

Inflation im frühen Universum

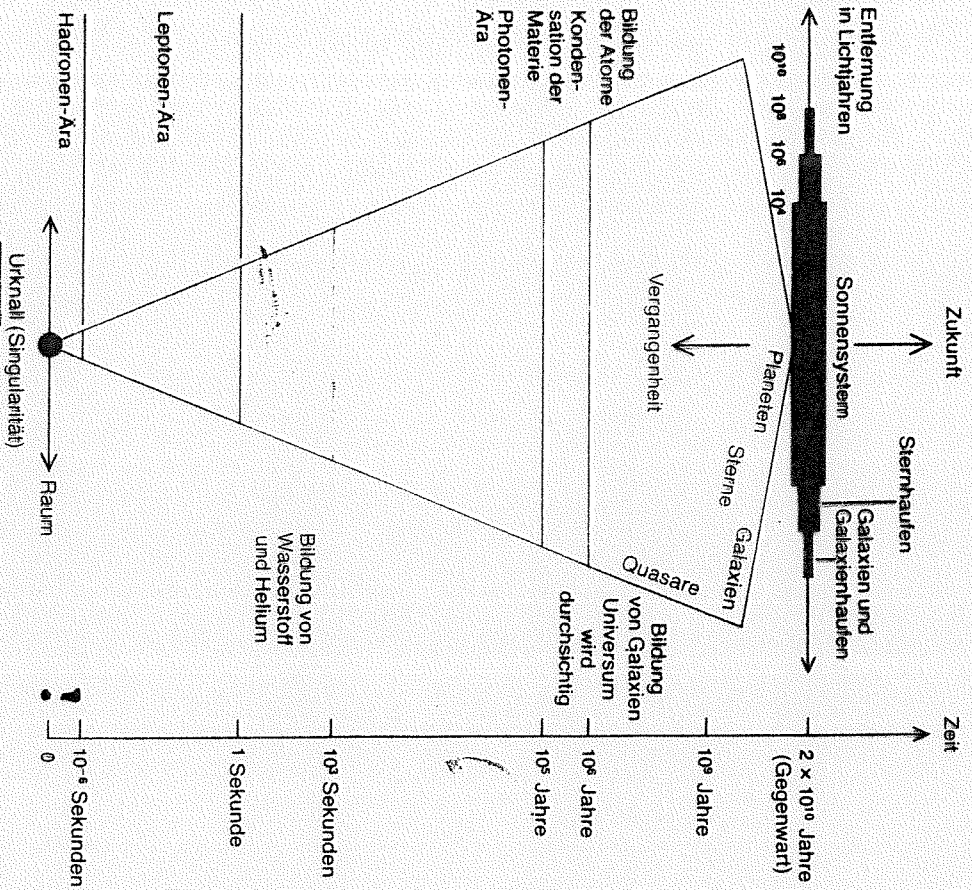
Carsten H. Greiner
(Universität Giessen)

- Standardmodell der Kosmologie
 - ◇ Friedmansche Weltmodelle
 - ◇ DeSitter-Lemaître Lösung
 - ◇ Probleme des **SM** ...!

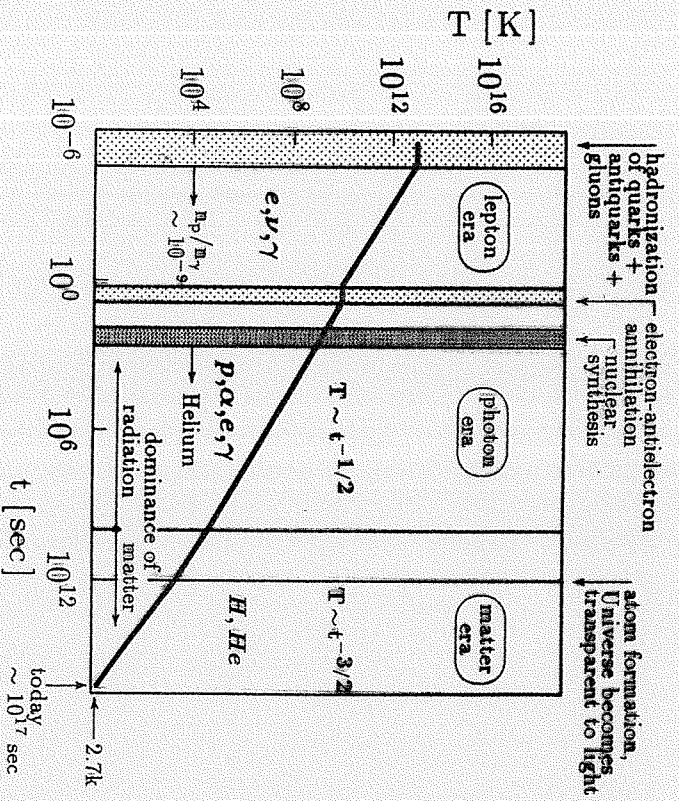
- Die Hypothese der Inflation
 - ◇ Große vereinheitlichte Theorie
 - ◇ Universum mit Symmetriebrechung
 - ◇ Lösen einiger Probleme

- Neue Fragestellungen
 - ◇ Fluktuationen in der CMBR
 - ◇ beschleunigtes Universum

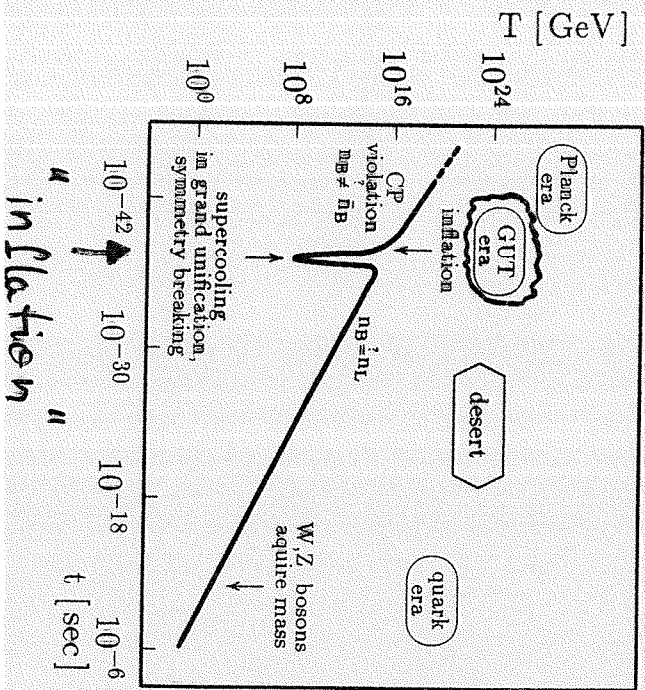
Motivation



Das kosmologische Standardmodell



altes SM

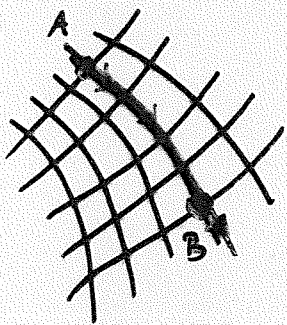


'neues' SM
Märchenära

inflation

Einsteins Allgemeine Relativitätstheorie

- Äquivalenzprinzip \leadsto schwere = träge Masse



$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\lambda\nu}^\mu [g_{\alpha\beta}] \frac{dx^\lambda}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

(Geodätenbewegung in
4-dim. gekrümmten Raum)

- Einsteinsche Feldgleichungen:

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu} [\rho c^2; P] - (\Lambda g_{\mu\nu})$$

$\underbrace{\hspace{10em}}$
kosmologische
Konstante

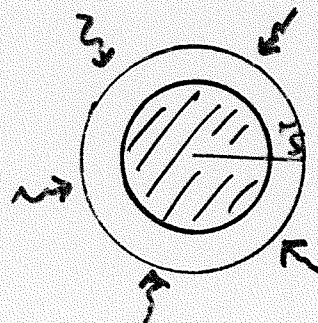
- ◇ $R_{\mu\nu} [g_{\alpha\beta}]$: Ricci-Tensor;
- ◇ $R := g_{\mu\nu} R^{\mu\nu}$: Krümmungsskalar;
- ◇ $T_{\mu\nu}$: Energie-Impuls-Tensor

- Beispiel: **Schwazschild-Lösung**

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \equiv c^2 d\tau^2$$

$$= \left(1 - \frac{r_S}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_S}{r}\right)} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

- ◇ $r_S := \frac{2G}{c^2} M_\odot$ - Schwarzschildradius
- ◇ Falls $r_S > R_\odot \leadsto$ schwarzes Loch!



Kosmologie

□ ΚΟΣΜΟΣ ↔ 'Ordnung'

↷ Entwicklung des Universums als Ganzes

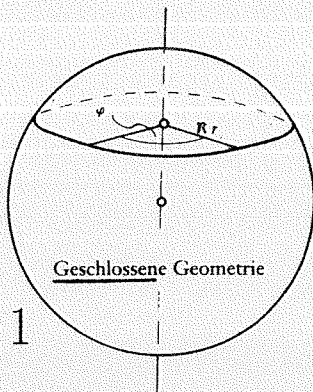
□ *Machsches Prinzip:*

Materieverteilung ↷ Raum-(Zeit-)Geometrie

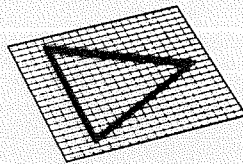
□ **kosmologisches Prinzip:** Physik in jedem Raumpunkt gleich

□ **Robertson-Walker-Metrik:**

$$ds^2 = c^2 dt^2 - \underbrace{R^2(t) \left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right)}_{dl^2}$$

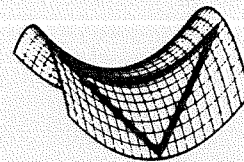


$k = 1$



Flache Geometrie

$k = 0$



Offene Geometrie

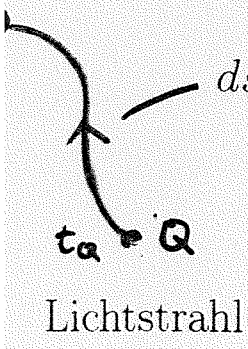
$k = -1$

□ Skalenfaktor $\underline{R}(t)$ ↷ **Rotverschiebung**

$$dL(t) = \underline{R}(t) dl \equiv \text{phys. Abstand}$$

$ds^2 = 0 \quad \rightsquigarrow \int_{t_Q}^{t_B} \frac{cdt}{R(t)} = |l_Q - l_B| = \text{const} \quad \rightsquigarrow \frac{\nu_B}{\nu_Q} = \frac{R(t_Q)}{R(t_B)}$

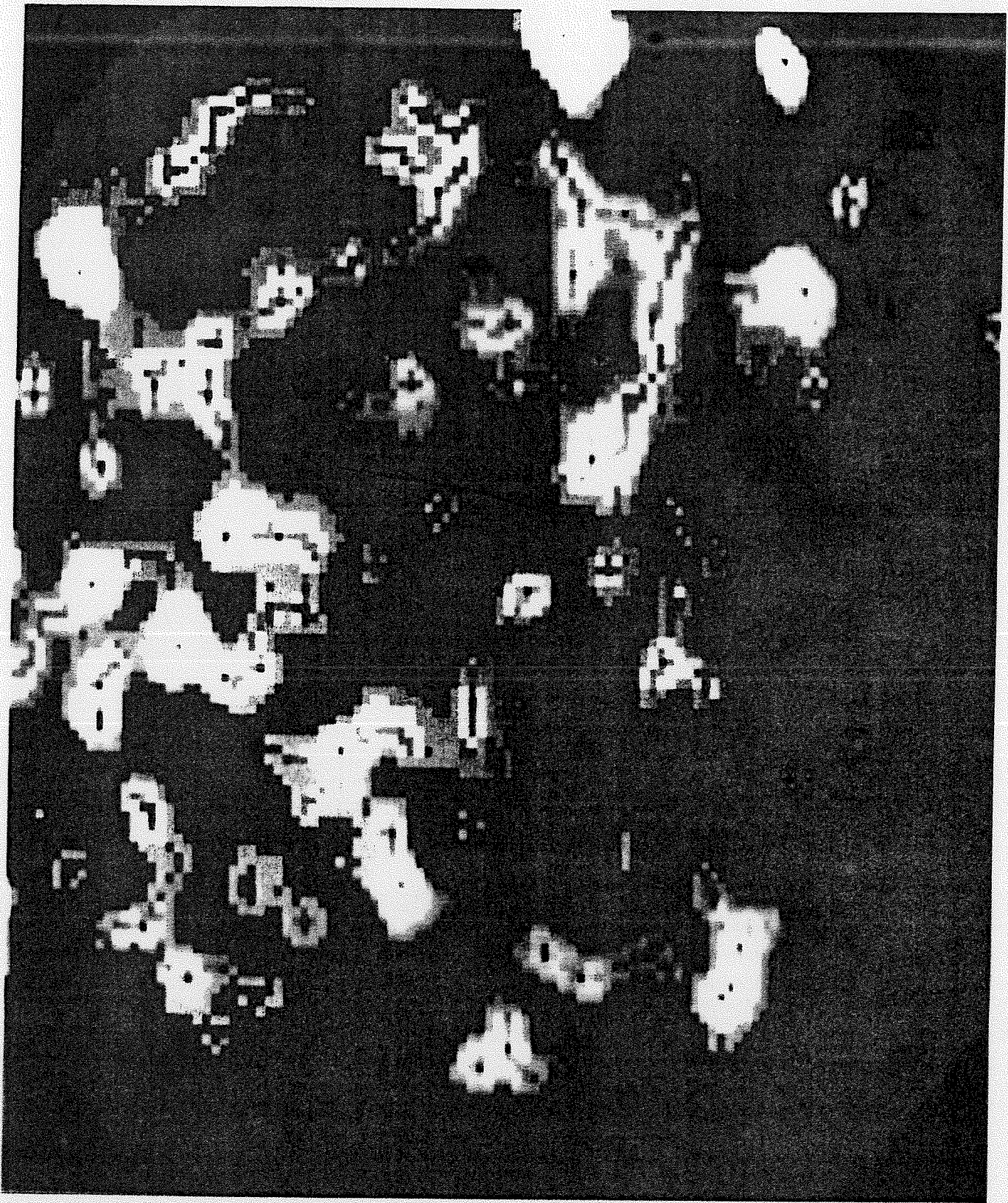
$$z = \frac{\lambda_B - \lambda_Q}{\lambda_Q} = \frac{R(t_B)}{R(t_Q)} - 1 \quad (> 0)$$



Verteilungsmuster von Galaxien:

$7 \cdot 10^9$ Galaxien

$\Delta\varphi \leq 100^\circ$, $\Delta L \leq 10^9 L_7$



!E: $1 \text{ pc} = 3,26 L_7 = 3,1 \cdot 10^{13} \text{ km} = 10^{-6} \text{ Mpc}$

$\phi_{\text{Galaxie}} \sim 1-100 \text{ kpc}$, $\phi_{\text{Haufen}} \sim 2-10 \text{ Mpc}$, $\phi_{\text{Super-Haufen}} \sim 50-100 \text{ Mpc} \ll \Delta \bar{V} \ll \phi_{\text{Unit.}} \sim 5 \cdot 10^4 \text{ Mpc}$

□ kosmologischer Dopplereffekt



$$z = \frac{1}{c} H_0 L + \frac{1}{2c^2} H_0^2 (1 + q_0) L^2 + \dots$$

- ◇ $H_0 = \frac{\dot{R}(t_0)}{R(t_0)} = (T_H)^{-1}$ - Hubble-‘Konstante’ = $75 \pm 25 \frac{\text{km}}{\text{s Mpc}}$
 $\hookrightarrow T_H \sim 1-2 \cdot 10^{10} \text{ J}$
- ◇ $q_0 = -\frac{\ddot{R}(t_0)R(t_0)}{(\dot{R}(t_0))^2} (> 0)$ ‘Verzögerungsparameter’
 $(0 < q_0 < 2)$

□ **Friedman-Gleichungen:**

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} - \left(\frac{\Lambda}{3}c^2\right) = \frac{8\pi G}{3c^2} \rho c^2$$

$$\frac{\ddot{R}}{R} - \left(\frac{\Lambda}{3}c^2\right) = \frac{4\pi G}{3c^2} (\rho c^2 + \underline{3P})$$

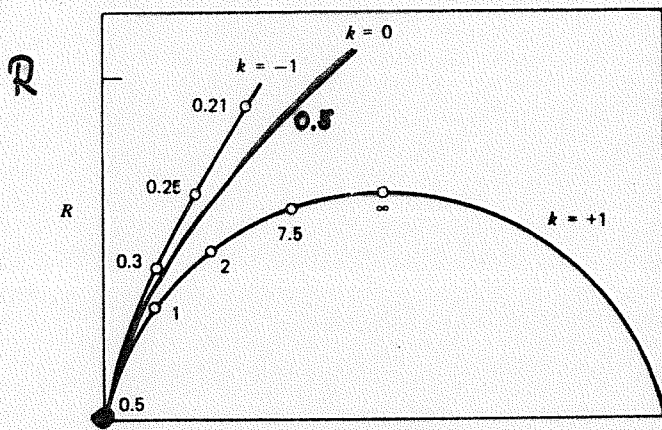
$$\rightsquigarrow (\dot{\rho}c^2) + 3\frac{\dot{R}}{R} (\rho c^2 + P) = 0 \quad \text{-isentrope Expansion}$$

□ Zustandsgleichung $P(\rho c^2)$:

$$\rightarrow \begin{cases} P \sim \frac{1}{3} \rho c^2 & \text{strahlungsdom. Univ.} \rightsquigarrow \rho_r \sim \frac{1}{R^4} \\ P \approx 0 & \text{materiedom. Univ.} \rightsquigarrow \rho_m \sim \frac{1}{R^3} \end{cases}$$

□ Weltmodelle ($\Lambda = 0$):

$$\frac{1}{c^2} \dot{R}^2 - \frac{D_0}{R} = (-k) \longleftrightarrow \frac{1}{2} m \dot{r}^2 - \frac{GMm}{r} = E = \text{const}$$



$$\begin{cases} E < 0 \equiv k = +1 & \text{geschl. Univ.} \\ E = 0 \equiv k = 0 & \text{flaches Univ.} \\ E > 0 \equiv k = -1 & \text{offenes Univ.} \end{cases}$$

"Urknall" : t

≙ Anfangssingularität

□ Beobachtung: $k = \frac{R^2}{c^2} \left(\frac{8\pi G}{3} \rho - H^2 \right) = \frac{R^2}{c^2} \frac{8\pi G}{3} (\rho - \rho_{cr})$

$$\underline{\Omega} := \frac{\rho}{\rho_{cr}} \rightarrow k = \frac{H^2 R^2}{c^2} (\Omega - 1)$$

◇ $\rho_0^{lum} \approx 0.02 \pm 0.01 \rho_{cr} \quad [\equiv n_N \sim 10^{-7} / \text{cm}^3]$

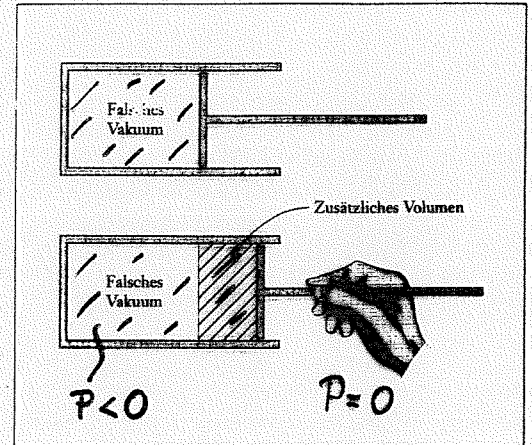
$\rightsquigarrow \Omega_0^{lum} \approx 0.02 \pm 0.01$

◇ **Aber:** $\Omega_{\text{Galaxienbew.}} \approx 0.1 - 0.2 \rightsquigarrow$ 'dunkle' Materie

DeSitter-Lemaître-Lösung

- $\Lambda_{(vac)} > 0$ kosmologische Konstante
- Einstein suchte nach statischem Universum
(war allerdings *instabil*)

$$\underbrace{\begin{pmatrix} \rho c^2 & & & \\ & P & & \\ & & P & \\ & & & P \end{pmatrix}}_{T_{\mu\nu}} \leftrightarrow \underbrace{\begin{pmatrix} \Lambda & & & \\ & -\Lambda & & \\ & & -\Lambda & \\ & & & -\Lambda \end{pmatrix}}_{\Lambda g_{\mu\nu}}$$

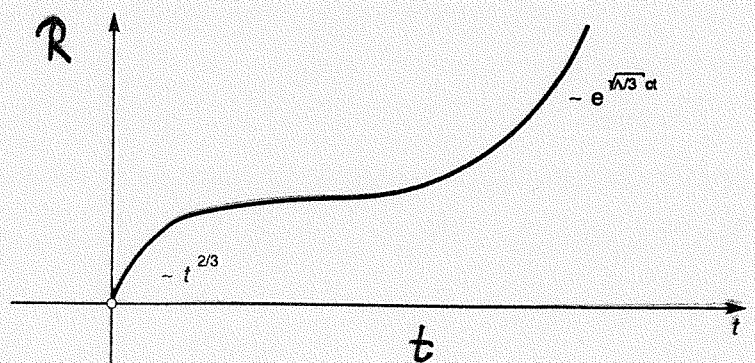


- Λ wirkt wie negativer Druck
 - ◇ $(\rho c^2 + 3P) \rightarrow -2\Lambda$
 - ◇ führt zu einer gravitativen Abstoßung

- Vernachlässigung von gewöhnlicher Materie:

$$\ddot{R}(t) - \frac{\Lambda c^2}{3} R(t) = 0 \Rightarrow R(t) = R_0 e^{ct\sqrt{\frac{\Lambda}{3}}}$$

- ◇ $H = \frac{\Lambda}{3} = const$,
- ◇ 1. FG $\leadsto \underline{k = 0}$,
i.e. Raum ist flach.



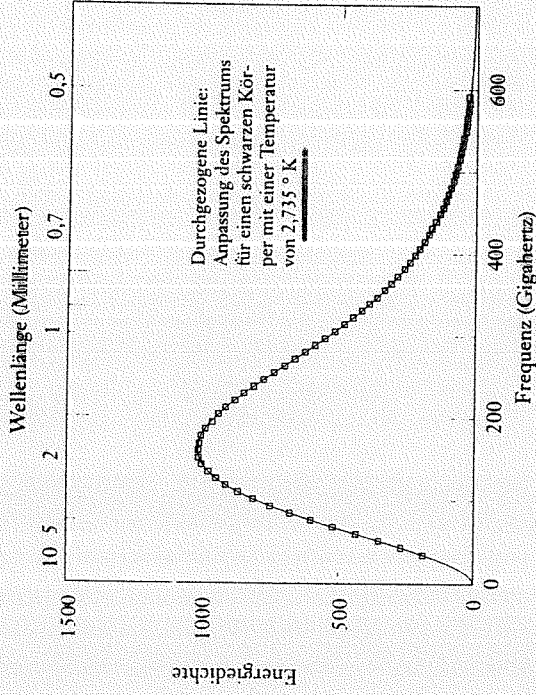
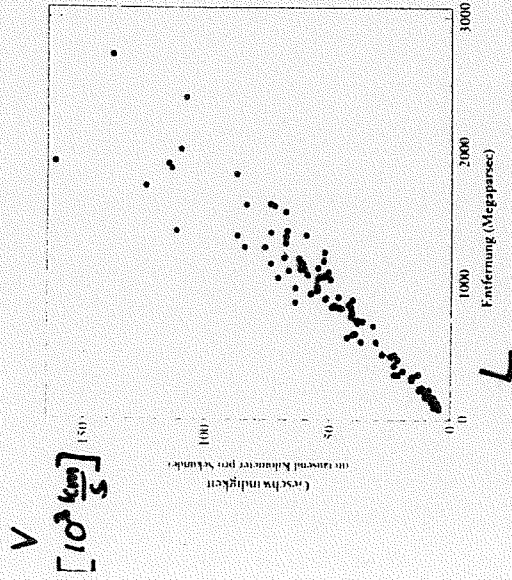
möglicher Ursprung von Λ ?!

Standpfeiler des SM

Hubble-Gesetz

kosm. Hintergrundstrahlung

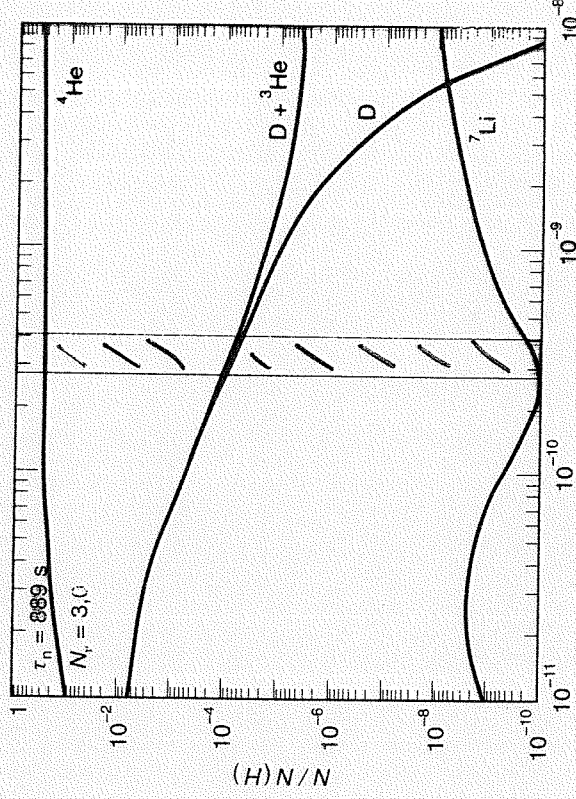
primordiale Nukleosynthese



$t = 10^6 \text{ } \tau, T = 3000 \text{ K}, Z = 10^3$

$\eta = \frac{n_B}{n_\gamma} \sim 3 \cdot 10^{-10}$

$t = 2 \cdot 10^3 \text{ sec}, T = 3 \cdot 10^8 \text{ } \mu, Z = 10^8$



Wo ist da das Problem ?!

...

Probleme des SM

□ Anfangssingularität \leadsto **Planck-Skala:**

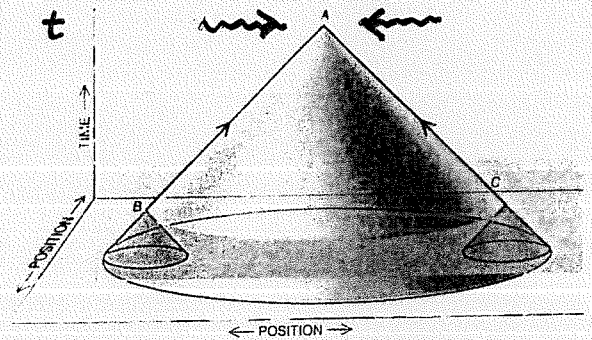
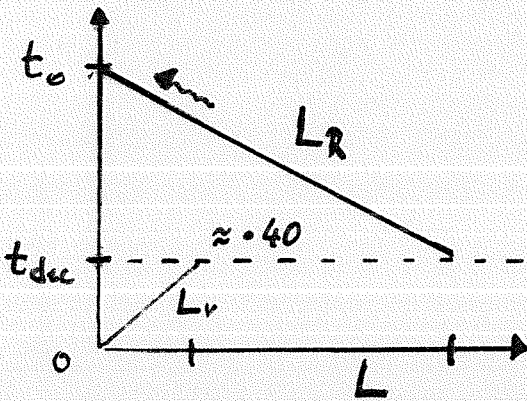
$$\diamond m_{Pl}c^2 = \left(\frac{\hbar c^5}{G}\right)^{\frac{1}{2}} \approx 10^{19} \text{ GeV} \rightarrow R_{Pl} \approx 10^{-33} \text{ cm}$$

$$\frac{Gm_{Pl}^2}{R_{Pl}} \approx \frac{Gm_{Pl}^2}{\frac{\hbar}{m_{Pl}c}} \approx m_{Pl}c^2 \text{ oder } \underline{r_S^{Pl}} = \frac{2Gm_{Pl}}{c^2} \approx \underline{R_{Pl}}$$

i.e. Gravitation und Quanteneffekte werden groß!

□ **Horizont-Problem:** Kausalität

γ -Entkopplung $\exists 10^5$ kausal nicht verbundene Gebiete!



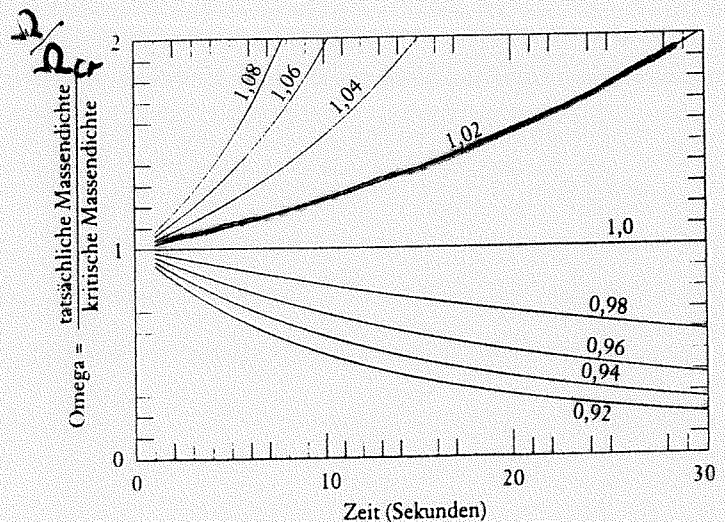
\diamond Warum ist dann die CMBR so isentrop?!

□ **Flachheits-Problem:** Warum ist $\Omega(t_0) \sim O(1)$?

$$\Omega(t) := \frac{\rho}{\rho_{cr}} \rightarrow x(t) = \frac{|\Omega-1|}{\Omega}$$

$$\dots \leadsto \underline{x(t) = \left(\frac{R(t)}{R(t_0)}\right)^2 x(t_0) \rightarrow 0}$$

$$x(t_{nuc}) \approx \underline{10^{-16}}; x(t_{Pl}) \approx 10^{-64}$$



\diamond extreme 'Feinabstimmung' im frühen Universum!

□ **Entropie:** $S_{Hor}(t_0) \approx \underline{10^{87}} \leftarrow S(t_{dec}) \approx \underline{10^{82}} \leftarrow S(t_{Pl}) \sim \underline{O(1)!}$

□ **Glattheits-Problem:** Warum gilt die Homogenität?

◇ Warum gilt das kosmologische Prinzip?!

□ **Struktur-Problem:** $\underline{\delta\phi_{causal}}(t = t_{equal}) \approx \underline{0.8^\circ!}$

◇ Wie bilden sich die grobkörnigen (Super-)Strukturen?

□ **Monopol-Problem:**

stabile und schwere (solitäre) Relikte aus der frühen Ursuppe sollten Masse um Faktor $\approx 10^8$ dominieren

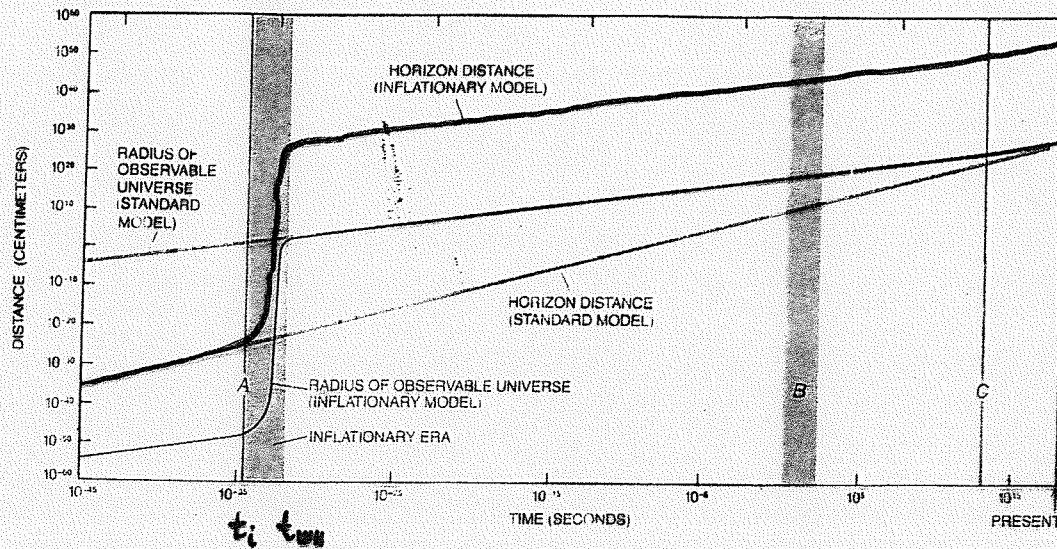
\leadsto Universum wäre schon lange kollabiert !

□ **Baryogenese:** $n_B/n_\gamma \sim 10^{-9}$, $n_{\bar{B}} \approx 0$

Woher kommt der Überschuß an Baryonen ?

⋮

Inflations-Hypothese ('Inflation Paradigm')



- ∃ eine temporäre und sehr frühzeitige Phase
(bei $t \sim 10^{-34}$ sec) nichtadiabatischer, exponentieller
Ausdehnung: $R(t) \sim e^{cHt}$

◇ ≡ kosmologischer 'Airbag'

◇ $\frac{R(t_{WH})}{R(t_i)} \sim e^{cH(t_{WH}-t_i)} \sim \underline{10^7 - 10^{10^{12}}}$!

◇ ... löst viele der Probleme

- Friedman-Glgn:

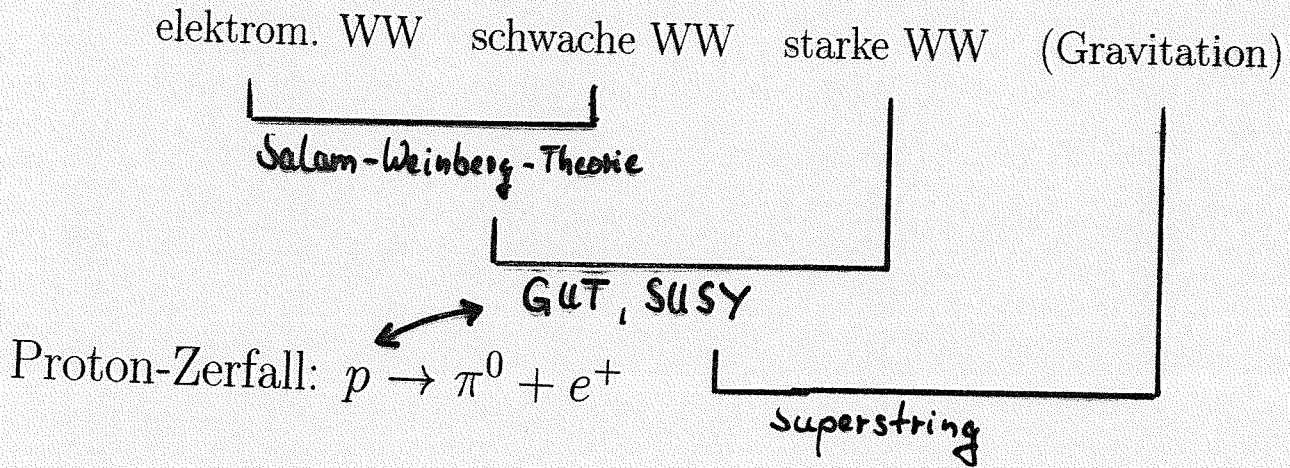
$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{kc^2}{R^2} = \frac{8\pi G}{3c^2} \rho_{infl} c^2 \approx H_{infl}^2 \stackrel{!}{=} const \rightsquigarrow \rho_{infl} c^2 \approx const$$

$$\rightsquigarrow \boxed{\rho_{infl} c^2 + P_{infl} \approx 0} \quad \text{!}$$

- Quantenfeldtheorie könnte so eine bizarres Verhalten
bewerkstelligen: \rightsquigarrow (kosm.) Phasenübergang

Große vereinheitlichte Theorie (GUT)

□ ∃ einheitliche Theorie aller 4 Wechselwirkungen:



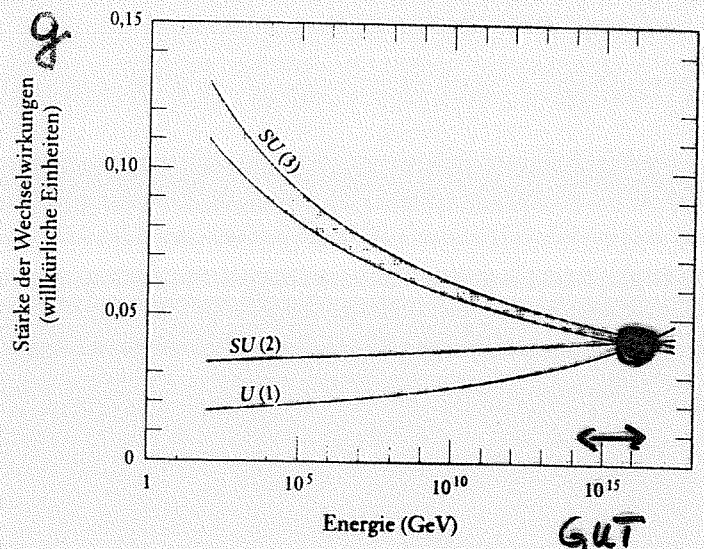
□ Kopplungskonstante:

$$SU(5) \supset (SU(3)_c \otimes (SU(2)_L \otimes (U(1)_Y)))$$

24 Eichbosonen

(γ -Photon, W^+ , W^- , Z^0 -Boson,
Gluonen (g), Leptoquarks (X))

$$\underline{M_X} c^2 \approx 10^{14} \text{ GeV}$$



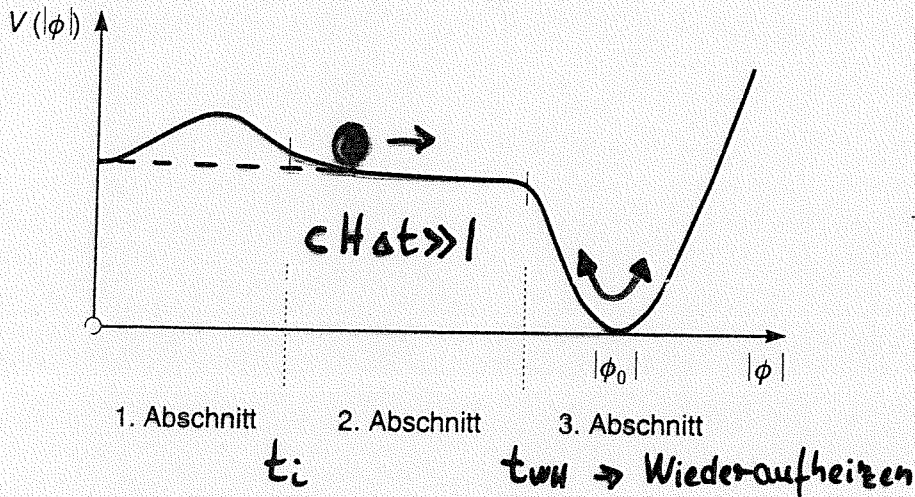
□ spontane Symmetriebrechung

generiert unterschiedliche Massen mittels Higgs-Felder

einige Details

□ sog. 'slow-roll'-Szenario

Skalen:



$$\underline{M} \approx \sigma_{vac} \approx M_X c^2$$

$$\approx k_B T_{GUT} \gtrsim \underline{10^{14} \text{ GeV}}$$

$$V(\phi \approx 0) := \rho c^2 \approx (T_{GUT})^4$$

$$\approx 10^{76} \text{ kg/m}^3$$

□ feldtheoretische Betrachtung: der negative Druck

$$\mathcal{L} := \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + \dots \rightarrow T_{\mu\nu}(\phi)$$

$$\rho c^2(\phi) = \underline{V(\phi)} + \left[\dot{\phi}^2/2 + (\nabla\phi)^2/(2R^2) \right] \hat{=} \underline{\Lambda}$$

$$P(\phi) = \underline{-V(\phi)} + \left[\dot{\phi}^2/2 - (\nabla\phi)^2/(6R^2) \right] \hat{=} \underline{-\Lambda}$$

und

$$\ddot{\phi} + 3cH\dot{\phi} + V'(\phi) + [\Gamma_\phi \dot{\phi}] = 0$$

□ e-folds:

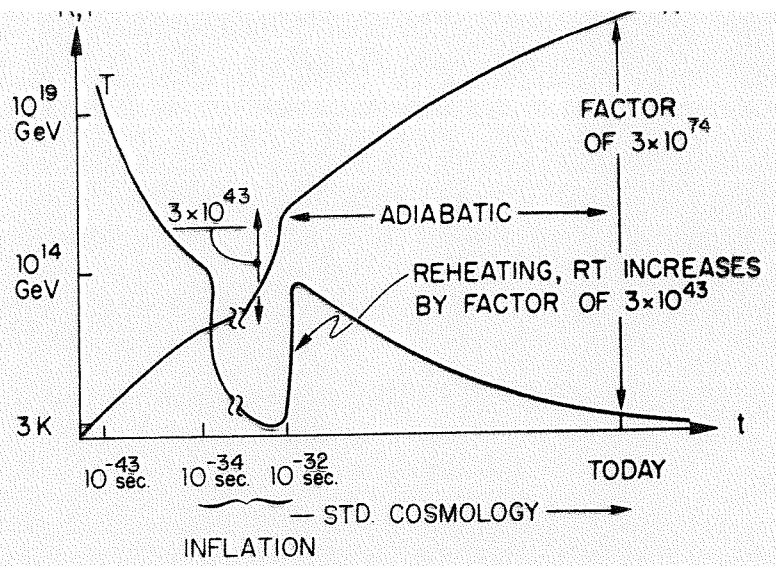
$$\ln \left(\frac{R_{WH}}{R_i} \right) \approx -\frac{8\pi}{m_{Pl}^2} \int_{\phi_i}^{\phi_{WH}} \frac{V(\phi)}{V'(\phi)} d\phi = \dots \underline{\geq 67}$$

□ Inflation: $cH_{GUT} \cdot \Delta t_{\text{Phasenübergang}} \gg 1$

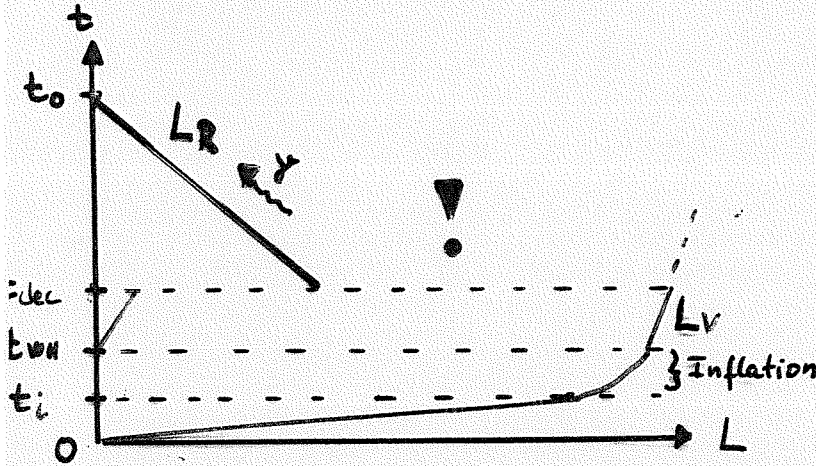
→ nur in sehr frühen Phase möglich, da dann

$$H^{-1} \sim 1/\sqrt{G\rho_{vac}} \text{ sehr klein.}$$

Lösen einiger Probleme



□ Horizont-Problem:



keine Diskrepanz $e^{cH\Delta t} \stackrel{!}{>} 75$

$$L_R = R(t_0) \int_{t_{dec}}^{t_0} \frac{dt'}{R(t')}$$

$$L_V = R(t_0) \int_{t_i}^{t_0} \frac{dt'}{R(t')}$$

□ Entropieproduktion während des Wiederaufheizens

$$\frac{S(t_{WH})}{S(t_i)} \approx \frac{T_{WH}^3}{T_i^3} \left(\frac{R(t_{WH})}{R(t_i)} \right)^3 \approx 10^{87}$$

$$\Rightarrow N = cH\Delta t > 67$$

□ Flachheits-Problem:

$$x(t) = \frac{|\Omega-1|}{\Omega} = \frac{3|k|c^2}{8\pi G R^2 \rho_{vac}}$$

$$\dots \rightsquigarrow x(t_{WH}) = \left(\frac{R(t_i)}{R(t_{WH})} \right)^2 x(t_i) \rightarrow e^{-2N} \equiv \underline{\underline{0}}$$

$$\rightsquigarrow \underline{k_{eff} \equiv 0}, \quad \text{d.h. } \boxed{\rho_0 \equiv \rho_{cr}(t_0)}$$

Wenn $R(t)$ (beliebig) groß wird, dann sehen wir nur (beliebig) kleinen Ausschnitt!

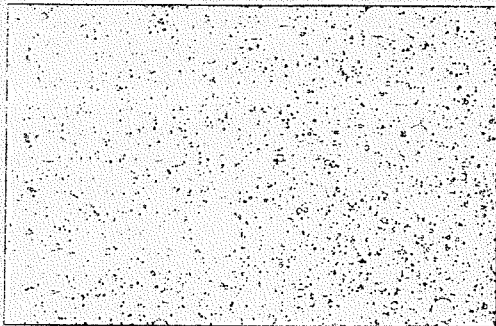
Inflationary universe: A possible solution to the horizon and flatness problems

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(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.



Alte Inflation

'Grateful Exit Problem'

Volume 108B, number 6

PHYSICS LETTERS

4 February 1982

! A NEW INFLATIONARY UNIVERSE SCENARIO: A POSSIBLE SOLUTION OF THE HORIZON, FLATNESS, HOMOGENEITY, ISOTROPY AND PRIMORDIAL MONOPOLE PROBLEMS

A.D. LINDE

Lebedev Physical Institute, Moscow 117924, USSR

Received 29 October 1981

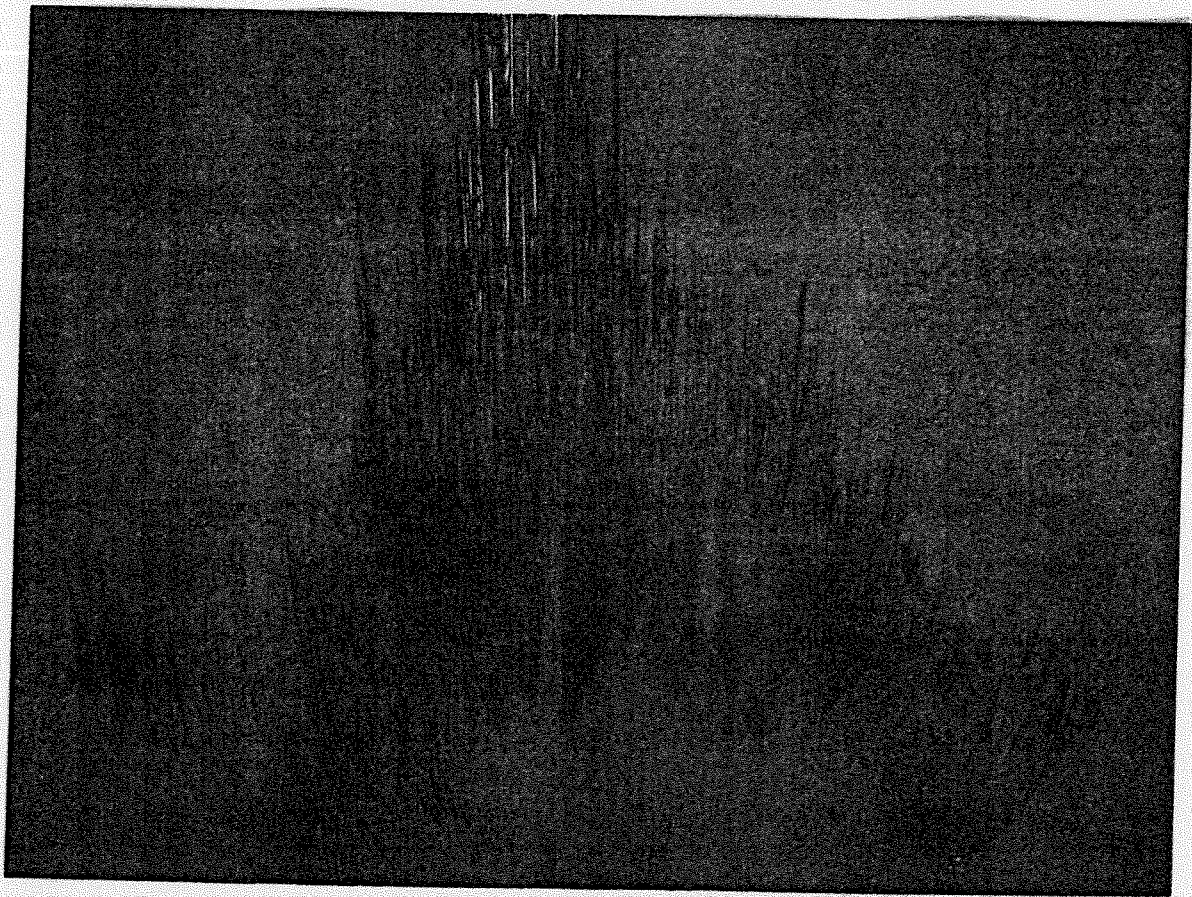
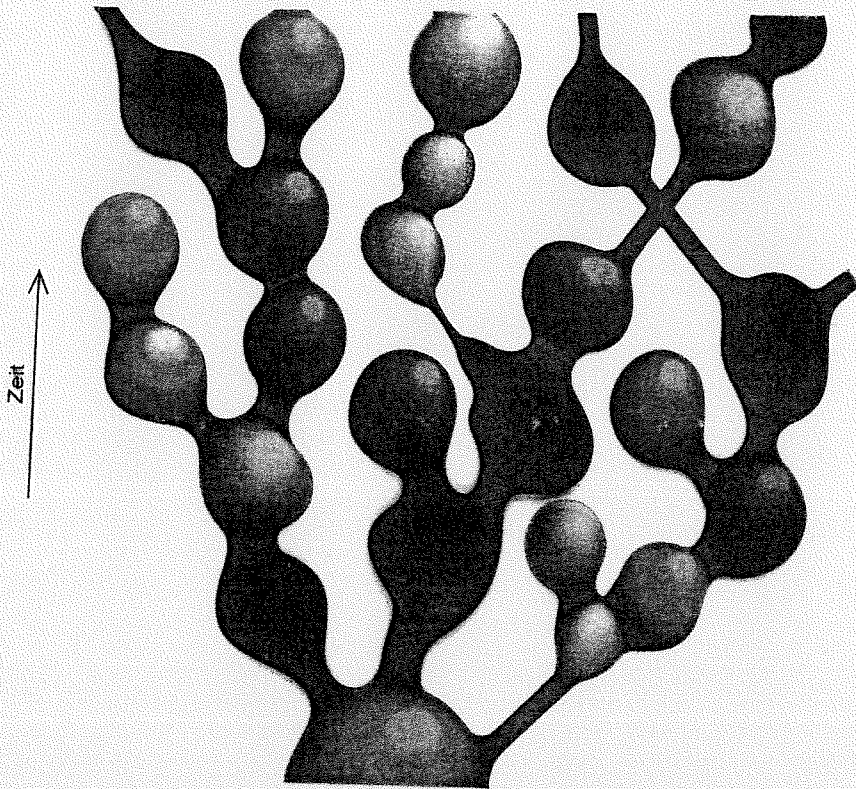
A new inflationary universe scenario is suggested, which is free of the shortcomings of the previous one and provides a possible solution of the horizon, flatness, homogeneity and isotropy problems in cosmology, and also a solution of the primordial monopole problem in grand unified theories.

Neue Inflation

□ noch mehr Inflation: chaotische Inflation, Hybrid-Inflation, doppelte Inflation, stochastische Inflation, 'everlasting' Inflation, offene Inflation, ...

◇ SUSY, Superstring, ...

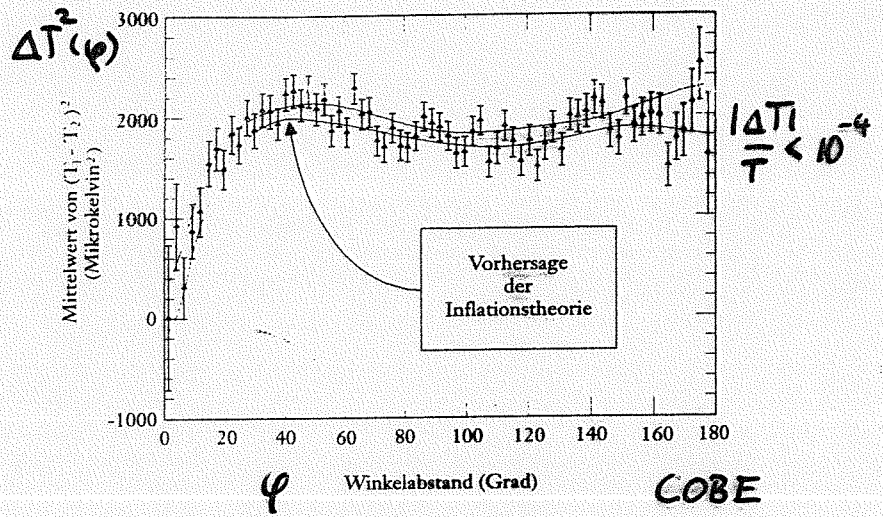
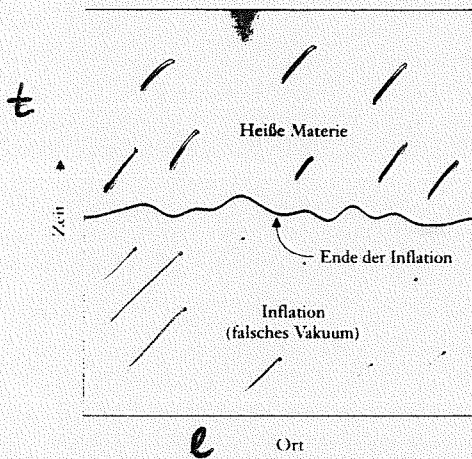
chaotische, fraktale Inflation



(A. Linde, *Scientific American*, Nov. 1984)

neue Fragestellungen und mögliche Tests :

- $\Omega_0 \approx 1, H_0 t_0 = 2/3$ ($H_0 t_0 \in [0.5, 2]$)
- primordiale (Quanten-)Fluktuationen



⇒ Auswirkungen auf CMBR

⇒ Auswirkungen auf Dichte-Fluktuationen $\frac{\delta \rho_{mat}}{\rho}$

Formation von Galaxien, Superhaufen, ...

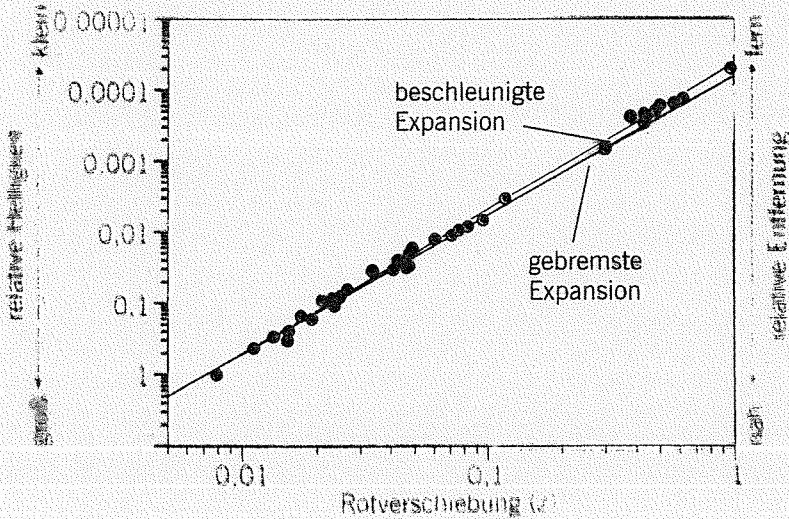
- Prozeß des Wiederaufheizens ... T_{WH}
- Baryogenese ...

□ Was konstituiert die fehlende Masse?

◇ HDM, CDM, kosm. Konstante (Λ CDM)

□ Spekulation über beschleunigtes Universum

durch Beobachtung sehr früher SN1A-Supernovae



$$\underline{\Omega_{mat} = \frac{1}{3}}, \quad \underline{\Omega_{\Lambda} = \frac{2}{3}} \dots ?$$

□ Was ist das Inflaton-Feld ϕ ?



Analogie:

$$k = 0 \quad \longleftrightarrow \quad E_{tot} = E_{mat} + U_{pot}^{grav} = 0$$

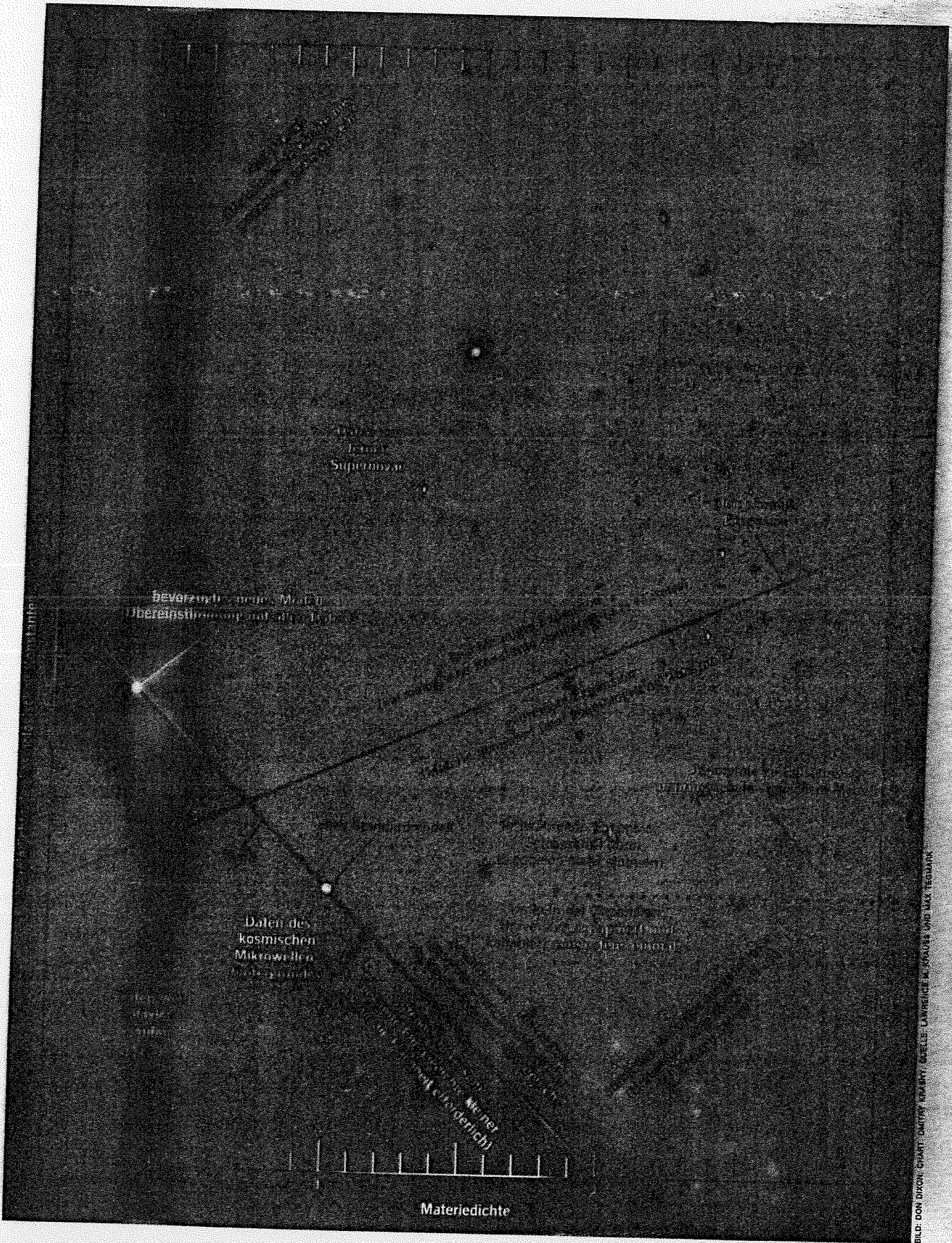


BILD: DON DIXON; CHART: DAVID KAGAN; QUELLE: LAWRENCE W. KRUMHOLTZ UND MAX TEGMARK