1. Theory of electromagnetic probes
   - The McLerran-Toimela formula

2. In-medium current-current correlator
   - Relation to chiral symmetry
   - QCD sum rules

3. Hadronic models for vector mesons
   - chiral symmetry constraints
   - Hadronic models for light vector mesons
   - Hadronic many-body theory (HMBT)

4. Transport theory and hydrodynamics
   - phase-space distribution
   - relativistic Boltzmann equation
   - the Boltzmann H theorem
   - hydrodynamics

5. References

6. Flash Talks

7. Quiz
Why Electromagnetic Probes?

- $\gamma, \ell^\pm$: only e. m. interactions
- reflect whole “history” of collision
- chance to see chiral symm. rest. directly?

Fig. by A. Drees (from [RW00])
Theory of electromagnetic probes
The McLerran-Toimela formula

- derivation of dilepton-production rate \([\text{MT85, GK91}]

\[
\frac{dR_{\ell^+\ell^-}}{d^4q} = \frac{dN_{\ell^+\ell^-}}{d^4x\,d^4q}
\]

- radiation of dileptons from thermalized strongly interacting particles with total pair four-momentum \(k\)
- dileptons escape fireball without any final-state interactions
- calculation exact concerning strong interactions
- leading-order \(O(\alpha^2)\) in QED
- implies assumption that leptons don’t suffer final-state interactions

\[
H_{\text{em}}^{(\text{int})} = e \int d^3\vec{x} \, J_\mu(t, \vec{x}) A^\mu(t, \vec{x}), \quad A^\mu(t, \vec{x}) = \frac{e\mu}{2\omega V} \exp(iq \cdot x)
\]

- \(J_\mu\): exact (wrt. strong interaction!)
  electromagnetic current operator of quarks or hadrons
  in the Heisenberg picture wrt. strong interactions
- \(e = \sqrt{4\pi\alpha}, \alpha \simeq 1/137\) written out explicitly
The McLerran-Toimela formula

Fermi’s golden rule $\Rightarrow$ transition-matrix element for process $|i\rangle \rightarrow |f'\rangle = |f\rangle + |\ell^+\ell^-(q)\rangle$

QED Feynman rules

$$S_{f'i} = \left\langle f \left| \int d^4x \, J_\mu(x) \right| i \right\rangle D^{\mu\nu}(x, x') e \bar{u}_\ell(x') \gamma_\nu \gamma_\nu v_\ell(x')$$
The McLerran-Toimela formula

- Fourier transformation: energy-momentum conservation $|f'\rangle = |f, \ell^+\ell^-(q)\rangle$

$$S_{f'i} = T_{f'i}(2\pi)^4 \delta^{(4)}(P_f + q - P_i)$$

- Fermi’s trick: Rate

$$R_{f'i} = \frac{|S_{f'i}|^2}{\tau V} = (2\pi)^4 \delta^{(4)}(P_f + q - P_i)|T_{f'i}|^2$$

- summing over $|f\rangle$ and polarizations of dilepton states

- averaging over initial hadron states: heat bath (grand canonical)

$$\rho = \frac{1}{Z} \exp[-\beta(H_{QCD} - \mu_B Q_{baryon})]$$
The McLerran-Toimela formula

- result (derivation see [GK91], Appendices)

\[
\frac{dR_{\ell^+\ell^-}}{d^4q} = -\frac{\alpha^2}{3\pi^3} \frac{q^2 + 2m_\ell^2}{(q^2)^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} g_{\mu\nu} n_B(q^0) \text{Im} \Pi_{\text{ret}}^{\mu\nu}(q)
\]

- em. current-current correlator

\[
i\Pi_{\text{ret}}^{\mu\nu}(q) := \int d^4x \exp(iq \cdot x) \langle [J^\mu(x), J^\nu(0)] \rangle_{T,\mu_B} \Theta(x^0)
\]

- written in (local) restframe of the medium
- in principle measureable: in linear response approximation Green’s function for lepton current running through medium
- \(q^2 = M^2 > 0\) invariant mass of dilepton
- probing medium with photons: same correlator for \(q^2 = M^2 = 0\)
- then correlator \(\leftrightarrow\) dielectric function \(\epsilon(\omega)\) in electrodynamics!
The McLerran-Toimela formula

- for real photons

\[ \omega \frac{dR}{d^3 \vec{q}} = -\frac{\alpha g_{\mu \nu}}{2\pi^2} \text{Im} \Pi_{\text{ret}}^{\mu \nu}(q)n_B(q^0), \quad q^0 = \omega = |\vec{q}| \]

- written in (local) restframe of the medium

- Phenomenological effective hadronic model: vector-meson dominance model

- em. current \( \propto V^\mu \) (with \( V \in \{ \rho, \omega, \phi \} \))

- Dilepton/photon rates: \( \propto A_V = -2 \text{Im} D^{\text{ret}}_V \) (vector-meson spectral function!)

- measuring in-medium vector-meson spectral function!?!
Em. current-current correlator
- photon and dilepton thermal emission rates given by same electromagnetic-current-correlation function \( J_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f \)

- McLerran-Toimela formula

\[
\Pi_{\mu \nu}^\prec(q) = \int d^4 x \exp(iq \cdot x) \langle J_\mu(0)J_\nu(x) \rangle_T = -2n_B(q_0) \text{Im} \Pi_{\mu \nu}^{(\text{ret})}(q)
\]

\[
q_0 \frac{dN_\gamma}{d^4 x d^3 \vec{q}} = -\frac{\alpha_{\text{em}}}{2\pi^2} g^{\mu \nu} \text{Im} \Pi_{\mu \nu}^{(\text{ret})}(q, u) \bigg|_{q_0 = |\vec{q}|} n_B(q \cdot u)
\]

\[
\frac{dN_{\ell^+\ell^-}}{d^4 x d^4 q} = -g^{\mu \nu} \frac{\alpha^2}{3q^2 \pi^3} \text{Im} \Pi_{\mu \nu}^{(\text{ret})}(q, u) \bigg|_{q^2 = M_{\ell^+\ell^-}^2} n_B(q \cdot u)
\]

- manifestly Lorentz covariant (dependent on four-velocity of fluid cell, \( u \))

- to lowest order in \( \alpha \): \( 4\pi \alpha \Pi_{\mu \nu} \sim \sum^{(\gamma)}_{\mu \nu} \)

- derivable from underlying thermodynamic potential, \( \Omega \)!
Vector Mesons and chiral symmetry

- **vector** and **axial-vector** mesons $\leftrightarrow$ respective current correlators

$$\Pi^{\mu\nu}_{V/A}(q) := \int d^4 x \exp(i q x) \langle J^\nu_{V/A}(0) J^{\mu}_{V/A}(x) \rangle_{\text{ret}}$$

- Ward-Takahashi Identities of $\chi$ symmetry $\Rightarrow$ Weinberg-sum rules

$$f_{\pi}^2 = -\int_0^\infty \frac{dq_0}{\pi p_0^2} [\text{Im} \Pi_V(q_0, 0) - \text{Im} \Pi_A(q_0, 0)]$$

- spectral functions of vector (e.g. $\rho$) and axial vector (e.g. $a_1$) directly related to **order parameter of chiral symmetry**!
Vector Mesons and chiral symmetry

- at high enough temperatures and or densities: melting of $\langle \bar{q} q \rangle$
- ⇒ spontaneous breaking of chiral symmetry suspended
- ⇒ chiral phase transition; chiral-symmetry restoration ($\chi$ SR)
- which scenario is right? microscopic mechanisms behind $\chi$ SR?

![Graph showing $-\text{Im} \Pi_{V,A}/(\pi s)$ vs. $s$ for $V[\tau \rightarrow 2n\pi \nu_\tau]$ and $A[\tau \rightarrow (2n+1)\pi \nu_\tau]$.]

- $\rho (770) + \text{cont.}$
- $a_1 (1260) + \text{cont.}$

---

Dropping Masses?

![Graph showing mass spectral function with $\rho$ and $a_1$ peaks and dropping mass curves.]

Melting Resonances?

![Graph showing mass spectral function with $\rho$ and $a_1$ peaks and melting resonance curves.]

from [Rap03]  
from [Rap05]
Scenarios for chiral symmetry restoration

- hadron spectrum must become \textit{degenerate} between chiral partners

\begin{itemize}
  \item models alone of little help (realization of $\chi_S$ not unique!)
    \begin{itemize}
      \item “vector manifestation” $\rho_{\text{long}} = \chi$ partner of $\pi \Rightarrow$ dropping mass
      \item “standard realization” $\rho = \chi$ partner of $a_1$, extreme broadening + little mass shifts
    \end{itemize}
  \item theory “shopping list”
    \begin{itemize}
      \item effective hadronic models (well constrained in vacuum!)
      \item \textit{and} concise evaluation in the medium!
      \item models for fireball evolution
        (blast-wave parametrizations, hydro, transport, and transport-hydro hybrids)
      \item must include partonic $\rightarrow$ phase transition $\rightarrow$ hadronic evolution
      \item precise $\ell^+\ell^-$ ($\gamma$) data from HICs \textit{mandatory!}
    \end{itemize}
\end{itemize}
QCD Sum Rules

- based on \([\text{LPM98}]\)
- calculate current correlator, e.g., the vector part of the em. current
  \[ j_\mu = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \]
- corresponds to the \(\rho\) meson!
- use pQCD to determine correlator
  \[ \Pi_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{k^2}\right)\Pi(q^2) \]

  in deep spacelike region, \(Q^2 = -q^2 \gg \Lambda_{\text{QCD}}\)
- related to time-like region \(\Rightarrow\) sum rule
  \[ \Pi(q^2) = \Pi(-Q^2) = \Pi(0) + c Q^2 + \frac{Q^4}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{s^2(s + Q^2 - i\epsilon)} \]
- dispersion relation: spectral function \(\text{Im} \Pi\)!
QCD Sum Rules

- left-hand side of sum rule
- pQCD + chiral models for baryon-pion interactions [see, e.g., [DGH92]]

\[
R(Q^2) := \frac{\Pi(k^2 = -Q^2)}{Q^2} = -\frac{1}{8\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \ln \left( \frac{Q^2}{\mu^2} \right) \\
+ \frac{1}{Q^4} m_q \langle \bar{q} q \rangle + \frac{1}{24Q^4} \left\langle \frac{\alpha_s}{\pi} F^{a \mu \nu}_{\mu \nu} F^{a \mu \nu} \right\rangle - \frac{112}{81Q^6} \kappa \langle \bar{q} q \rangle^2
\]

- additional cold-nuclear matter contributions

\[
\Delta R(Q^2) = \frac{m_N}{4Q^4} A_2 \rho_N - \frac{5m_N^3}{12Q^6} A_4 \rho_N
\]

- \(A_{2,4}\) from parton-distribution functions
- also condensates medium-modified (in low-density approximation)

\[
\langle \bar{q} q \rangle = \langle \bar{q} q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q} \rho_N,
\]

\[
\left\langle \frac{\alpha_s}{\pi} F^{a \mu \nu}_{\mu \nu} F^{a \mu \nu} \right\rangle = \left\langle \frac{\alpha_s}{\pi} F^{a \mu \nu}_{\mu \nu} F^{a \mu \nu} \right\rangle_{\text{vac}} - \frac{8}{9} m_N^{(0)} \rho_N
\]
- QCD Sum Rules
- right-hand side of sum rule
- use hadronic models to fit measured vector-current correlator
- e.g., ALEPH/OPAL data of $\tau \rightarrow \nu + 2n\pi$

![Graph showing the behavior of $-\text{Im} \Pi_V/(\pi M^2)$ versus $M^2$ (GeV^2) for different processes with data points from ALEPH at LEP.](image)
QCD Sum Rules

- typical result from [LPM98]

\[ \rho_N = \rho_0 \]
\[ \kappa = 236 \]

S. Leupold, W. Peters, U. Mosel

- possible medium effects on \( \rho \) meson
  - dropping mass, unchanged/small width
  - unchanged mass, broadened spectrum (large width)
Weinberg Sum Rules

- **vector** and **axial-vector** mesons ↔ respective current correlators

\[
\Pi_{V/A}^{\mu\nu}(q) := \int d^4x \exp(iq x) \langle J_{V/A}^\nu(0) J_{V/A}^\mu(x) \rangle_{\text{ret}}
\]

- Ward-Takahashi Identities of \(\chi\) symmetry \(\Rightarrow\) Weinberg-sum rules

\[
f_\pi^2 = - \int_0^\infty \frac{dq_0^2}{\pi q_0^2} \left[ \text{Im} \Pi_V(q_0, 0) - \text{Im} \Pi_A(q_0, 0) \right]
\]

- **spectral functions** of vector (e.g. \(\rho\)) and axial vector (e.g. \(a_1\)) directly related to **order parameter** of chiral symmetry!
Weinberg Chiral Sum Rules

- Chiral-Sum-Rule analysis by Hohler and Rapp \cite{HR14}
- using detailed in-medium models for vector-meson spectral functions
- construct axial-vector-meson spectral functions

![Graph showing spectral functions at different temperatures](image)

- compatible with chiral-symmetry restoration
Hadronic models
Effective hadronic models: chiral-symmetry constraints

- different realizations of *chiral symmetry*
- equivalent only on shell ("low-energy theorems")
- model-independent conclusions only in *low-temperature/density limit* (chiral perturbation theory) or from *lattice-QCD calculations*
- QCD sum rules: allow dropping-mass or melting-resonance scenario
- use *phenomenological hadronic many-body theory* (HMBT) to assess medium modifications of vector mesons
  - build models with *hadrons* as effective degrees of freedom
  - based on *(chiral) symmetries*
  - constrained by data on cross sections, branching ratios,... in the vacuum
  - in-medium properties assessed by *many-body (thermal) field theory*
Realistic hadronic models for light vector mesons

- CERES data: pion-$\rho$ model too simplistic
- many approaches to more realistic models
  - gauged linear $\sigma$-model + vector-meson dominance [Pis95, UBW02]
    - gauge-symmetry breaking $\Rightarrow$ pions still in physical spectrum!
  - massive Yang-Mills model; gauged non-linear chiral model with explicitly broken gauge symmetry [Mei88, LSY95]
  - hidden local symmetry: Higgs-like chiral model [BKU+85, HY03]
    - allows for vector manifestation or usual manifestation (with $a_1$)
- here: phenomenological model by Rapp, Wambach, et al [RW99a]
Hadronic many-body theory

- Phenomenological HMBT \({\text{[RW99a]}}\) for vector mesons
- \(\pi\pi\) interactions and baryonic excitations

Baryon (resonances) important, even at RHIC with low net baryon density \(n_B - n_{\bar{B}}\)

- reason: \(n_B + n_{\bar{B}}\) relevant (CP inv. of strong interactions)
The meson sector (vacuum)

- most important for $\rho$-meson: pions

![Graph showing $|F_\pi(q^2)|^2$ and $\delta_1$ vs. $q^2$ and $M_{\pi\pi}$](image)
The meson sector (matter)

- Pions dressed with N-hole-, $\Delta$-hole bubbles
- Ward-Takahashi $\Rightarrow$ vertex corrections mandatory!
The meson sector (contributions from higher resonances)
The baryon sector (vacuum)

- $P = 1$-baryons: $p$-wave coupling to $\rho$:
  - $N(939), \Delta(1232), N(1720), \Delta(1905)$

- $P = -1$-baryons: $s$-wave coupling to $\rho$:
  - $N(1520), \Delta(1620), \Delta(1700)$
Photoabsorption on nucleons and nuclei

\[ \sigma_N, \sigma_A \] vs. \( q_0 \) [MeV]

- \( \gamma p/\gamma n \) averaged

- N(1520)
- N(1720)

- Sn (Frascati)
- Pb (Frascati)
- Pb (Saclay)
- U (averaged)
In-medium spectral functions and baryon effects

-Im $D_{\rho}$ (GeV$^{-2}$)

$\rho(770)$

$M$ (GeV)

T=0
T=120 MeV
T=150 MeV
T=175 MeV

baryon effects important

- large contribution to broadening of the peak
- responsible for most of the strength at small $M$

[R. Rapp, J. Wambach 99]
In-medium spectral functions and baryon effects

-\text{Im} D_\omega (\text{GeV}^{-2})

M (GeV)

\omega (782)

\begin{itemize}
  \item baryon effects important
    \begin{itemize}
      \item large contribution to broadening of the peak
      \item responsible for most of the strength at small \( M \)
    \end{itemize}
\end{itemize}

[R. Rapp, J. Wambach 99]
baryon effects important
- large contribution to broadening of the peak
- responsible for most of the strength at small $M$
In-medium spectral functions and baryon effects

- baryon effects important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small $M$

[R. Rapp, J. Wambach 99]
Dilepton rates: Hadron gas ↔ QGP

- in-medium hadron gas matches with QGP
- similar results also for $\gamma$ rates
- “quark-hadron duality”?  
- hidden local symm.+baryons?

[BKU$^+$85, HY03, HS06, HSW08]
Transport theory and hydrodynamics
Phase-space distribution

- classical many-body system of relativistic particles
- all particles are on their mass shell: \( E = E_p := \sqrt{\vec{p}^2 + m^2} \)
- Boltzmann equation \([dvv80, CK02, Hee15]\):
  dynamical equation for phase-space distribution function \( f(t, \vec{x}, \vec{p}) \)
- relativistic covariance of phase-space distribution
  - \( f(t, \vec{x}, \vec{p}) \) defined as Lorentz scalar quantity
  - particle number \( N \): \( dN = d^3 \vec{x} d^3 \vec{p} f(t, \vec{x}, \vec{p}) \)
  - particle-number four-vector current \((N^\mu) = (n, \vec{N})\)

\[
N^\mu = \int_{\mathbb{R}^3} d^3 \vec{p} \frac{p^\mu}{E_p} f(t, \vec{x}, \vec{p})
\]

- flow-velocity of fluid cell (“Eckart frame”)

\[
\vec{v}_{\text{Eck}}(x) = \frac{\vec{N}(x)}{N^0(x)}, \quad u^\mu_{\text{Eck}} = \frac{N^\mu}{\sqrt{N_\mu N^\mu}} = \frac{N^\mu}{n_0}
\]

- \( n_0 \): particle density in local fluid (Eckart) restframe
Relativistic Boltzmann equation

- particles moving along trajectories \((\vec{x}(t), \vec{p}(t))\)
- for infinitesimal time step \(dt\)

\[
dN(t + dt) = f(t + dt, \vec{x} + dt \vec{v}, \vec{p} + dt \vec{F}) d^6 \xi(t + dt), \quad d^6 \xi = d^3 \vec{x} d^3 \vec{p}
\]

- Jacobian for phase-space volume

\[
d^6 \xi(t + dt) = d^6 \xi(t) \det \left( \frac{\partial (\vec{x} + dt \vec{v}, \vec{p} + dt \vec{F})}{\partial (\vec{x}, \vec{p})} \right) = d^6 \xi(t) \left(1 + dt \hat{\nabla}_p \cdot \vec{F}\right) + \mathcal{O}(dt^2)
\]

- total change of \(dN\)

\[
dN(t + dt) - dN(t) = d^6 \xi(t) dt \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\partial (\vec{F} \cdot f)}{\partial \vec{p}} \right]
\]
Relativistic Boltzmann equation

- covariance: \( d\tau = dt \sqrt{1 - \vec{v}^2} \) proper time, \( \vec{v} = \vec{p}/E_p, \sqrt{1 - \vec{v}^2} = m/E_p \)

\[
\begin{align*}
\frac{dt}{d\tau} \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} \right] &= d\tau \frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} \Rightarrow \text{covariant!}
\end{align*}
\]

- covariant equation of motion for point particle

\[
\frac{dp^\mu}{d\tau} = K^\mu, \quad p_\mu p^\mu = m^2 = \text{const} \Rightarrow
\]

\[
K^0 = \frac{\vec{p}}{E_p} \cdot \vec{K} \Rightarrow \frac{d\vec{p}}{dt} = \vec{F} = \vec{K} \frac{m}{E_p}
\]

\[
\frac{E_p}{m} \nabla_p (\vec{F} f) = \frac{\partial}{\partial p^\mu} (K^\mu f) \Rightarrow \text{covariant!}
\]

\[
\frac{dN(t + dt) - dN(t)}{d^6\xi} = dt \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\partial (\vec{F} f)}{\partial \vec{p}} \right] = d\tau \left[ \frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} + \frac{\partial (K^\mu f)}{\partial p^\mu} \right]
\]
Relativistic Boltzmann equation

- change of particle number due to collisions
- short-range interactions: collisions at one point (local) in space
- invariant cross section

\[
dN_{\text{coll}}(p'_1, p'_2 \leftarrow p_1, p_2) = d^4 x \frac{d^3 \vec{p}_1}{E_1} \frac{d^3 \vec{p}_2}{E_2} \frac{d^3 \vec{p}'_1}{E'_1} \frac{d^3 \vec{p}'_2}{E'_2} f_1 f_2 W(p'_1, p'_2 \leftarrow p_1, p_2),
\]

\[
d\sigma = \frac{W(p'_1, p'_2 \leftarrow p_1, p_2) d^4 x \frac{d^3 \vec{p}_1}{E_1} \frac{d^3 \vec{p}_2}{m} \frac{d^3 \vec{p}'_1}{E'_1} \frac{d^3 \vec{p}'_2}{E'_2} f_1 f_2}{d^4 x d^3 \vec{p}_1 \nu_{\text{rel}} f_1 d^3 \vec{p}_2 f_2},
\]

\[
d\sigma = \frac{d^3 \vec{p}'_1}{E'_1} \frac{d^3 \vec{p}'_2}{E'_2} \frac{W(p'_1, p'_2 \leftarrow p_1, p_2)}{I}, \quad I = \sqrt{(p_1 \cdot p_2)^2 - m^4}
\]

- important: \(\nu_{\text{rel}}\) is velocity of particle 1 in rest frame of particle 2
- from relativistic covariance (or unitarity of S-matrix!) \(\Rightarrow\) detailed-balance relation

\[
W(p'_1, p'_2 \leftarrow p_1, p_2) = W(p_1, p_2 \leftarrow p'_1, p'_2)
\]

- Boltzmann equation (manifestly covariant form)

\[
p^\mu \frac{\partial f}{\partial x^\mu} + m \frac{\partial (K^\mu f)}{\partial p^\mu} = \frac{1}{2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}'_1}{E'_1} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}'_2}{E'_2} W(p'_1, p'_2 \leftarrow p, p_2)(f'_1 f'_2 - f f_2)
\]

- collision integral: “gain minus loss”
input from quantum mechanics: particle in a cubic box (periodic boundary cond.)

\[ \mathbf{\hat{p}} = \frac{2\pi}{L} \mathbf{\hat{n}}, \quad \mathbf{\hat{n}} \in \mathbb{Z}^3 \]

\[ \Delta^6 \xi_j = L^3 \Delta^3 \mathbf{\hat{p}} \] ("microscopically large, macroscopically small")

contains \( G_j \) single-particle states (\( g \): degeneracy due to spin, isospin, ...)

\[ G_j = g \frac{\Delta^6 \xi_j}{(2\pi)^3} \]

statistical weight for \( N_j \) particles in \( \Delta^6 \xi_j \):

factor \( 1/N_j! \): indistinguishability of particles

\[ \Delta \Gamma_j = \frac{1}{N_j!} G_j^{N_j} \]

entropy a la Boltzmann

\[ S = \sum_j \ln \Delta \Gamma_j \approx \sum_j [N_j \ln G_j - N_j (\ln N_j - 1)] \]

\[ = - \int d^3 \mathbf{x} d^3 \mathbf{p} f(x, p) \{ \ln[(2\pi)^3 f(x, p)/g] - 1 \} \]
The Boltzmann H theorem

- \( H = \text{greek Eta: Boltzmann's notation for entropy} \)
- covariant description of entropy: entropy four-flow

\[
S^\mu(x) = - \int_{\mathbb{R}^3} \frac{d^3 \vec{p}}{E} p^\mu f(x, p) \{ \ln[(2\pi)^3 f(x, p)/g] - 1 \}
\]

- Boltzmann equation + symmetries of \( W(p_1' p_2' \leftarrow p_1 p_2) \)

\[
\frac{\partial S^\mu}{\partial x^\mu} := \zeta = + \frac{1}{4} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}}{E} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_1'}{E_1'} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2'}{E_2'} f f_2 \\
\quad \times \left[ \frac{f_1' f_2'}{f f_2} - \ln \left( \frac{f_1' f_2'}{f f_2} \right) - 1 \right] W(p_1' p_2' \leftarrow p, p) \geq 0
\]

- (on average) entropy can never decrease with time!
- equilibrium \( \iff \) \( S \) maximal!
- bracket must vanish \( \Rightarrow \) Maxwell-Boltzmann distribution

\[
f_{eq}(x, p) = \frac{g}{(2\pi)^3} \exp \left[ -\beta(x) \left( u(x) \cdot p - \mu(x) \right) \right], \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}
\]

- \( \beta = 1/T \): inverse temperature, \( u \): fluid four-velocity, \( \mu \): chemical potential
- temperature, chemical potential are Lorentz scalars!
in the limit of very small mean-free path: system in local thermal equilibrium
switch to ideal hydrodynamics description
forget about “particles” ⇒ fluid description
equations of motion for \( \vec{v}(t, \vec{x}) \): conservation laws

\[
\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu \nu} = 0
\]

\( N^\mu \): net-baryon number, \( T^{\mu \nu} \): energy-momentum tensor

ideal hydrodynamics

\[
N^\mu = n u^\mu, \quad T^{\mu \nu} = (\epsilon + P) u^\mu u^\nu - P \eta^{\mu \nu}
\]

\[
\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu \nu} = 0
\]

\( n \): proper net-baryon density, \( \epsilon \): proper energy density, \( P \): pressure
5 equations of motion, 6 unknowns: \( \vec{v}, n, \epsilon, P \)
need also equation of state \( \epsilon = \epsilon(P) \)

hadron-resonance gas EoS (low energies)
lQCD based cross-over phase transition (high energies)
http://dx.doi.org/10.1103/PhysRevLett.54.1215

http://dx.doi.org/10.1007/978-3-0348-8165-4


http://dx.doi.org/10.1016/0550-3213(91)90459-B


http://dx.doi.org/10.1016/S0370-1573(03)00139-X

http://dx.doi.org/10.1016/S0375-9474(97)00634-9

https://doi.org/10.1016/0370-2693(95)80022-P

http://dx.doi.org/10.1016/0370-1573(88)90090-7
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<th>Reference</th>
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<th>Journal Details</th>
<th>DOI</th>
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<tr>
<td>[Rap03]</td>
<td>R. Rapp, Dileptons in high-energy heavy-ion collisions</td>
<td>Pramana 60 (2003) 675</td>
<td><a href="http://dx.doi.org/10.1007/BF02705167">http://dx.doi.org/10.1007/BF02705167</a></td>
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http://dx.doi.org/10.1007/s100500050364


http://dx.doi.org/10.1103/PhysRevLett.88.042002

http://link.aps.org/abstract/PRL/v18/p507
Flash Talks
Summarize the basic steps to derive the McLerran-Toimela formula (slides 5-7, private notes to be distributed)

What are the basic ideas behind QCD and chiral sum rules (slides 15-17, [LPM98, Wei67])

What's the main mechanism behind in-medium modifications of the \( \rho \) meson? (slides 24-32, [RW99b])

Explain (intuitively) what's behind the Boltzmann equation (slides 35-37, [Hee15])
Quiz
1. Which important “theoretical quantity” can be assessed by observing electromagnetic probes in HICs (and elementary reactions)?

2. What is chiral-symmetry restoration (χSR) and in which ways could it be realized in nature?

3. What have em. probes in heavy-ion collisions to do with chiral symmetry?

4. What can we learn from QCD and chiral sum rules about χSR?

5. What’s basic assumption of the vector-meson dominance (VMD) model?

6. What tell effective hadronic models about the medium modification of light vector mesons and the related χSR?

7. Why are baryon-vector-meson interactions important even at high collision energies, where $\mu_B \sim 0$ (nearly 0 net-baryon density)?