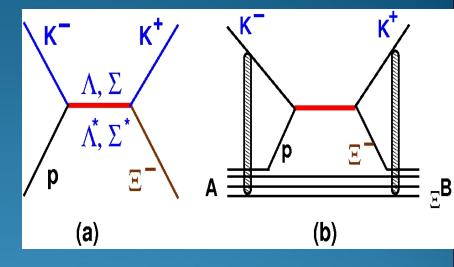
## S=-1 Hypernuclei with Heavy ion beams: The HYPHI Project at GSI Darmstadt



Heavy ion induced reaction at relativistic energies. NN -> LKN : Energy threshold ~ 1.6 GeV. Heavy ion beams with energies > 1.6 GeV/nucleon

## Strangeness S=-2 hypernuclei via (K-,K+) reaction JPARC, Japan

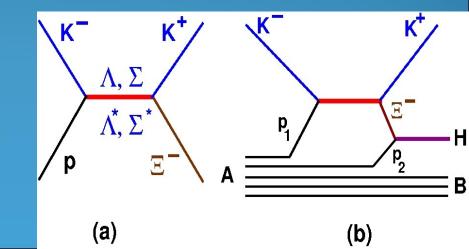
## Production of $\Xi$ hypernuclei



#### Production of $\Lambda\Lambda$ hypernuclei

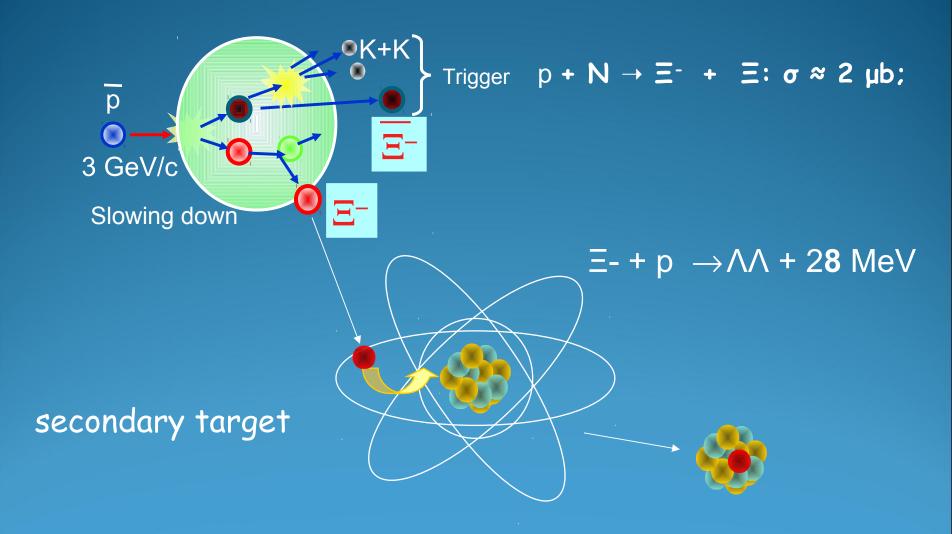
 $\Xi^-$  + p  $\longrightarrow \land \land$  in nuclear orbit

#### Production of H dibaryon

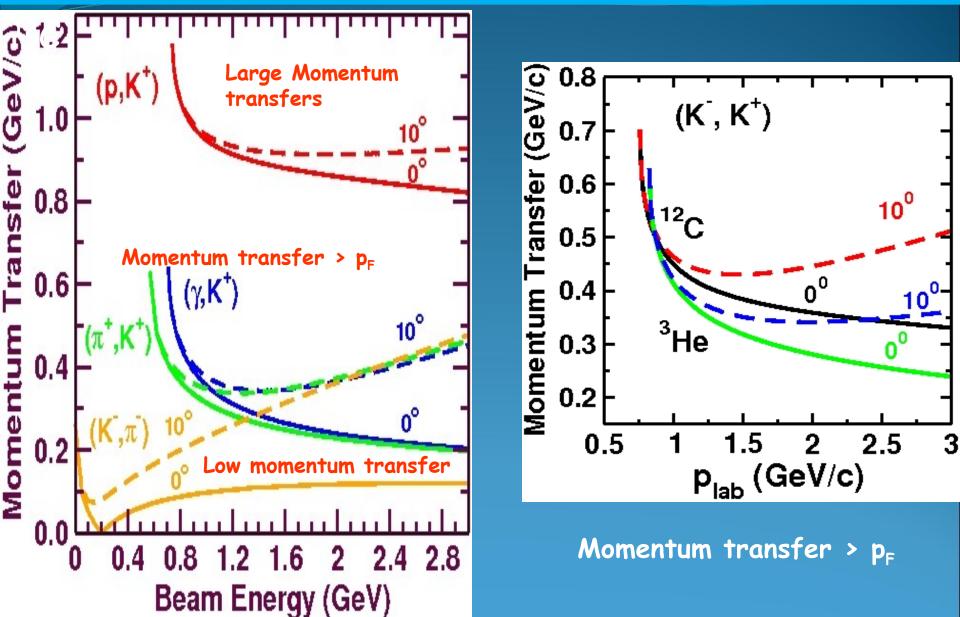


## Double $\Lambda$ -Hypernuclei at PANDA, FAIR

Two step production mechanism



## KINEMATICS



**Description of Hypernuclear Production Reactions** with elementary probes

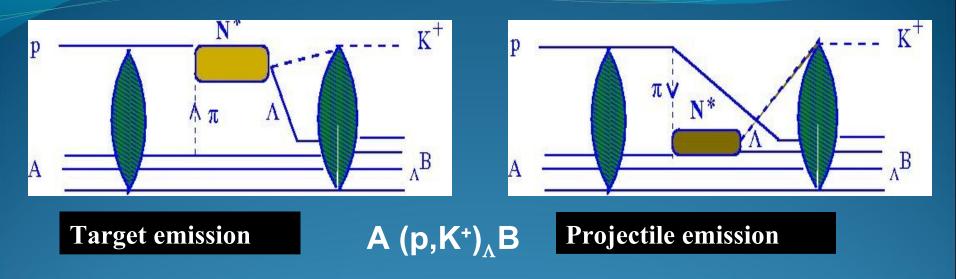
We need

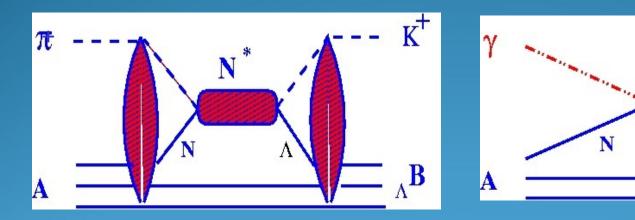
\* Dynamics of the probe  $(\pi +, \gamma, \text{ or } K^-)$ 

Description of the production dynamics

\* Dynamics of the hyperon in the nuclear matter

#### Production processes for reactions leading to S=-1 hypernuclei





A  $(\pi^+, K^+)_{\Lambda}B^*$ 

**Α** (γ,**K**<sup>+</sup>)<sub>Δ</sub>**Β**′

N

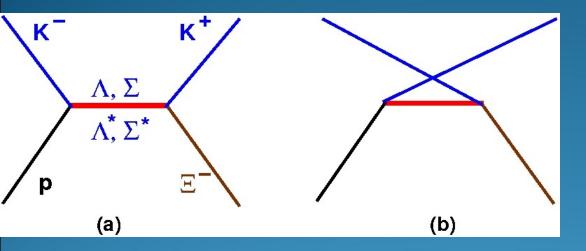
K

<sub>∧</sub>В

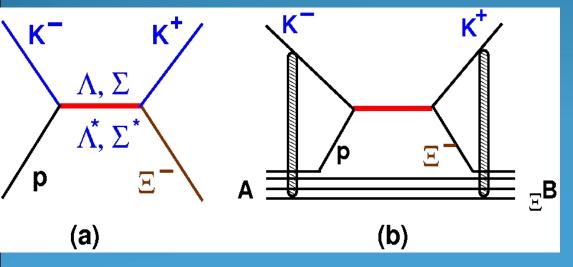
N\* (1650), N\*(1710), N\*(1720) baryonic resonances.

#### Production process of Cascade (S=-2) hypernuclei

(K-,K+) Reaction leads to the transfer of two units of Charge and strangeness



S-channel and u-channel diagrams for elementary reaction



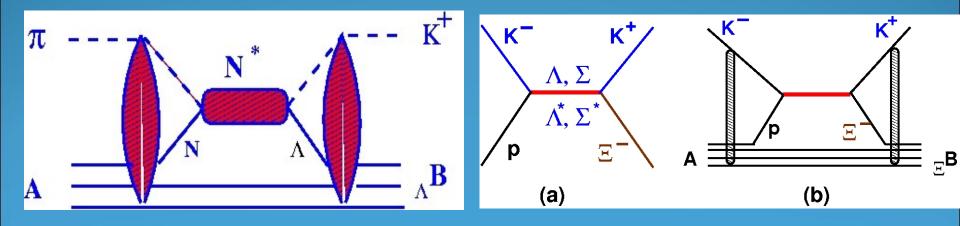
Cascade Hypernuclear production in s-channel

#### Covarient Description of A $(h\gamma, K^+)_{Y}B$ reaction, Effective Lagrangian model

Effective Lagrangians at Meson-baryon-Resonance vertices

Coupling constants, form-factors (from the description of elementary reaction)

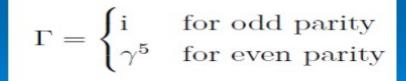
- Propagators for resonances (spin-1/2, spin-3/2)
- Bound state nucleon (hole) and hyperon (particle) spinors
- \* Initial and final state interactions (distorted waves).
- \* Medium modification effects of Resonances



#### Spin-1/2 Resonances

#### Pseudo-vector coupling:

$$\mathcal{L}_{\pi NR}^{PV} = -\frac{g_{\pi NR}}{m_{R} \pm m_{N}} \bar{\psi}_{R} \gamma^{\mu} \Gamma \partial_{\mu} (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_{\pi}) \psi_{N} + h. c. ,$$
$$\mathcal{L}_{RK\Lambda}^{PV} = -\frac{g_{RK\Lambda}}{m_{R} \pm m_{\Lambda}} \bar{\psi}_{R} \gamma^{\mu} \Gamma \partial_{\mu} \phi_{K} \psi_{\Lambda} + h. c. ,$$



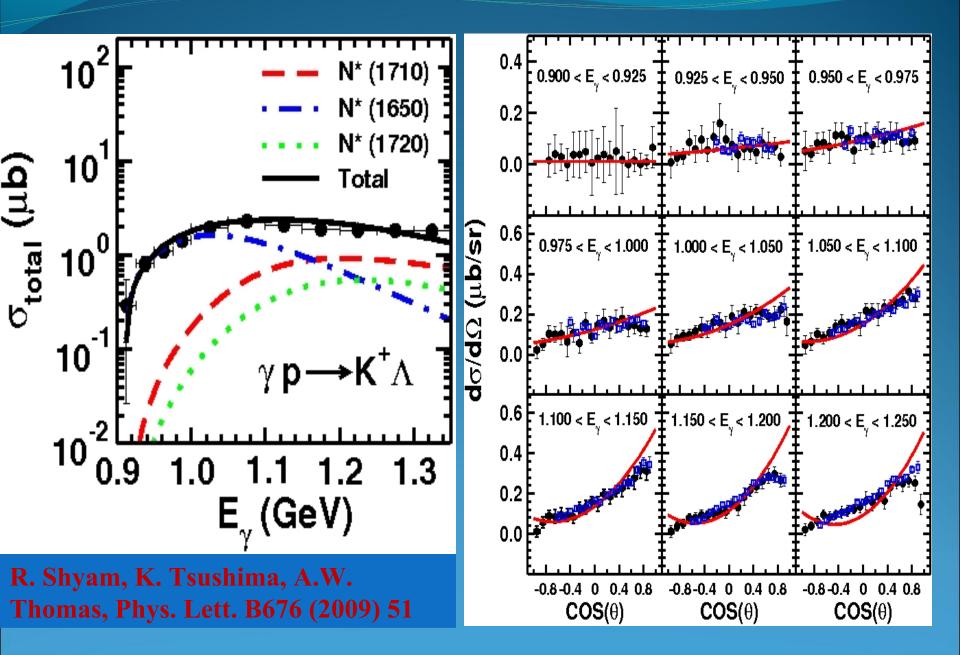
#### Spin-3/2 Resonances

$\mathcal{L}_{\pi NR} = \frac{g_{\pi NR}}{m_{\pi}} \bar{\psi}_{R}^{\mu} \partial_{\mu} (\boldsymbol{\tau} \cdot \boldsymbol{\phi}_{\pi}) \psi_{N} + h. c. ,$	
$\mathcal{L}_{\mathrm{RKA}} = \frac{g_{\mathrm{RKA}}}{m_{\mathrm{K}}} \bar{\psi}_{\mathrm{R}}^{\mu} \partial_{\mu} \phi_{\mathrm{K}} \psi_{\mathrm{A}} + \mathrm{h.~c.} \ .$	

## Coupling constants, form-factors at different vertices

## from the description of elementary reaction

## Effective Lagrangian model for $\gamma p \rightarrow K \Lambda$ reaction

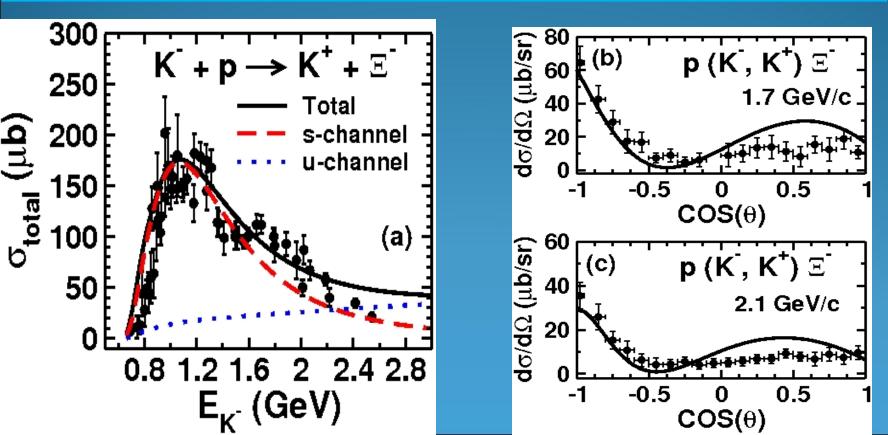


# p (K⁻,K⁺) Ξ⁻

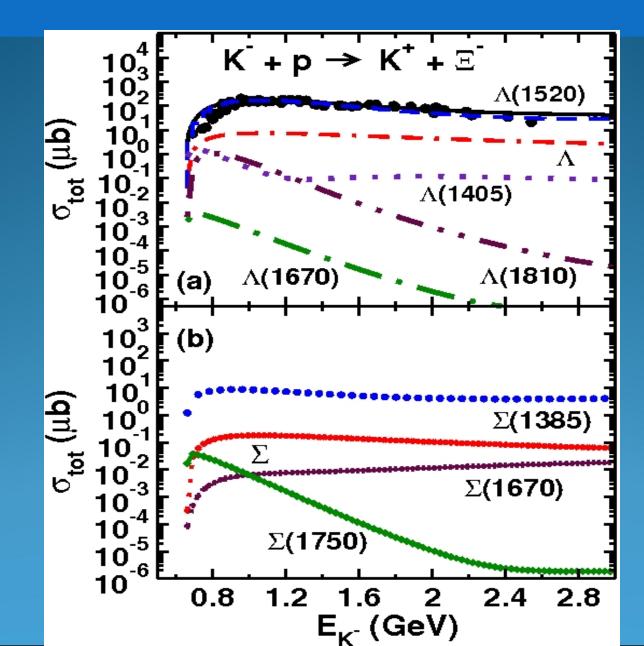
 $\Lambda(1116), \Lambda(1180), \Lambda(1405), \Lambda(1520), \Lambda(1670), \Lambda(1890), \Sigma(1189), \Sigma(1385), \Sigma(1670), \Sigma(1750)$ 

From SU(3) model, old experimental determinations

R. Shyam, Olaf Scholten and A.W. Thomas, Phys. Rev. C84 (2011) 042201(R)



#### Elementary reactions for $\Xi^-$ production, Role of resonances



# BOUND STATE SPINORS

$$\psi(p) = \delta(p_0 - E) \begin{pmatrix} f(k) \mathscr{Y}_{\ell 1/2j}^{m_j}(\hat{p}) \\ -ig(k) \mathscr{Y}_{\ell' 1/2j}^{m_j}(\hat{p}) \end{pmatrix},$$

#### l and j are the quantum numbers of the bound states

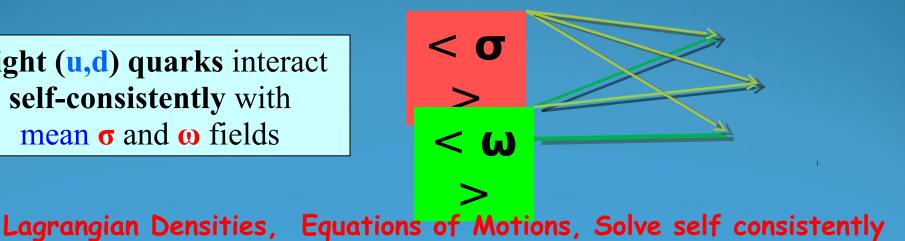
## **Bound States Spinors**

## <u>Phenomenological Model</u>

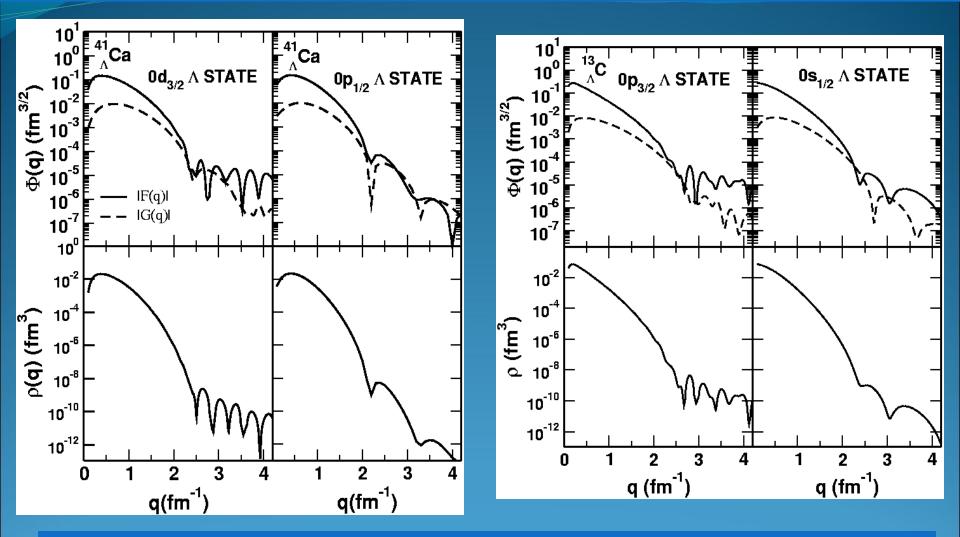
Solve Dirac equation with scalar and vector fields having WS radial shapes, depths are searched to reproduce the BE of a given state.

**OMC model** P.Guichon, Phys. Lett. B 200 (1988) 235 **Review, PPNP 58, 1 (2007)** 

Light (u,d) quarks interact self-consistently with mean  $\sigma$  and  $\omega$  fields



## Bound Hypernuclear wave spinors

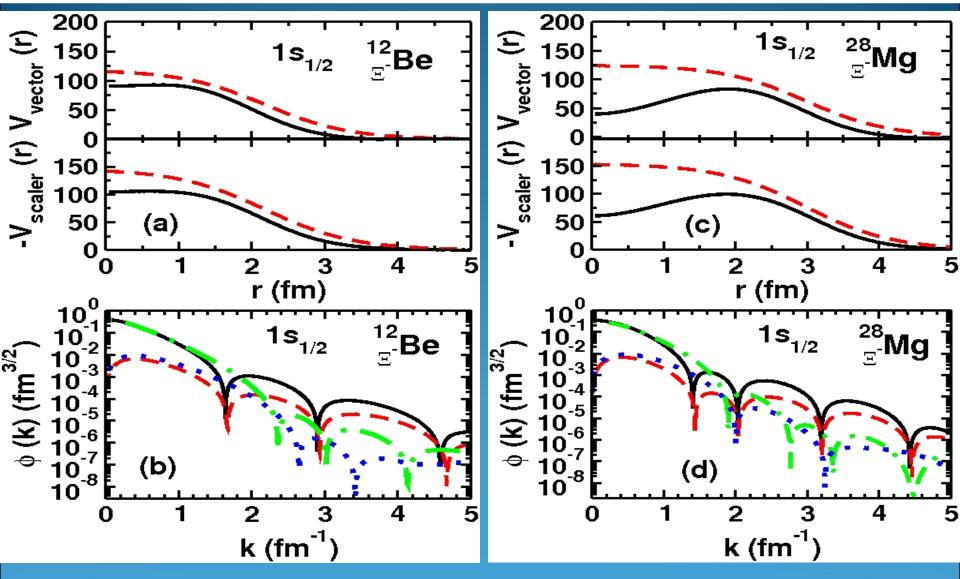


In the region of the momentum transfer of interest, the lower component of the spinor is not negligible.

#### **Cascade bound states**

nomenological

#### quark-meson Coupling models

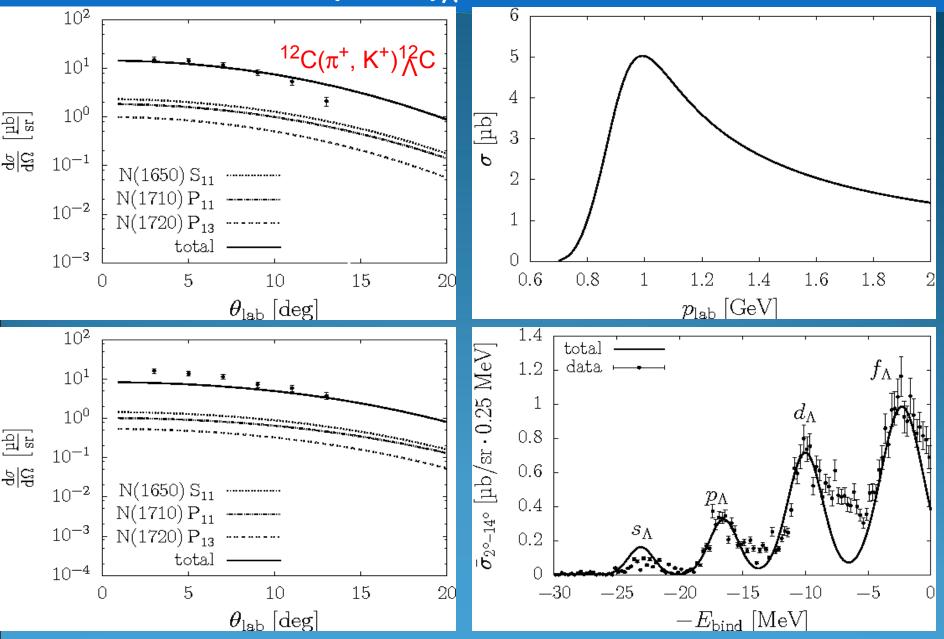


# Some Results for the cross sections $A(\pi^+, K^+)_A B$ reaction

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{\mathrm{K}}} = \frac{1}{16\pi^2} \frac{m_{\mathrm{A}}m_{\mathrm{B}}}{s} \frac{|\boldsymbol{p}_{\mathrm{K}}(E_{\mathrm{K}})|}{|\boldsymbol{p}_{\pi}|} |\mathcal{M}(E_{\mathrm{K}} = \tilde{E}_{\mathrm{K}})|^2$$

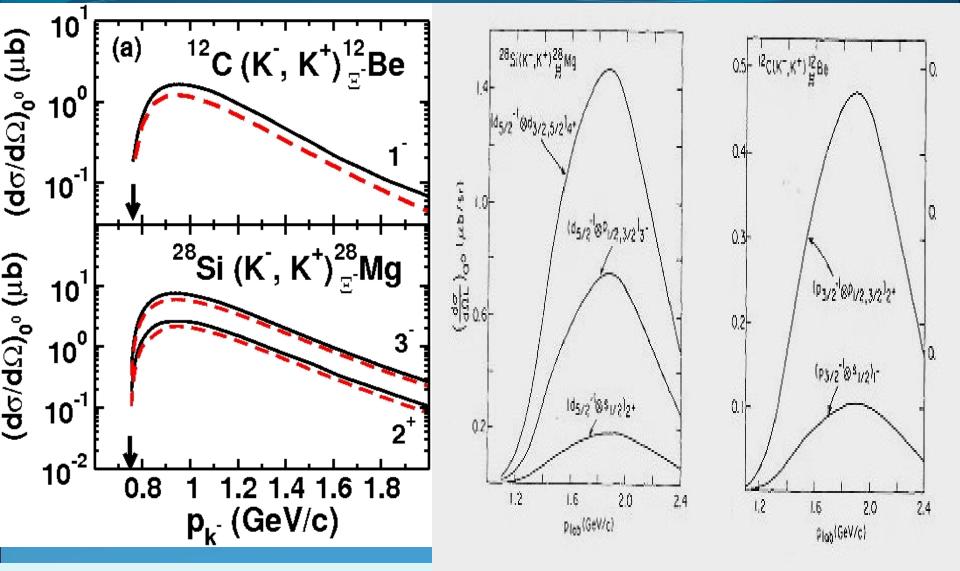
$$\mathcal{M} = \int \frac{\mathrm{d}^4 k_{\mathrm{N}}}{(2\pi)^4} \int \frac{\mathrm{d}^4 k_{\Lambda}}{(2\pi)^4} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \widehat{\phi}_{\mathrm{K}}^*(p-k_{\Lambda}) \bar{\psi}_{\Lambda}(k_{\Lambda}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_1 \,\mathrm{i} \frac{\gamma \cdot p + m_{\mathrm{R}}}{p^2 - m_{\mathrm{R}}^2 + \mathrm{i}\epsilon} \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N}}) \Psi_{\mathrm{N}}(k_{\mathrm{N}}) \Gamma_2 \widehat{\phi}_{\pi}(p-k_{\mathrm{N$$

## $A(\pi^+, K^+)_A B$ reactions



S. Bender, R. Shyam and H. Lenske, Nucl. Phys. A839 (2010) 51

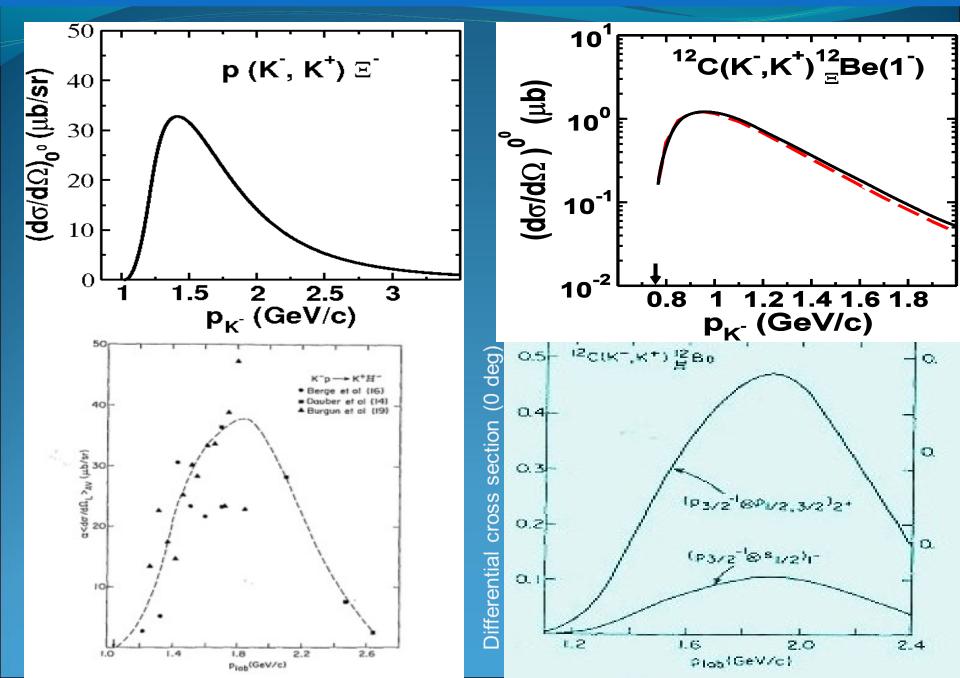
# Cross section for **Z**-hypernuclear production



R. Shyam, K. Tsushima and A.W. Thomas, Nucl. Phys. A 881 (2012) 255

Dover and Gal, Ann. Phys. 146 (1983) 256

#### **Difference between Old and New Results**



# **The H dibaryon**

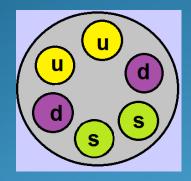
#### Perhaps a Stable Dihyperon\*

PRL 38 (1977) 195

R. L. Jaffe†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,<sup>‡</sup> Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 (Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the  $\Delta(1236)$  bind six quarks to form a stable, flavor-singlet (with strangeness of -2)  $J^P = 0^+$  dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H\*) with  $J^P = 1^+$  at 2335 MeV should appear as a bump in AA invariant-mass plots. Production and decay systematics of the H are discussed.



Compact 6q object (single hadron)
Not loosely bound s-wave states of two baryons like deuteron.

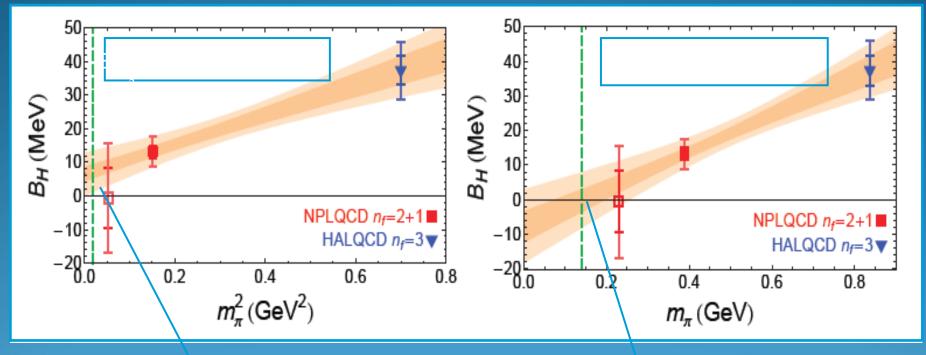
Large binding in Jaffe's prediction is because of the strong attractive color magnetic interaction

**Corrections: center-of-mass motion, pionic cloud etc. reduce this to much lower values** 

## **Recent Lattice QCD results**

NPLQCD:  $m_{\pi} = 390 \text{ MeV}, BH = 13.2 \pm 1.8 \pm 4.0 \text{ MeV}$ 

HALQCD:  $m_{\pi} = 837$  MeV, BH =  $37.4 \pm 4.4 \pm 7.3$  MeV PRL 106, 162001 (2011) PRL 106, 162002 (2011)



PRL 107, 092004 (2011)

BH = 7.4 ± 2.1 ± 5.8 MeV

 $BH = -0.2 \pm 3.3 \pm 7.3 MeV$ 

J.K. Ahn et al., proposal at JPARC

**"Search for H-dibaryon with Large Acceptance Hyperon Spectrometer"** 

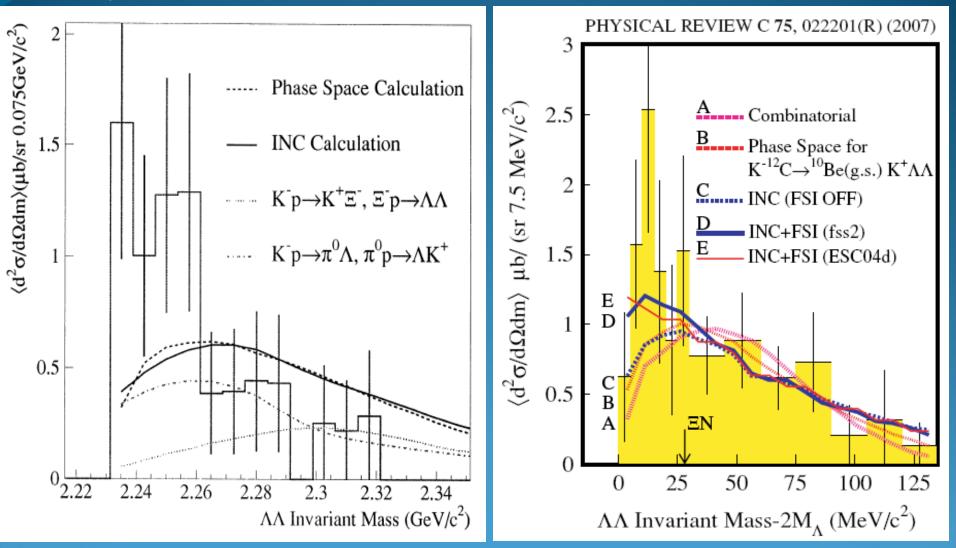
available at the URL

http://nuclpart.kek.jp/pac/1207/pdf/15thPAC.120714.P42.JKAhn.pd f

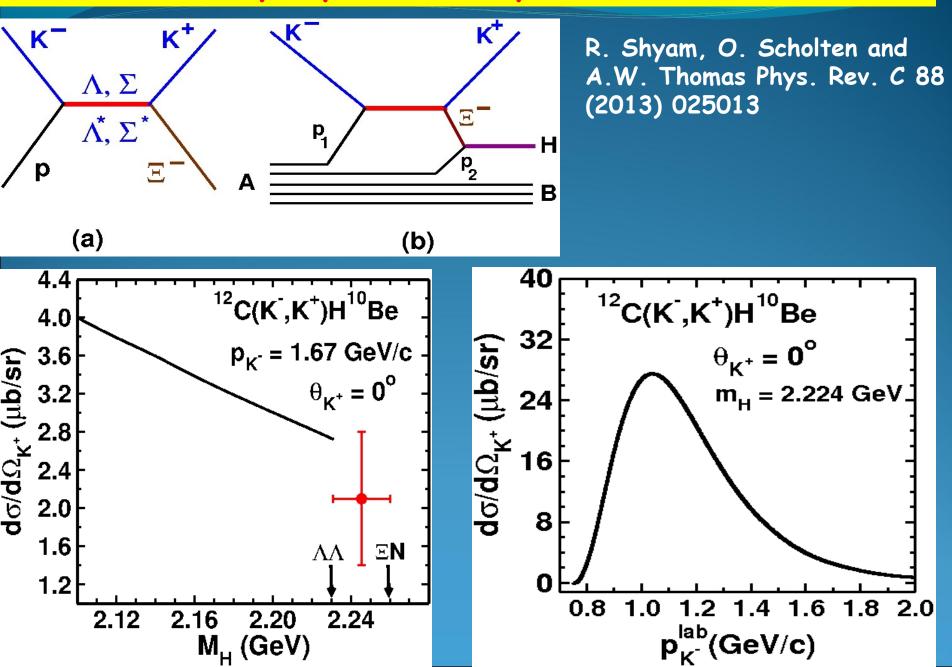
(K<sup>-</sup>, K<sup>+</sup>) reaction on nuclei offers a promising way of producing a H dibaryon system.

## (K<sup>-</sup>, K<sup>+</sup>) reaction on <sup>12</sup>C nucleus

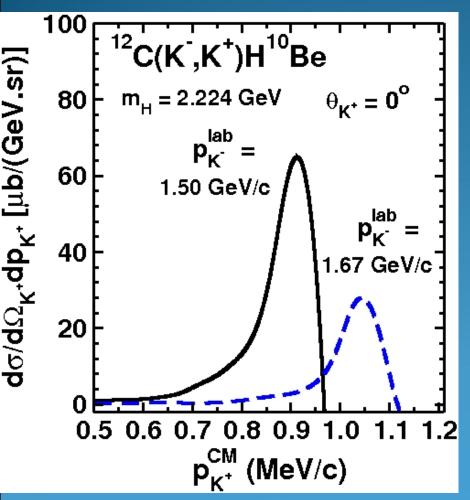
#### Phys. Lett B 444, 267 (1998)

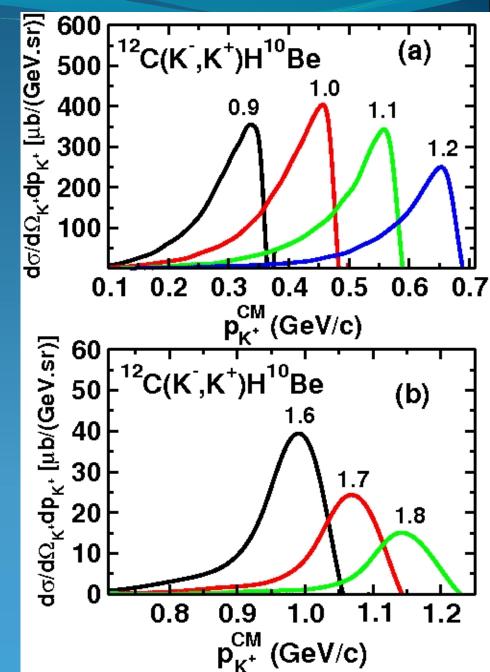


#### H Dibaryon production by $(K^-, K^+)$ reaction



# Momentum spectrum of K<sup>+</sup>





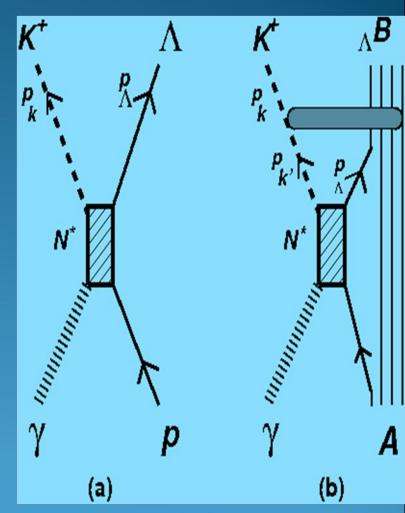
# $(\gamma, K^{+})$ reaction on Nuclei

K<sup>+</sup> is weakly absorbing so reaction occurs deep in the nuclear interior.

A proton is converted into a  $\Lambda$ , produces neutron rich hypernuclei.

Unnatural parity states strongly excited

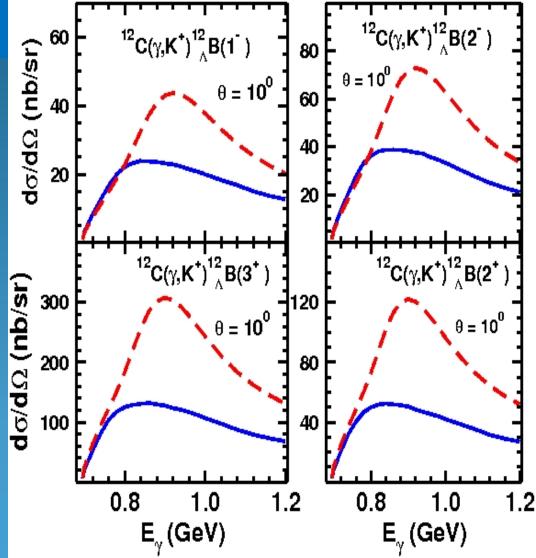
 $\gamma p \rightarrow \Lambda K^+$  reaction well understood within an effective Lagrangian picture



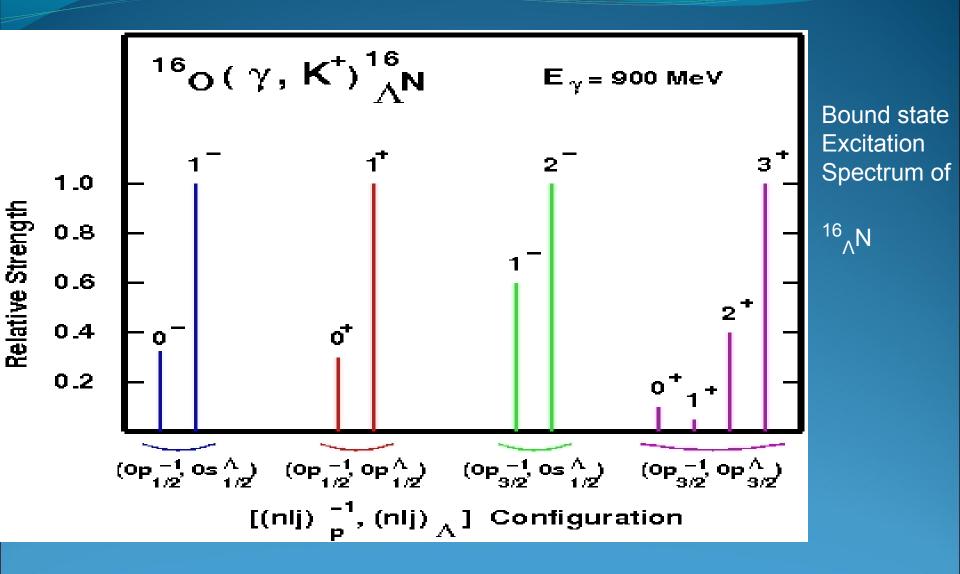
Excitations of N\*(1650), N\*(1710), N\*(1720) resonances.

## Differential cross sections: ${}^{12}C(\gamma, K^+)_{\Lambda}B$

R. Shyam, K. Tsushima, A.W. dơ/dΩ (nb/sr) ⁴ 8 Thomas, Phys. Lett. B676 (2009) 51  $1^{-}, 2^{-} \Rightarrow (1\overline{p}_{3/2}^{p}, 1s_{1/2}^{\wedge})$  $2^+, 3^+ \Longrightarrow (1p_{3/2}^{-p}, 1p_{3/2}^{\wedge})$ 00 00 00/00 00 00 00 Phenomenological model **Quark Meson Coupling model** 0.8 1.0



#### Unnatural parity states of highest J strongly excited



R. Shyam, H. Lenske and U. Mosel, Phys. Rev. C 77, (2008) 052201 (R)

### SUMMARY AND OUTLOOK

 $A(h\gamma, K^+)_A B$  reactions provide mutually complimentary information about the hypernuclear spectrum.

A fully covariant description of these reactions is desirableand is possible.

Tighter constraints on the models of  $\Lambda$ -N interaction

 $(\gamma, K^+), (\gamma^*, K^+)$  strongly excite the unnatural parity stretched states.

A new description of the cascade hypernuclear production via (K<sup>-</sup>, K<sup>+</sup>) reaction that is based on the mechanism of hyperon resonance excitation and decay. New calculations differ significantly from the older one.

New Measurements are needed for some key quantities to resolve the differences between the two calculations.

#### Summary

**1. Cross sections in 12C(K-,K+)H 10Be reactions is found to be more than an order of magnitude larger than those calculated previously on a 3He target.** 

2. At the beam momentum of 1.67 GeV/c, the magnitude of this Cross section for a H mass very close to the = threshold is comparable to the upper limit of the H production cross section estimated in a measurement of the of the 12C(K-,K+) X = = reaction at the same beam momentum

3. Cross section around 1.0 GeV/c beam momentum is expected to be an order of magnitude larger than that at 1.8 GeV/c.

# $\ell_{\text{KBR1/2}} = -g_{\text{KBR1/2}} \Psi_{R1/2} \left[ \bigotimes i \bigotimes \Phi_{K} + ((1-\Im)/M) \square \gamma_{\mu} (\partial^{\mu} \Phi_{K}) \right] \Psi_{B}$

 $\boxtimes = \boxtimes_5$  even parity resonance, 1 odd parity

resonance

 $\boldsymbol{\ell}_{\text{KBR3/2}} = -\boldsymbol{g}_{\text{KBR3/2}} \boldsymbol{\Psi}_{R3/2} \partial_{\mu} \boldsymbol{\Phi}_{K} \boldsymbol{\Psi}_{B} + \boldsymbol{h.c}_{.}$ 

# BOUND STATE SPINORS

$$\psi(p) = \delta(p_0 - E) \begin{pmatrix} f(k) \mathscr{Y}_{\ell 1/2j}^{m_j}(\hat{p}) \\ -ig(k) \mathscr{Y}_{\ell' 1/2j}^{m_j}(\hat{p}) \end{pmatrix},$$

l and j are the quantum numbers of the hyperon bound states

# BOUND STATE SPINORS

### A mean field approach

Dirac Eq. in momentum space

$$p \psi(p) = m_N \psi(p) + F(p)$$

with

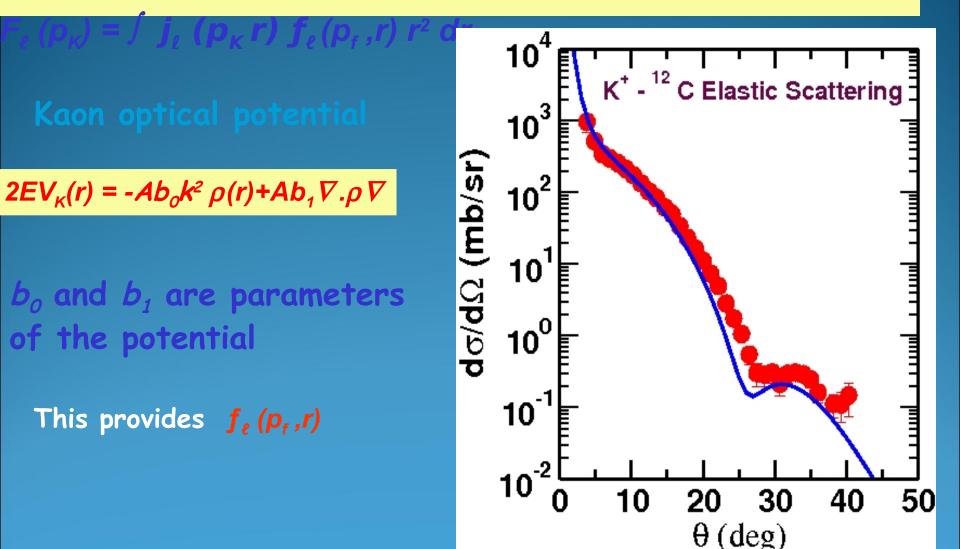
$$\begin{split} F(p) &= \delta(p_0 - E) \left[ \int d^3 p' V_s(-\vec{p}') \psi(\vec{p} + \vec{p}') \\ &- \gamma_0 \int d^3 p' V_v^0(-\vec{p}') \psi(\vec{p} + \vec{p}') \right] \end{split}$$

spinors

$$\psi(p) = \delta(p_0 - E) \begin{pmatrix} f(|\vec{p}|) \mathcal{Y}_{\ell,1/2,j}^{m_j}(\hat{p}) \\ -ig(|\vec{p}|) \mathcal{Y}_{\ell',1/2,j}^{m_j}(\hat{p}) \end{pmatrix}$$

## scattering states

## $\Phi_{pf}(p_{\kappa}) = \delta (p_{\kappa}^{o} - E_{\kappa}) \sum_{\ell m} (-)^{\ell} Y_{\ell m}(p_{f}) Y^{*}_{\ell m}(p_{\kappa}) F_{\ell}(p_{\kappa})$



Intermediate state (R)	LIJ	M (GeV)	Width (GeV)	<b>g</b> <sub>krn</sub>	<b>g</b> <sub>kre</sub>
Λ		1.116	0.0	-16.750	10.132
Σ		1.189	0.0	5.580	-13.500
Λ(1405)	<b>S01</b>	1.406	0.050	1.585	-00.956
Λ(1670)	<b>S01</b>	1.670	0.035	0.300	-00.182
Λ(1810)	<b>P01</b>	1.180	0.150	2.800	02.800
Λ(1890)	<b>P03</b>	1.890	0.100	0.800	00.800
Λ(1520)	<b>D03</b>	1.520	0.016	-27.46	-16.610
Σ(1750)	<b>S11</b>	1.750	0.090	0.500	00.500
Σ(1385)	<b>P13</b>	1.383	0.036	-6.22	-06.220
Σ(1670)	<b>D13</b>	1.670	0.060	2.80	02.800