A Theory of Modern ART

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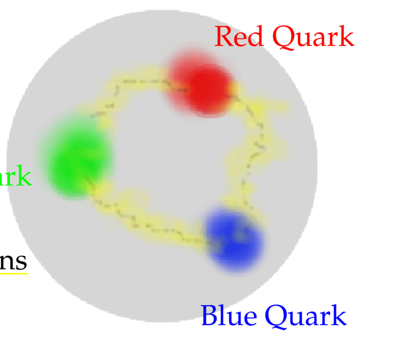
QuantumCromoDynamic

The QuantumCromoDynamic (QCD) is a $SU(3)_{(c)}$–Yang-Mills Gauge Theory. This force acts between particles which have a Color-Charge. The interaction particle is a massless Gluon which has itself a Color-Charge.
Barionic particles are composed of three valence quarks.



All of the fundamental particles (quarks and Gluons) inside this barion have color charges. The most important property of the QCD-Interaction is the fact, that the gluonic interaction is so strong for low momentum that the quarks are confined inside the barion. The property of CONFINEMENT curves the color-space in such a strong way that no color charges can be found by an outside observer. This means that the different color charges of the fundamental particles cancel each other, so that the resulting barion seems to be white!

- A, B : SU(3)–Color indices
- $G_{\beta A}{}^{B}$: Gluons
- $D_{\beta A}{}^{B} = \partial_\beta 1_A{}^B + i g\, G_{\beta A}{}^B$: The covariant derivative contains two color indices (A, B) and one spacetime index β
- $\psi^f_A = \begin{pmatrix}\psi^f_r\\ \psi^f_g\\ \psi^f_b\end{pmatrix}$: The Quarks are described by a Dirac-Spinor having one color and one flavour index

Yang-Mills Theory ⇔ Allgemeine Relativitäts Theorie

Four Interactions are presently known:

- The Electrodynamic Force
  This Force acts between charged or magnetized Particles and is described by a U(1)–Yang-Mills Gauge Theory.
- The Week Nuclear Force
  is responsible for the β-decay of nuclei. It is described by a SU(2)–Yang-Mills Gauge Theory.
- The Strong Nuclear Force
  This Force is the strongest Force, it binds the hadrons inside a nuclei and is also responsible for the confinement of Quarks. The fundamental description of this Force is known as the Quantum Cromo Dynamic which is a SU(3)–Yang-Mills Gauge Theory.
- Gravity
  The gravitational Force is the weakest Force. In comparison to all the other Forces it is not possible to describe this Interaction by a Yang-Mills Gauge Theory. The gravitational Force was reformulated by Einstein as a curvature of SpaceTime and he described it within the Allgemeine Relativitäts Theorie (Genearal Relativity).

It is mathematically possible to unify the Electrodynamic, the Week and the Strong Force within the Grand Unified Theory. This GUT theory is a SU(5)– Yang-Mills Gauge Theory.
The important expressions of Yang-Mills Gauge Theories and General Relativity (ART) have similarities and differences:

Yang-Mills Theory	ART (General Relativity)
$D_{\beta a}{}^{b} = \partial_\beta 1_a{}^b + i g A_{\beta a}{}^b$	$D_\beta v^\alpha = \partial_\beta v^\alpha + \Gamma^\alpha_{\sigma\beta} v^\sigma$
$F_{\alpha\beta a}{}^{b} = \frac{1}{ig}[D_{\alpha a}{}^c, D_{\beta c}{}^b] = A_{\beta a}{}^b{}_{\mid\alpha} - A_{\alpha a}{}^b{}_{\mid\beta} + \frac{1}{ig}[A_{\alpha a}{}^c, A_{\beta c}{}^b]$	$R^\delta{}_{\mu\alpha\beta} v^\mu = [D_\alpha, D_\beta] v^\delta$; $R^\delta{}_{\mu\alpha\beta} = \Gamma^\delta_{\mu\alpha\mid\beta} - \Gamma^\delta_{\mu\beta\mid\alpha} + \Gamma^\delta_{\nu\beta}\Gamma^\nu_{\mu\alpha} + \Gamma^\delta_{\nu\alpha}\Gamma^\nu_{\mu\beta}$
$\mathcal{L}_{YM} = \frac{1}{4} F_{\mu\nu a}{}^b F^{\mu\nu}{}_b{}^a$	$\mathcal{L}_G = R + \underbrace{(c_1 R_{\mu\nu} R^{\mu\nu} + ...)}_{\equiv 0 \text{ for ART}}$

Allgemeine Relativitäts Theorie (General Relativity)

The field equations of the gravitational force (Einstein-Equation) follow from the variational principle:

$$\delta_{g_{\mu\nu}} \int (R + \kappa \mathcal{L}_{Matter}) \sqrt{-g}\, d^4x = 0$$

- $\mathcal{L}_{Matter}$: Lagrangian density of all matter and fields (The gravitational field itself is the only exception; it is not included here).
- R : This invariant is called the Ricci-Scalar. It follows from the Ricci-Tensor $R_{\mu\nu}$ by summation over the spacetime indices. It describes roughly how strong the spacetime is curved.
- $\sqrt{-g}$: Square root of the determinant of the spacetime metric $g_{\mu\nu}$.
- $\delta_{g_{\mu\nu}}$: Variation of the metric $g_{\mu\nu}$.

⇓ Einstein–Equation ⇓

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi\kappa\, T_{\mu\nu}$$

The Einstein–Equation means the following:

How does the spacetime curves	if we have somewhere an amount of energy-momentum.
How do the gravitational fields look like $(g_{\mu\nu}, \Gamma^\mu_{\alpha\beta})$	if we have somewhere an amount of energy-momentum.

QCD-motivated Models of the Strong Interaction

For high Temperatures ($T > 170 MeV$) or high densities ($\rho_B > 3\,\rho_0$) hadronic matter is expected to be deconfined. In such a deconfined matter (Quark-Gluon Plasma) the quarks and gluons are quite free; they are not restricted to their own hadron.
To calculate the properties of compact stars one needs the Equation of State (EoS) of hadronic matter at low temperatures ($T < 1 MeV$) and high densities. Until now it is not possible to extract an EoS strait forward from QCD; therefore we took the following effective QCD-motivated models:

Models for Hadronic Matter
- Relativistic Mean Field Models (RMF) TM1, TM1YY NLZ, NLZYY
- SU(3) Chiral Hadron Model (CHM)

Models for Quark Matter
- MIT-Bag Model
- Nambu-Jona-Lasinio Model (NJL)
- Massive Quasi Particle Model (MQM)

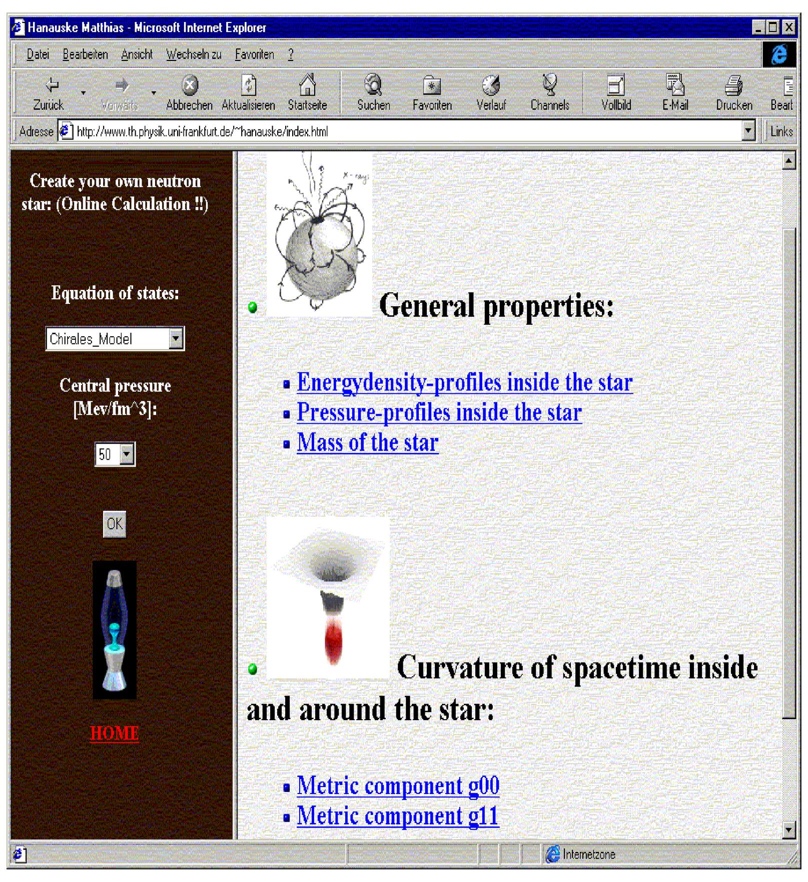
Models for Hybrid Matter
The Quark-Hadron Phase Transition
- All Models where the Gibbs Construction is possible

Models for Matter with other types of Phase Transitions
- Phase Transition to Hyperon Matter

ONLINE– Create your own Compact Star –ONLINE

Calculating properties of compact stars is now easy:
All you need is to choose an EoS from the left hand side and a computer code which binds the star by gravity (curvature).

Equation of State + Tollman - Oppenheimer - Volkoff Equation



http:// www.th.physik.uni-frankfurt.de/~hanauske/index.html

Spacetime inside a Compact Star

We have to solve the full Einstein Equation:
$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi\kappa T_{\mu\nu}$$
We restrict the system to be static and spherically symmetric. Moreover the star should not rotate and should not have a charge.

We can make the following Ansatz of the metric:

$$g_{\mu\nu} = \begin{pmatrix} e^{\nu(r)} & 0 & 0 & 0 \\ 0 & -e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

Ansatz of the Energy-Momentum Tensor:

- static and spherical symmetric : $T^{\mu\nu}(x^\alpha) \to T^{\mu\nu}(r)$
- no sheer forces: $T^{\mu\nu} \equiv 0$, if $\mu \neq \nu$
- EMT of an ideal fluid

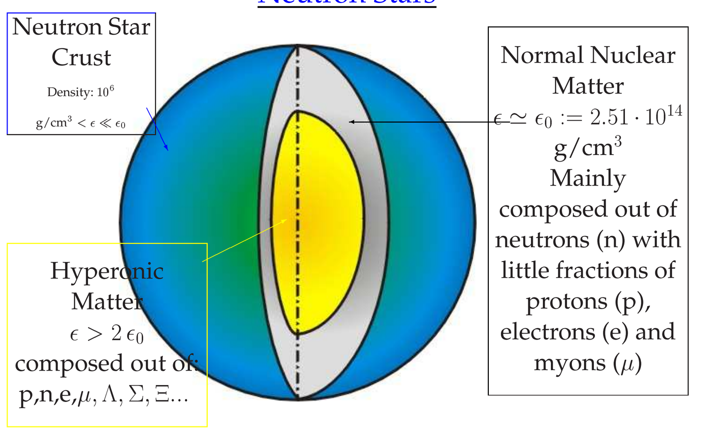
$$T^{\mu\nu} = -p\, g^{\mu\nu} + (p + \epsilon)\, u^\mu u^\nu$$
$$u^\mu = \frac{dx^\mu}{d\tau} \text{ : local 4-velocity}$$
$$= \left(\frac{1}{\sqrt{g_{tt}}}, 0, 0, 0\right) \text{ : fluid with no motion}$$

Inserting $T^{\mu\nu}$ and $g_{\mu\nu}$ in the Einstein Equation one results in a system of differential equations called the

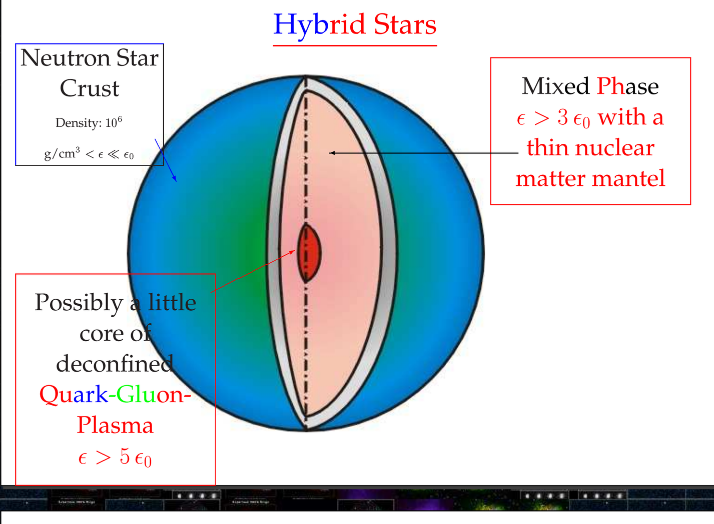
Tollman - Oppenheimer - Volkoff Equation

$$g_{rr} = -\left(1 - \frac{2m(r)}{r}\right)^{-1}$$
$$m(r) := 4\pi \int_{r'=0}^{r'=r} \epsilon(r') r'^2\, dr', \quad m(R) = M$$
$$\frac{d\nu}{dr} = \frac{m(r) + 4\pi r^3 p(r)}{r\,(r - 2m(r))}, \quad \nu(R) = \ln\left(1 - \frac{2M}{r}\right)$$
$$\frac{dp}{dr} = -(p(r) + \epsilon(r))\frac{d\nu}{dr}, \quad p(R) = 0$$

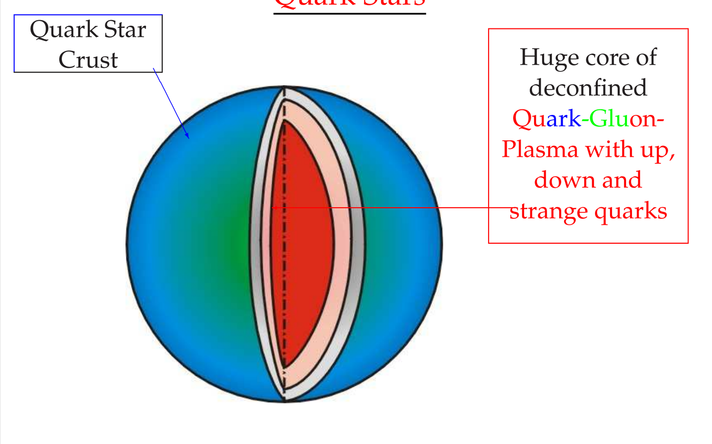
Neutron Stars



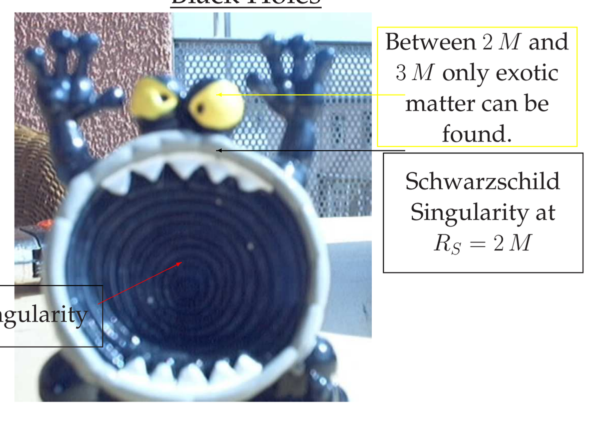
Hybrid Stars



Quark Stars

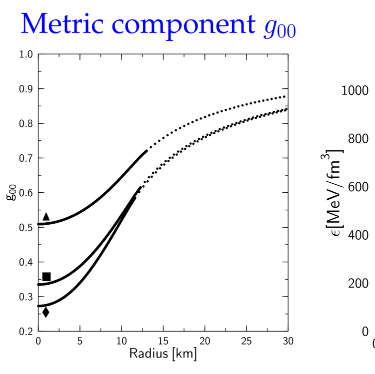


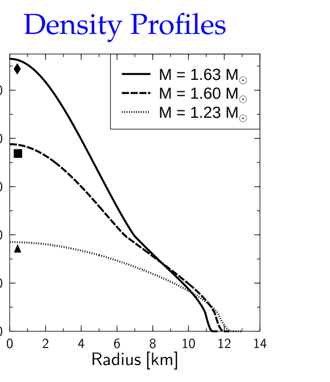
Black Holes



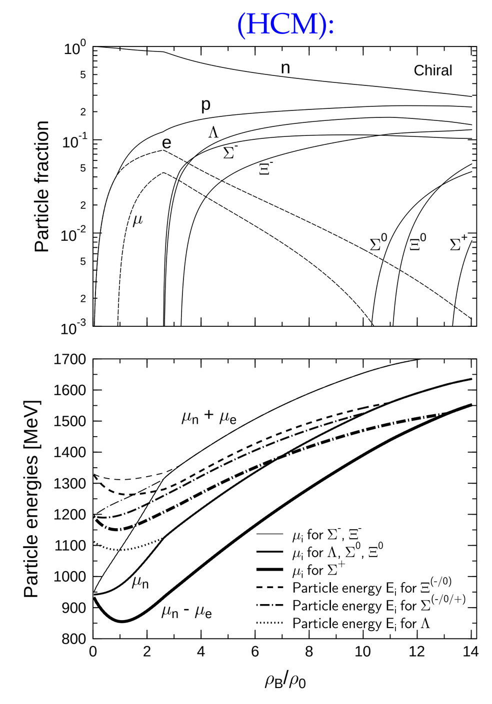
Properties of Neutron Stars

Metric component g_{00} — Density Profiles

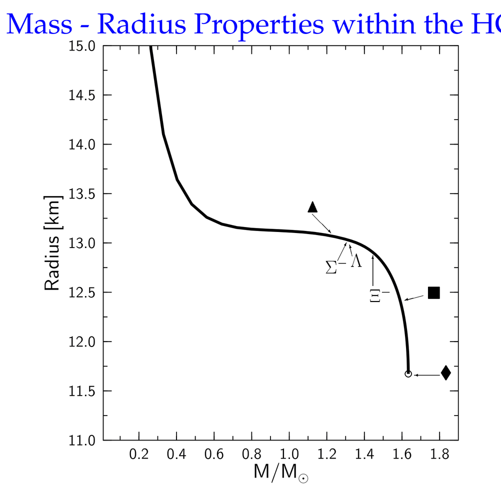




Particle Cocktail within the Hadronic Chiral Model (HCM):



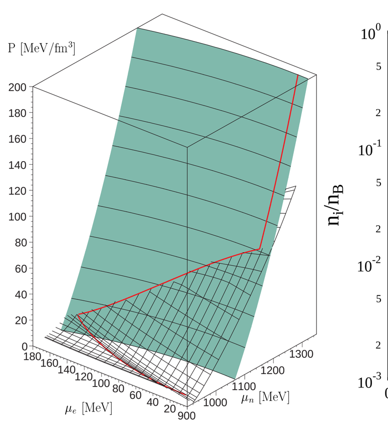
Mass - Radius Properties within the HCM:

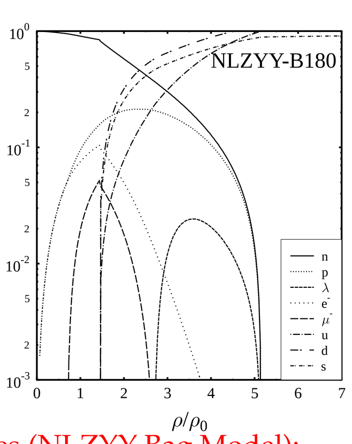


Neutron Stars have radii of $R \approx 11 - 14$ km and Masses $M < 1.65 M_\odot$

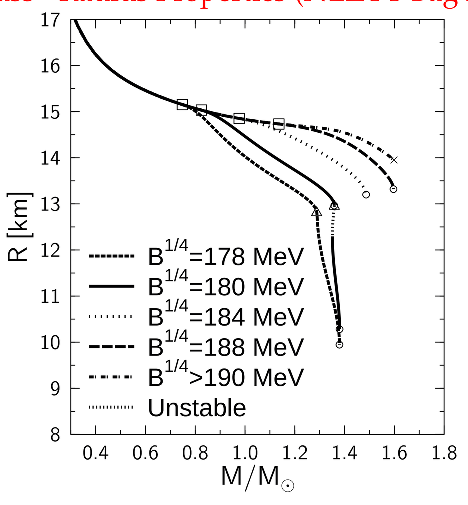
Properties of Hybrid Stars

Gibbs Construction — Particle Cocktail



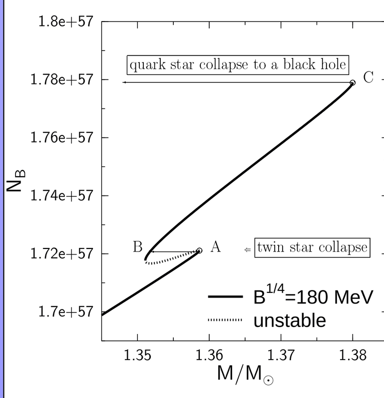


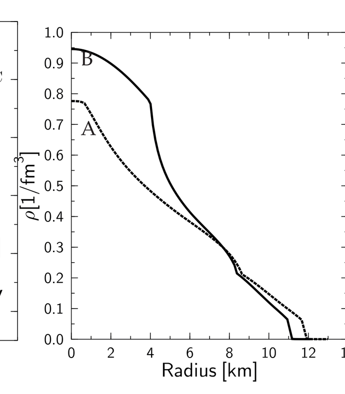
Mass - Radius Properties (NLZYY-Bag Model):



Twin Star Properties (NLZYY-Bag180 Model):

Total number of Barions — Twin star Profiles

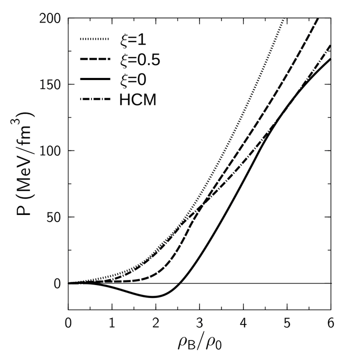


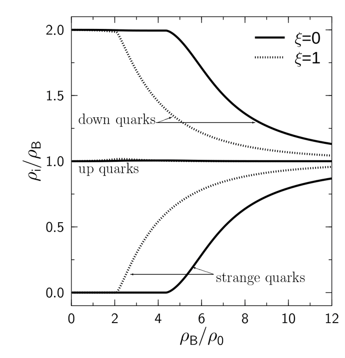


In the interior of Hybrid Stars occurs a Quark-Hadron phase transition. Within a small parameter range Twin Stars are possible. A Twin Star collapse could emit enough energy to explain Gamma-Ray burst; Gravitational wave emission is also possible.

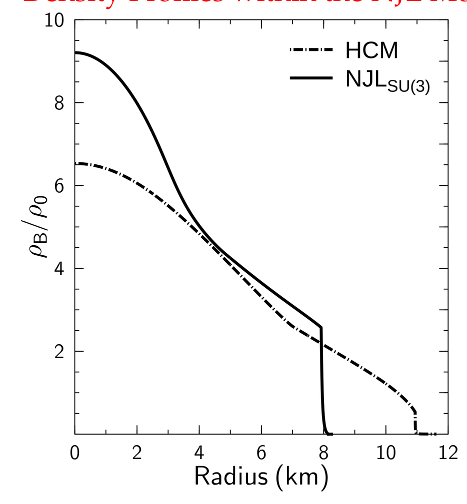
Properties of Strange Quark Stars within the NJL Model

Equation of State — Particle Cocktail

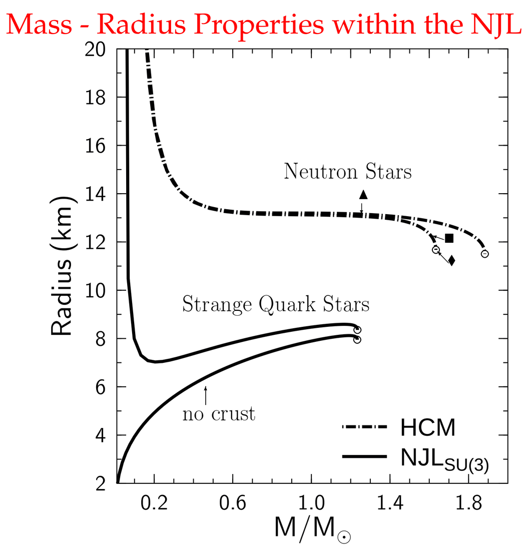




Density Profiles within the NJL Model:



Mass - Radius Properties within the NJL Model:



Quark Stars are much smaller than Neutron Stars $R \approx 8$ km and their limiting Mass is smaller $M < 1.3 M_\odot$

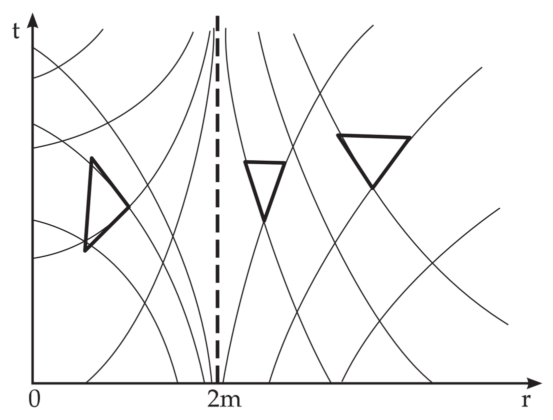
Properties of Black Holes

The Schwarzschild Black Hole
Restrictions: Spherical symmetry, no rotation, no charge and $T_{\mu\nu} = 0$
The Einstein Equation reduces under these restrictions to $R_{\mu\nu} = 0$ and one can find a coordinate system with the following general metric:

$$g_{\mu\nu} = \begin{pmatrix} e^{\nu(r,t)} & 0 & 0 & 0 \\ 0 & -e^{\lambda(r,t)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

Inserting this metric into the Einstein Equation one can proof the following results:
$\lambda(r,t)$ and $\nu(r,t)$ are not time dependent (Birkhoff) ; $\lambda(r) = -\nu(r)$, $e^{\nu(r)} = 1 - \frac{2M}{r}$ (M is the mass of the black hole) ; Schwarzschild singularity $R_S := 2M$; Real (intrinsic) singularity at r=0

Spacetime Diagram of the Schwarzschild metric



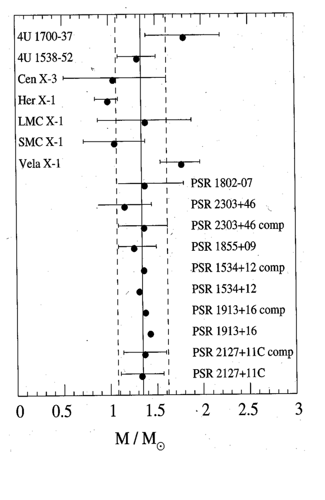
At the Schwarzschild singularity space and time changes into each other. The result is that all fields and matter with $r < 2M$ need to fall into the real singularity at r=0; no way to get out!
Neutron, Hybrid and Quark Stars with masses greater than the maximum mass M_{max} are unstable and need to collapse to Twin Stars or Black Holes. During the fast collapse more and more matter in the stars interior deconfines.
Unfortunately it looks like that during the stage when the quarks mean to get really free and deconfined they have crossed the event horizon, they can not show their color charge to the outer world – the Black Hole has macroscopically confined the Quark-Gluon-Plasma
A Black Hole with a typical mass of a Neutron or Quark Star has a Schwarzschild radius of $R \approx 4$ km.

Astrophysical Observables for the Quark-Gluon Plasma

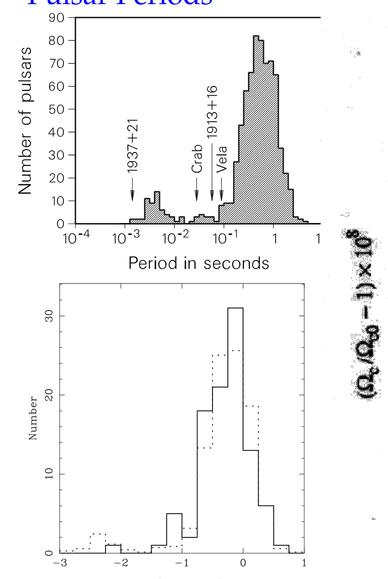
Quark Stars are smaller, the observation of a Twin Star would be the proof of having a phase transition, the observation of a sub millisecond Pulsar, the 'Spin-Up' effect, the observed 'Spin-Down' Effect in LMXB, Gamma-Ray Burst and gravitational wave emission from a Twin Star collapse, Supernova–Hypernova

Observed Astrophysical Objects

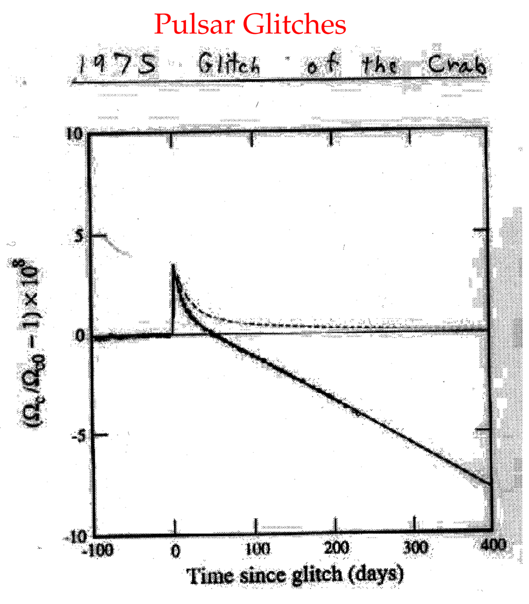
Neutron Star masses



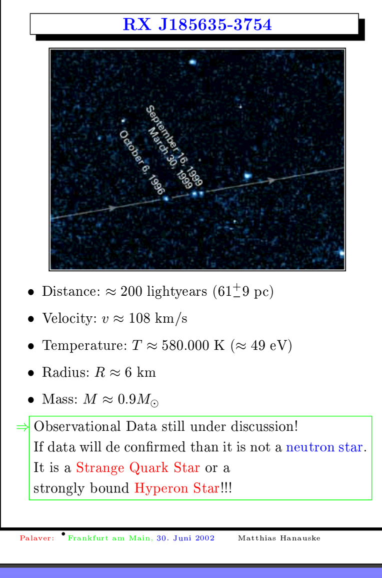
Pulsar Periods



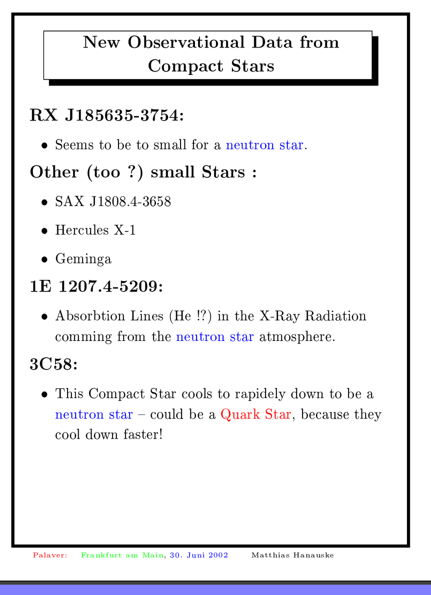
Pulsar Glitches



Quark Star obseved !?!



Other Quark Star Candidates



X-Ray Image of Quark Star Candidates