The QCD static potential at NLO

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Introduction

static potential

$\rightarrow$ dominant interaction between heavy q-$\bar{q}$ at low energy

$T = 0$

- attractive
- coulomb-like at small r (linearly rising at large r)

$T \neq 0$

- the short-distance potential screened
- yukawa-like with screening mass $\propto T$

proposed signal for QGP formation:

suppression of heavy q-$\bar{q}$ bound state production at high T

T. Matsui and H. Satz, Phys. Lett. B **178**, *416-422 (1986).*

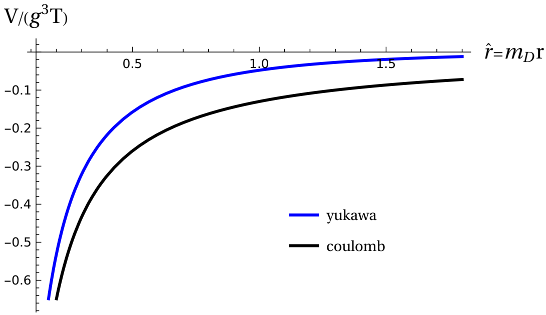
$C_F = (N_c^2 - 1)/(2N_c)$

debye mass $= m_D = gT\hat{m}_D$ with $\hat{m}_D^2 = (N_c + N_f/2)/3$

$$\mathrm{Re}[V_{coulomb}] = -\frac{g^2 C_F}{4\pi}\frac{1}{r}$$

$$\mathrm{Re}[V_{yukawa}] = -\frac{g^2 C_F}{4\pi}\frac{e^{-m_D r}}{r}$$

$$\mathrm{Re}[V_{nlo}] = ?$$



dissociation

idea: if Im[V] $\sim$Re[V] where screening becomes important

$\rightarrow$ bound states disappear because decay *(become wide resonances)*

- not because V is screened *(too shallow to support them)*

M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP **03**, *054 (2007).*

to study this we need to calculate the imaginary potential at nlo

calculational method

MEC, C. Manuel and J. Soto, Phys. Rev. Lett. 134, 011905 (2025)

- thermalized plasma

- $M_q \gg$ all other physical scales

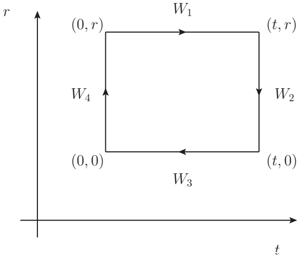
$\rightarrow$ static $q\bar{q}$ are (unthermalised) probe particles

- *we considered bottomonium* $M = 4676\ MeV$

- coulomb gauge

- dimensional regularization

potential obtained from real time QCD (rectangular) wilson loop



$W(t,r) = \frac{1}{N_c}\left\langle \mathcal{P}\exp\left(ig\int A_\mu(z)dz^\mu\right)\right\rangle$

$V(r) = \lim_{t\to\infty}\frac{i}{t}\ln[W(t,r)]$

in coulomb gauge $G_{0i}=0 \to$ in limit $t\to\infty$ lines on sides set to 1

$q\bar{q}$ couple to A_0 on 11 branch of CTP contour

lo real potential

contract A_0 on W_1 and W_3

$$W_{1a}(t,r) = \frac{(ig)^2}{N_c}\left\langle \int_0^t dx_0 A_0(x_0,\vec{r}) \int_t^0 dy_0 A_0(y_0,\vec{0}) \right\rangle$$

use $\langle A_0(x_0,\vec{r})A_0(y_0,\vec{0})\rangle = -iG_{00}(x_0-y_0,\vec{r}) \equiv -iG(x_0-y_0,\vec{r})$

$$W_{1a}(t,r) = -ig^2 C_F \int_0^t dx_0 \int_0^t dy_0 \int \frac{d^4p}{(2\pi)^4} e^{-i(p_0(x_0-y_0)-\vec{p}\cdot\vec{r})} G(p_0,\vec{p})$$

identity: $\lim_{t\to\infty} \int_0^t dx_0 \int_0^t dy_0\, e^{ip_0(x_0-y_0)} = t\,2\pi\delta(p_0) + \mathcal{O}(t^0)$

$$\begin{aligned} V(r) &= g^2 C_F \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{r}} G_{\rm htl}(0,p) \\ &= -g^2 C_F \int \frac{d^3p}{(2\pi)^3}\, \frac{e^{i\vec{p}\cdot\vec{r}}}{p^2+m_D^2} = -\frac{\alpha C_F}{r} e^{-m_D r} \end{aligned}$$

constant contribution

contract two A_0's on W_1 (or W_3)

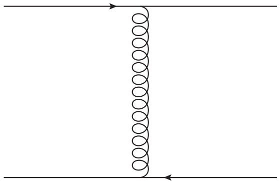
$$W_{1b}(t) = 2ig^2 C_F \int_0^t dx_0 \int_0^{x_0} dy_0 \int \frac{d^4p}{(2\pi)^4} e^{-ip_0(x_0-y_0)} G(p_0,\vec{p})$$

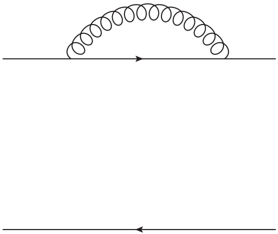
$\rightarrow \qquad V = -\alpha C_F m_D$

combining we have

$$\mathrm{Re}\,[V_{\mathrm{lo}}(r)] = -\alpha C_F \left(\frac{e^{-m_D r}}{r} + m_D \right)$$

corresponding figures:





LO complex potential

time ordered propagator (using: $n(p_0) = 1/(e^{p_0/T} - 1) \to \frac{T}{p_0} + \ldots$)

$$G(p_0, \vec{p}) = \frac{1}{2}\left(G^{\rm ret}(p_0, \vec{p}) + G^{\rm adv}(p_0, \vec{p}) + (1 + 2n(p_0))\left[G^{\rm ret}(p_0, \vec{p}) - G^{\rm adv}(p_0, \vec{p})\right]\right)$$

htl propagator: $G_{\rm lo}(0, p) = -\frac{1}{m_D^2 + p^2} + \frac{i\pi T m_D^2}{p(p^2 + m_D^2)^2}$

$$V_{\rm lo}(\vec{r}) = -g^2 C_F \int \frac{d^3 p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{r}} G_{\rm lo}(0, p) + \text{const}$$

coordinate space potential is ($\hat{r} = r m_D$)

$$\mathrm{Re}[V_{\rm lo}] = -\frac{g^2 C_F}{4\pi} m_D \left(1 + \frac{e^{-\hat{r}}}{\hat{r}}\right)$$

$$i\mathrm{Im}[V_{\rm lo}] = i\frac{g^2 C_F T}{4\pi}\left(\frac{2 I_2(\hat{r})}{\hat{r}} - 1\right)$$

defined dimensionless integrals $I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \sin(\hat{p}\hat{r})\, (\hat{p}^2 + 1)^{-j}$

dissociation

recall:

bound states disappear because decay *(become wide resonances)*

- not because V is screened *(too shallow to support them)*

Q: what is the scale where we expect this to happen?

propagator: $G_{\rm lo}(0,p) = -\frac{1}{m_D^2+p^2} + \frac{i\pi T m_D^2}{p(p^2+m_D^2)^2}$

if $p \sim g^a T$ then $\mathrm{Re}[\tilde{V}_{1LO}(p)] \sim \frac{g^{2-2a}}{T^2}$ and $\mathrm{Im}[\tilde{V}_{1LO}(p)] \sim \frac{g^{4-5a}}{T^2}$

for a resonance to exist need $\mathrm{Im}[\tilde{V}] < \mathrm{Re}[\tilde{V}] \Rightarrow 0 < a < 2/3$

$a = 2/3$ parametrically scale we expect quarkonium to dissociate

real time static potential beyond LO in equilibrium

we consider $p \sim g^a T$ with $1/2 < a < 2/3 \Rightarrow m_D \ll p \ll T$

• upper bound on p: from condition $\mathrm{Re}\,\tilde{V}_{\mathrm{lo}}(p) \sim \mathrm{Im}\,\tilde{V}_{\mathrm{lo}}(p)$

$\rightarrow$ bound state decays

• lower bound on p: require p "semi-hard"

- calculation of next-to-leading order potential is simplified

consequences: $V(r)$ valid for $r\, m_D \ll 1 \ll r\,T$

1st part: large r $\rightarrow$ momenta smaller than the semi-hard scale

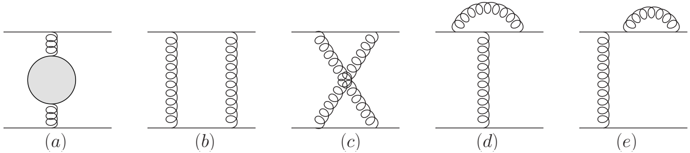
2nd part: small r $\rightarrow$ momenta larger than the temperature motivation

- a check of the idea of quarkonium dissociation
- provides wider set of physically motivated forms of the potential

- to use as input for methods to extract V from lattice correlators

how to calculate static potential beyond leading order

- expand W to higher order in g
- dress the propagator in the LO contribution



- iterate the LO potential (not shown)

determine how to dress lines/vertices for $p \sim g^a T$ with $\frac{1}{2} < a < \frac{2}{3}$

comment about power counting

calculation of NLO HTL n-point functions

$\rightarrow$ follow prescription ...

for the static potential there are two important differences

1. fermion lines have the form $\frac{1}{p_0 \pm i\eta}$ ($M_q \gg$ all other scales)
2. external frequencies are taken to zero

$\Rightarrow$ external momenta don't flow through the diagram

** power counting is different from standard thermal field theory

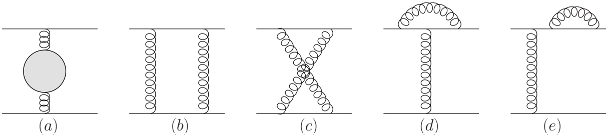


diagram (a) - include corrections to HTL self-energy

- power correction to HTL gluon bubble

- one loop gluon bubble with loop momenta semi-hard

- can be done with bare lines and vertices

- bose-einstein distributions $\sim T/p_0$ *since* $p \ll T$

- no quark loop (pauli blocking)

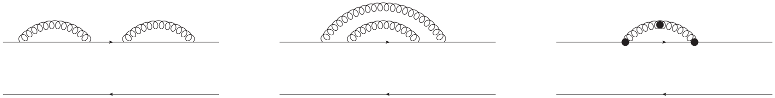
diagrams $(bcde) =$ ladder diagrams

- HTL propagators and bare vertices

also: additional contributions from static quark self-energies

$\rightarrow$ *constant contributions that we have not calculated*

- *method to include these contributions explained in a minute*



- we take into account corrections to LO

real part: larger than g^2 & imag part: larger than g^{2-a}

- denominators $\sim p^2 + m_D^2$ kept unexpanded *(damped approximation)*

$\rightarrow$ extends region that coordinate space potential is valid

NLO momentum space potential

$$\mathrm{Re}[\tilde{V}_2^{(0)}(p)] = -\frac{g^4 N_c C_F}{16\pi}\frac{1}{p^2+m_D^2}\left(1-\frac{3\pi^2}{16}\right)\frac{T}{m_D}$$

$$\mathrm{Re}[\tilde{V}_{1\mathrm{HTL}}(p)] = \frac{g^4 p T N_c C_F}{4\left(m_D^2+p^2\right)^2}$$

$$\mathrm{Re}[\tilde{V}_2^{(1)}(p)] = \frac{g^4 T N_c C_F}{16}\frac{m_D}{3\pi\left(m_D^2+p^2\right)^2}\left(1-\frac{\pi^2}{16}\right)$$

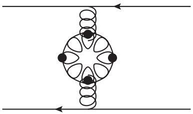
$$i\mathrm{Im}[\tilde{V}_{1\mathrm{HTL}}(p)] = -\frac{7ig^4 T^2 N_c C_F}{6\pi\left(m_D^2+p^2\right)^2}$$

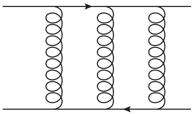
$$i\mathrm{Im}[\tilde{V}_{1\mathrm{POW}}(p)] = \frac{ig^4 p T C_F\left(2N_c-N_f\right)}{8\pi\left(m_D^2+p^2\right)^2}$$

$$i\mathrm{Im}[\tilde{V}_2^{(1)}(p)] = \frac{g^4 T N_c C_F}{16}\frac{iTm_D}{p\left(m_D^2+p^2\right)^2}\left(1-\frac{3\pi^2}{16}\right).$$

verify consistency of the calculation $(p \sim g^a T$ and $1/2 < a < 2/3)$

the biggest contributions that we did not calculate





HTL corrections: $\text{Re}[V] \sim g^{6-5a}$ and $\text{Im}[V] \sim g^{6-6a}$

2 loop contributions: $\text{Re}[V] \sim g^{4-2a}$ and $\text{Im}[V] \sim g^{4-2a}$

real part: for $0 < a < 2/3$ the biggest neglected contro is $\sim g^{4-2a}$

from previous slide: smallest calculated (kept) is $\text{Re}[V_2^{(1)}] \sim g^{5-4a}$

must require $4 - 2a > 5 - 4a \to a > 1/2$

imaginary part gives $a > 1/3 \Rightarrow$ combining $1/2 < a < 2/3$

coordinate space potential beyond leading order

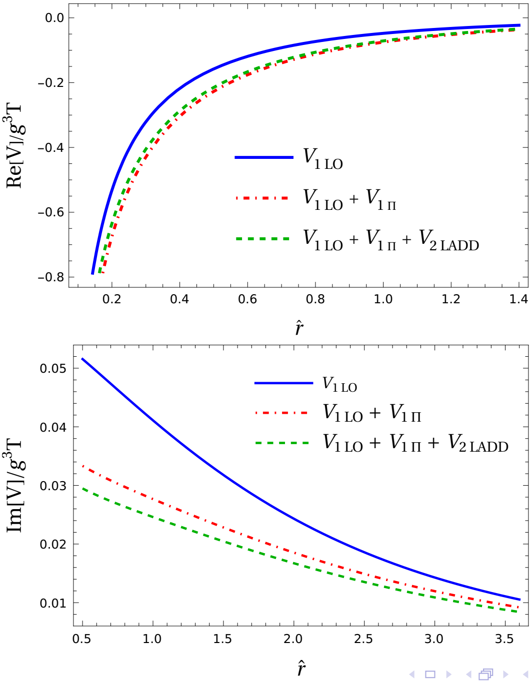
$\hat{r} = r m_D$ and $m_D = g T \hat{m}_D$

$I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \sin(\hat{p}\hat{r})\, (\hat{p}^2 + 1)^{-j}$

$$V_{1\mathrm{lo}} = -\frac{g^2 C_F}{4\pi\hat{r}}\left(m_D e^{-\hat{r}} - 2iT\, I_2(\hat{r})\right)$$

$$\mathrm{Re}[V_{\mathrm{nlo}}] = \frac{g^4 N_c C_F T}{64\pi^2\hat{r}}\left\{8\left(I_2(\hat{r}) - I_1(\hat{r})\right) + \frac{e^{-\hat{r}}}{16}\left(3\pi^2 - 16 + \frac{\hat{r}}{6}\left(16 - \pi^2\right)\right)\right\}$$

$$i\mathrm{Im}[V_{\mathrm{nlo}}] = -i\frac{g^3 C_F\, T}{16\pi^2\hat{m}_D}\left\{\frac{3\pi^2 - 16}{32\,\hat{r}} I_2(\hat{r}) + \frac{7}{3}\, N_c e^{-\hat{r}} - \frac{2g\,\hat{m}_D}{\pi\hat{r}}\left(N_c - \frac{N_f}{2}\right)\left(I_1(\hat{r}) - I_2(\hat{r})\right)\right\}$$


Re[V]/g^3T
0.0
−0.2
−0.4
−0.6
−0.8
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\hat{r}$
$V_{1\,\mathrm{LO}}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi} + V_{2\,\mathrm{LADD}}$
Im[V]/g^3T
0.05
0.04
0.03
0.02
0.01
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$\hat{r}$
$V_{1\,\mathrm{LO}}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi} + V_{2\,\mathrm{LADD}}$

soft contributions

want to include contributions from $p \sim m_D$ *that we haven't calculated*

- since $pr \sim m_D r \sim g^{1-a} < 1 \rightarrow$ can expand the exponential

$$V_{\rm soft}(r) = \int \frac{d^3p}{(2\pi)^3} \left(1 + i\vec{p}\cdot\vec{r} - \frac{1}{2}(\vec{p}\cdot\vec{r})^2 + \cdots\right) \tilde{V}(p)$$

keep terms that are $\geq$ smallest contributions in analytic result

odd powers zero by symmetry in an isotropic system

- add contributions:

$\text{Re}[V] = C + g^3 q_0 T$

$\text{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$

coefficients obtained by fitting to lattice results

C is a global constant that adjusts the origins of the energies a check: expanded momentum space potential

$$V_{2\,\exp}(p) = -\frac{g^4 C_F N_c T}{16\pi m_D p^2}\Bigg\{\left[1-\frac{3\pi^2}{16}+\frac{4\pi m_D}{p}+\frac{m_D^2}{p^2}\left(\frac{5\pi^2}{24}-\frac{4}{3}\right)\right] + i\frac{\pi T m_D}{p^2}\left[\frac{56}{3\pi}-\left(1-\frac{3\pi^2}{16}\right)\frac{m_D}{p}-\left(1-\frac{N_f}{2N_c}\right)\frac{4p}{\pi T}\right]\Bigg\}$$

fourier transform

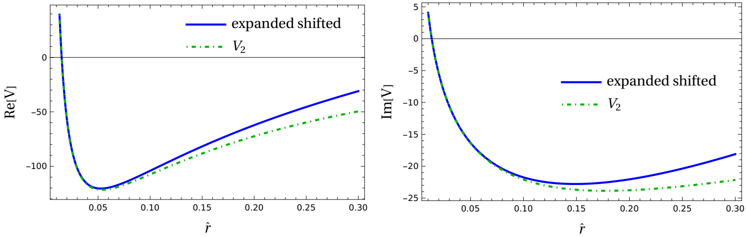
$$V_{2\,\exp}(r) = -\frac{g^4 C_F N_c T}{16\pi m_D}\Bigg\{\left[\left(1-\frac{3\pi^2}{16}\right)\frac{1}{4\pi r}-\frac{m_D}{\pi}L(r)-\left(\frac{5\pi^2}{24}-\frac{4}{3}\right)\frac{r m_D^2}{8\pi}\right] - i\pi T m_D\left[\frac{7r}{3\pi^2}-\left(1-\frac{3\pi^2}{16}\right)\frac{m_D r^2}{24\pi^2}(1-L(r))-\frac{1}{\pi^3 T}\left(1-\frac{N_f}{2N_c}\right)L(r)\right]\Bigg\}$$

$L(r) = -1/\epsilon + \gamma + \log\left[\pi(r\mu)^2\right]$

$\rightarrow 1/\epsilon$ poles could be absorbed into the parameters of $V_{2\,\mathrm{soft}}$

- damped approximation regulates these poles

- reshuffles part of the soft contribution into the semi-hard one figure shows NLO potential and expanded approximation to it - shifted so it matches the NLO potential at $r = 0.01$ fm



also note: constant terms that correspond to contributions from the quark self-energies do not need to be calculated because they are absorbed into the constants (q_0, i_0)

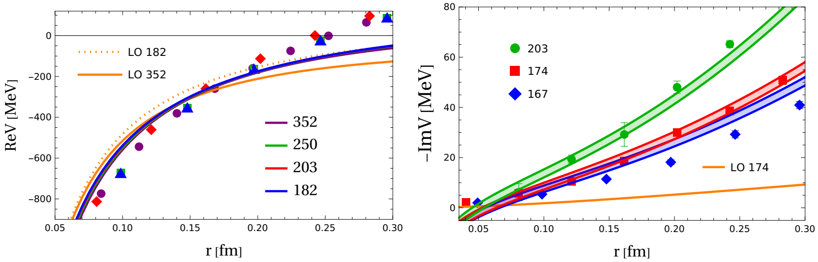
lattice calculation: A. Bazavov, D. Hoying, O. Kaczmarek, R. N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf and J. H. Weber, [arXiv:2308.16587 [hep-lat]].

use $g = 1.8$ from fit to $T = 0$ lattice data

find (C, q_0, i_0, i_2) with fit to all available T and $r \in (0.02, 0.3)$ fm

- real part of potential varies little with T (like data)
- imaginary part gets big contro from soft region

– solid bands are uncertainties in fitted coefficients inherited from lattice data



physical effects of NLO corrections to the potential

correct method to include dissipation

- couple system to a bath and do a partial trace over bath dof

simpler ad-hoc way to include dissipation in SE *(Landau&Lifschitz QM vol 3)*

- phenomenological method to describe unstable, decaying states
- imaginary part of V related to the average lifetime of the state

schrödinger equation is

$$i\hbar\frac{\partial\Psi(t,\vec{r})}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(t,\vec{r}) + V(\vec{r})\Psi(t,\vec{r})$$

separate: $\Psi(t,\vec{r}) = \varphi(t)\psi(\vec{r}) = e^{-i\theta t}\psi(\vec{r}) = e^{-i(E-i\Gamma/2)t}\psi(\vec{r})$

note: $\Psi^\dagger(t,\vec{r})\Psi(t,\vec{r}) = e^{-\Gamma t}\psi^\dagger(\vec{r})\psi(\vec{r})$

$\Rightarrow$ Γ has interpretation of a damping rate

- write the laplacian in spherical coordinates
- assume spherically symmetric potential $\rightarrow \psi(\vec{r}) = Y_{l0}(\theta, 0) R_{nl}(r)$
- define $u_{nl}(r) = r\, R_{nl}(r)$
- separate real/imag $V(r) = V_R(r) + iV_I(r)$ and $u(r) = u_R(r) + iu_I(r)$

$$-\frac{\hbar^2}{2m}\left(\frac{d^2u_R(r)}{dr^2} - \frac{l(l+1)}{r^2}u_R(r)\right) + [V_R(r)u_R(r) - V_I(r)u_I(r)] = Eu_R(r) + \frac{\Gamma}{2}u_I(r)$$

$$-\frac{\hbar^2}{2m}\left(\frac{d^2u_I(r)}{dr^2} - \frac{l(l+1)}{r^2}u_I(r)\right) + [V_R(r)u_I(r) + V_I(r)u_R(r)] = -\frac{\Gamma}{2}u_R(r) + Eu_I(r)$$

$$-\frac{\hbar^2}{2m}\left(\frac{d^2u_R(r)}{dr^2}-\frac{l(l+1)}{r^2}u_R(r)\right)+[V_R(r)u_R(r)-V_I(r)u_I(r)]=Eu_R(r)+\frac{\Gamma}{2}u_I(r)$$

- should solve two coupled equations
- will make the approximation that we can separate re/im parts
- use Re$[V]$ to find binding energy and wavefunction

$$-\frac{\hbar^2}{2m}\left(\frac{d^2u_R(r)}{dr^2}-\frac{l(l+1)}{r^2}u_R(r)\right)+V_R(r)u_R(r)=Eu_R(r)$$

- calculate the decay width as expectation value $\Gamma=-2\langle\mathrm{Im}[V]\rangle$

use $M_Q = 2m$ and define the scaled variables

$$\bar{r} = r/a \quad \text{with} \quad a = 4\pi/(g^2 C_F M_Q)$$
$$\hat{V} = M_Q a^2 V \quad \text{and} \quad \hat{E} = M_Q a^2 E$$

schrödinger equation takes the form

$$-\frac{d^2 u(\bar{r})}{d\bar{r}^2} + \frac{l(l+1)}{\bar{r}^2} u(\bar{r}) + \hat{V}(\bar{r}) u(\bar{r}) = \hat{E} u(\bar{r})$$

centrifugal term dominates when $r \to 0$

boundary conditions $u(\bar{r}_0) = \bar{r}_0^{l+1}$ and $u'(\bar{r}_0) = (l+1)\bar{r}_0^{l}$

method:

maximum of the potential is its value as $\bar{r} \to \infty$

minimum is found numerically

$\to$ solve using trial eigenvalue $E_{\rm try} = (E_{\rm min} + E_{\rm max})/2$

- search for a normalizable solution with the right # nodes

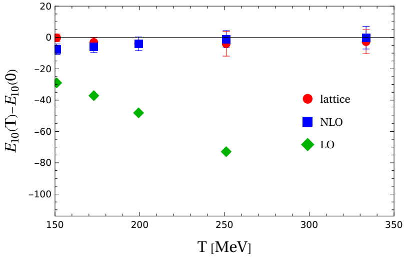
adjust the trial energy either up or down based on

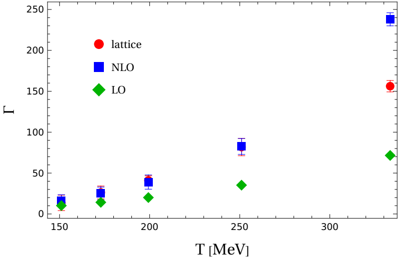
- solution diverges to positive or negative infinity
- # number of nodes is even or odd

lattice calculation : *R. Larsen, S. Meinel, S. Mukherjee and P. Petreczky, Phys. Lett. B* **800**, *135119 (2020).*

find soft coefficients by fitting to all available temperatures

- error bars from fitting to upper/lower values





⇒ reasonable description of data for both $E_{\rm bind}$ and Γ

fitted soft contribution

recall: contribution to $V(r)$ from $p \sim m_D$

$\mathrm{Re}[V] = C + g^3 q_0 T$

$\mathrm{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$

find values of coefficients by fitting to 2 sets of lattice data

in our calculation all scales are explicit

$\rightarrow$ expect same size for all numerical coefficients

$(q_0, i_0, i_2) = (0.027, \quad -0.019 \pm 0.001, 0.194 \pm 0.002)$

$(q_0, i_0, i_2) = (0.044 \pm -0.002, -0.026 \pm 0.009, 0.052 \pm 0.002)$

i_2 from the first fit is significantly larger

C=219 MeV from first calculation

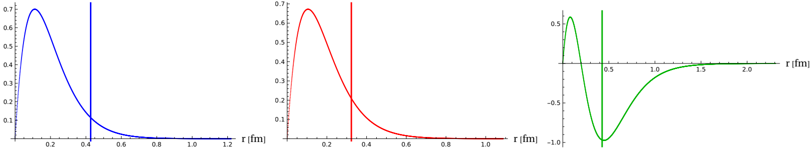
in second the coulomb binding energy is subtracted (C plays no role) dissociation:

- bound states disappear because decay (become wide resonances)
- not because V is screened *too shallow to support them*

Q: how to define dissociation temperature in our calculation?

$\sim$ *temperature where adjacent peaks merge ...*

but: problem with excited states



T=151 MeV and n=1 | T=200 MeV and n=1 | T=151 MeV and n=2

vertical line is value of r for which $rm_D = 1$

alternate definition:

$T_{\rm diss} \approx$ temperature where ground state $E_{\rm bind} = \Gamma = -2\langle {\rm Im} V \rangle$

- define $E_{\rm bind}$ as eigenvalue of V with threshold set to 0

lo result: $T_{\rm diss} = 193.2$ MeV

nlo result: $T_{\rm diss} = 151.8 \pm 1.2$ MeV $\leftarrow$ using first fit

*** unphysical result from outlying value for i_2*

nlo result: $T_{\rm diss} = 225 \pm 10$ MeV $\leftarrow$ using second fit

improvement 1

'self-energy' part of the potential from $G = -\frac{1}{p^2+\Pi_{00}}$

real part of self-energy is $\lim_{p_0\to 0}\Pi_{00} = m_D^2 - \frac{1}{4}g^2 pTN_c$

$$\text{expansion:}\quad G = -\frac{1}{p^2+m_D^2} - \frac{1}{p^2+m_D^2}\left[\frac{1}{4}g^2 pTN_c\right]\frac{1}{p^2+m_D^2}$$

$\rightarrow$ fourier transform can easily be done analytically

alternative: do fourier transform on the unexpanded potential

$$V(r) = -g^2 C_F \int \frac{d^3p}{(2\pi)^3}\frac{e^{i\vec{p}\cdot\vec{r}}}{p^2+m_D^2-\frac{1}{4}g^2 pTN_c}$$

$\rightarrow$ extend the region of validity of the potential

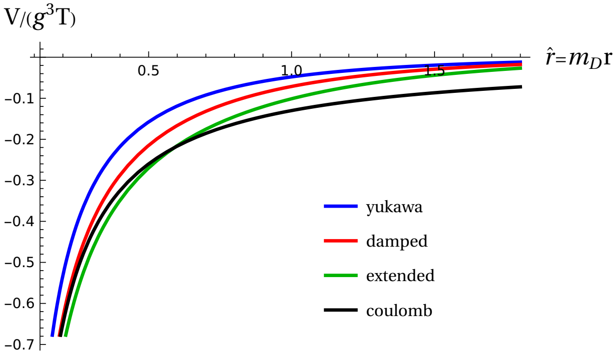
- *same idea as the yukawa potential*

effect:

yukawa potential is screened (shallower) than coulomb

nlo damped potential is deeper (promotes binding)

'extended' potential is even deeper



improvement 2

our method to solve SE:

solve real part of equation for E and $\psi_R(r)$

$\rightarrow$ find width $\Gamma = -2\langle \mathrm{Im} V \rangle$

more accurately: solve real/imag parts of SE self-consistently

$$-\frac{\hbar^2}{2m}\left(\frac{d^2 u_R(r)}{dr^2} - \frac{l(l+1)}{r^2} u_R(r)\right) + [V_R(r)u_R(r) - V_I(r)u_I(r)] = E u_R(r) + \frac{\Gamma}{2} u_I(r)$$
$$-\frac{\hbar^2}{2m}\left(\frac{d^2 u_I(r)}{dr^2} - \frac{l(l+1)}{r^2} u_I(r)\right) + [V_R(r)u_I(r) + V_I(r)u_R(r)] = -\frac{\Gamma}{2} u_R(r) + E u_I(r)$$

$\rightarrow$ search 2d space for (E,Γ) so $(u_R(r),\ u_I(r))$ are normalizable preliminary results:

both changes slightly increase the dissociation temperature
most of the effect is from improvement 1

$T_{\rm diss} = 171 \pm 2$ MeV $\longleftarrow$ from first fit
$T_{\rm diss} = 231 \pm 8$ MeV $\longleftarrow$ from second fit

conclusions

- calculated beyond-lo corrections to momentum space potential
- when the typical momentum transfer p satisfies $m_D \ll p \ll T$
- relevant region to obtain dissociation T for heavy quarkonium
- we include soft contributions $p \lesssim m_D$
- have universal form because we can expand exponential in f-transform
- coefficients from fitting to lattice data
- reasonable description of lattice data (LO fails)
- *identify an inconsistency between 2 different sets of lattice data*
- results provide useful inputs for the Bayesian methods required in the effort to determine the potential from euclidean lattice data