Neutron and strange quark stars*

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Abstract

We investigate the properties of charge–neutral β–equilibrium cold quark matter within the Nambu–Jona–Lasinio model. The calculations are carried out for different ratios of coupling constants characterizing the vector and scalar 4–fermion interaction, $\xi \equiv G_V/G_S$. It is shown that for $\xi < 0.4$ matter is self bound and for $\xi < 0.65$ it has a first order phase transition of the liquid gas type. The characteristics of the quark stars are calculated for $\xi = 0, 0.5$ and 1. For $\xi = 1$ the properties of quark stars show behaviors typical for neutron stars.

I. INTRODUCTION

The direct application of QCD at moderate temperatures and nonzero baryon densities is not possible at present. Therefore, more simple effective models respecting some basic

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*We dedicate this paper to Professor Marcos Moshinsky on the occasion of his 80^{th} birthday. His outstanding contributions to nuclear and mathematical physics were recognized by our Johann Wolfgang Goethe Universität in Frankfurt am Main by awarding to him the doctor of natural philosophy honoris causa in the year 2000.

symmetry properties of QCD are commonly used. The Nambu–Jona-Lasinio (NJL) model [1,2] which is dealing with constituent quarks and respects chiral symmetry is one of the most popular model of this kind. In recent years this model has been widely used for describing hadron properties (see reviews [3,4]), phase transitions in dense matter [5–7] and multiparticle bound states [8–10]. This model has been also applied for studying the Equation of State (EoS) of β–equilibrated matter appropriate for stars [11,12]. Unfortunately, the repulsive vector interaction which is very important in dense baryon–rich environment [6,9] was not taken into account there. The main goal of our paper is to study the influence of this interaction on the EoS and the structure of quark stars. We assume that these stars are composed of pure quark matter, similar to previous studies using a simplified EoS of the MIT bag model [13,14].

II. β–EQUILIBRIUM QUARK MATTER WITHIN THE NJL MODEL

Below we use the SU(3)–flavour version of the NJL model proposed in Ref. [15], but including not only the scalar but also the vector 4–fermion interaction. The color singlet part of the Lagrangian in the mean field approximation can be written as [10] ($\hbar = c = 1$)

$$\mathcal{L} = \sum_f \overline{\psi}_f \left(i\partial\!\!\!/ - m_f - \gamma_0 G_V \rho_f\right) \psi_f - \frac{G_S}{2} \sum_f \rho_{Sf}^2 + \frac{G_V}{2} \sum_f \rho_{Vf}^2 + 4K \prod_f \rho_{Sf} + \mathcal{L}_e \,. \tag{1}$$

Here ψ_f is the field operator of quarks with flavour $f = u, d, s$ and $\rho_{Sf} =< \overline{\psi}_f \psi_f >$ and $\rho_f =< \overline{\psi}_f \gamma_0 \psi_f >$ are their scalar and vector densities where angular brackets denote quantum–statistical averaging. G_S, G_V and K in Eq. (1) are respectively the coupling constants of scalar, vector and flavour mixing interactions. The constituent quark masses, m_f, are determined from the coupled set of gap equations [10]. The last term in Eq. (1) is the leptonic part of the Lagrangian. Below we take into account only electrons and treat them as ideal gas of massless particles.

Within this model chiral condensates $< \overline{\psi}_f \psi_f >$ are given by the scalar densities of quarks occupying both the negative and positive energy states. The divergent contributions of negative energy states of the Dirac sea are regularized by introducing a 3–momentum cut–off $\Theta(\Lambda - p)$ in momentum space integrals. In the following we use for the model parameters m_{0f}, G_S, K, Λ the set presented in Ref. [15].

We assume that the quark matter is in chemical equilibrium with respect to the strong and weak interactions. If neutrinos accompanying weak processes escape freely ($\mu_\nu = 0$), one obtains the following conditions $\mu_d = \mu_s = \mu_u + \mu_e$ relating the chemical potentials μ_i of various particle species $i = u, d, s, e$.

The chemical potentials, constituent quark masses and particle densities ρ_i are found by simultaneously solving all the field equations under conditions of fixed baryon and zero charge densities. To obtain the EoS of quark matter, one can calculate the energy density $\epsilon = T^{00}$ directly from the Lagrangian (1).

Fig. 1 presents the pressure as a function of the baryon density for three values of the ξ parameter: $\xi = 0, 0.5$ and 1. The calculations are compared with the predictions of the Hadron Chiral Model (HCM) [16,17]. Comparison of the NJL model predictions for different ξ reveals a strong sensitivity to this parameter. At $\xi = 0$ one can see two zero pressure points at nonzero baryon densities. They correspond to a local maximum and a local minimum in the energy per baryon ϵ/ρ_B. In the considered case the local minimum corresponds to a bound state of quark matter. This means that finite droplets of such matter can exist in mechanical equilibrium with vacuum even without gravitational force.

At $\xi \sim 0.5$ the pressure curve $P = P(\rho_B)$ does not cross zero, but still contains parts unstable with respect to the baryon and charge density fluctuations. This implies the possibility of a first order phase transition of the liquid–gas type. In the case of isospin–symmetric matter similar phase transitions have been studied in Ref. [10,18]. The Gibbs conditions in the mixed phase are applied for the case of two chemical potentials associated with the baryon number and electric charge [19].

III. QUARK STAR PROPERTIES

In this section we construct the models of compact stars composed of pure quark matter. The gravitational field is described in a standard way by using a spacetime–dependent metric $g_{\mu\nu}(x^\alpha)$ obeying the Einstein's equations [20]. We consider only spherically symmetric configurations and neglect all dynamical effects, like oscillation and rotation. To describe the low–density ($\epsilon < \epsilon_{\rm drip} = 4.3 \cdot 10^{11}$ g/cm^3) nuclear crust we use the EoS suggested in Ref. [22].

The calculated mass–radius relations are presented in Fig. 2. One can see that the maximum masses predicted by the NJL model depend strongly on the relative strength of the scalar and vector interactions ξ. At $\xi = 0$ the maximum mass has a quite low value $M_{\rm max} = 1.23\, M_\odot$ with a high central baryon density $\rho_c^{\rm max} = 9.2\, \rho_0$ and a small radius $R_{(M_{max})} \simeq 8.4$, which is typical for quark matter stars [23,13]. In the case where the crust is neglected the curves for self–bound quark matter ($\xi < 0.4$) start from the origin and at small R one has $M \propto R^3$. If we include the crust following the standard prescriptions [13] we see that the low–mass stars acquire an extended mantel of crust, which may reach hundreds of kilometers. On the other hand, for quark stars with high masses (near to the maximum mass) the inclusion of the crust leads to a relatively small increase of the radii of about several hundred meters.

For higher values of ξ the EoS becomes stiffer (see Fig. 1) which results in increasing maximum mass and radius. On the other hand, the central density $\rho_c^{\rm max}$ of the maximum mass star decreases with ξ. For $\xi = 1$ the maximum mass star has $M_{\rm max} = 1.60\, M_\odot$, $\rho_c^{\rm max} = 7.8\, \rho_0$ and $R_{(M_{\rm max})} \simeq 11.2$.

In Fig. 2 we compared our results with predictions for neutron stars obtained within the HCM. The curve with the larger maximum mass corresponds to the calculation where hyperons where neglected whereas the lower one shows the results with the inclusion of hyperons. One can see that at higher ξ the NJL results for quark stars become close to the predictions of the HCM (with inclusion of hyperons) for neutron stars.

ACKNOWLEDGMENTS

This work has been supported by the RFBR Grant No. 00–15–96590, the Alexander von Humboldt Foundation, the Graduiertenkolleg "Experimentelle und Theoretische Schwerionenphysik", GSI, BMBF, DFG, and the Hessische Landesgraduiertenförderung.

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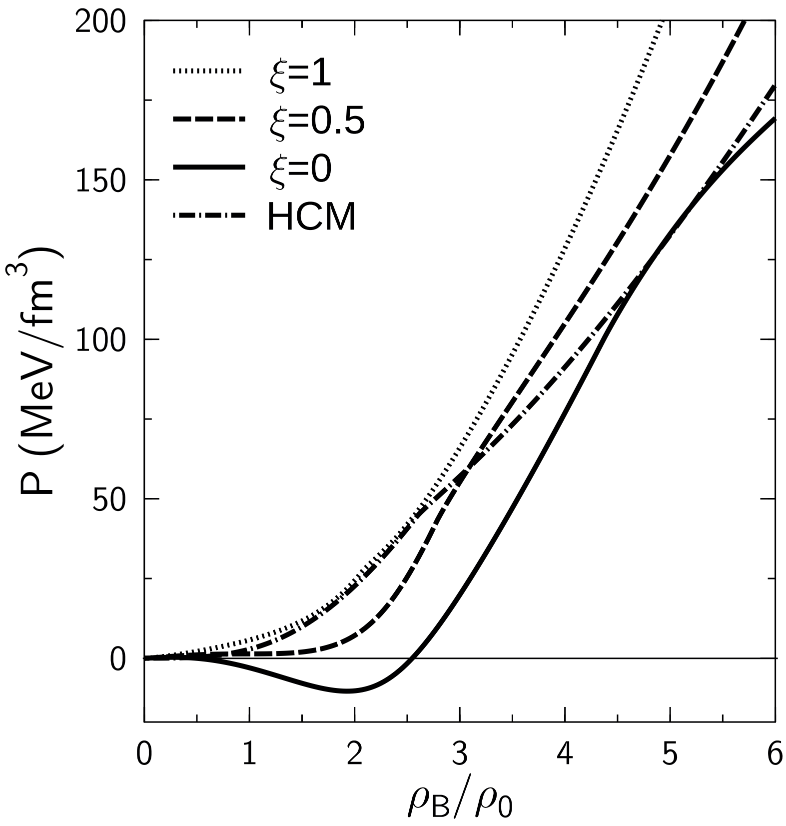

FIG. 1. Pressure as a function of the baryon density (in units of $\rho_0 = 0.15$ fm^{-3}) calculated within the NJL model for different ratios of vector and scalar coupling constant ξ. In the case of $\xi = 0$ the matter is self–bound that is signaled by the presence of negative pressures. Dashed–dotted line show predictions of the HCM (with hyperons).

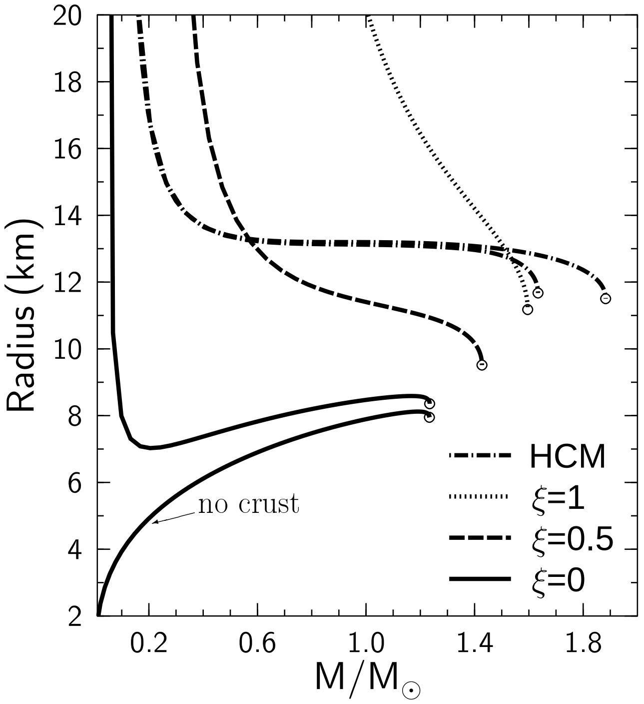

FIG. 2. Mass–radius curves predicted by the NJL model for three values of the parameter ξ. For comparison, the predictions of the HCM with and without hyperons are also shown. The results for self–bound stars (solid lines) are presented for two calculations with and without crust.