Turbulent thermalization of the Quark Gluon Plasma

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RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

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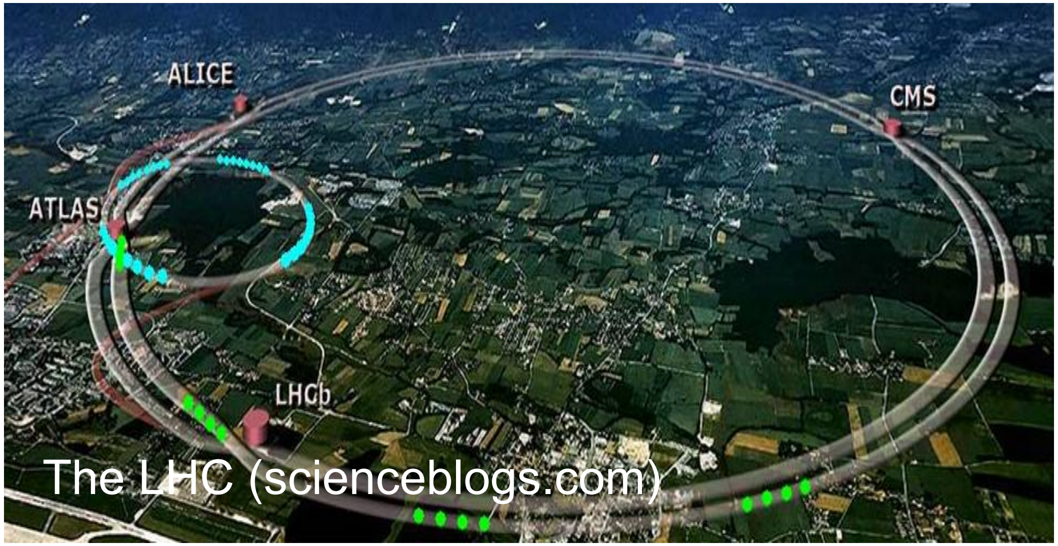
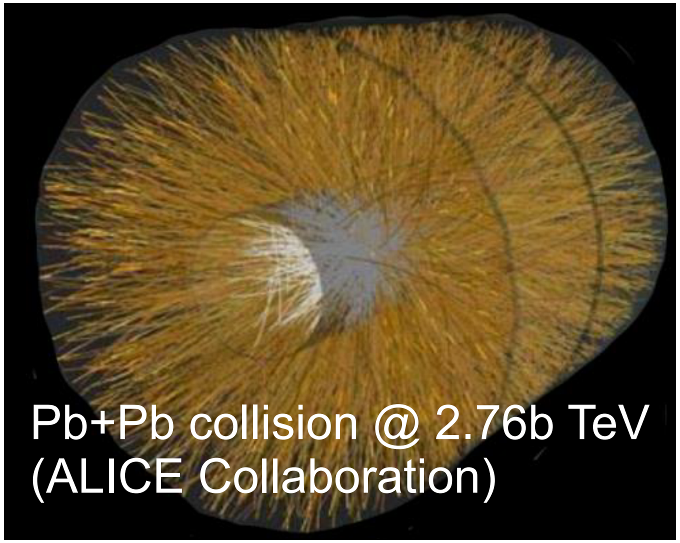
Motivation

Relativistic heavy-ion collision experiments at RHIC and LHC





Can we understand the complex dynamics in an ***ab-initio*** approach to heavy-ion collisions?

Heavy-ion collisions

Conjectured space-time evolution of a heavy-collision based on phenomenological models and experimental information

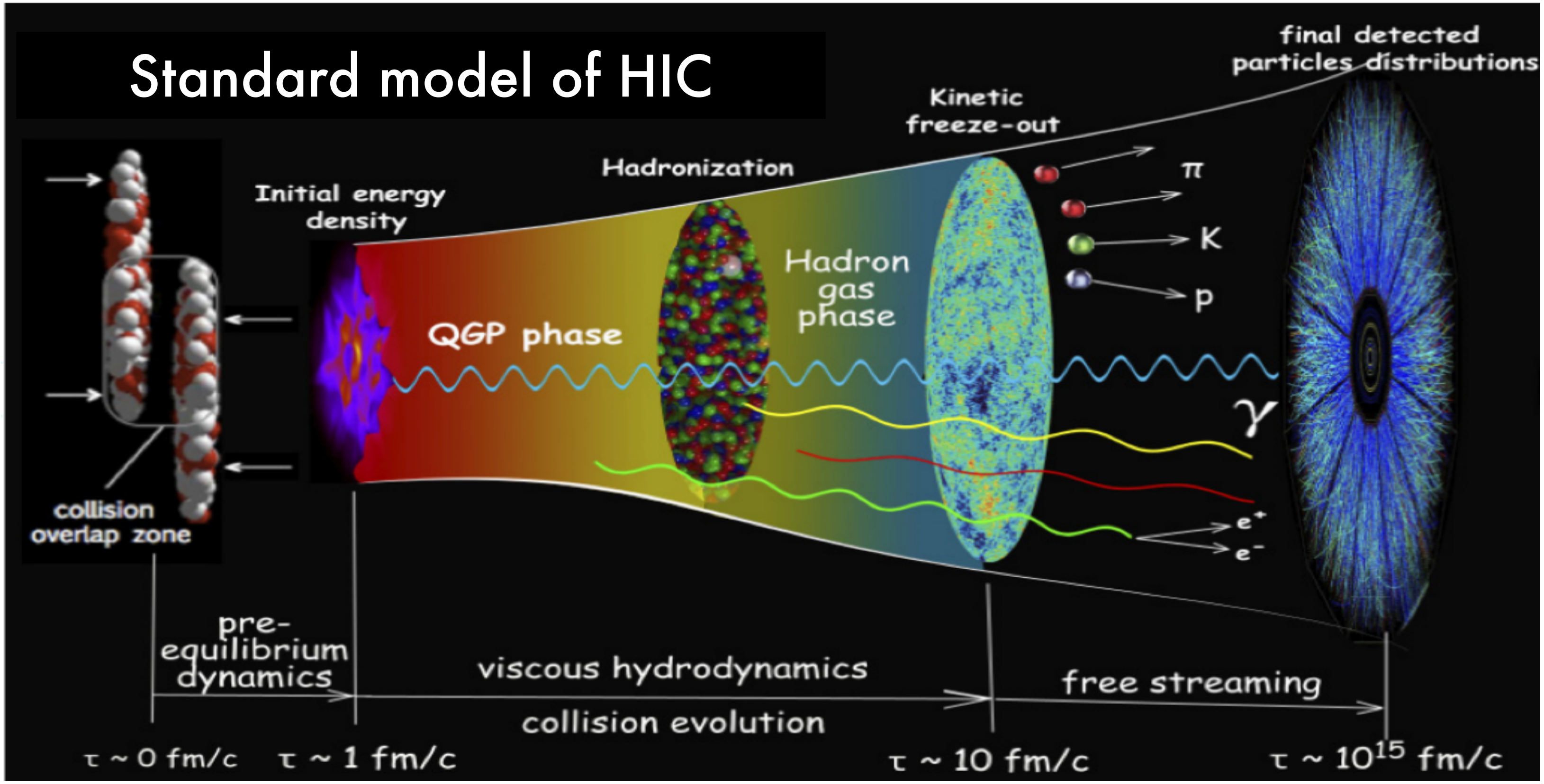
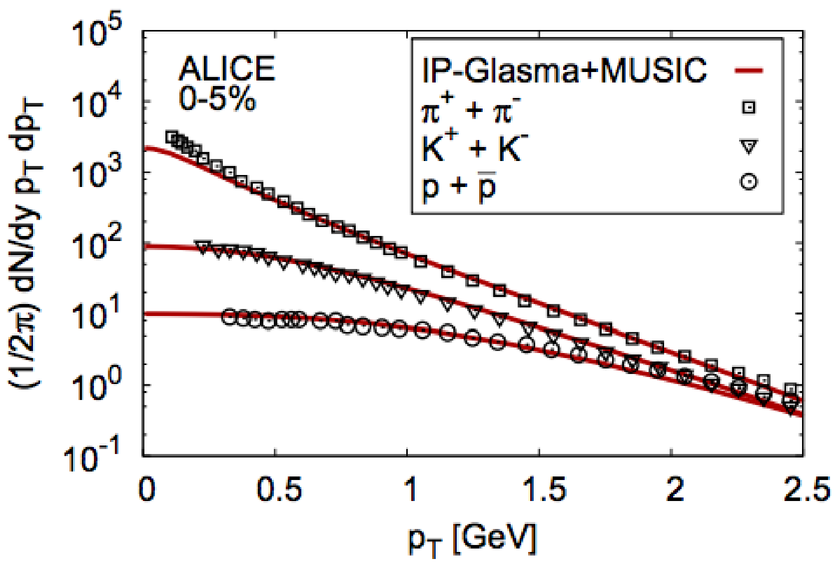
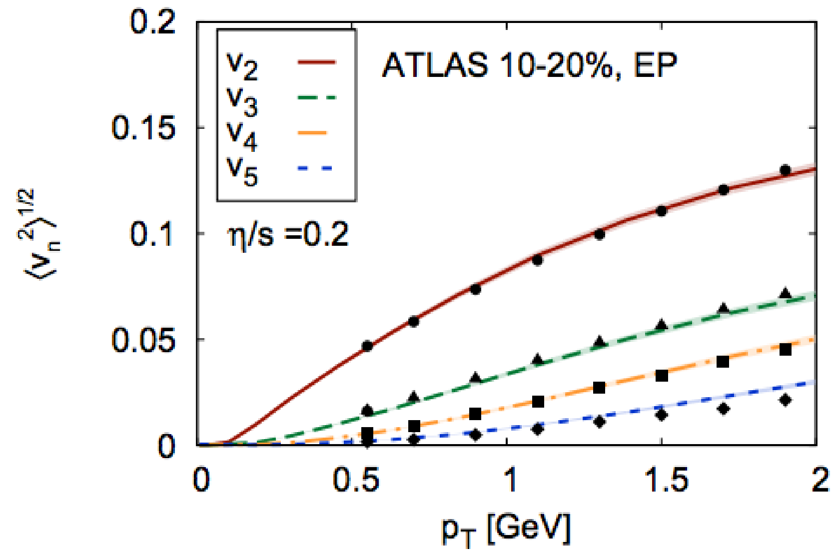


Fig. by P. Sorensen and C. Shen

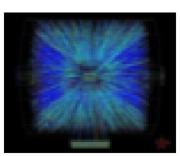
Heavy-ion collisions

Hydrodynamic simulations versus experiment





Schenke et al. PRL 110 (2013) 012302



A large variety of data at RHIC and LHC can be explained based on this standard model

When and to what extend is isotropization/thermalization achieved?
Can this be understood within an *ab-initio* approach?

Heavy-ion collisions

Progress in a first-principle understanding from two limiting cases

Holographic thermalization:

a) strong coupling? *Heller, Janik, Witaszczyk; Chesler, Yaffe ...*

Sizeable anisotropy at transition to hydrodynamic regime

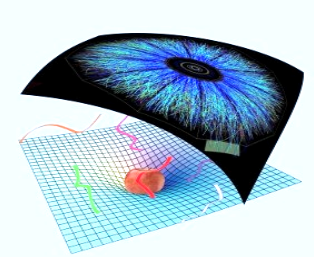


Fig. from strings.net.technion.ac.il

Turbulent thermalization:

b) weak coupling but highly occupied? *CGC: McLerran, Venugopalan ...*

Energy density of gluons with typical momentum Q_s (at time ~$1/Q_s$)

$$\epsilon \sim \frac{Q_s^4}{\alpha_s}$$ i.e. 'occupation numbers' $$n(p \lesssim Q_s) \sim \frac{1}{\alpha_s}$$

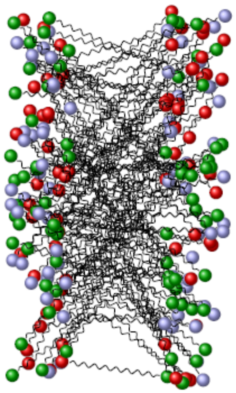


Fig. by T. Epelbaum

Non-equilibrium dynamics

Solve ***initial value problem*** in QCD

- ***Initial conditions:***

Based on ***color glass condensate*** (CGC) description of heavy ion collisions ***(n(p)~1/α)***

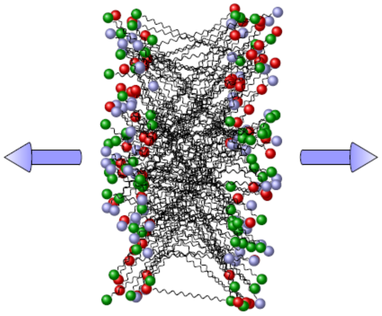


Fig. by T. Epelbaum

- ***Non-equilibrium dynamics:***

- Classical-statistical lattice simulations (numerical studies) **(n(p)>>1)**

- Kinetic theory **(n(p)<~1/α)** (analytic discussion)

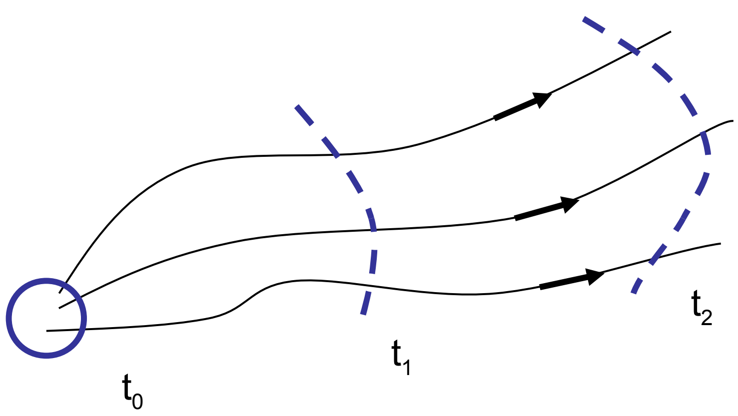
First principle

Intuitive picture

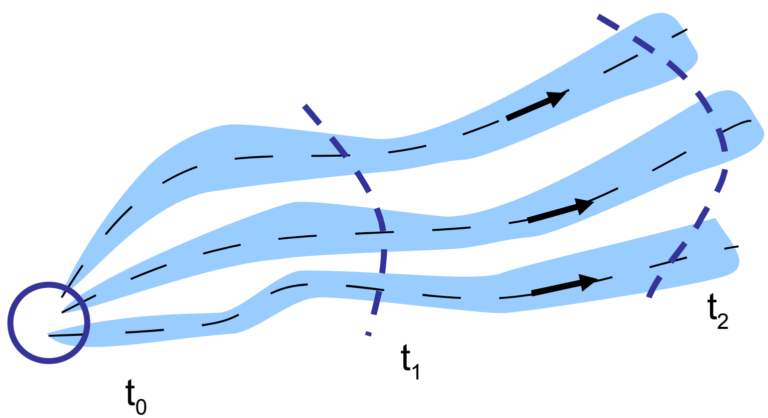
Classical-statistical (real-time) lattice gauge theory

Correspondence principle: for high occupation numbers the quantum dynamics is well described by an ensemble of classical fields

- Start with a set of initial conditions at t_0
- Solve classical evolution equations on the lattice
- Average over phase space trajectories to calculate expectation values of observables



Classical-statistical evolution



Quantum evolution

Non-equilibrium dynamics

Initial state:
Far from equilibrium

Non-equilibrium dynamics

Final state:
Thermal equilibrium

How is thermal equilibrium achieved?

Turbulent Thermalization

Non-equilibrium phenomena may be shared by a large class of strongly correlated many-body systems

I) Thermalization in scalar field theory – Cosmology

(Micha, Tkachev PRD 70 (2004) 043538)

II) Thermalization in Yang-Mills theory in Minkowski space

(Berges,SS,Sexty PRD 86 (2012) 074006; SS PRD 86 (2012) 065008)

III) Thermalization in heavy-ion collisions at ultra-relativistic energies – weak coupling, large nuclei

(Berges,Boguslavski,SS,Venugopalan arXiv:1303.5650 [hep-ph])

Turbulent thermalization - Cosmology

Space time evolution of the Big Bang

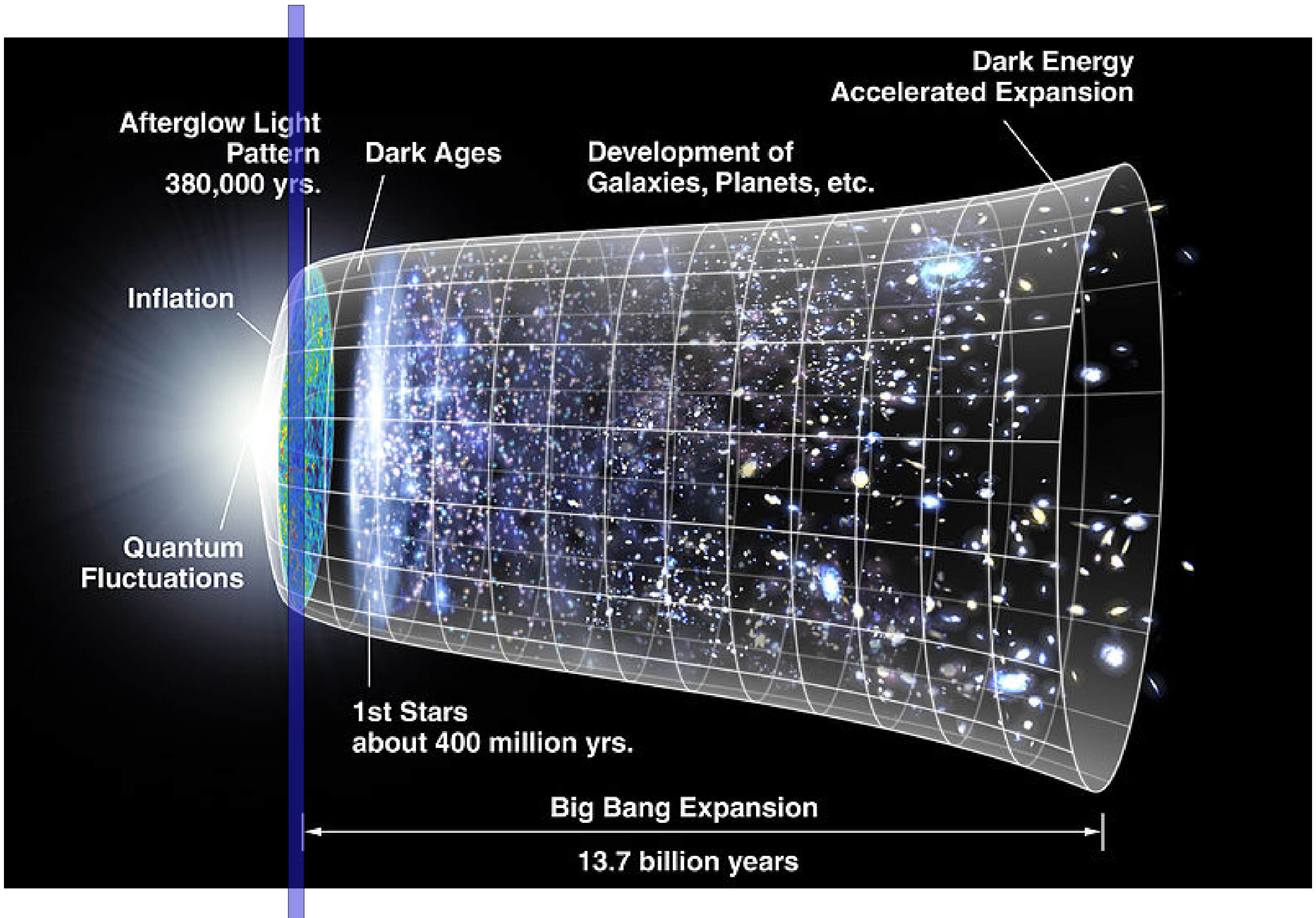
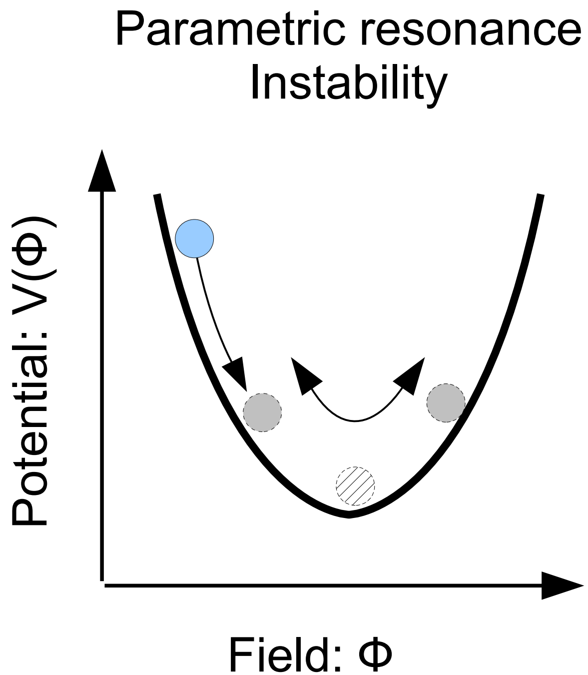


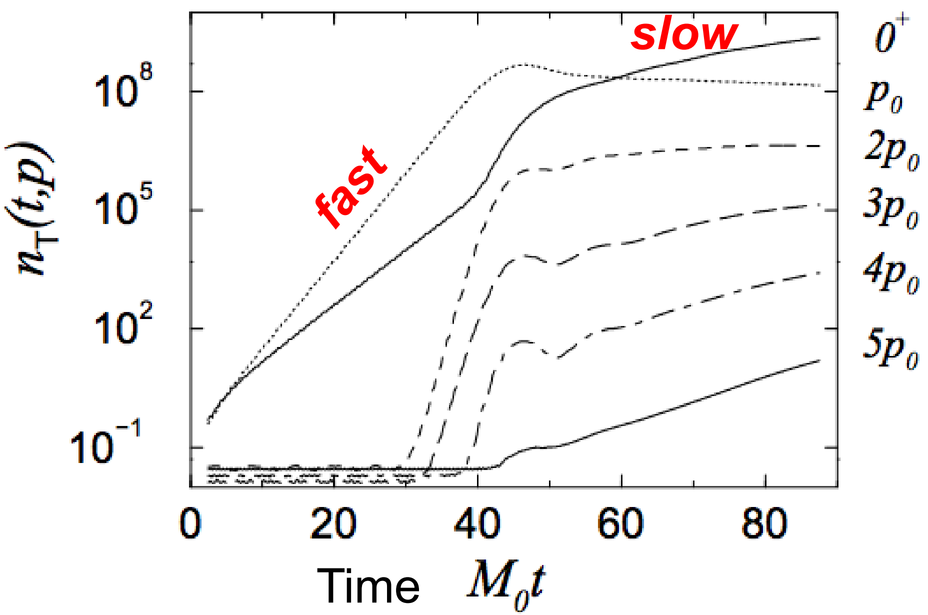
Fig by NASA

Turbulent thermalization - Cosmology

A model for thermalization of the early universe:

- Reheating after inflation – scalar field theory ($\lambda\Phi^4$)
- Homogenous background field Φ_0 + vacuum fluctuations



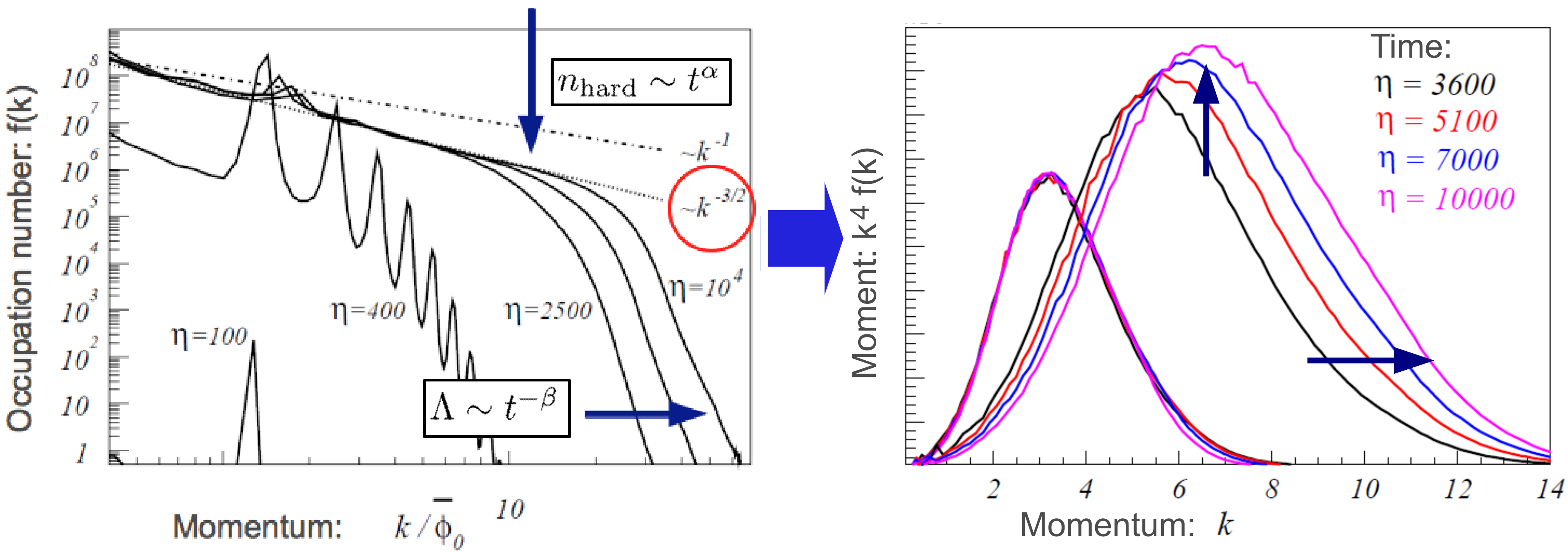


(Berges, Serreau PRL 91 (2003) 111601)

Turbulent thermalization - Cosmology

Non-thermal fixed point

Self-similar evolution



- The thermalization process is described by a ***quasi-stationary evolution*** with ***scaling exponents***

Dynamic: α=-4/5 β=-1/5 Spectral: κ=-3/2

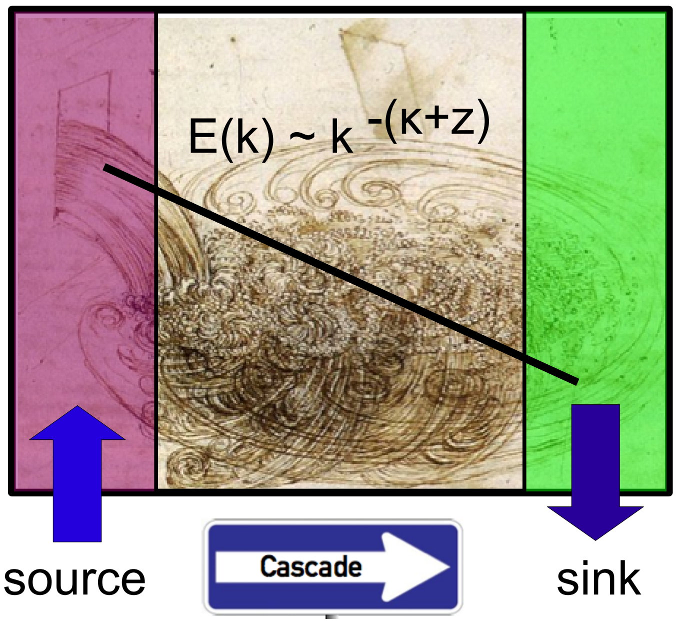
(Micha, Tkachev PRD 70 (2004) 043538)

Wave turbulence

"Driven" Turbulence – Kolmogorov wave turbulence

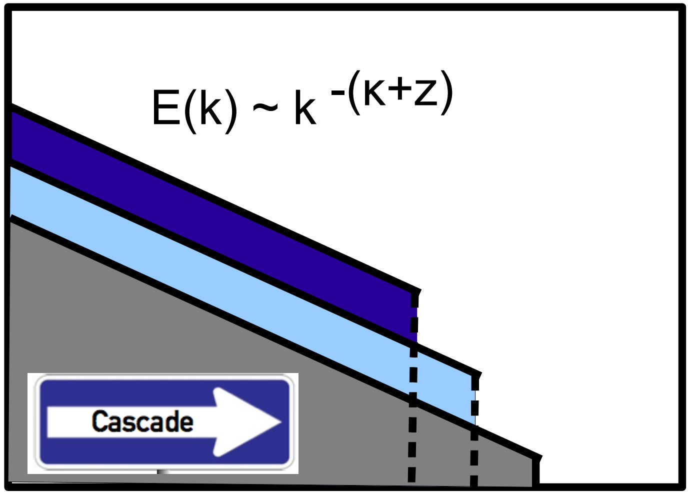
vs.

"Free" Turbulence – Turbulent Thermalization



▪ ***Stationary solution*** with universal ***non-thermal*** scaling exponents

Uriel Frisch, *Turbulence. The Legacy of A. N. Kolmogorov. (CUP, 1995)*

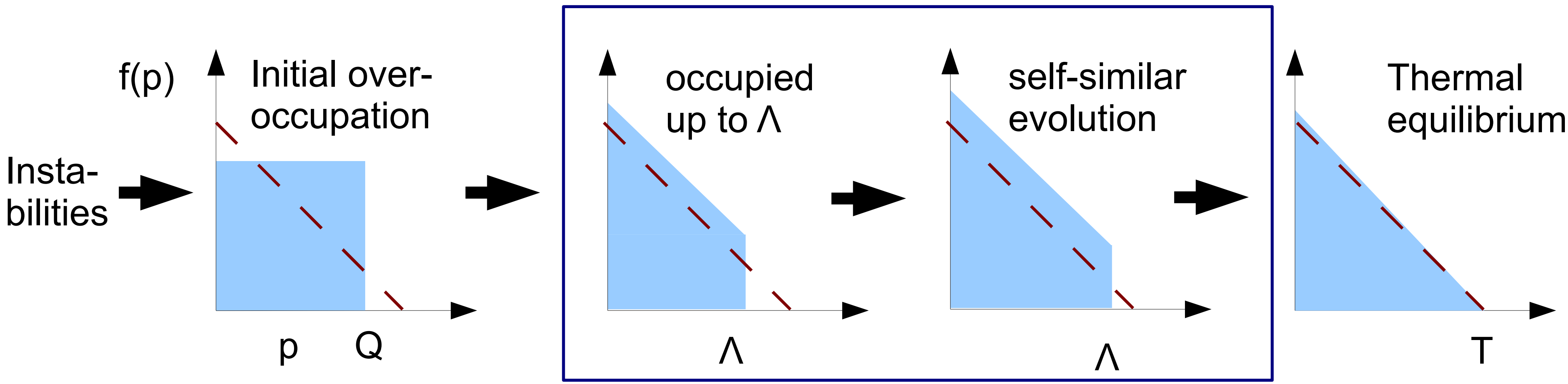


closed system

▪ ***quasi-stationary solution*** with universal ***non-thermal spectral exponents***

▪ ***Self-similar evolution*** with universal ***dynamical scaling exponents***

Evolution in kinetic theory



- Search for ***self-consistent scaling solutions***

$$f(p,t) = t^{\alpha}\ f_S(t^{\beta} p) \qquad \partial_t f(p,t) = C[f](p,t)$$

- Fixed point equation + Scaling relation

$$\alpha f_S(p) + \beta \partial_p f_S(p) = C[f_S](p) \qquad \alpha - 1 = \mu(\alpha, \beta)$$

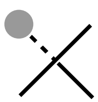
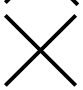
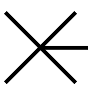
(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

Evolution in kinetic theory

- The ***self-similar evolution*** is characterized by scaling solutions in time with ***universal scaling exponents*** fixed by

$$\alpha - 1 = \mu(\alpha, \beta) \qquad + \ \textit{conservation laws}$$

Dynamic scaling relation

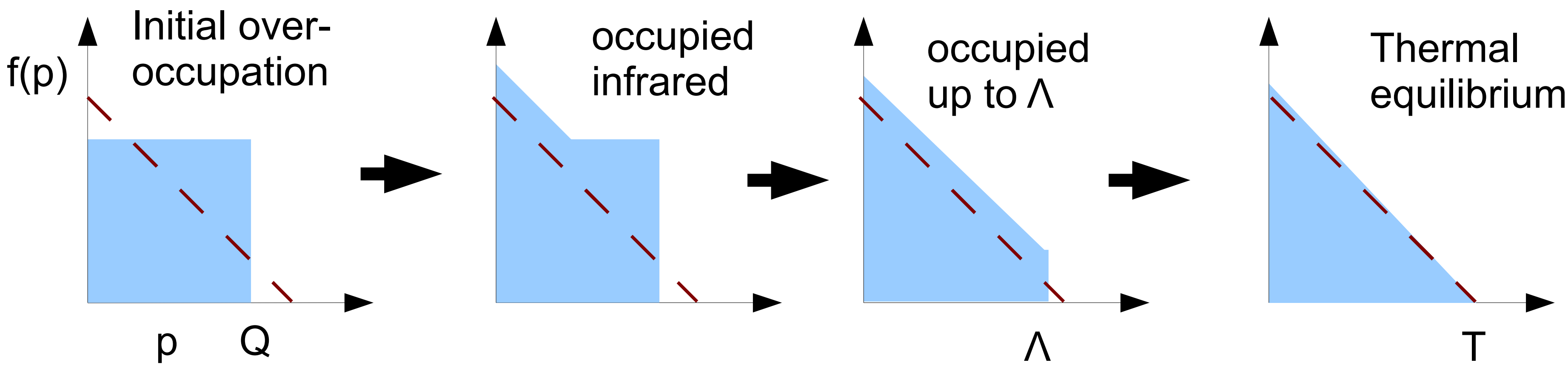
Interaction		***Spectral Shape (Exponent κ)***	***Λ evolution (Exponent α)***	***Occupancy evolution (Exponent β)***
	***2<->1+soft***	***3/2***	***-1/5***	***-4/5***
	***2<->2***	***4/3***	***-1/7***	***-4/7***
	***2<->3***	***??***	***-1/7***	***-4/7***
(gauge theory)				

- ***Scalar theory:*** turbulent cascade is driven by ***2<->(1+soft)*** interaction and leads to a ***transient condensate*** formation

(Cosmology: Micha, Tkachev PRD 70 (2004) 043538)

SU(2) gauge theory – Static Box

- Consider homegenous and ***isotropic*** systems which are initially ***highly occupied*** and initially characterized by a single momentum scale Q



How does thermalization proceed? What are the relevant processes? Turbulence? Bose Einstein Condensation?

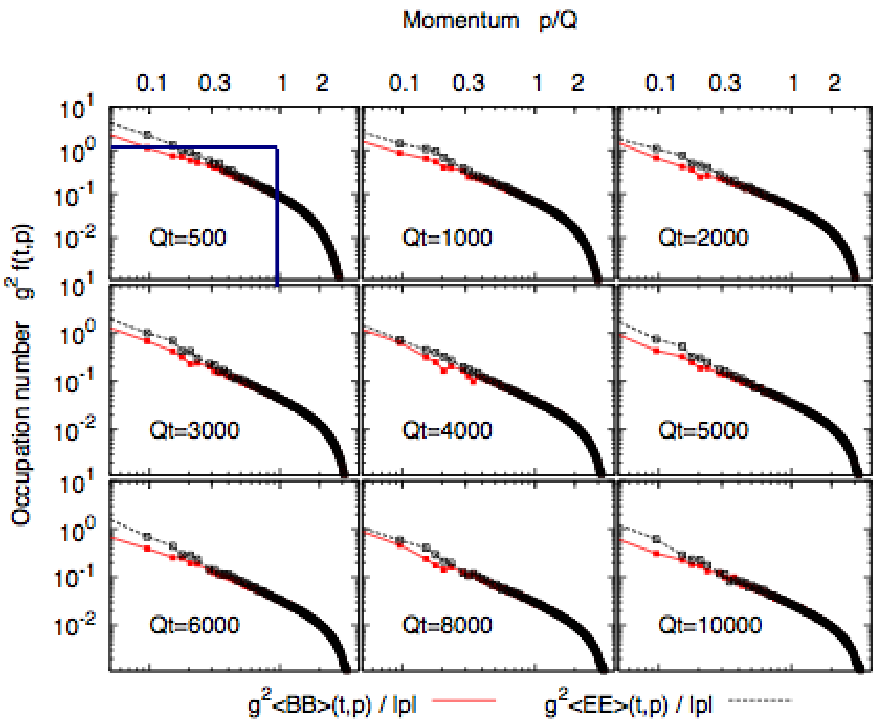
(c.f. Kurkela, Moore JHEP 1112 (2011) 044; Blaizot et al. Nucl.Phys. A873 (2012) 68-80)

Occupation number

- Non-perturbative and non-equilibrium calculation; occupation number is a ***gauge dependent*** quantity

- Chose ***temporal axial + Coulomb type gauge*** to fix the gauge freedom

- Define occupation number from ***equal time correlation functions***



$$\langle EE\rangle(t,\mathbf{p}) = \int_{\mathbf{x}\text{-}\mathbf{y}} \frac{\langle E_i^a(t,\mathbf{x})E_i^a(t,\mathbf{y})\rangle}{(N_c^2-1)(d-1)} e^{-i\mathbf{p}(\mathbf{x}\text{-}\mathbf{y})}$$

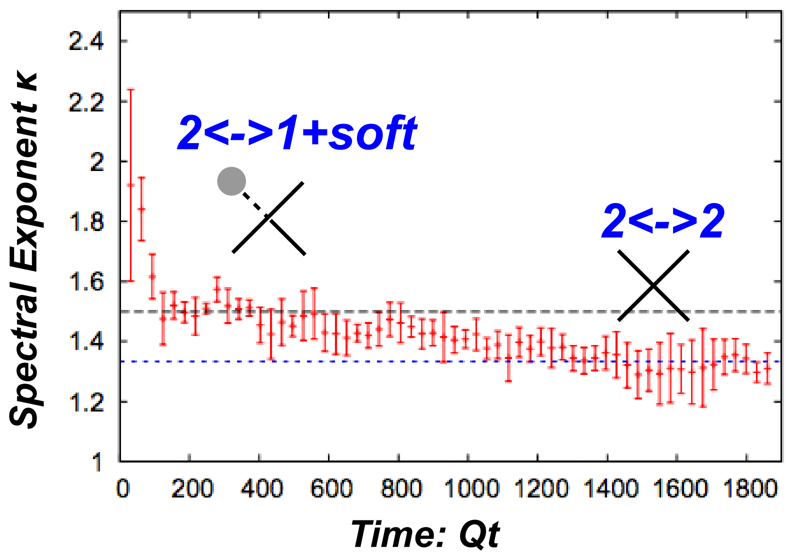
$$f(t,\mathbf{p}) = \frac{\langle EE\rangle(t,\mathbf{p})}{|\mathbf{p}|}$$

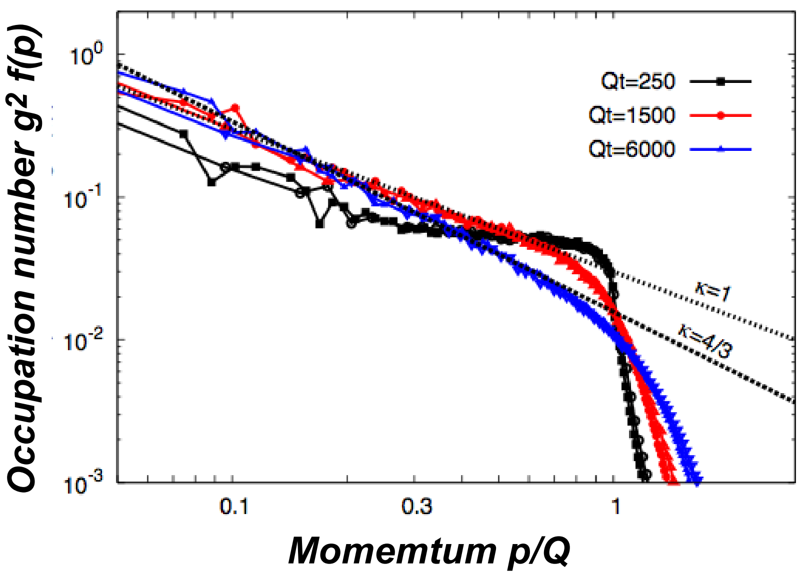
(Berges, Scheffler, Sexty PLB 681 (2009) 362-366; Berges,SS,Sexty arXiv:1203.4646 (2012); SS arXiv:1207.1450 (2012); Kurkela, Moore arXiv:1207.1663 (2012))

Turbulent Spectra

- Spectrum at ***early times*** consistent with ***ϰ=3/2*** (as in scalar theory possible sign of condensation); at ***late times*** always closer to ***ϰ=4/3***

- ***Inelastic processes*** much more efficient in gauge theory as compared to scalars; The appearance of ***ϰ=3/2*** also depends on initial conditions;





(Berges, Scheffler, Sexty PLB 681 (2009) 362-366; Berges,SS,Sexty arXiv:1203.4646 (2012); SS arXiv:1207.1450 (2012); Kurkela, Moore arXiv:1207.1663 (2012))

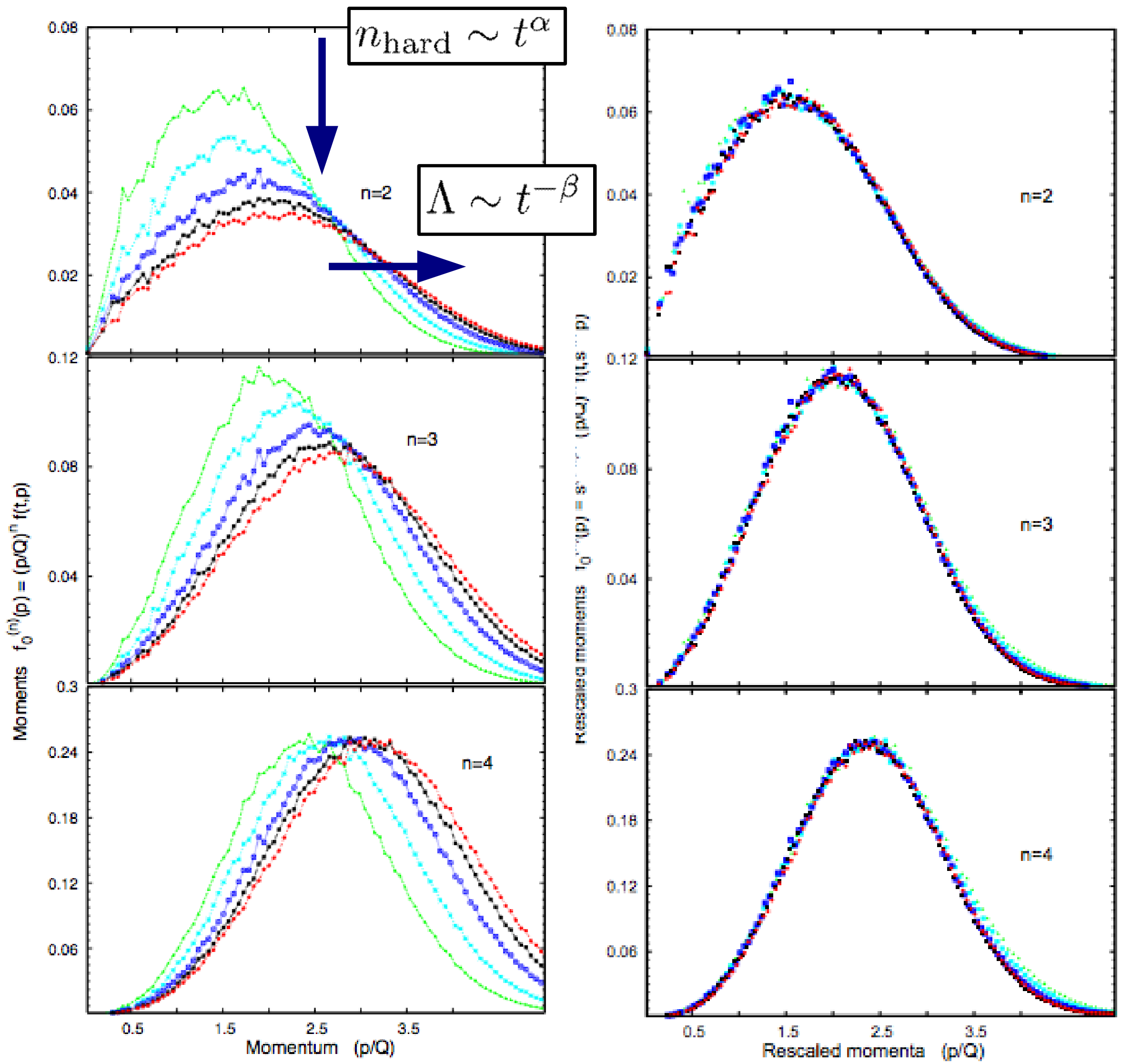
Dynamic Scaling

- Evolution at late times shows a ***self-similar*** behavior with dynamic scaling exponents

α = -4/7

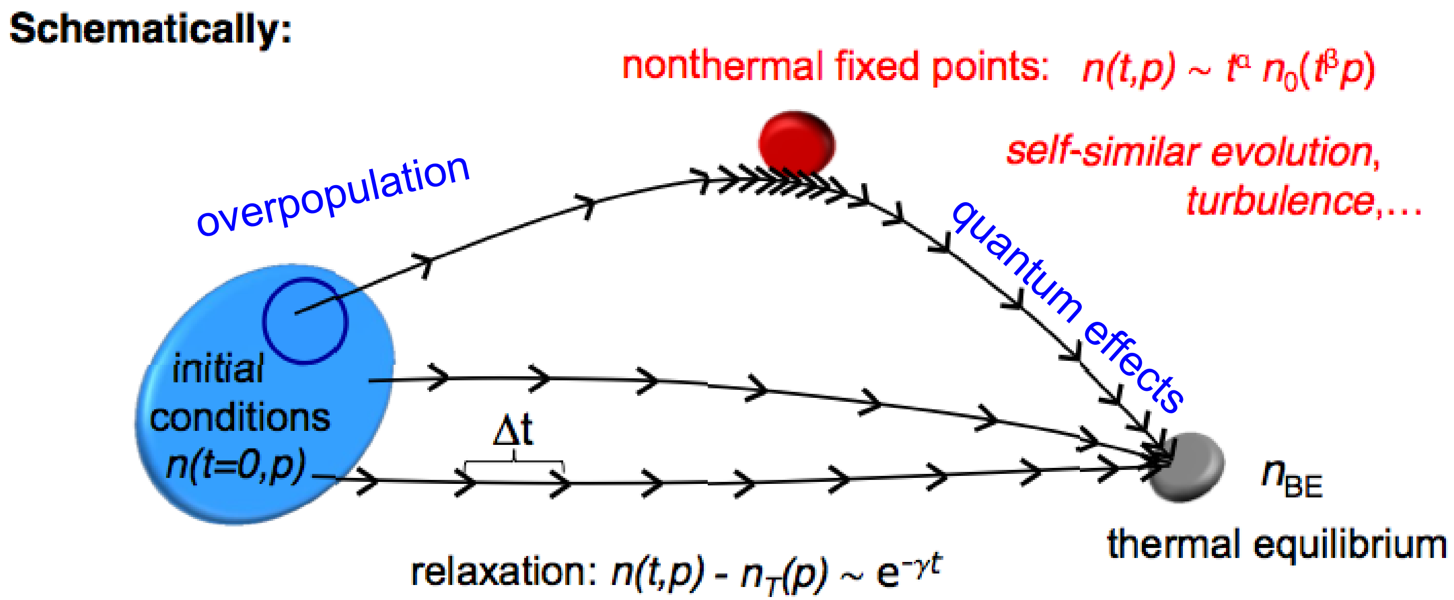
β = -1/7

- Consistent with ***elastic and inelastic*** scattering processes



Turbulent thermalization

Thermalization of over-occupied systems proceeds as a ***turbulent cascade*** with a self-similar evolution associated to the presence of a ***non-thermal fixed point***



Does this picture hold for heavy-ion collisions in the limit of weak coupling and large nuclei?

Heavy-Ion collisions at weak coupling

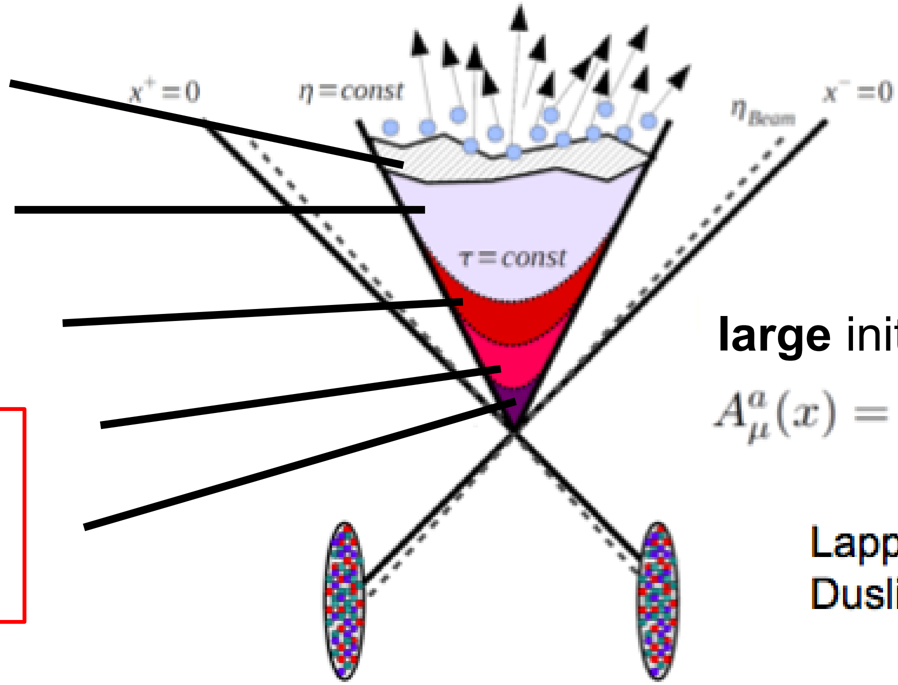
Experimental measurements

Hadronization and freeze out

(3+1)D Viscous hydrodynamics

Thermalization

Strong YM Fields



large initial fields – **boost invariant**

$$A^a_\mu(x) = \langle \hat{A}^a_\mu(x) \rangle \sim \mathcal{O}(1/g)$$

Lappi, McLerran, Dusling, Gelis, Venugopalan,…

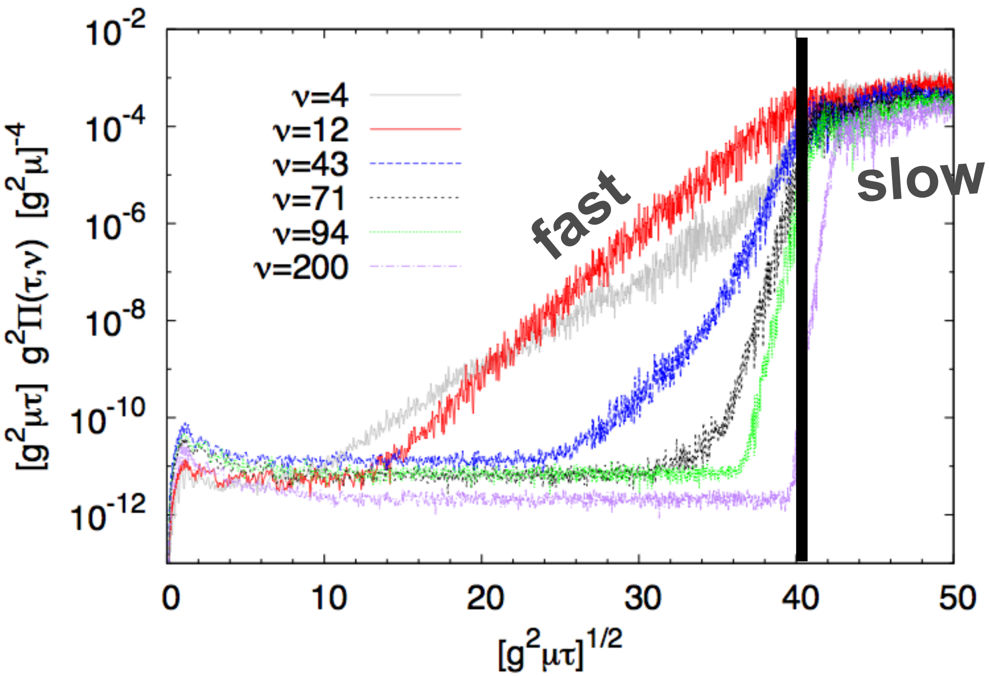
- **small** initial (vacuum) fluctuations:

$$F^{ab}_{\mu\nu}(x,y) = \frac{1}{2}\left\langle \left\{ \hat{A}^a_\mu(x), \hat{A}^b_\nu(y) \right\} \right\rangle - A^a_\mu(x) A^b_\nu(y)$$
$$\sim \mathcal{O}(1)$$

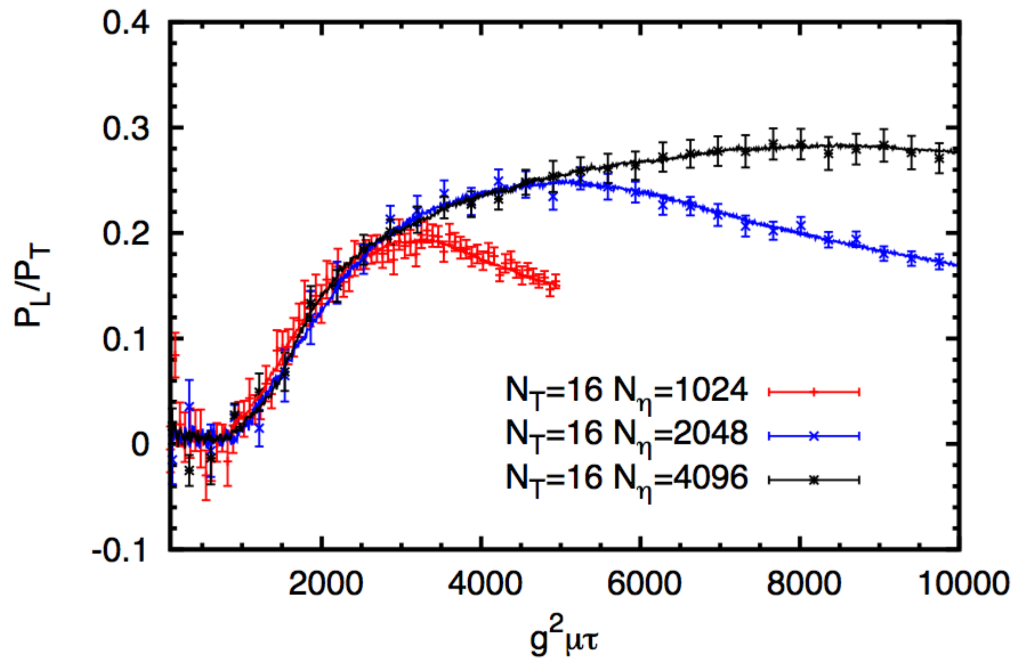
→ ***instability!***

Mrowczynski; Rebhan, Romatschke, Strickland; Arnold, Moore, Yaffe …

Expanding systems - Early times



Thermalization? Turbulence?
Non-thermal fixed points?



Plasma Instabilities - vacuum fluctuations grow exponentially in time

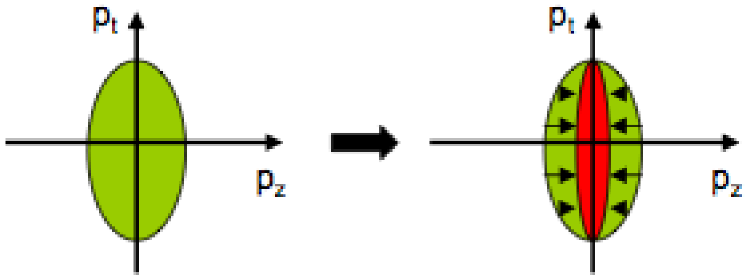
Not sufficient to isotropize the system

Romatschke,Venugopalan (2006);
Fukushima, Gelis (2011);

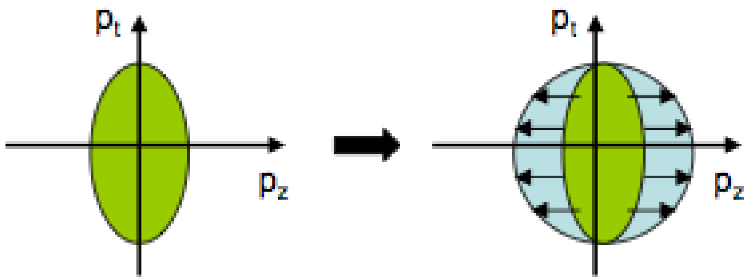
Berges, Schlichting, PRD 87 (2013) 014026

Expanding systems - Thermalization

- The ***longitudinal expansion*** renders the system ***anisotropic*** on large time scales. There is a natural ***competition*** between ***interactions*** and the longitudinal expansion



Longitudinal Expansion:
- red-shift of longitudinal momenta p
  → increase of anisotropy
- dilution of the system



Interactions:
- isotropize the system

Expanding systems - Thermalization

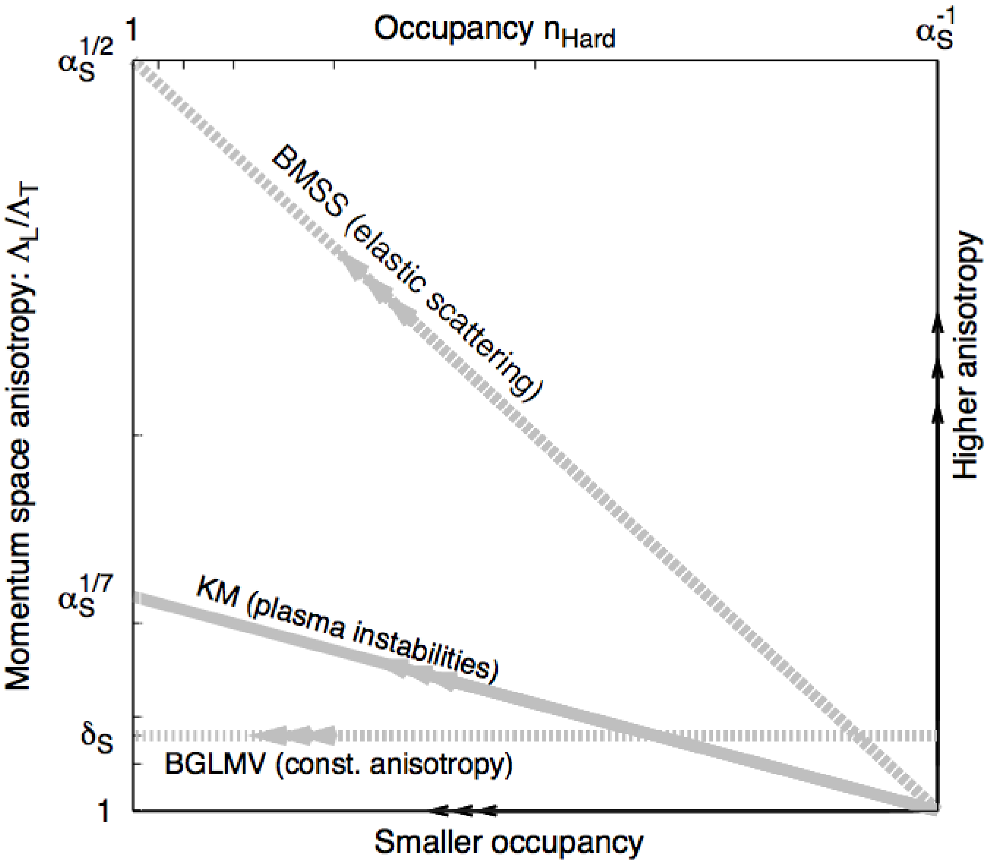
- Different scenarios of how thermalization proceeds have been proposed in the literature

Baier et al. (BMSS),
PLB 502 (2001) 51-58

Kurkela, Moore (KM),
JHEP 1111 (2011) 120

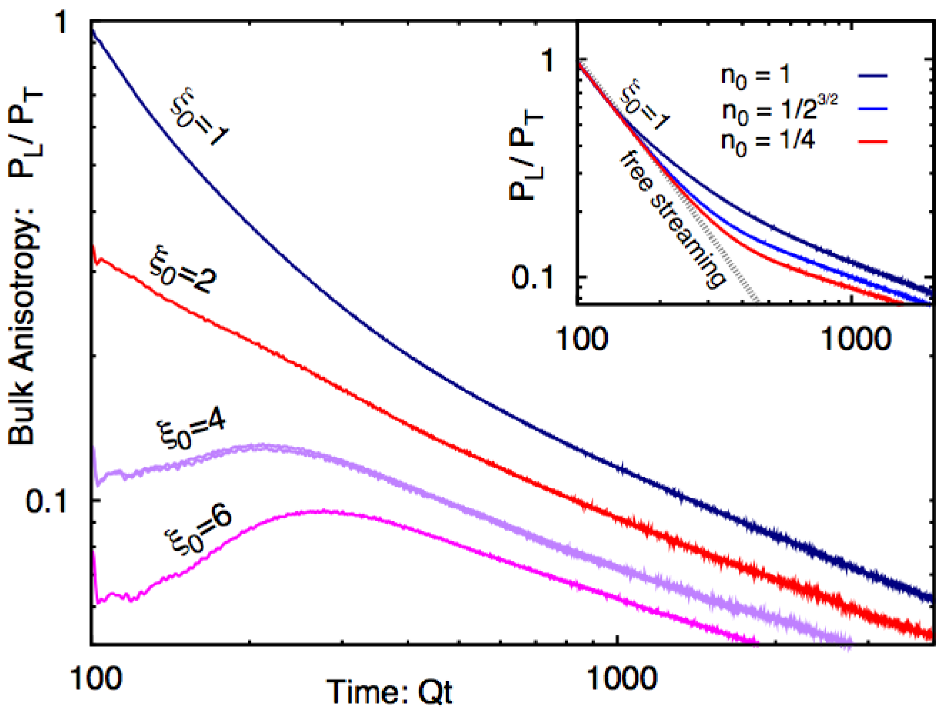
Blaizot et al. (BGLMV),
Nucl. Phys. A 873 (2012) 68-80

Classical-statistical lattice simulations can be used to determine which solution is realized!



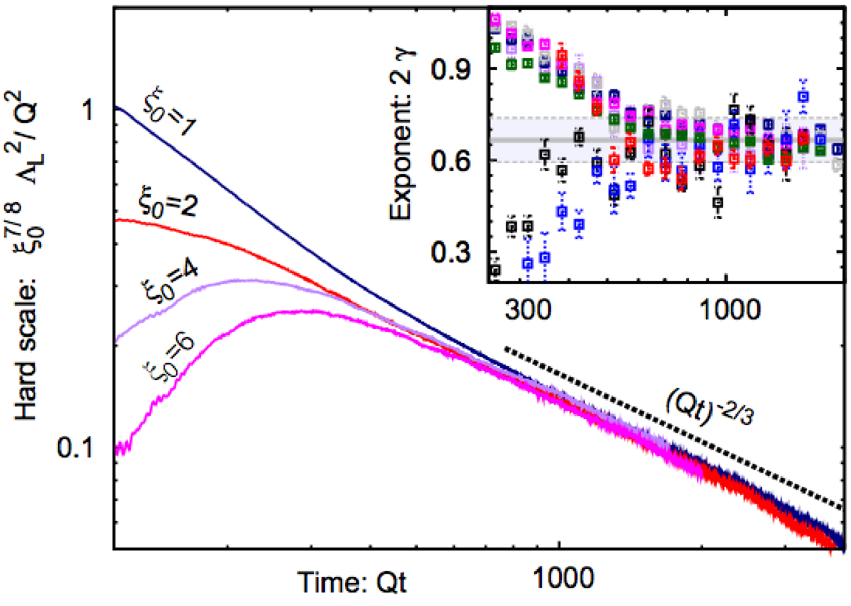
Expanding systems - Anisotropy

- The ***anisotropy*** of the system ***increases*** due to the longitudinal expansion.
- The system remains ***strongly interaction*** throughout the entire evolution.
- At late times, the evolution becomes ***insensitive*** to the details of the ***initial conditions***



(Berges,Boguslavski,SS,Venugopalan arXiv:1303.5650 [hep-ph])

Expanding systems - Scaling



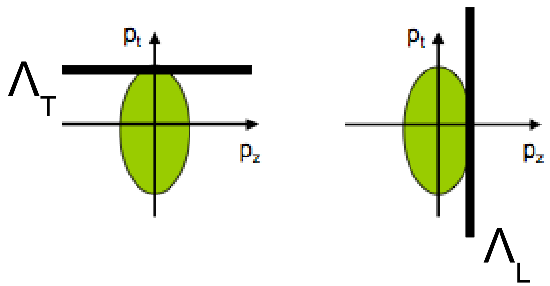
- The typical ***longitudinal momentum*** of hard excitations exhibits a ***universal scaling*** behavior

$$\Lambda_L^2/Q^2 \sim (Qt)^{-2\gamma}$$

$$2\gamma = 0.67 \pm 0.07$$

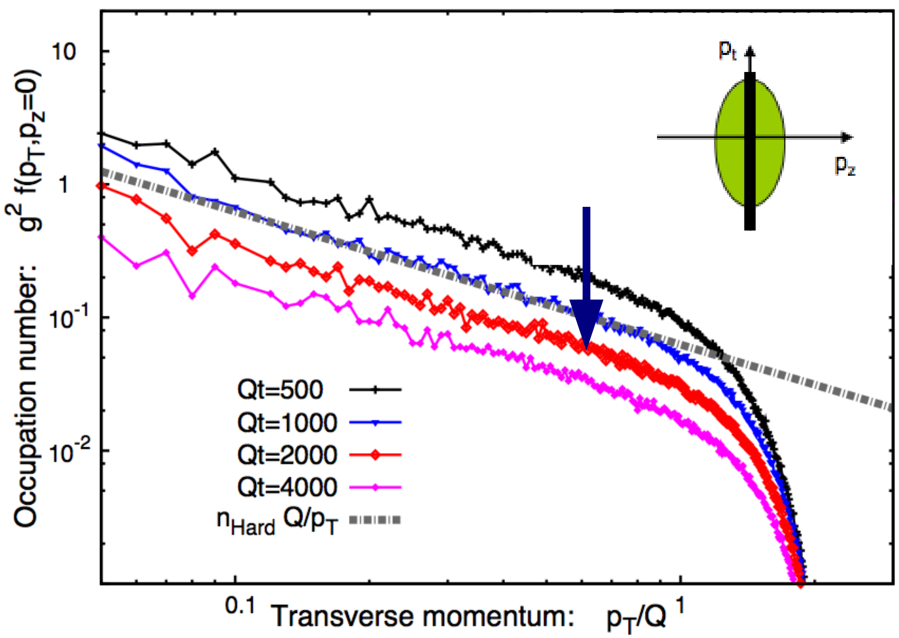
- The typical ***transverse momentum*** of hard excitations remains approximately ***constant***

$$\Lambda_T^2/Q^2 \sim (Qt)^{-2\beta} \qquad 2\beta \simeq 0$$

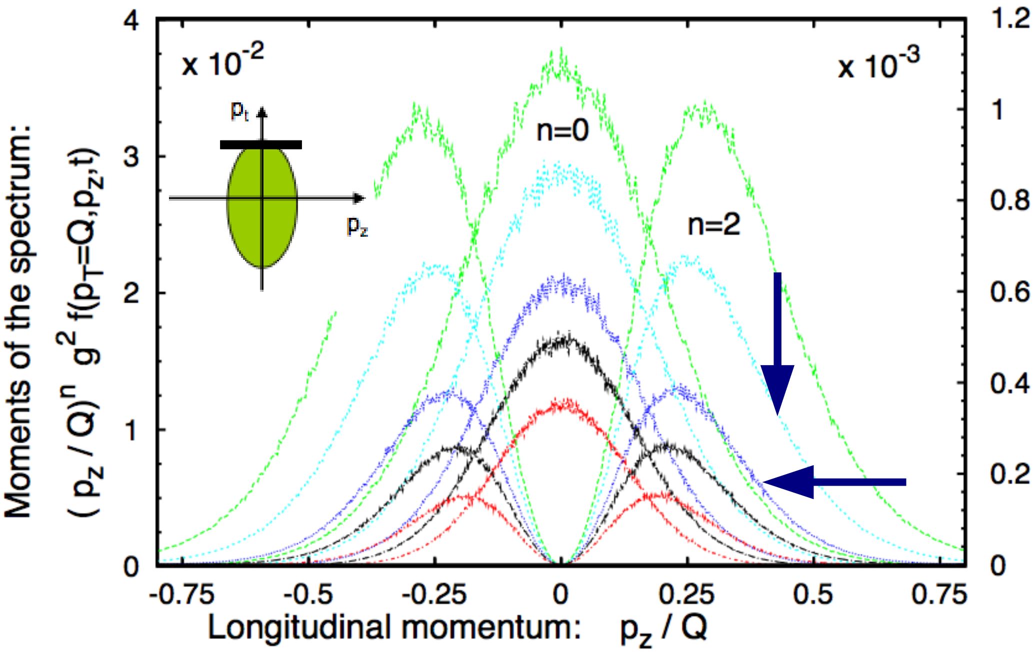


Expanding systems - Spectrum

Transverse spectrum



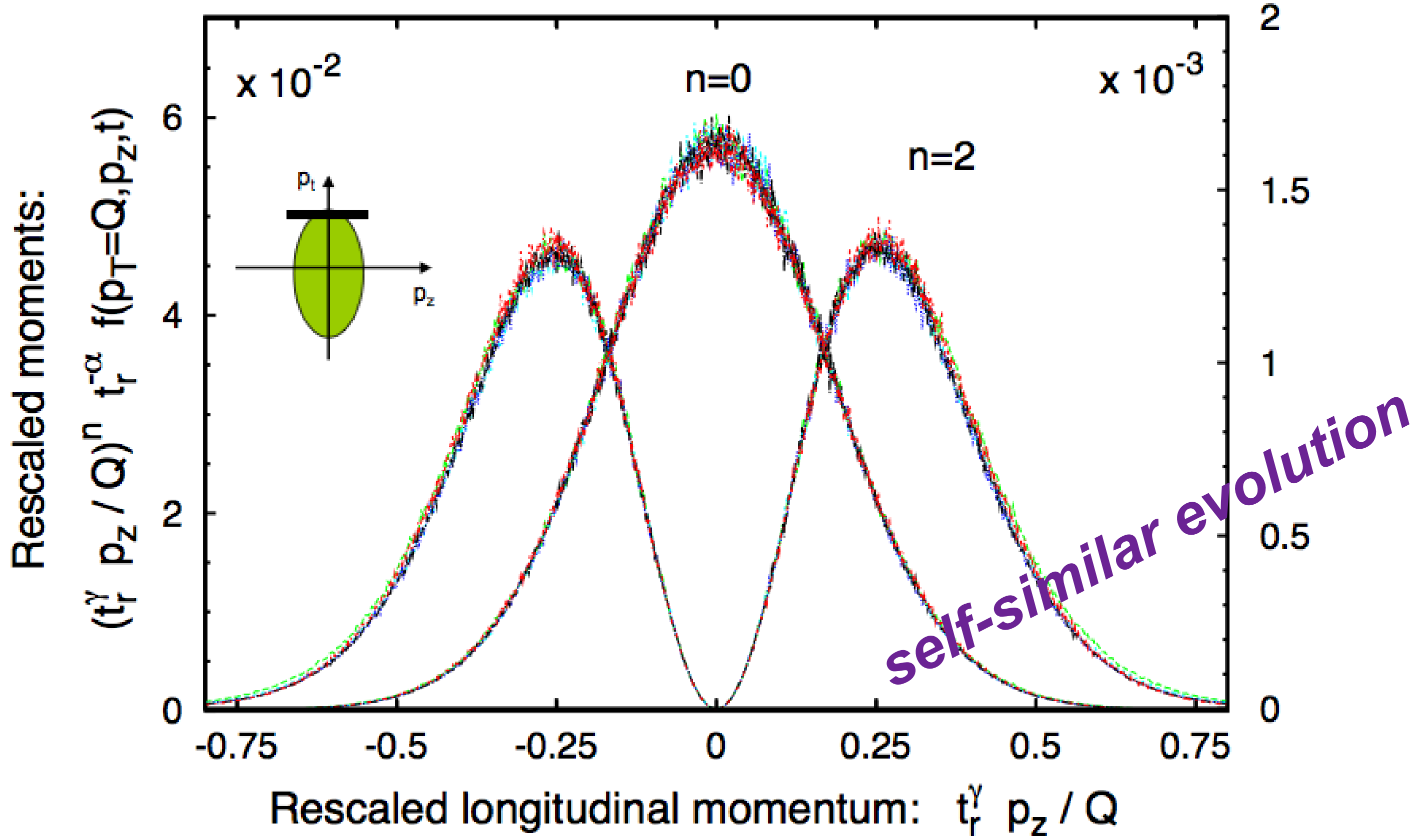
Longitudinal spectrum



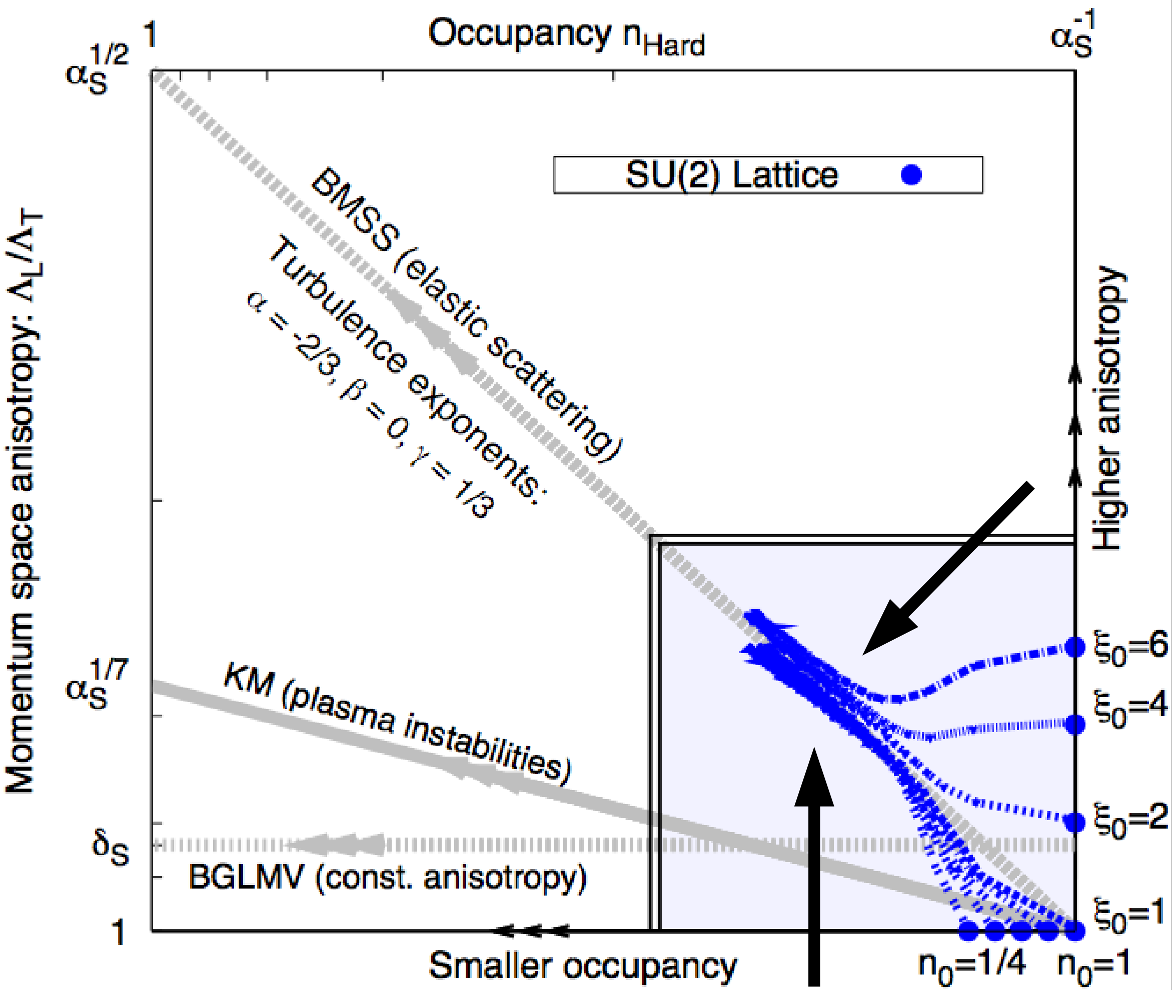
- Thermal like T/p_T spectrum with decreasing amplitude
- Decrease of the characteristic longitudinal momentum

Expanding systems - Self-similarity

- The spectrum of hard excitations shows a ***self-similar evolution*** characteristic of ***wave turbulence***



The attractor solution



- Universal scaling behavior for different initial conditions
- Self-similar evolution can be characterized by the scaling exponents α, β, γ

$$f(p_\perp, p_z, t) = t^\alpha f_S(t^\beta p_\perp, t^\gamma p_z)$$

- Characteristic of wave turbulence
- Qualitative agreement with “bottum-up” thermalization scenario

Conclusion & Outlook

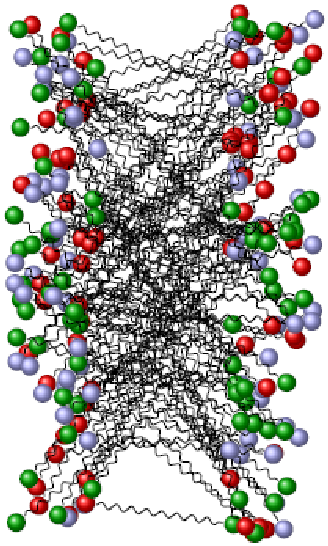
- ***Classical-statistical lattice simulations*** can be used to study the non-equilibrium dynamics from first principles in weak coupling limit.

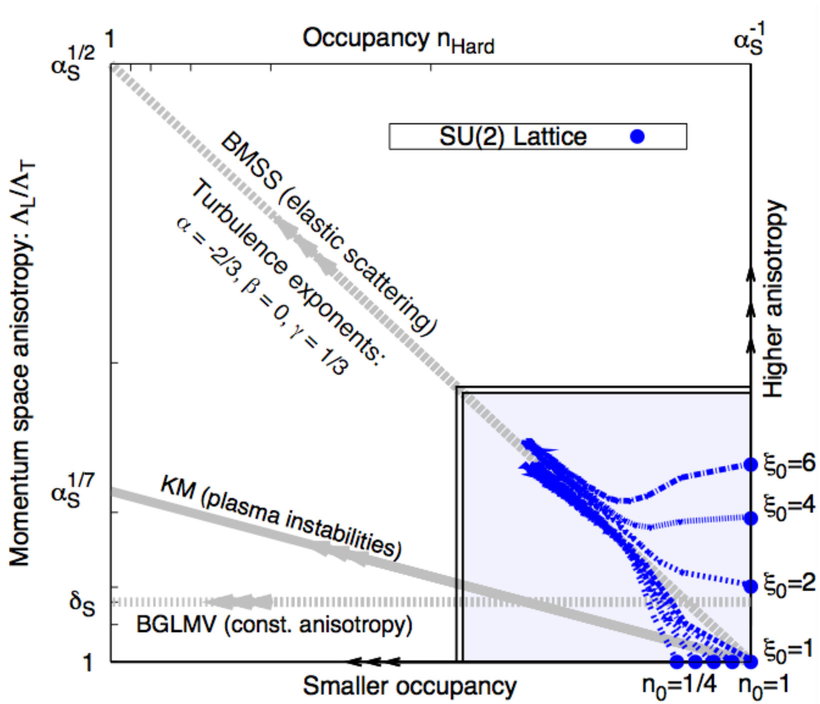
- The ***thermalization process*** in the classical-statistical regime is governed by ***non-thermal fixed points***, where the system exhibits a self-similar evolution. The ***universal scaling exponents*** can be determined from a kinetic theory analysis.

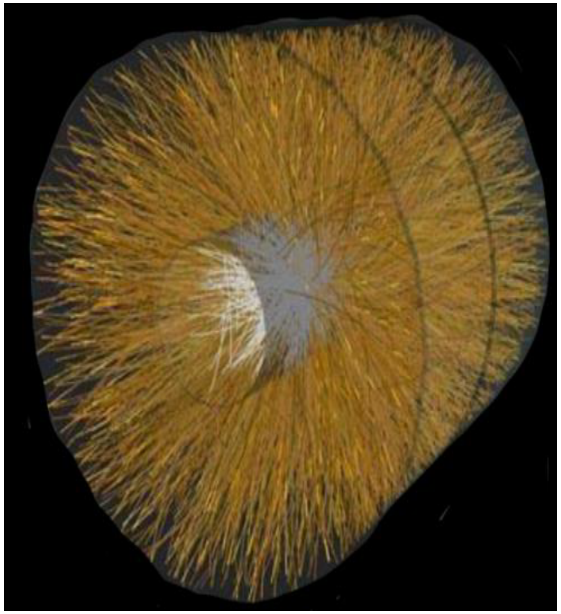
- Generic feature of strongly correlated many-body systems across different energy scales *('big bang', 'little bang', 'ultracold bang')*

- In the weak-coupling limit there is ***no evidence for fast isotropization*** in the classical regime.
(Phenomenological consequences at realistic values of the coupling?)

- To study the complete thermalization at realistic values of the coupling, simulations need to include ***quantum evolution effects***
(include Quarks, study the quantum regime within a kinetic description)

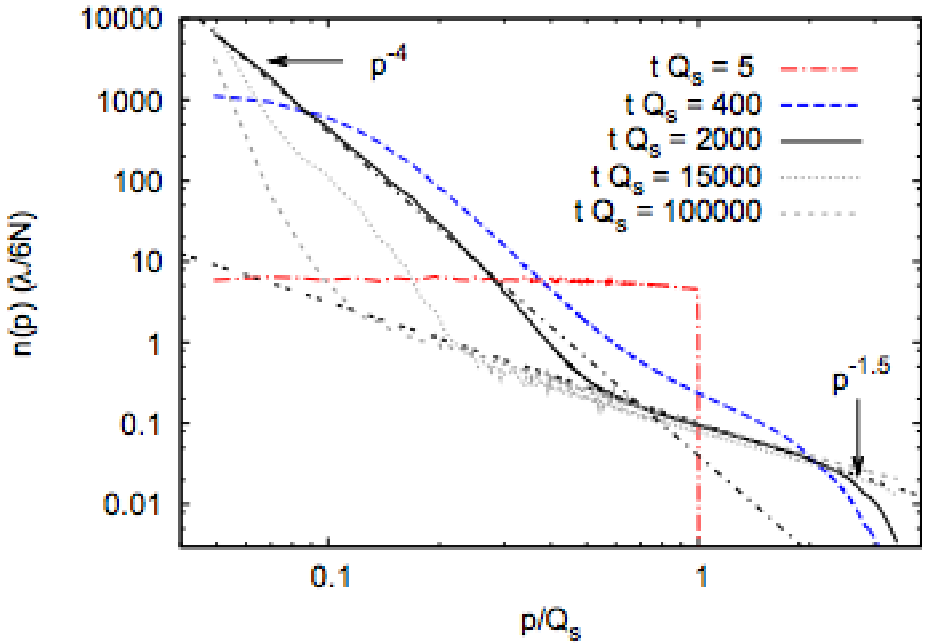


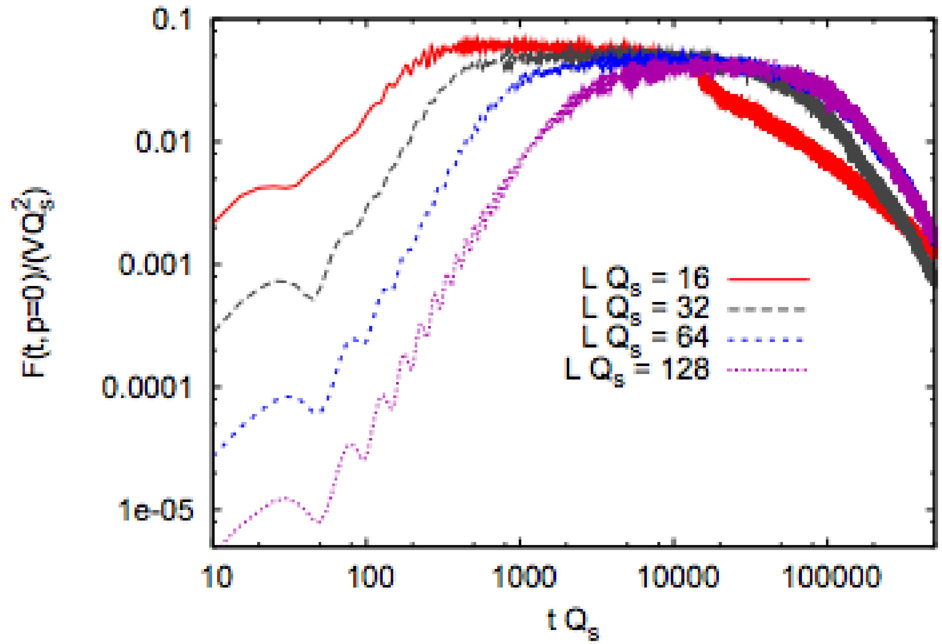

Occupancy n_{Hard}
1
α_S^{-1}
$\alpha_S^{1/2}$
SU(2) Lattice
BMSS (elastic scattering)
Turbulence exponents:
$\alpha = -2/3, \beta = 0, \gamma = 1/3$
Higher anisotropy
Momentum space anisotropy: Λ_L/Λ_T
$\alpha_S^{1/7}$
KM (plasma instabilities)
δ_S
BGLMV (const. anisotropy)
1
Smaller occupancy
$\xi_0=6$
$\xi_0=4$
$\xi_0=2$
$\xi_0=1$
$n_0=1/4$
$n_0=1$



Thank you!

Backup – Bose Condensation





(Berges, Sexty Phys.Rev.Lett. 108 (2012) 161601)