Gravitational collapse and the quantum
Horizons, Hawking radiation and all that*

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*= don’t expect anything new yet...
1. SEMICLASSICAL collapse and Hawking radiation
   1.1 What is a horizon?
   1.2 What is a particle in QFT?
   1.3 QFT+horizon: Hawking radiation as a tidal effect
   1.4 Backreaction: Hawking radiation as “self-tunneling”
2. BEYOND semiclassical?
   2.1 Classicalization
   2.2 GUPs (from QM to QFT?)
3. Outlook
1) Gravitational collapse

Standard semiclassical picture of gravitational collapse

Classical background: classical matter and “geometrical” space-time*

*Prototype background: 

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \]

Quantum foreground: “radiation”
1) Gravitational collapse

$|0; t = +\infty\rangle$

$e^{-\frac{i}{\hbar} \int \hat{H} \, dt}$

$|0; t = -\infty\rangle$

$|0; t = +\infty\rangle = \sum \text{excitations} = \text{Hawking radiation}$

Q1) Background: horizon?  A1) Trapping horizon

Q2) Foreground: particle?  A2) QFT
1) Gravitational collapse

Semiclassical picture: classical background + quantum foreground

\[ R_{\mu\nu} + \frac{1}{2} R g_{\mu\nu} = 8\pi G \left( T_{\mu\nu} + \langle \hat{T}_{\mu\nu} \rangle \right) \]

In Schwarzschild \( \langle 0_H|\hat{T}_{\mu\nu}|0_H \rangle \sim \frac{1}{M^2} \) finite and "small" down to horizon* (in Unruh vacuum = with radiation)

\[ T_{\mu\nu} \sim 0 \quad \text{Small = globally} \quad \int d^3x \langle \hat{T}_{\mu\nu} \rangle \ll M \]

perturbatively \( \langle \hat{T}_{\mu\nu} \rangle \ll M_p^{-2} \)

Backreaction large when \( M \gg M_p \)

End of story and talk...

*Around a static star \( \langle 0_B|\hat{T}_{\mu\nu}|0_B \rangle \sim \frac{1}{r - 2M} \) (= radiation better than nothing)
1.1) Horizons

Naive concept: where escape velocity = speed of light
In GR: many definitions (often mathematical and hard to figure...)

1) Event horizon: global (teleological) concept

Star interior = regular space
Star exterior = Schwarzschild
1.1) Horizons

Space

Time

Trapping surface: local concept = naive definition

Star interior = regular space

\(\kappa < 0\)

\(\kappa = 0\)

\(\kappa > 0\)

Space

Time

Star exterior = Schwarzschild

\[ds^2 = \gamma_{ij} dx^i dx^j + R^2(x^i) d\Omega^2\]

(Expansion of null geodesics)

\((-\theta_+ \theta_- = \kappa = \gamma^{ij} \partial_i R \partial_j R\) (\(\kappa \sim \dot{R}^2 - 1 \sim R - 2M\))

(\(\dot{R} = \partial_\tau R\))

Areal radius

Proper time of freely falling observer
2) Trapping horizon: (space-null-time) sequence of trapping surfaces

Dynamical black hole
1.2) Particles

Quantum field theory in a nutshell

1) Solve (classical) wave equation:
\[ \Box \Phi = 0 \]

2) Organize solutions into vector (formal Hilbert) space:
\[ \Phi = \sum \phi \left[ a_{\vec{k}} \phi_{\vec{k}} + a^\dagger_{\vec{k}} \phi^*_{\vec{k}} \right] \]
\[ (\phi_{\vec{k}} | \phi_{\vec{k}'} ) = \delta_{\vec{k} \vec{k}'}, \]

3) Lift (normal mode) solutions (excitations) to operators:
\[ a_{\vec{k}} \mapsto \hat{a}_{\vec{k}} \quad a^\dagger_{\vec{k}} \mapsto \hat{a}^\dagger_{\vec{k}} \]

4) Build (probabilistic Hilbert) Fock space of quantum states:
\[ |\vec{k}; n \rangle \propto \left( \hat{a}^\dagger_{\vec{k}} \right)^n |\vec{k}, 0 \rangle \quad \hat{a}_{\vec{k}} |\vec{k}, 0 \rangle = 0 \]

So particles = field excitations or...?
1.2) Particles

Naive concept: particle = localized object

Example: free scalar field in 1+1 (Fourier transform in “formal solutions”)

\[ \phi_k = e^{-i \omega t + i k x} \quad \omega^2 = k^2 + m^2 \]

Time evolution preserves (physically sensible) “packets”:

\[ \Phi = \sum_{a \ few} \phi_k \]

Initial packet

Later packet!

\( t = 0 \)  \hspace{1cm} \text{localized object}

\( t = 1 \)  \hspace{1cm} \text{localized object}
1.2) Particles

The Newton-Wigner operator

1-particle states:

\[ |\phi\rangle = \int_{-\infty}^{+\infty} \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2\omega}} e^{-i\omega t} \phi(p) \hat{a}^\dagger_p |0\rangle \]

\[ \omega = \sqrt{\vec{p} \cdot \vec{p} + m^2} \]

\[ \langle \phi | \psi \rangle = \int_{-\infty}^{+\infty} \frac{d^3 p}{(2\pi)^{3/2} 2\omega} \phi^*(p) \psi(p) \equiv (\phi, \psi) \]

Position operator:

\[ \hat{Q}_i = i\hbar \left( \frac{\partial}{\partial p_i} - \frac{p_i}{2\omega^2} \right) \]

\[ \left[ \hat{Q}_i, \hat{Q}_j \right] = 0 \]

\[ \left[ \hat{Q}_i, \hat{P}_j \right] = i\hbar \delta_{ij} \]

\[ \left[ \hat{Q}_i, \hat{J}_j \right] = i\epsilon_{ij}^k \hat{Q}_k \]

Wave-packet

SO(3,1)

\[ \hat{P}^\mu = p^\mu \]

\[ \hat{J}_i = -i\epsilon_{ij}^k p^j \frac{\partial}{\partial p^k} \]

\[ \hat{K}_i = i\omega \frac{\partial}{\partial p^i} \]
1.2) Particles

Example: free scalar field in 1+1

1-particle at rest in x=0:
\[ \phi_0(p) = N e^{-\frac{p^2}{2\Delta^2}} \]  
\[ (\phi_0, \hat{Q} \phi_0) = 0 \]

1-particle at rest in x=\bar{x} :
\[ \phi_{\bar{x}} = e^{-i \bar{x} \hat{P}} \phi_0 \]  
\[ (\phi_{\bar{x}}, \hat{Q} \phi_{\bar{x}}) = \bar{x} \]

1-particle with speed \( \beta \):
\[ \phi_{\beta} = e^{+i \beta \hat{K}} \phi_0 \]  
\[ (\phi_{\beta}, \hat{P} \phi_{\beta}) = \beta \omega \]

“Orthogonality”:

\[ (\phi_0, \phi_{\bar{x}}) \approx e^{-\frac{\bar{x}^2}{4\ell^2}} \left[ 1 + \mathcal{O}(\Delta^2/m^2) \right] \]
\[ \ell = \Delta^{-1} \]
\[ \lambda_m = m^{-1} \]

\[ (\phi_0, \phi_{\beta}) \approx e^{-\beta^2 \frac{m^2}{\Delta^2}} \]

But: what if...?
1.2) Particles

Example: free scalar field in 1+1

2-particle states:

\[ |\phi, \psi\rangle = \int_{-\infty}^{+\infty} \frac{dp, dq}{4\pi \sqrt{\omega(p)\omega(q)}} e^{-i [\omega(p)+\omega(q)] t} \phi(p) \psi(q) \hat{a}_p \hat{a}_q^\dagger |0\rangle \]

\[ \langle \phi, \psi | \hat{Q} | \phi, \psi \rangle = \int_{-\infty}^{+\infty} \frac{dp}{4\pi \omega(p)} \phi^*(p) \left[ \hat{Q} \phi(p) \right] \]

\[ + \int_{-\infty}^{+\infty} \frac{dq}{4\pi \omega(q)} \phi^*(q) \psi(q) \times \int_{-\infty}^{+\infty} \frac{dp}{4\pi \omega(p)} \psi^*(p) \left[ \hat{Q} \phi(p) \right] \]

"Should vanish...?"

Interacting theory does not preserve particle number...

And: what if...?
Counter-example: toy scalar field in 1+1

\[ \phi_k = e^{-i \omega t + i k x - k^* |x|} \quad \omega^2 = k^2 + m^2 \]

Time evolution does not preserve “packets”:

\[ \Phi = \sum_{\text{a few}} \phi_k \]
Quantum field theory in curved space-time

5) Usually, normal modes are **not** plane waves (everywhere)

   Packets not preserved
   (particles lose identity)

6) Normal modes (excitations) depend on observer

   Fock space depends on observer
   (observation dependent vacuum)
   (+possible non-unitary issue)
7) **Horizon** = “boundary condition”: only ingoing modes exist

\[
\phi_{\tilde{k}} \sim e^{-i \omega t - ik \rho}
\]

\[
\phi_k \sim e^{-i \omega t \pm ik \rho}
\]

**Tidal forces** cause in-falling packets to lose outgoing modes

[W. Unruh, PRD 51 (1985) 2827]

+ occurs to quantum collapsing shells: R.C. et al, PRD 64 (2001) 104012
1.3) QFT + Horizons

\[ u = \infty \]
\[ u < \infty \]
\[ u = t - r \]
\[ v = t + r \]
\[ u < v_0 \]
\[ v > v_0 \]
\[ v < v_0 \]

OUT Fock space:
\[ \phi_\omega \sim \begin{cases} e^{4 M i \omega \ln(v_0-v)} & v < v_0 \\ 0 & v > v_0 \end{cases} \]

Bogoliubov transformations:
\[ \beta_\omega \omega' \sim \int_{-\infty}^{v_0} \phi_\omega \psi_{\omega'} \, dv \]
\[ N_\omega = \sum_{\omega'} |\beta_\omega \omega'|^2 \sim \frac{1}{e^{8 \pi M \omega} - 1} \]

IN Fock space:
\[ \psi_\omega \sim \begin{cases} e^{-i \omega u} & u < 1 \\ e^{-i \omega v} & v > 1 \end{cases} \]

(asymptotic modes + boundary condition) determine “particle” content

Modes pile up (tidal effect)

u increases BR to TL
v increases BL to TR
a) Trans-Planckian problem:

\[ N_\omega = \sum_{\omega'} |\beta_{\omega \omega'}|^2 \sim \frac{1}{e^{8\pi M \omega} - 1} \]

UV cut-off or modified dispersion relations?

Classically \( \omega_H \sim M^{-1} \) spreads over \( d_H \sim M \): “averaged blue-shift”?

[R.C., CQG 19 (2002) 2453]

b) Finite frequency blue-shifts to trans-Planckian near horizon:

\[ \omega_r \sim \frac{\omega_\infty}{\sqrt{r - 2M}} \]

Point-like particle

Quantum mechanical tunneling for particles:

\[ \phi_k \sim e^{-i\omega t \pm ikr} \]

\[ \omega < V_0 \]

Transmission amplitude:

\[ p = \sqrt{2m(E - V_0)} \]

(classically forbidden path)
Horizon quantum tunneling (or particle self-tunneling)

Quantum mechanical tunneling across horizon does not work

Without backreaction

Transmission probability: \( P \sim e^{-\beta_H \omega} \)

With backreaction

(“Particle opens its own exit door”)

\( \beta_H = 8 \pi M \)
2) Semiclassical and beyond?

**Semiclassical gravitational collapse:**

1) Trapping horizons Hawking radiate (tidal effect)

2) Hawking radiation = backreaction

**Beyond semiclassical gravitational collapse:**

1) Collapsing matter is quantum

2) Gravity is quantum?
The hoop conjecture (Thorne, 1972):

A black hole forms whenever the impact parameter $b$ of two colliding objects (of negligible spatial extension) is shorter than the radius of the would-be-horizon (roughly, the Schwarzschild radius, if angular momentum can be neglected) corresponding to the total energy $M$ of the system, that is for

$$b \lesssim \frac{2 \ell_p M}{m_p}$$

Classicalization (Dvali, 2010):

At high (~Planckian) energy, quantum particle scatterings lead to formation of “classicalons” and quantum degrees of freedom disappear (no UV divergences). For gravity, “classicalons” = black holes = BEC of gravitons
Example: Gaussian packets

\[ \rho_{\pm}(x, y) = \frac{\rho_0}{\pi \ell^2} \exp \left\{ -\frac{(x \pm b)^2 + y^2}{\ell^2} \right\} \]
\[ = \frac{\rho_0}{\pi \ell^2} \exp \left\{ -\frac{r^2 \pm 2 br \cos(\theta) + b^2}{\ell^2} \right\} = \rho_{\pm}(r, \theta) \]

(Spherically symmetric)

mass function:

\[ M(r) = \frac{4\pi}{3} \int_0^r \rho(t, \bar{r}) \bar{r}^2 d\bar{r} \]

(Outer) horizon!

\[ 2M(r) = r \]

No BH!

\[ r = 2\ell_p \frac{M(r)}{M_p} \]
2.1) Classicalization?

1) **Threshold** for BH formation:
   (Packet width)

\[ M_0 \simeq M_p \left( \frac{\ell}{\ell_p} \right) \]

2) **Particle interpretation:**

IN or OUT?
(Full dynamics will tell...?)
“Classicalization” \(\iff\) Formation of **classical bound** state
(gravity is always practically classical)

**QFT on a self-consistently evolved classical background**

[M. Reuter, PRD 57 (1998) 971]

or

**Semiclassical QFT propagators**

[R.C. arXiv:0806.0501v3]

or

**Space-time non-commutativity**

[Nicolini and Spallucci]

or

**Generalized Uncertainty Principle**

[F. Scardigli, PLB 452 (1999) 39 and many others]
“Measuring very short lengths requires much energy: a BH is produced and precision reduces”

$$\Delta L = L_{QM} + L_G = 2 \left( \ell_p \frac{L \ m_P}{M} \right)^{1/2} + \ell_p \frac{M}{m_P}$$

$$\frac{\Delta L_{\text{min}}}{L} \approx \left( \frac{\ell_p}{L} \right)^{2/3}$$

Minimum length!
2.2) GUP in QFT?

\[ \Delta x \gtrsim \frac{\mathcal{L}_P m_P}{\Delta p} + \alpha \mathcal{L}_P \frac{\Delta p}{m_P} \]

**Deformed Poincare algebra**

[M. Maggiore, PRD 49 (1994) 5182]

\[ \hat{X}_i = \hbar \cosh \left( \frac{p_0}{2\kappa} \right) \frac{\partial}{\partial p_i} - \frac{p_i}{8\kappa^2 \sinh^2(p_0/2\kappa)} \]

\[ [\hat{X}_i, \hat{X}_j] = -\frac{\hbar^2}{4\kappa^2} i \varepsilon_{ij}^k \hat{J}_k \]

\[ [\hat{X}_i, \hat{P}_j] = i \hbar \delta_{ij} \cosh \left( \frac{p_0}{2\kappa} \right) \]

**Modified canonical QFT**

[V. Husain et al, arXiv:1208.5761]

\[ [\hat{x}, \hat{p}] = i \hbar f \left( \frac{\hat{p}}{2\kappa} \right) \quad \text{QM} \]

\[ [\hat{\phi}_k, \hat{\pi}_k] = i \hbar f \left( \frac{\hat{\pi}_k}{2\kappa} \right) \quad \text{QFT} \]

**pro)** Modified NW operator

**con)** What about fields (particle states)?

**pro)** Workable approach

**con)** What about localization?
2.2) GUP in QFT?

Deformed Hilbert space

“Physical fields could be differentiable functions which possess merely a finite density of degrees of freedom.”

\[ \frac{\Box \psi}{\psi} \sim E^2 - p^2 < \Lambda^2 \]

Covariant cut-off* (virtual states)

*Trans-Planckian modes do not sample...
Graviton exchanges
(Sum over graphs)

\[ \sum \]

\[ G_{\text{Minkowski}}(k) \]

Gravity = quantum spin-2 field

Propagation on curved space

\[ G_{p-k}(k) \]

Gravity = classical geometry

Does gravity ever go quantum?

2.2) GUP in QFT?
Does gravity ever go quantum?
3) Outlook

a) we do not yet understand black hole formation at quantum level (scattering of quantum particles)
b) we do not understand what happens with scatterings about the Planck scale
c) we do not understand what happens with black hole evaporation about the Planck scale
d) given a)-c), do we really understand gravitational collapse at all?*

Let’s try GUP from QM to QFT as a working tool

*Trapping horizon likely starts forming at Planckian scale from the core and expands...
Pauli, long ago [around 1930], suggested that gravity could act as a regulator for the UltraViolet divergences* that plague Quantum Field Theory by providing a natural cut-off at the Planck scale. Later on, classical divergences in the self-mass of point-like particles were indeed shown to be cured by gravity [Arnowitt, Deser and Misner, Phys. Rev. Lett. 4 (1960) 375], and the general idea has since then resurfaced in the literature many times.
In spite of that, Pauli’s ambition has never been fulfilled.

* same as classicalization
Thanks!