Evolutionäre Spieltheorie und Unternehmensnetzwerke der Softwarebranche

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Vortrag im Foko 2011

18th January 2011

Motivation: "Institute for Advance Study" in Princeton (1933 -1950)

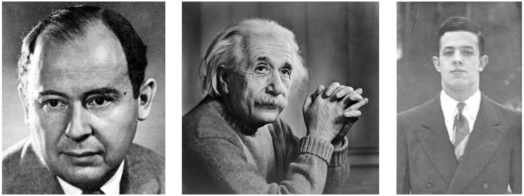

Figure: Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

Johann (John) von Neumann. Zur Theorie der Gesellschaftsspiele. *Mathematische Annalen*, 100:295–300, 1928.

J. von Neumann. *Mathematische Grundlagen der Quantenmechanik*. Springer, 1932.

J. von Neumann and O. Morgenstern. *The Theory of Games and Economic Behaviour*. Princeton University Press, 1947.

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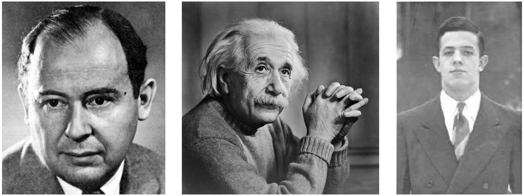

Figure: Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

Quantum Entanglement and the "EPR-Paradoxon":

A. Einstein, B. Podolsky, and N. Rosen. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, 47:777–780, 1935.

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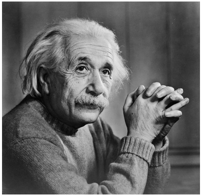



Figure: Johann von Neumann, Albert Einstein und John Forbes Nash Jr.

John F. Nash Jr. Equilibrium Points in N-person Games. *Proceedings of the National Academy of Sciences*, 36:48–49, 1950.

John F. Nash Jr. The Bargaining Problem. *Econometrica*, 18:155–162, 1950.

John F. Nash Jr. Non-Cooperative Games. *The Annals of Mathematics*, 54(2):286–295, 1951.

Research Questions of the Talk

Mathematical description of Quantum Game Theory

What are the main mathematical concepts of quantum game theory? How are the theories (Game Theory and Quantum Theory) unified?

Results for Quantum Games within different game classes

What are the main differences between classical and quantum game theory. Is the underlying Nash equilibrium structure of (2 player)-(2 strategy) games changed within a quantum game theory-based analysis?

Quanten-Spieltheorie und Unternehmensnetzwerke der Softwarebranche

Wie kann man die Quanten-Spieltheorie auf die evolutionäre Entwicklung der Unternehmensnetzwerke der Softwarebranche anwenden?

Classical (2 person)-(2 strategy) game

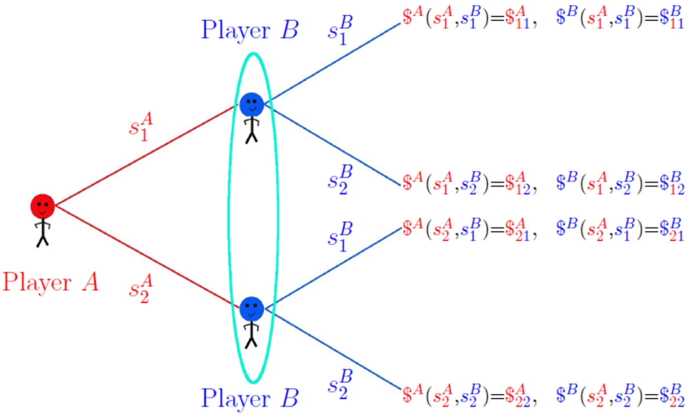


Figure: Game tree of a (2 person)-(2 strategy) game with payoff for player A ($\$^A$) and player B ($\$^B$).

Definition of a (2 player)-(2 strategy) game Γ

An unsymmetric (2×2) game Γ is defined as ...

(2×2) Game: $\Gamma := \left(\{A, B\}, \mathcal{S}^A \times \mathcal{S}^B, \hat{\$}_A, \hat{\$}_B\right)$

Set of pure strategies of player A and B: $\mathcal{S}^A = \left\{s_1^A, s_2^A\right\}$, $\mathcal{S}^B = \left\{s_1^B, s_2^B\right\}$

Set of mixed strategies of player A and B: $\tilde{\mathcal{S}}^A = \left\{\tilde{s}_1^A, \tilde{s}_2^A\right\}$, $\tilde{\mathcal{S}}^B = \left\{\tilde{s}_1^B, \tilde{s}_2^B\right\}$

Payoff matrix for player A: $\hat{\$}_A = \begin{pmatrix} \$_{11}^A & \$_{12}^A \\ \$_{21}^A & \$_{22}^A \end{pmatrix}$

Payoff matrix for player B: $\hat{\$}_B = \begin{pmatrix} \$_{11}^B & \$_{12}^B \\ \$_{21}^B & \$_{22}^B \end{pmatrix}$

The mixed strategy payoff function $\tilde{\$}^{\mu}$ of player $\mu = A, B$

Normalizing conditions for the mixed strategies of player μ:

$$\tilde{s}_1^{\mu} + \tilde{s}_2^{\mu} = 1 \; \forall \, \mu = A, B \quad \tilde{s}_1^{\mu}, \tilde{s}_2^{\mu} \in [0, 1]$$

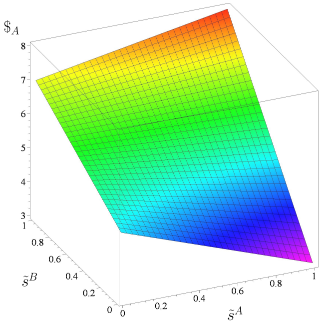
The mixed strategy payoff function reduces to:

$$\begin{aligned} & \tilde{\$}^{\mu} : ([0,1] \times [0,1]) \to \mathbb{R} \\ \tilde{\$}^{\mu}(\tilde{s}^A, \tilde{s}^B) \; = \; & \$^{\mu}_{11} \tilde{s}^A \tilde{s}^B + \$^{\mu}_{12} \tilde{s}^A (1 - \tilde{s}^B) + \\ & + \$^{\mu}_{21} (1 - \tilde{s}^A) \tilde{s}^B + \$^{\mu}_{22} (1 - \tilde{s}^A)(1 - \tilde{s}^B) \end{aligned}$$

, where $\tilde{s}^A := \tilde{s}_1^A$, $\tilde{s}^B := \tilde{s}_1^B$, $\tilde{s}_2^A = 1 - \tilde{s}_1^A$ and $\tilde{s}_2^B = 1 - \tilde{s}_1^B$

The mixed strategy payoff function $\tilde{\$}^\mu$ of player $\mu = A, B$

Mixed strategy payoff function $\tilde{\$}^A\left(\tilde{s}^A, \tilde{s}^B\right)$ of player A
$(\$^A_{11} = 8,\ \$^A_{12} = 3,\ \$^A_{21} = 7,\ \$^A_{22} = 5)$



Payoff $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$ as a function of $\tilde{s}^A, \tilde{s}^B \in [0,1]$:

$$\tilde{\$}^\mu : ([0,1] \times [0,1]) \rightarrow \mathbb{R}$$

$$\tilde{\$}^\mu(\tilde{s}^A, \tilde{s}^B) = \$^\mu_{11}\tilde{s}^A\tilde{s}^B + \$^\mu_{12}\tilde{s}^A(1-\tilde{s}^B) + +\$^\mu_{21}(1-\tilde{s}^A)\tilde{s}^B + \$^\mu_{22}(1-\tilde{s}^A)(1-\tilde{s}^B)$$

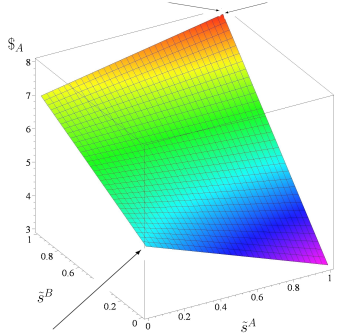
, where $\tilde{s}^A := \tilde{s}^A_1$, $\tilde{s}^B := \tilde{s}^B_1$, $\tilde{s}^A_2 = 1 - \tilde{s}^A_1$ and $\tilde{s}^B_2 = 1 - \tilde{s}^B_1$

A\B	s^B_1	s^B_2
s^A_1	(8,8)	(3,7)
s^A_2	(7,3)	(5,5)

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu \left(\tilde{s}^A, \tilde{s}^B\right)$

Pure Nash equilibrium $\left(\tilde{s}^A = 1, \tilde{s}^B = 1\right)$



Pure Nash equilibrium $\left(\tilde{s}^A = 0, \tilde{s}^B = 0\right)$

A strategy combination $(\tilde{s}^{A*}, \tilde{s}^{B*})$ is called a Nash equilibrium, if:

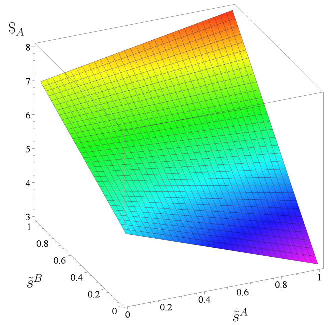
$$\tilde{\$}^A(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^A(\tilde{s}^{A}, \tilde{s}^{B*}) \quad \forall \ \tilde{s}^A \in [0,1]$$
$$\tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B*}) \geq \tilde{\$}^B(\tilde{s}^{A*}, \tilde{s}^{B}) \quad \forall \ \tilde{s}^B \in [0,1]$$

A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

$$\left.\frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A}\right|_{\tilde{s}^B = \tilde{s}^{B\star}} = 0 \quad \forall \ \tilde{s}^A \in [0,1] \ , \ \tilde{s}^{B\star} \in \,]0,1[$$
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall \ \tilde{s}^B \in [0,1] \ , \ \tilde{s}^{A\star} \in \,]0,1[$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

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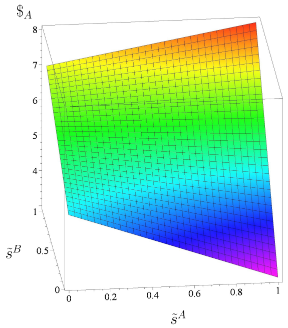
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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A strategy combination $(\tilde{s}^{A\star}, \tilde{s}^{B\star})$ is called an interior (mixed strategy) Nash equilibrium, if:

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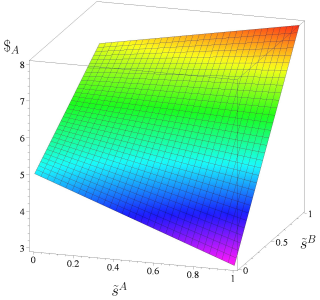
$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall\ \tilde{s}^B \in [0,1] \ ,\ \tilde{s}^{A\star} \in]0,1[$$

Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



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$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall\ \tilde{s}^B \in [0,1]\ ,\ \tilde{s}^{A\star} \in]0,1[$$

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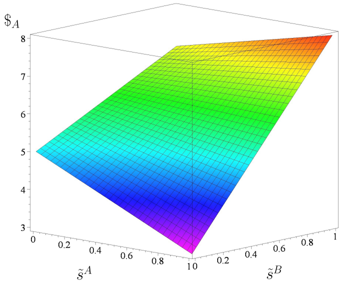

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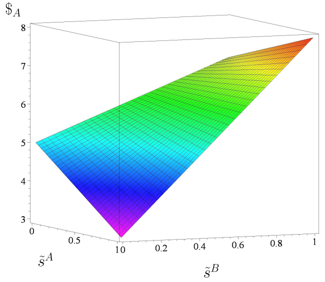
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^{\mu}\left(\tilde{s}^A, \tilde{s}^B\right)$



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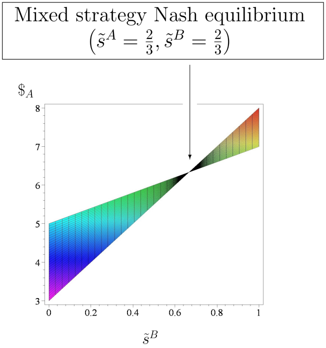
Nash equilibria (NE)

Nash equilibria and $\tilde{\$}^\mu\left(\tilde{s}^A, \tilde{s}^B\right)$



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$$\left.\frac{\partial \tilde{\$}^B(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^B}\right|_{\tilde{s}^A = \tilde{s}^{A\star}} = 0 \quad \forall\ \tilde{s}^B \in [0,1] \ ,\ \tilde{s}^{A\star} \in]0,1[$$

Evolutionary Game Theory

Replicatordynamics: The dynamical behavior of a population of players

$$\frac{dx_i^A(t)}{dt} = x_i^A(t)\left[\sum_{l=1}^{m} \$_{il}^A\, x_l^B(t) - \sum_{l=1}^{m}\sum_{k=1}^{m} \$_{kl}^A\, x_k^A(t)\, x_l^B(t)\right]$$

$$\frac{dx_i^B(t)}{dt} = x_i^B(t)\left[\sum_{l=1}^{m} \$_{li}^B\, x_l^A(t) - \sum_{l=1}^{m}\sum_{k=1}^{m} \$_{lk}^B\, x_l^A(t)\, x_k^B(t)\right]$$

The two population vectors $\vec{x}^A$ and $\vec{x}^B$ have to fulfill the normalizing conditions of a unity vector

$$x_i^\mu(t) \geq 0 \quad \text{and} \quad \sum_{i=1}^{m} x_i^\mu(t) = 1 \quad \forall\, i = 1, 2, ..., m\,,\ t \in \mathbb{R},\ \mu = A, B$$

Replicatordynamics of (2×2) games

Replicatordynamics of unsymmetric (2×2) games

$$\frac{dx(t)}{dt} = \left(\left(\$^A_{11} + \$^A_{22} - \$^A_{12} - \$^A_{21} \right) \left(x(t) - (x(t))^2 \right) \right) y(t) + \left(\$^A_{12} - \$^A_{22} \right) \left(x(t) - (x(t))^2 \right) =: g_A(x, y)$$

$$\frac{dy(t)}{dt} = \left(\left(\$^B_{11} + \$^B_{22} - \$^B_{12} - \$^B_{21} \right) \left(y(t) - (y(t))^2 \right) \right) x(t) + \left(\$^B_{12} - \$^B_{22} \right) \left(y(t) - (y(t))^2 \right) =: g_B(x, y)$$

Replicatordynamics of symmetric (2×2) games

$$\frac{dx}{dt} = x \left[\$_{11}(x - x^2) + \$_{12}(1 - 2x + x^2) + \$_{21}(x^2 - x) + \$_{22}(2x - x^2 - 1) \right]$$

$$= x \left[(\$_{11} - \$_{21})(x - x^2) + (\$_{12} - \$_{22})(1 - 2x + x^2) \right] =: g(x)$$

with: $x = x(t) := x_1(t) \quad \rightarrow \quad x_2(t) = (1 - x(t))$

Payoff Transformation and Game classes

Nash equivalent games

The set of Nash equilibria, the dynamical behavior of evolutionary games and the existence of evolutionary stable strategies (ESS) are unaffected by positive affine payoff transformations and by additionally added constants, where the strategy choice of the other players are fixed (see e.g. Weibull(1995)[**?**]). In the following the second kind of payoff transformation will be used to transform the payoff matrices in order to classify the games into different categories.

Symmetric payoff matrix after payoff transformation

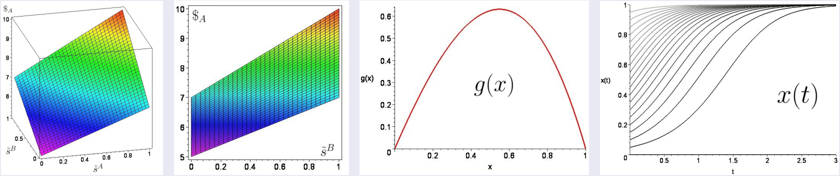
A\B	s_1^B	s_2^B
s_1^A	$(\$_{11},\$_{11})$	$(\$_{12},\$_{21})$
s_2^A	$(\$_{21},\$_{12})$	$(\$_{22},\$_{22})$

$\Longrightarrow$

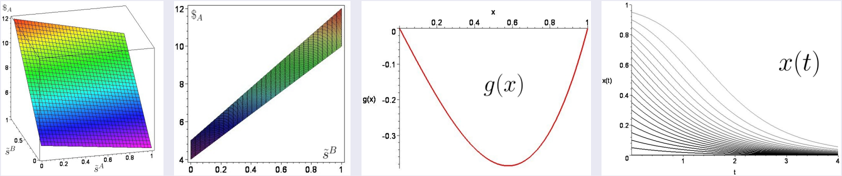
A\B	*Trafo* s_1^B	*Trafo* s_2^B
Trafo s_1^A	$(\underbrace{\$_{11}-\$_{21}}_{:=a},\underbrace{\$_{11}-\$_{21}}_{:=a})$	(0,0)
Trafo s_2^A	(0,0)	$(\underbrace{\$_{22}-\$_{12}}_{:=b},\underbrace{\$_{22}-\$_{12}}_{:=b})$

Symmetric (2×2) games: Dominant Class $(a < 0,\ b > 0)$ or $(b < 0,\ a > 0)$

Dominant Game: a=3, b=-2, one pure NE and one ESS (s_1^A, s_1^B)

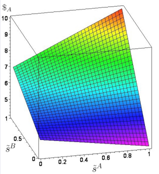


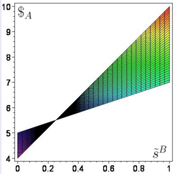
Prisoner's Dilemma: a=-2, b=1, one pure NE and one ESS (s_2^A, s_2^B)

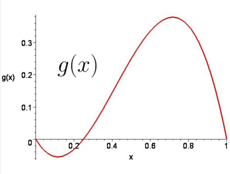


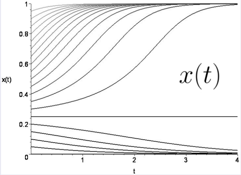
Coordination $(a, b > 0)$ and Anti-Coordination $(a, b < 0)$ Class

Coordination game: a=3, b=1, two pure and one interior NE at $\tilde{s}^\star = \frac{1}{4}$, two ESS $((s_1^A, s_1^B)$ and $(s_2^A, s_2^B))$

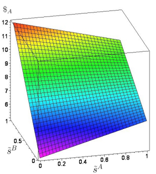


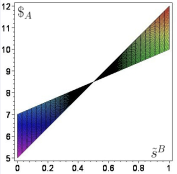


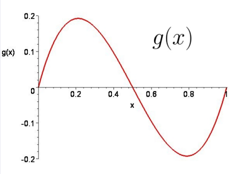


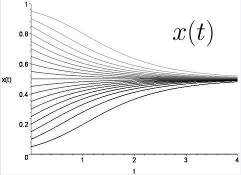


Anti-Coordination game: a=-2, b=-2, two pure asymmetric NE and one interior NE at $\tilde{s}^\star = \frac{1}{2}$, one ESS $(\tilde{s}^{A\star} = \frac{1}{2}, \tilde{s}^{B\star} = \frac{1}{2})$





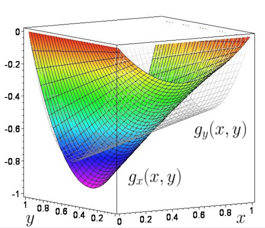




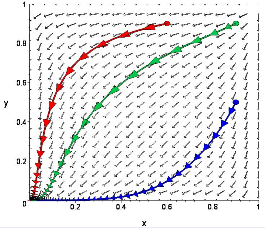
Game classes of unsymmetric (2 player)-(2 strategy) games

Corner Class (one ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

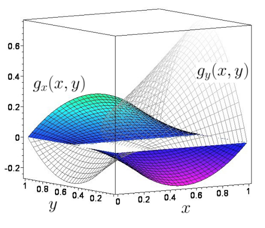


Phase diagram of xy-trajectories:

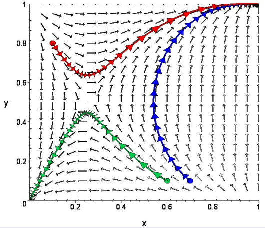


Saddle Class (two ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

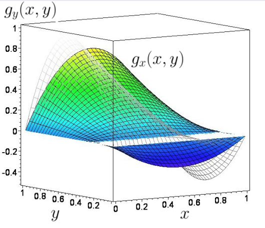


Phase diagram of xy-trajectories:

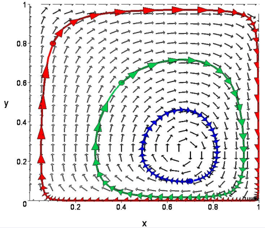


Center Class (no ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):



Phase diagram of xy-trajectories:



Quanten-Spieltheorie und Unternehmensnetzwerke der Softwarebranche

Spieltheoretische Formulierung des evolutionären Hubs- und Spoke Spiels

- Menge der Spieler unterteilt sich in zwei Gruppen. A:Hubs und B:Spokes
- Hub-Strategien:
  s_1^A:=Opportunist sein (z.B Aufnahme der Funktionalität ins eigene Portfolio)
  s_2^A:=kein Opportunist sein (z.B nur Bereitstellung der Plattform)
- Spoke-Strategien:
  s_1^B:=keinerlei Partnerschaftmit dem Hub
  s_2^B:=Partnerschaftmit dem Hub
- Auszahlungsmatrix für Hub und Spoke

Auszahlungsmatrix der Hubs und Spokes

Allgemeine Auszahlungsmatrix

Spoke\Hub	Kein Opportunist	Opportunist
Loose C.	$(\$^S_{11}, \$^H_{11})$	$(\$^S_{12}, \$^H_{12})$
Tight C.	$(\$^S_{21}, \$^H_{21})$	$(\$^S_{22}, \$^H_{22})$

$$
\begin{aligned}
\$^S_{11} &:= R_L + R_\Delta - (I_L + I_\Delta + F_L) \\
\$^S_{12} &:= \alpha R_L + R_\Delta - (I_L + I_\Delta + F_L) \\
\$^S_{21} &:= R_T - (I_T + F_T) \\
\$^S_{22} &:= \alpha R_T - (I_T + F_T) \\
\$^H_{11} &:= \beta R_L + F_L + T \\
\$^H_{12} &:= (1 - \alpha + \beta) R_L + F_L \\
\$^H_{21} &:= \beta R_T + F_T + T \\
\$^H_{22} &:= (1 - \alpha + \beta) R_T + F_T
\end{aligned}
$$

Auszahlungsmatrix der Hubs und Spokes

Festlegung einiger Parameter

$$R_L = 10 < R_T = 14\,, \quad I_L = 5 < I_T = 7$$

$$F_L = 2 < F_T = 3\,, \quad R_\Delta = 4\,, \quad I_\Delta = 2$$

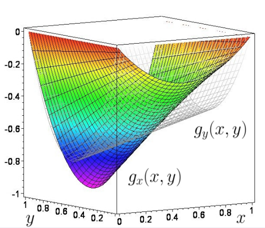
Spoke\Hub	Kein Opportunist	Opportunist
Loose C.	$(5\,,\,10\beta + 2 + T)$	$(10\alpha - 5\,,\,10\beta + 12 - 10\alpha)$
Tight C.	$(4\,,\,14\beta + 3 + T)$	$(14\alpha - 10\,,\,14\beta + 17 - 14\alpha)$

$$\begin{aligned} a^S &:= 1 > 0 \\ b^S &:= 4\alpha - 5 < 0 \\ a^H &:= 10\,(\alpha - 1) + T \\ b^H &:= 14\,(1 - \alpha) - T \end{aligned} \tag{1}$$

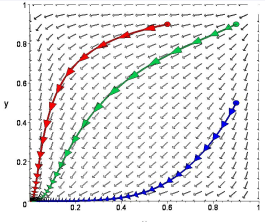
Auszahlungsmatrix der Hubs und Spokes definiert die Spielklasse

Corner Class (one ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

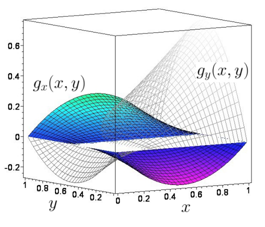


Phase diagram of xy-trajectories:

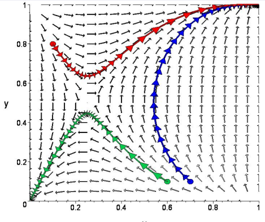


Saddle Class (two ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):

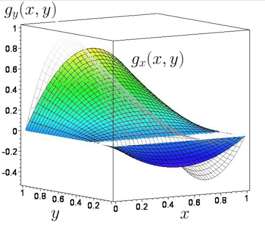


Phase diagram of xy-trajectories:

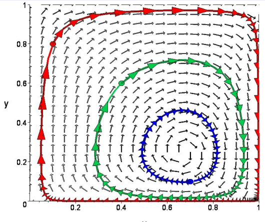


Center Class (no ESS)

$g_x(x, y)$ (colored) and $g_y(x, y)$ (wired):



Phase diagram of xy-trajectories:



PHD-Thesis: Evolutionäre Quanten Spieltheorie im Kontext sozio-ökonomischer Systeme

Articles of my cumulative PHD-Thesis

- Article 0: Evolutionary Quantum Game Theory
- Article 1: Quantum Game Theory and Open Access Publishing
- Article 2: Evolutionary Quantum Game Theory and Scientific Communication
- Article 3: Doves and hawks in economics revisited: *An evolutionary quantum game theory-based analysis of financial crises*
- Article 4: Experimental Validation of Quantum Game Theory
- Article 5: Evolutionary Game Theory and Complex Networks of Scientific Information

Summary

Summary of the talk

Quantum game theory is a mathematical and conceptual amplification of classical game theory. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. If this strategy entanglement is large enough, then, additional Nash-equilibria can occur, previously present dominant strategies could become nonexistent and new evolutionary stable strategies can appear.
Within this talk the framework of Quantum Game Theory was described in detail. The formal mathematical model, the different concepts of equilibria and the various classes of quantum games have been defined, explained and visualized to understand the main ideas of Quantum Game Theory. Additionally some applications where discussed at the end of the talk.