Deconfinement and Equation of State in QCD

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- Introduction (Symmetries of QCD, Computational challenges etc.)
- Deconfinement and Color Screening
- QCD Equation of State in the Continuum Limit at Zero Net Baryon Density
- Taylor expansion: Equation of State at Non-Zero Baryon Density
- Taylor expansion: Fluctuations and Correlations of Conserved Charges and Deconfinement
- Conclusions
Lattice QCD at \( T > 0 \) now and then

Lattice QCD calculations at \( T > 0 \) around 2002:

\[
T_c \approx 173 \text{MeV}
\]

for both chiral transition and deconfinement transition (in terms of Polyakov loop)

Problems:

\[
N_\tau = 4 : a \equiv 1/(N_\tau a) = 1/(4T)
\]

\[
m_\pi = (500 - 800) \text{MeV}
\]

Continuum limit and physical masses are needed

\[
N_\tau \to \infty
\]

\[
m_\pi = 140 \text{MeV}
\]

\[
\text{costs} \sim N_\tau^{11}
\]

\[
\sim 1/m_\pi^3
\]

2014: Calculations using Highly Improved Staggered Quark (HISQ) formulations

⇒ Largely reduced discretization effects, continuum extrapolation possible

\[
m_\pi = 160 \text{MeV}
\]

Fluctuations of conserved charges: new look into deconfinement and QGP properties
Symmetries of QCD at T>0

- **Chiral symmetry**: \( m_{u,d} \ll \Lambda \)
  
  \[ SU_A(2) \text{ symmetry } \psi \rightarrow e^{i\phi T^a \gamma_5} \psi \quad \psi_{L,R} \rightarrow e^{i\phi L,R T^a} \psi_{L,R} \]
  
  \[ \langle \bar{\psi}\psi \rangle = \langle \bar{\psi}_L \psi_R \rangle + \langle \bar{\psi}_R \psi_L \rangle \neq 0 \]
  
  \[ \langle \bar{\psi}\psi \rangle = 0 \]

  \( U_A(1) \) is broken by anomaly

- **Center (Z3) symmetry**: invariance under global gauge transformation
  
  \[ A_\mu(0, x) = e^{i2\pi N/3} A_\mu(1/T, x), \quad N = 1, 2, 3 \]

  Exact symmetry for infinitely heavy quarks
  
  \[ \langle L \rangle = 0 \]

  Polyakov loop:
  
  \[ L = \text{tr} P e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})} \]

  LQCD calculations with staggered quarks suggest crossover, e.g. Aoki et al, Nature 443 (2006) 675

  Evidence for 2\(^{nd}\) order transition in the chiral limit
  
  \[ SU_A(2) \sim O(4) \]

  relation to spin models

  \( U_A(1) \) restoration ?

  Center symmetry does not seem to play any role in QCD
The notion of the transition temperature is only useful if it can be related to the critical temperature in the chiral limit: fit the lattice data on the chiral condensate with scaling form + simple Ansatz for the regular part

\[ M_b = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} = h^{1/\delta} f_G(t/h^{1/\beta \delta}) + f_{M, reg}(T, H) \]

\[ f_{reg}(T, H) = (a_1(T - T_c^0) + a_2(T - T_c^0)^2 + b_1)H \]

\[ t = \frac{1}{t_0} \left( \frac{T - T_c^0}{T_c^0} + \frac{\mu_q^2}{T^2} \right), \quad H = \frac{m_l}{m_s}, h = \frac{H}{h_0} \]

6 parameter fit: \( T_c^0, t_0, h_0, a_1, a_2, b_1 \)

\[ T_c = (154 \pm 8 \pm 1(\text{scale})) \text{MeV} \]
Deconfinement and color screening

Onset of color screening is described by Polyakov loop

\[ L = \text{tr} \mathcal{P} e^{ig \int_0^{1/T} d\tau A_0(\tau, \vec{x})} \]
\[ \exp(-F_{Q\bar{Q}}(r, T)/T) = \frac{1}{9} \langle \text{tr} L(r) \text{tr} L^\dagger(0) \rangle \]

\[ F_{Q\bar{Q}}(r \to \infty, T) = 2F_Q(T) \]

\[ \Rightarrow L_{\text{ren}} = \exp(-F_Q(T)/T) \]

2+1 flavor QCD, continuum extrapolated:

Free energy of static quark anti-quark pair shows Debye screening at high temperatures

\[ F_{Q\bar{Q}} - T \log(\theta) [\text{GeV}] \]

Pure glue ≠ QCD!
Calculations with HISQ action agree with the calculations performed with stout action (WB, Borsanyi et al., JHEP 1504 (2015) 138)
Polyakov and gas of static-light hadrons

\[ Z_{Q\bar{Q}}(T)/Z(T) = \sum_n \exp(-E_{n}^{Q\bar{Q}}(r \to \infty)/T) \]

Energies of static-light mesons:

\[ E_{n}^{Q\bar{Q}}(r \to \infty) = M_n - m_Q \]

Free energy of an isolated static quark:

\[ F_Q(T) = -\frac{1}{2} (T \ln Z_{Q\bar{Q}}(T) - T \ln Z(T)) \]

Megias, Arriola, Salcedo, PRL 109 (12) 151601

Bazavov, PP, PRD 87 (2013) 094505

Ground state and first excited states are from lattice QCD

Michael, Shindler, Wagner, arXiv1004.4235
Wagner, Wiese, JHEP 1107 016, 2011

Higher excited state energies are estimated from potential model

Gas of static-light mesons only works for \( T < 145 \) MeV
The entropy of static quark

\[ S_Q = -\frac{\partial F_Q}{\partial T} \]

At low \( T \) the entropy \( S_Q \) increases reflecting the increase of states the heavy quark can be coupled to.

At high temperature the static quark only “sees” the medium within a Debye radius, as \( T \) increases the Debye radius decreases and \( S_Q \) also decreases.

The onset of screening corresponds to peak is \( S_Q \) and its position coincides with \( T_c \).
\[
\frac{\Theta_{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = T \frac{d}{dT} \left( \frac{p}{T^4} \right)
\]

\[
\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \frac{\Theta_{\mu\mu}(T')}{T'^5},
\]

\[
\frac{\Theta_{\mu\mu}(T)}{T^4} = \frac{\varepsilon - 3p}{T^4} = R_\beta \{ \langle S_G \rangle_0 - \langle S_G \rangle_T \} - R_\beta R_m \{ 2m_l(\langle \bar{q}q \rangle_0 - \langle \bar{q}q \rangle) + m_s(\langle \bar{s}s \rangle_0 - \langle \bar{s}s \rangle_T) \}
\]

\[
R_\beta(\beta) = -a \frac{d\beta}{da}, \quad R_m = \frac{1}{m_q(\beta)} \frac{dm_q(\beta)}{d\beta}, \quad \beta = 10/g^2
\]


The peak height is much reduced compared to the asqtad and p4 \( N_\tau = 8 \) calculations.

Agreement with p4 and asqtad calculations for \( T > 350 \) MeV.

Small cutoff effects for HISQ except for \( N_\tau = 6 \).
Perform spline interpolation of all the $N_{\tau} > 6$ data with spline coefficients of the form $a + b/N_{\tau}^2$, stabilize the spline demanding that $\varepsilon - 3p$ is given by HRG at $T = 130$ MeV. Set the lower integration limit to $T_0 = 130$ MeV and take $p_0 = p^{HRG}(T = 130$ MeV) → $p(T)$.


Hadron resonance gas (HRG): Interacting gas of hadrons = non-interacting gas of hadrons and hadron resonances (virial expansion, Prakash & Venugopalan)

HRG agrees with the lattice for $T < 145$ MeV

$T_c = (154 \pm 9)$ MeV

$\varepsilon_c \simeq 300$ MeV/fm$^3$

$\varepsilon_{low} \simeq 180$ MeV/fm$^3$ ↔ $\varepsilon_{nucl} \simeq 150$ MeV/fm$^3$

$\varepsilon_{high} \simeq 500$ MeV/fm$^3$ ↔ $\varepsilon_{proton} \simeq 450$ MeV/fm$^3$
How Equation of state changed since 2002

• Much smoother transition to QGP
• The energy density keeps increasing up to 450 MeV instead of flattening
The high temperature behavior of the trace anomaly is not inconsistent with weak coupling calculations (EQCD) for $T > 300$ MeV.

For the entropy density the continuum lattice results are below the weak coupling calculations for $T < 500$ MeV.

At what temperature can one see good agreement between the lattice and the weak coupling results?
QCD thermodynamics at non-zero chemical potential

Taylor expansion:

\[
\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left( \frac{\mu_B}{T} \right)^i \cdot \left( \frac{\mu_Q}{T} \right)^j \cdot \left( \frac{\mu_S}{T} \right)^k
\]

hadronic

\[
\frac{p(T, \mu_u, \mu_d, \mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left( \frac{\mu_u}{T} \right)^i \cdot \left( \frac{\mu_d}{T} \right)^j \cdot \left( \frac{\mu_s}{T} \right)^k
\]

quark

\[
\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T, V, \mu_a, \mu_b, \mu_c)\bigg|_{\mu_a=\mu_b=\mu_c=0}
\]

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

\[
\chi_X^2 = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2)
\]

\[
\chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)
\]

information about carriers of the conserved charges ( hadrons or quarks )

probes of deconfinement
Equation of state at non-zero baryon density

Taylor expansion up to 4\textsuperscript{th} order for net zero strangeness $n_S = 0$ and $r = n_Q/n_B = Z/A = 0.4$

Moderate effects due to non-zero baryon density up to $\mu_B/2 = 2 \leftrightarrow \sqrt{s} \sim 20\text{GeV}$

Energy density at freeze-out is independent of $\mu_B$
Deconfinement: fluctuations of conserved charges

\[
\chi_B = \frac{1}{VT^3} \left( \langle B^2 \rangle - \langle B \rangle^2 \right)
\]
baryon number

\[
\chi_Q = \frac{1}{VT^3} \left( \langle Q^2 \rangle - \langle Q \rangle^2 \right)
\]
electric charge

\[
\chi_S = \frac{1}{VT^3} \left( \langle S^2 \rangle - \langle S \rangle^2 \right)
\]
strangeness

Ideal gas of massless quarks:

\[
\chi_{SB}^B = \frac{1}{3} \quad \chi_{SB}^Q = \frac{2}{3} \quad \chi_{SB}^S = 1
\]
conserved charges carried by light quarks

Conserved charges are carried by massive hadrons

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,
Deconfinement: fluctuations of conserved charges

\[ \chi_4^B = \frac{1}{VT^3}(\langle B^4 \rangle - 3\langle B^2 \rangle^2) \]  
\[ \chi_4^Q = \frac{1}{VT^3}(\langle Q^4 \rangle - 3\langle Q^2 \rangle^2) \]  
\[ \chi_4^S = \frac{1}{VT^3}(\langle S^4 \rangle - 3\langle S^2 \rangle^2) \]  

baryon number

electric charge

strangeness

Ideal gas of massless quarks:

\[ \chi_4^B_{SB} = \frac{2}{9\pi^2} \quad \chi_4^Q_{SB} = \frac{4}{3\pi^2} \quad \chi_4^S_{SB} = \frac{6}{\pi^2} \]

BNL-Bielefeld: talk by C. Schmidt
BW: talk by Borsanyi

@ Confinement X conference

conserved charges are carried by massive hadrons

conserved charges carried by light quarks
Deconfinement of strangeness

Partial pressure of strange hadrons in uncorrelated hadron gas:

\[
P_S = \frac{p(T) - p_{S=0}(T)}{T^4} = M(T) \cosh \left( \frac{\mu_S}{T} \right) + \\
B_{S=1}(T) \cosh \left( \frac{\mu_B - \mu_S}{T} \right) + B_{S=2}(T) \cosh \left( \frac{\mu_B - 2\mu_S}{T} \right) + B_{S=3}(T) \cosh \left( \frac{\mu_B - 3\mu_S}{T} \right)
\]

\[
v_1 = \chi_3^B - \chi_1^B
\]

\[
v_2 = \frac{1}{3} (\chi_4^S - \chi_2^S) - 2\chi_1^B - 4\chi_2^B - 2\chi_3^B
\]

- \(v_1\) and \(v_2\) do vanish within errors at low \(T\)
- \(v_1\) and \(v_2\) rapidly increase above the transition region, eventually reaching non-interacting quark gas values

Bazavov et al, PRL 111 (2013) 082301
Deconfinement of strangeness (cont’d)

Using the six Taylor expansion coefficients related to strangeness

\[
\chi_2^S, \chi_4^S, \chi_{13}^S, \chi_{22}^S, \chi_{31}^S
\]

it is possible to construct combinations that give

\[
M(T), B_{S=1}(T), B_{S=2}(T), B_{S=3}(T)
\]

up to terms \( c_1 v_1 + c_2 v_2 \)

Bazavov et al, PRL 111 (2013) 082301

Hadron resonance gas descriptions breaks down for all strangeness sectors above \( T_c \)

\[ \Rightarrow \text{Strangeness deconfines at } T_c \]
What about charm hadrons?

We could introduce chemical potential for charm quarks and study the derivatives of the pressure with respect to the charm chemical potential. Bazavov et al, PLB737 (2014) 210

$m_c \gg T$ only $|C|=1$ sector contributes

In the hadronic phase all $BC$-correlations are the same!

Hadronic description breaks down just above $T_c$ ⇒ open charm deconfines above $T_c$

The description in terms of un-correlated gas of hadrons breaks down at $T_c$ for all $BX$-correlations, $X=Q,S,C$
"Missing" strange baryons have to be included to obtain a good agreement between HRG and the lattice results.

Bazavov et al, PLB737 (2014) 210, PRL 113 (2014) 072001

Many baryons predicted by quark model (QM) and LQCD are missing from PDG.
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

- Lattice results converge as the continuum limit is approached
- Good agreement between lattice and the weak coupling approach for 2\textsuperscript{nd} order quark number fluctuations
- For 4\textsuperscript{th} order the weak coupling results are in reasonable agreement with lattice

Bazavov et al, PRD88 (2013) 094021
• The value chiral transition temperature is now well established in the continuum
\[ T_c = 154(9) \text{ MeV} \]
• Equation of state are known in the continuum limit up to \( T = 400 \text{ MeV} \)
• Hadron resonance gas can describe various thermodynamic quantities at low temperatures
• Deconfinement transition can be studied in terms of fluctuations and correlations of conserved charges, it manifest itself as a abrupt breakdown of hadronic description that occurs around the chiral transition temperature
• The approach to the weakly interacting quark gluon gas for \( T > T_c \) is rather slow and the matter is strongly interacting for \( T < 300 \text{ MeV} \) with no apparent quasi-particle composition
• For \( T > (300-400) \text{ MeV} \) weak coupling expansion works well for certain quantities (e.g. quark number susceptibilities), more work is needed to connect lattice and weak coupling results
• Comparison of lattice and HRG results for certain strangeness and charm correlations hints for existence of yet undiscovered excited baryons
Continuum results obtained with stout and HISQ action agree reasonably well given their errors (some tension for the entropy density)

Even in the transition region the speed of sound is not much smaller than the HRG speed of sound (the EoS is never really soft)
Domain wall Fermions and $U_A(1)$ symmetry restoration

Domain Wall Fermions, Bazavov et al (HotQCD), PRD86 (2012) 094503

$$\chi_i = \int d^4 x G_i(x)$$

chiral:

$$\chi_\pi = \chi_\delta + \chi_{\text{disc}}$$

axial:

$$\chi_\pi = \chi_\delta$$

$$\chi_\delta = \chi_\pi - \chi_{5,\text{disc}}$$

$$\chi_{\text{disc}} = \chi_5$$

$$\chi_{\text{disc}} = -\chi_{5,\text{disc}}$$

Peak position roughly agrees with previous staggered results

axial symmetry is effectively restored $T>200$ MeV!
Improved staggered calculations at finite temperature

**low T region**

$T < 200 \text{ MeV}$

$O(a_s^n (a \Lambda_{QCD})^2)$ errors

$a > 0.125 \text{ fm}$

hadronic degrees of freedom

improvement of the flavor symmetry is $\rightarrow$ fat links important

**cutoff effects are different in**

$$a = 1/(TN_T)$$

$N_T = 8$

for #flavors < 4

rooting trick

$$detD \rightarrow (detD)^{n_f/4}$$

**high-T region**

$T > 200 \text{ MeV}$

$O((aT)^2)$ errors

$a < 0.125 \text{ fm}$

quark degrees of freedom

quark dispersion relation
The Highly Improved Staggered Quark (HISQ) Action

**HISQ action**

two levels of gauge field smearing with re-unitarization

Follana et al, PRD75 (07) 054502

**Smearing level 1**

\[
U_\mu(x) = e^{iga A_\mu(x)}
\]

\[= U_\mu^{fat7} \rightarrow \tilde{U}_\mu\] = \[
\frac{U_\mu^{fat7}}{\sqrt{U_\mu^{fat7} U_\mu^{fat7\dagger}}}\]

**Smearing level 2**

projection onto U(3) improves flavor symmetry

Hasenfratz, arXiv:hep-lat/0211007

3-link (Naik) term to improve the quark dispersion relation + asqtad smearing

asqtad