Relativistic Hydrodynamics in the Context of the Hadron-Quark Phase Transition in Compact Stars

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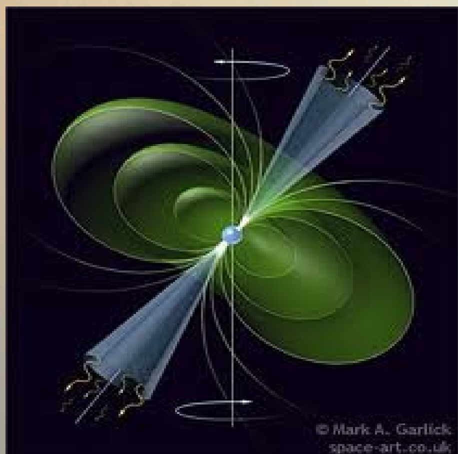
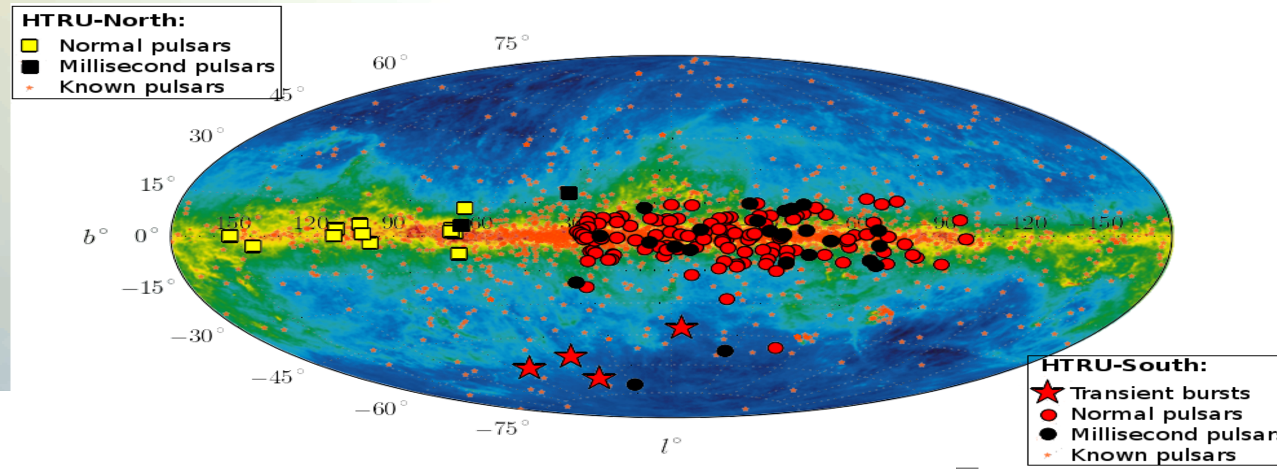
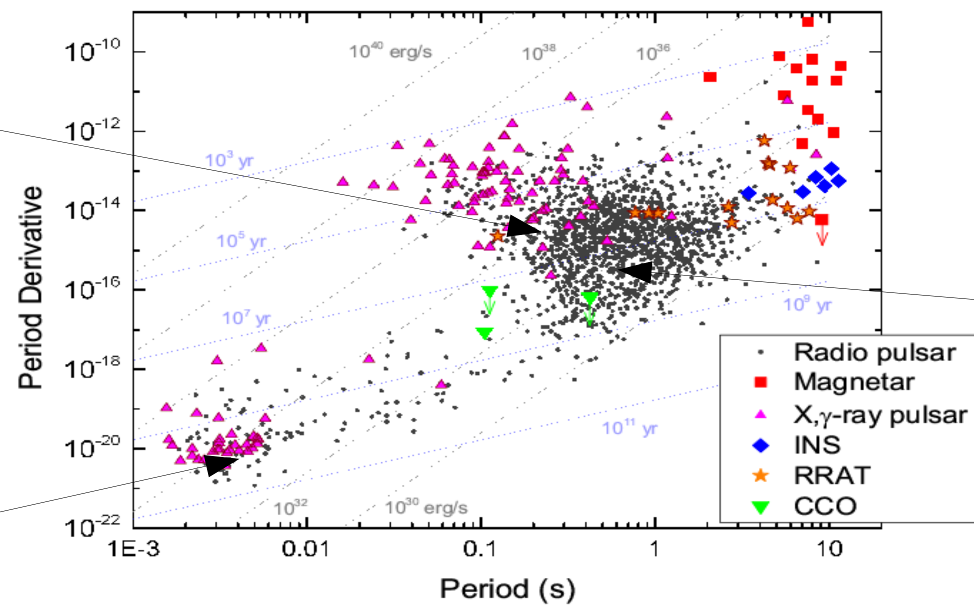
1. Introduction
2. The Hadron-Quark Phase Transition in the Interior of Compact Stars
3. Mergers of two Hybrid Stars
4. Summary

Pulsars ⇔ Compact Stars







PSR B0531+21 (33.5 ms)

Crab Pulsar

PSR B0329+54 (0.715 s)

PSR B1937+21 (1.56 ms)

Talk by M.Kramer
(GR 1.2)

General Relativity and Quantum Cromodynamics

ART	Yang-Mills-Theories
$D_\beta v^\alpha = \partial_\beta\, v^\alpha + \Gamma^\alpha_{\sigma\beta}\, v^\sigma$	$D_\beta{}_a{}^b = \partial_\beta\, 1_a{}^b + ig\, A_\beta{}_a{}^b$
$R^\delta{}_{\mu\alpha\beta}\, v^\mu = [D_\alpha, D_\beta]\, v^\delta$ $R^\delta{}_{\mu\alpha\beta} = \Gamma^\delta_{\mu\alpha\|\beta} - \Gamma^\delta_{\mu\beta\|\alpha} + \Gamma^\delta_{\nu\beta}\Gamma^\nu_{\mu\alpha} + \Gamma^\delta_{\nu\alpha}\Gamma^\nu_{\mu\beta}$	$F_{\alpha\beta}{}_a{}^b = \frac{1}{ig}\left[D_\alpha{}_a{}^c, D_\beta{}_c{}^b\right]$ $= A_\beta{}_a{}^b{}_{\|\alpha} - A_\alpha{}_a{}^b{}_{\|\beta} + \frac{1}{ig}\left[A_\alpha{}_a{}^c, A_\beta{}_c{}^b\right]$
$\mathcal{L}_G = R + \underbrace{(c_1\, R_{\mu\nu}R^{\mu\nu} + \dots)}_{\equiv 0 \text{ for ART}}$	$\mathcal{L}_{YM} = \frac{1}{4} F_{\mu\nu}{}_a{}^b\, F^{\mu\nu}{}_a{}^b$

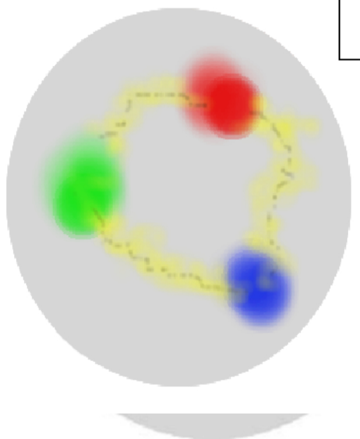
QuantumCromoDynamic:

$(SU(3)_{(c)})$- Color Yang-Mills-Gauge Theory)

$$D_\beta{}_A{}^B = \partial_\beta\, 1_A{}^B + ig\, G_\beta{}_A{}^B$$

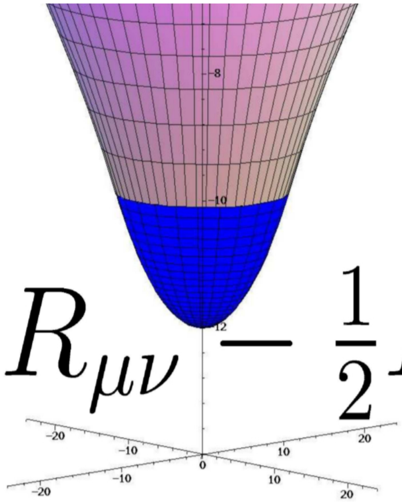
$_A,{}^B$ =red, green, blue

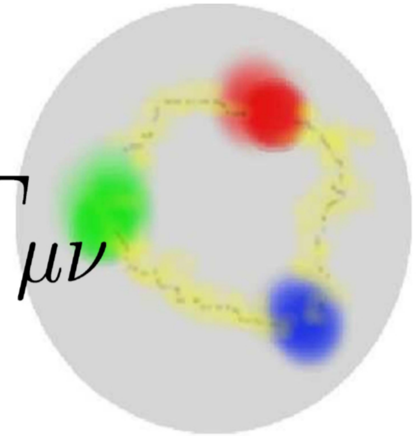
$$\psi^f_A = \begin{pmatrix} \psi^f_r \\ \psi^f_g \\ \psi^f_b \end{pmatrix}$$



Confinement

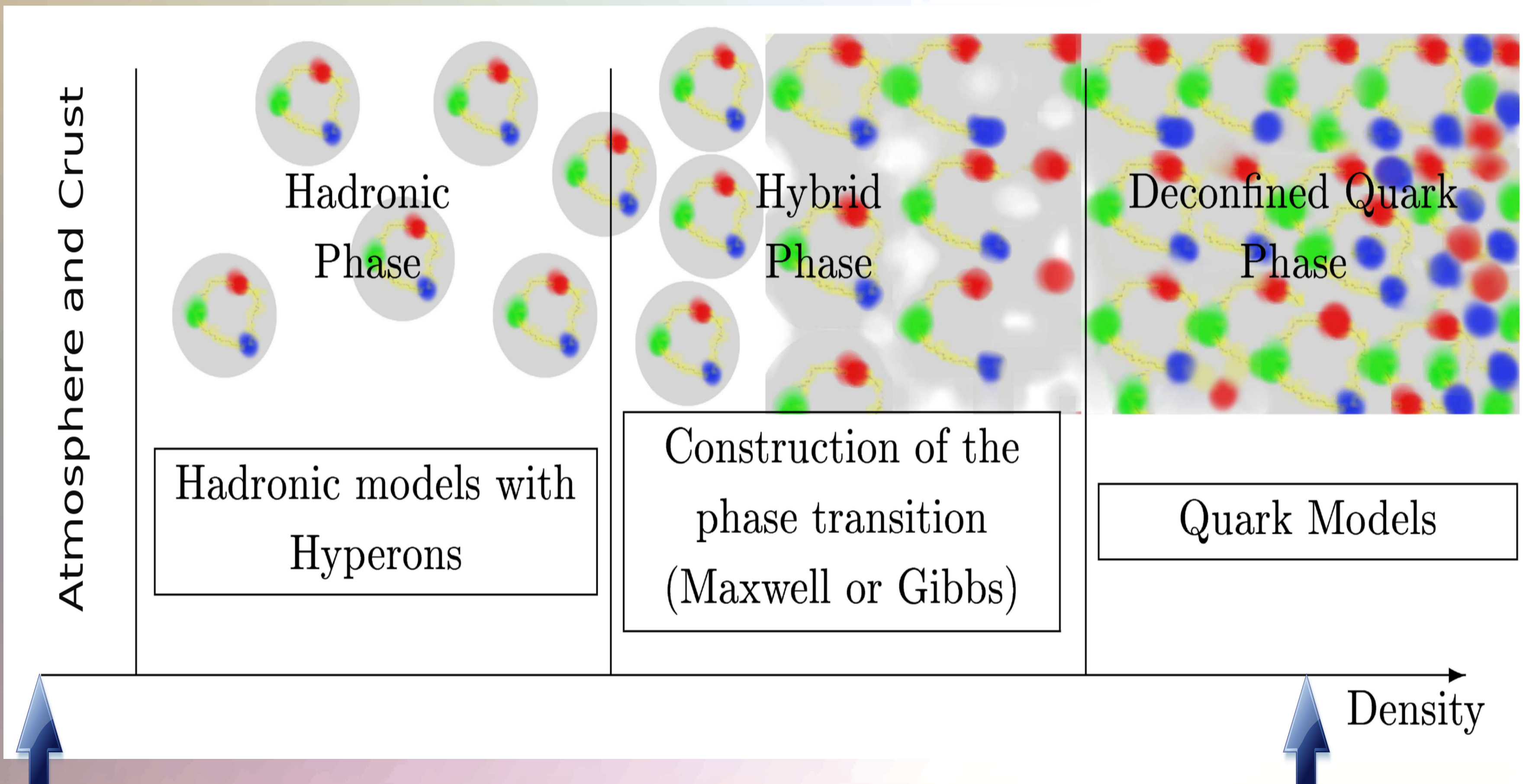
chiral symmetry, ...





$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}$$

The QCD – Phase Transition and the Interior of a Hybrid Star

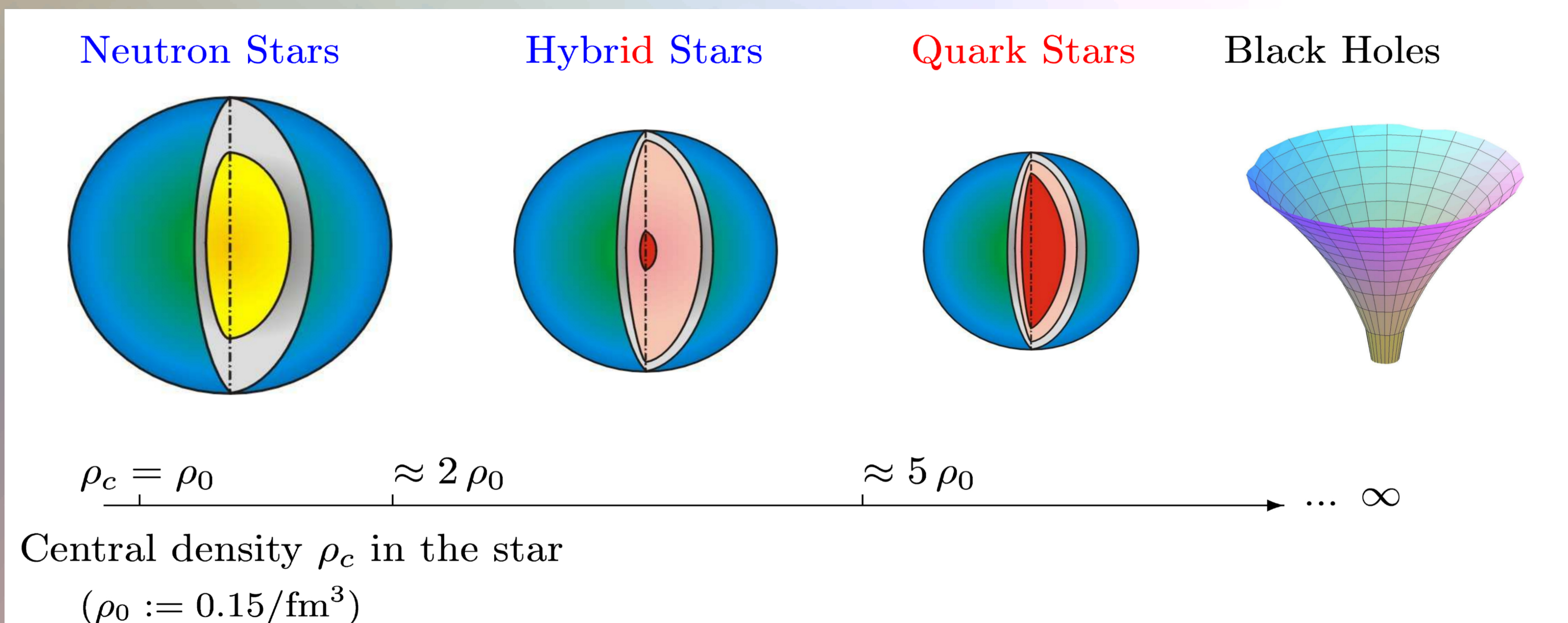


Surface of the Star

Center of the Star

The Compact Star Zoo

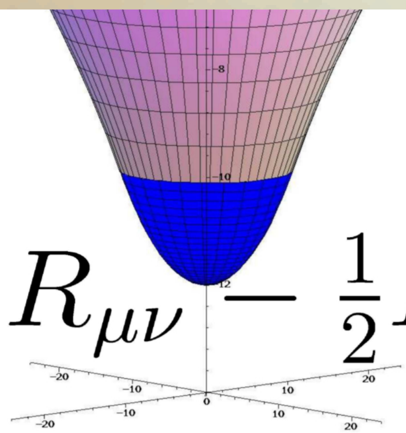
Neutron stars with and without hyperons, quark stars and strange quark stars, hybrid stars with color superconducting quark matter, hybrid stars with Bose-Einstein condensates of antikaons, ...

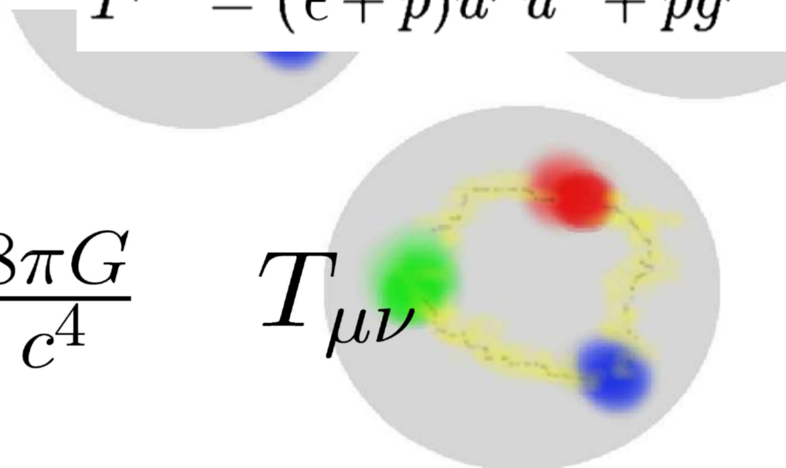


Compact Stars

(Simplest model: Nonrotating, spherical symmetric, static, ideal fluid)

$$T^{\mu\nu} = (\epsilon + p)u^{\mu}u^{\nu} + pg^{\mu\nu}$$





$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Space-Time Metric :

$$g_{\mu\nu} = \begin{pmatrix} e^{\nu(r)} & 0 & 0 & 0 \\ 0 & -\left(1 - \frac{2m(r)}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix}$$

Tolman-Oppenheimer-Volkoff Equation

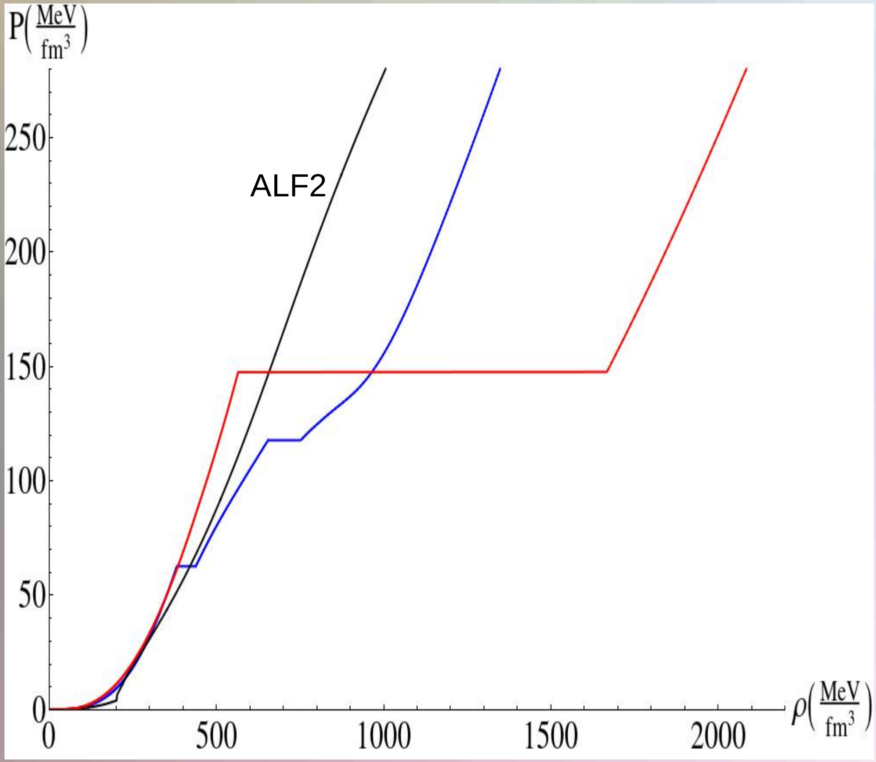
$$\frac{dP}{dr} = -\frac{(\epsilon + P)4\pi r^3 + m}{r(r - 2\,m)}$$

$$m(r) = \int_0^r 4\pi \tilde{r}^2 \epsilon(\tilde{r})\; d\tilde{r}$$

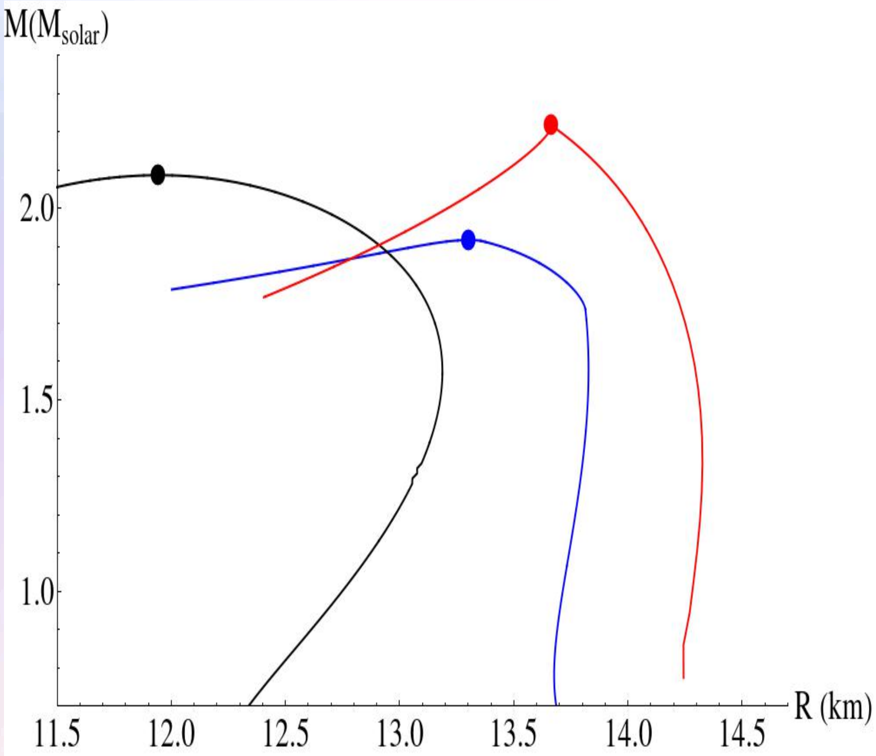
$$\frac{d\nu}{dr} = \frac{8\pi P r^3 + 2\,m}{r(r - 2\,m)} \quad ,$$

The Equation of State (EoS)

The construction of an realistic EoS, which includes a hadron-quark phase transition, depends on the underlying effective model of the hadron and quark interaction.



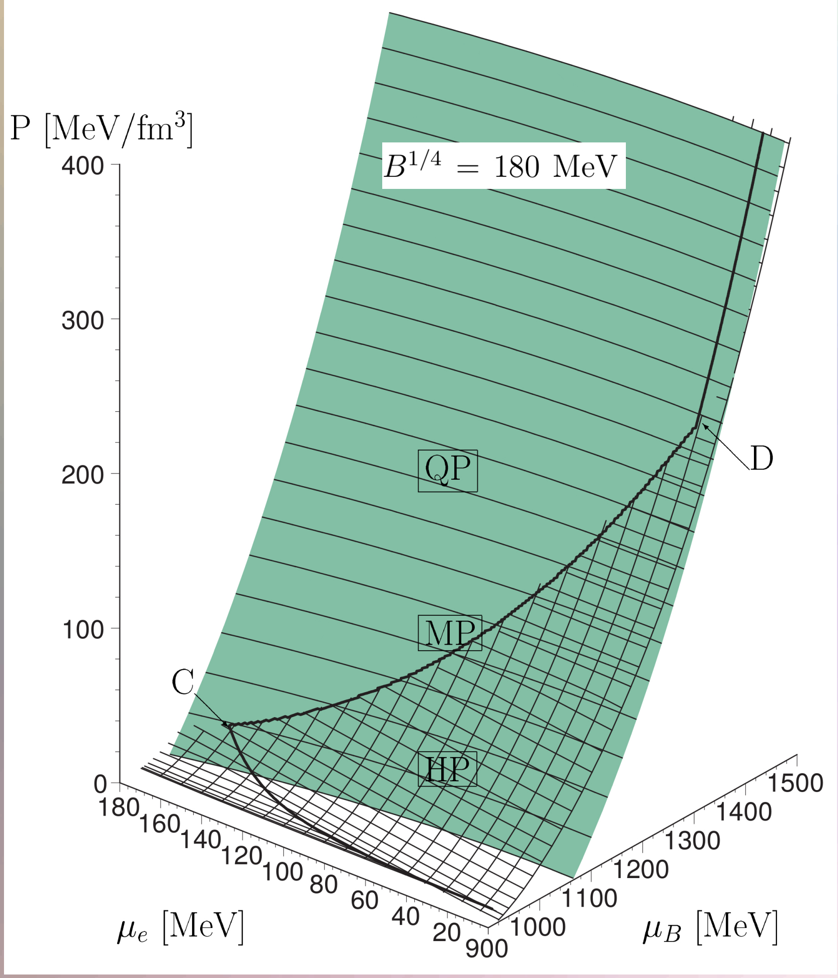
Selection of current EoS



Star properties (Mass vs. Radius). Points correspond to the corresponding maximum mass of the EoS.

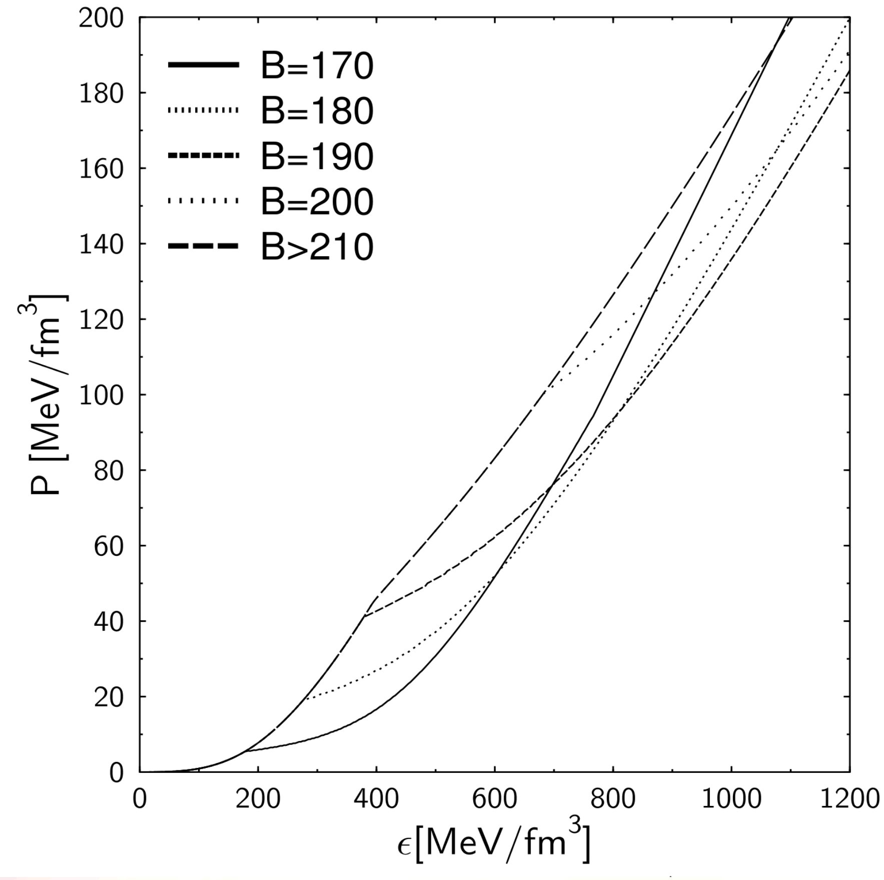
The Gibbs Construction

Hadronic and quark surface:



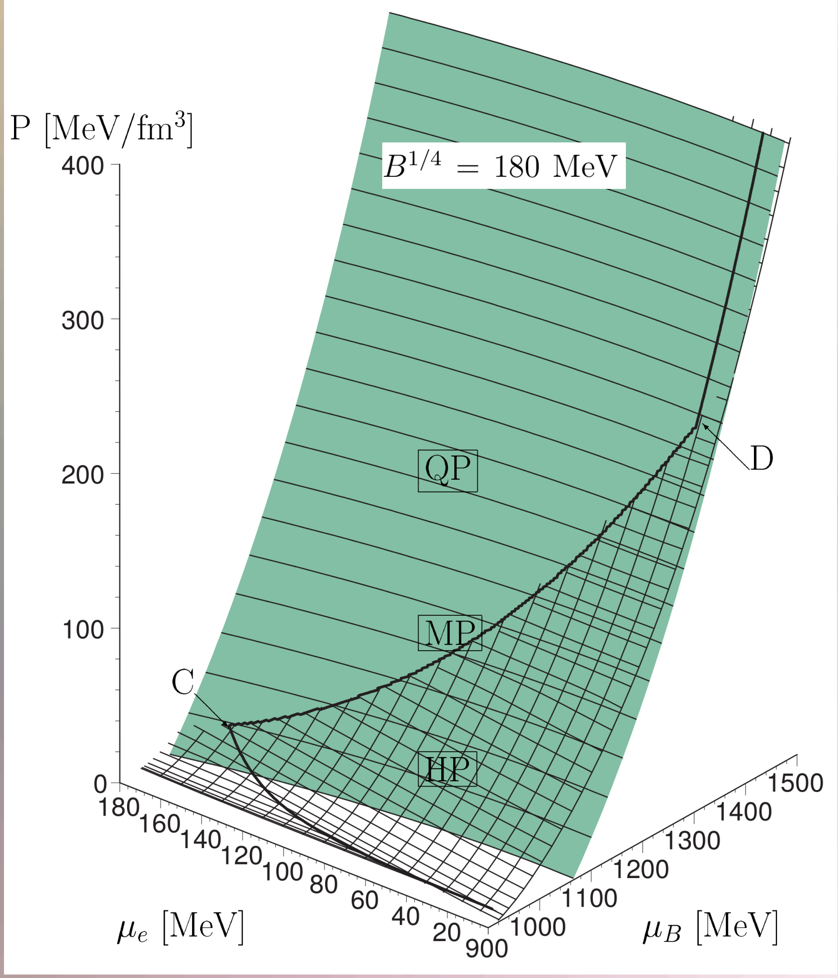
Charge neutrality condition is only globally realized

$$\rho_e := (1-\chi)\rho_e^H(\mu_B,\mu_e) + \chi\rho_e^Q(\mu_B,\mu_e) = 0\,.$$

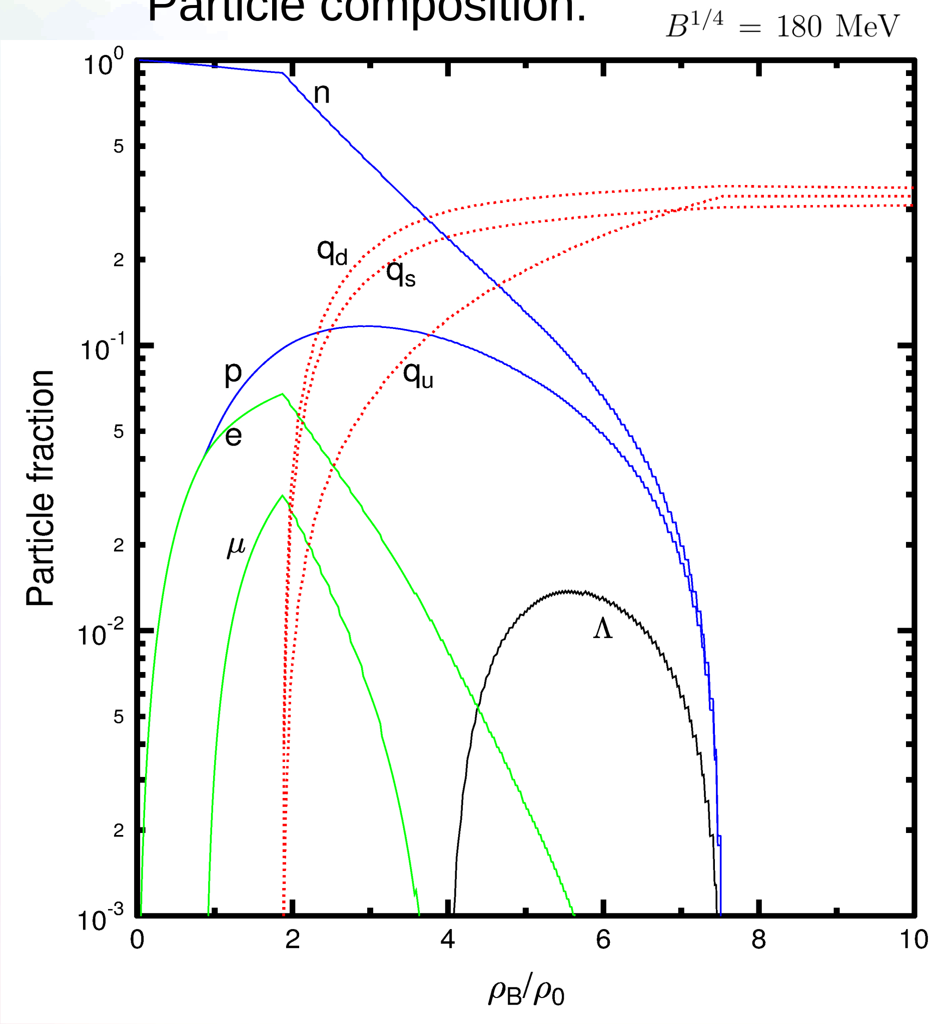


The Gibbs Construction

Hadronic and quark surface:



Particle composition:



The Maxwell Construction

If the surface tension is large -> mixed phase disappears

-> sharp boundary between the hadron and quark phase
->Maxwell construction should be used

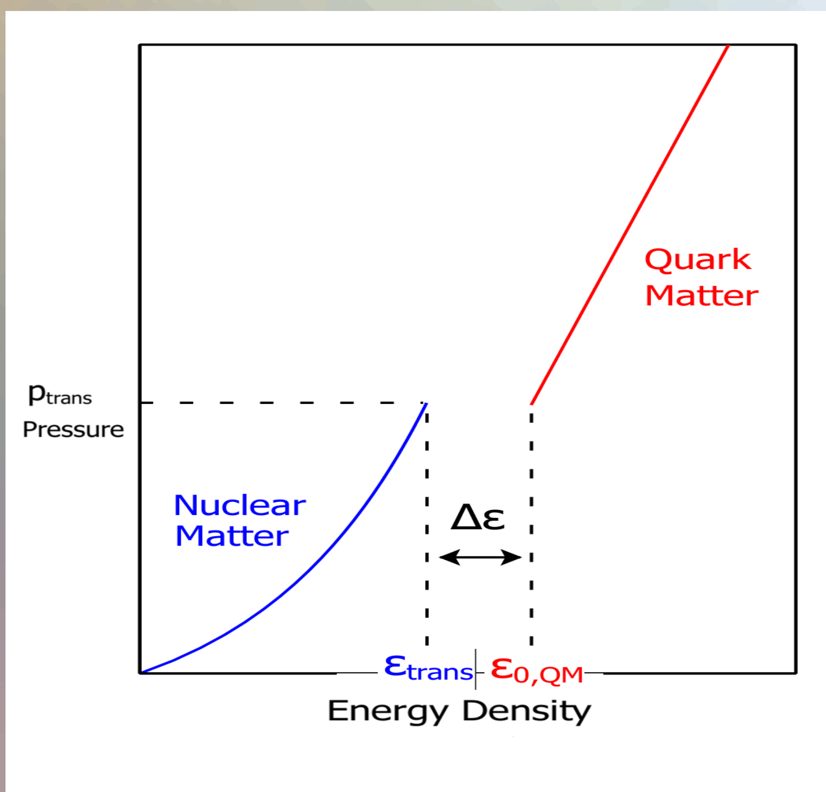
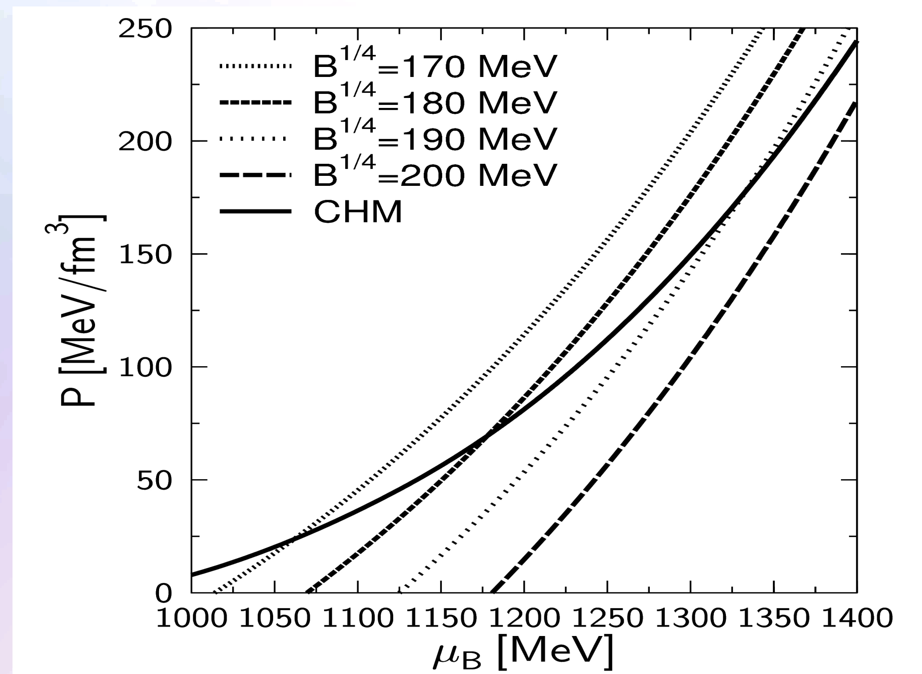


Image from M.G. Alford, S. Han, and M. Prakash, Phys. Rev. D 88, 083013 (2013)

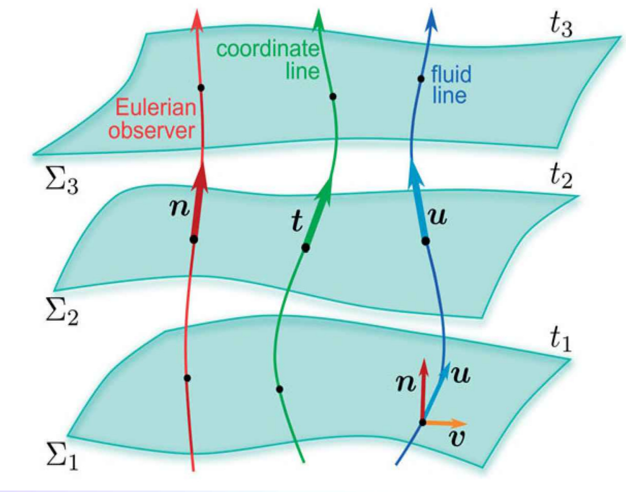


Pressure and baryon chemical potential stays constant, while the density and the charge chemical potential jump discontinuously during the phase transition.

Relativistic Hydrodynamics and Numerical General Relativity

(3+1)-Split, ADM equations,BSSNOK, Valencia formulation, high resolution shock capturing methods ….

Talk by B.Brügmann
(GR 9.2)

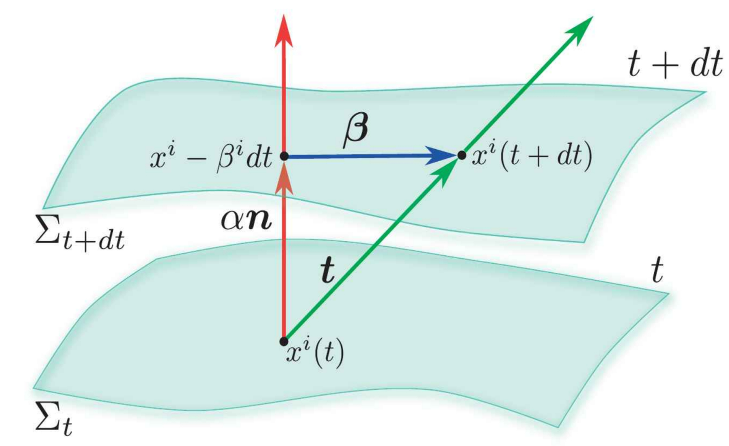


$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

$$\nabla_\mu (\rho u^\mu) = 0\,, \quad \nabla_\nu T^{\mu\nu} = 0\,.$$

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \beta_i \beta^i & \beta_i \\ \beta_i & \gamma_{ij} \end{pmatrix}$$

(3+1) decomposition of spacetime



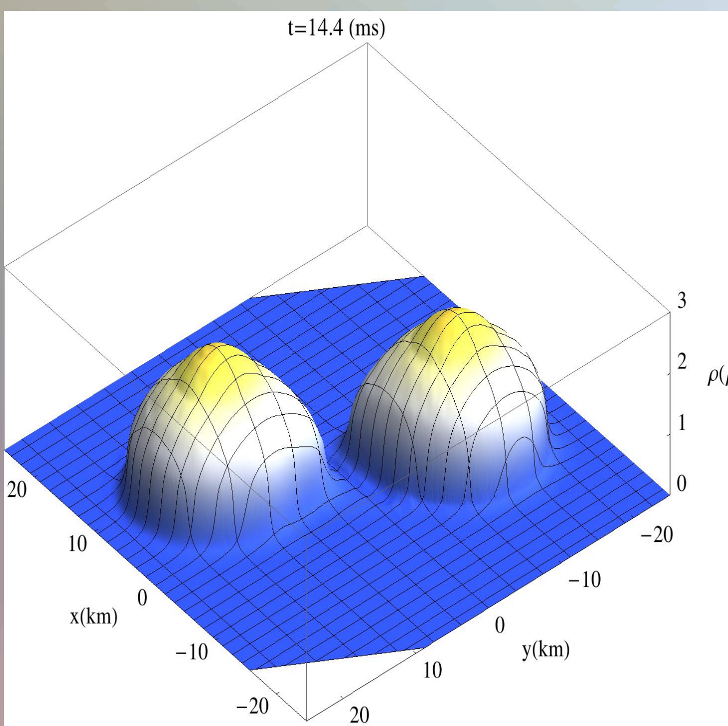
$$d\tau^2 = \alpha^2(t, x^j) dt^2$$

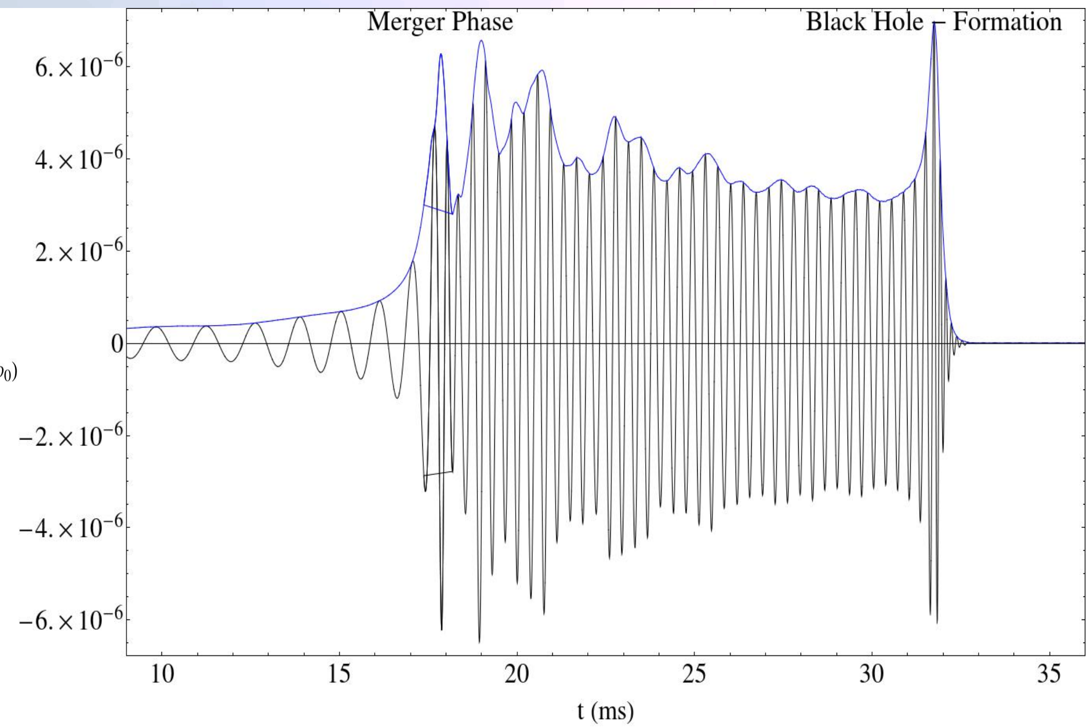
$$x^i_{t+dt} = x^i_t - \beta^i(t, x^j) dt$$

All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

Mergers of two Hybrid Stars

The following results represent a hybrid star merger calculation (EOS: ALF2 (APR-Hadronic model +Gibbs construction of MIT-Bag+CFL-Phase), see Alford et.al.,APJ 629,2005) where the EOS was fitted with a piecewise polytrope (see K.Takami, L.Rezzolla, and L.Baiott, "Constraining the Equation of State of Neutron Stars from Binary Mergers", arXiv:1403.5672).





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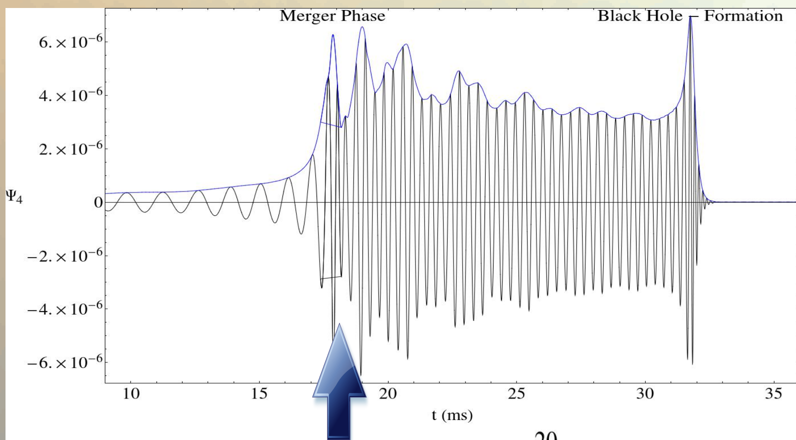
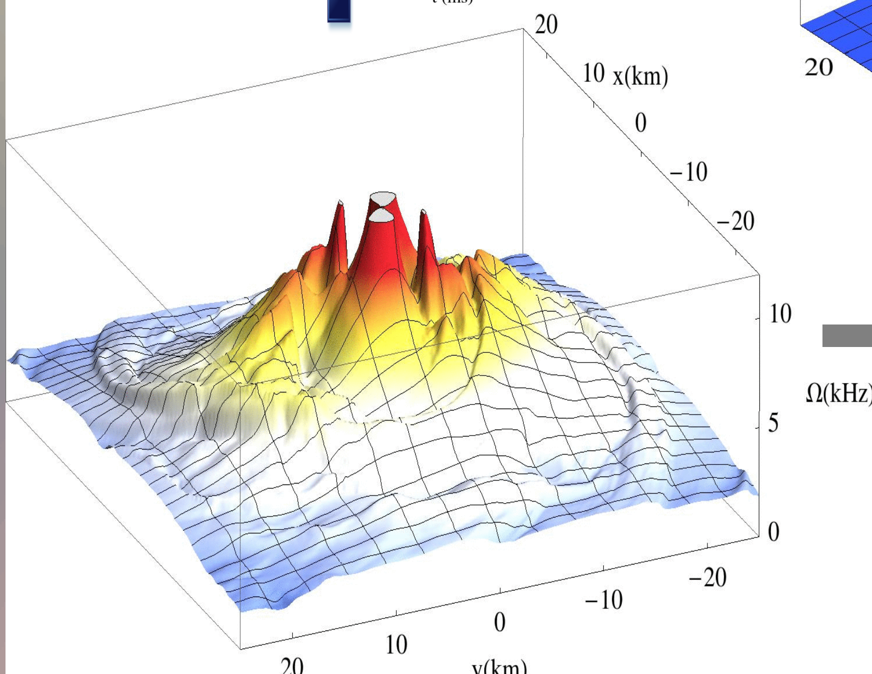
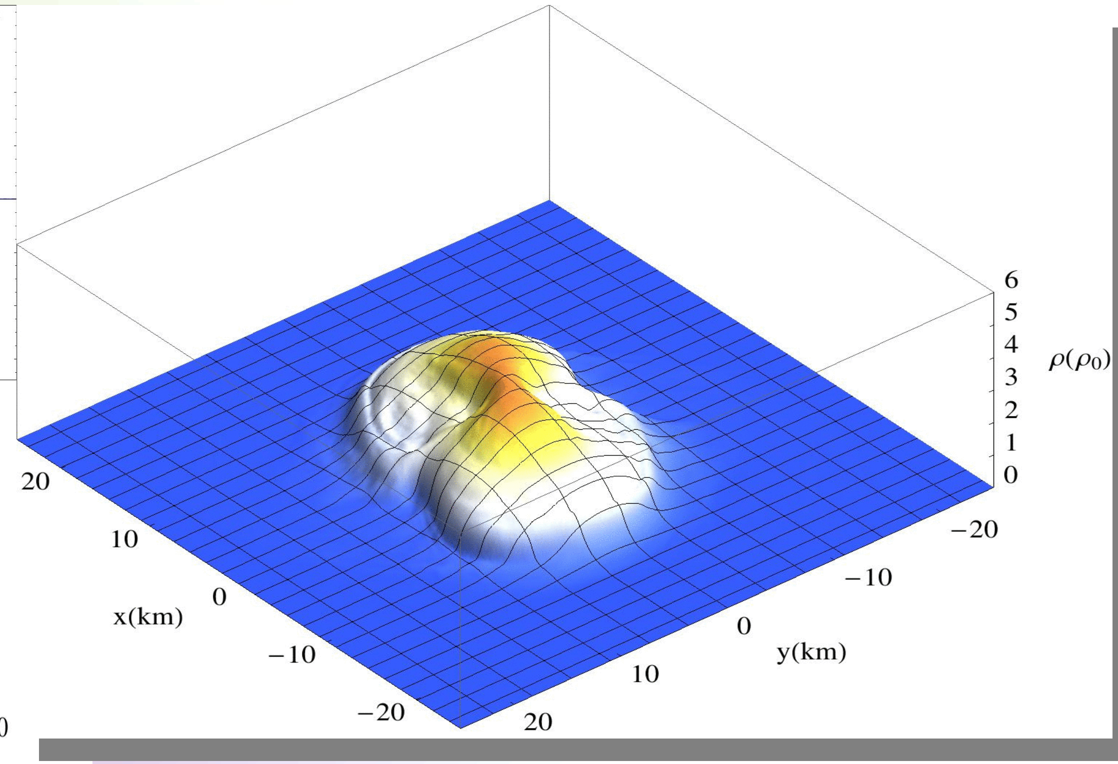






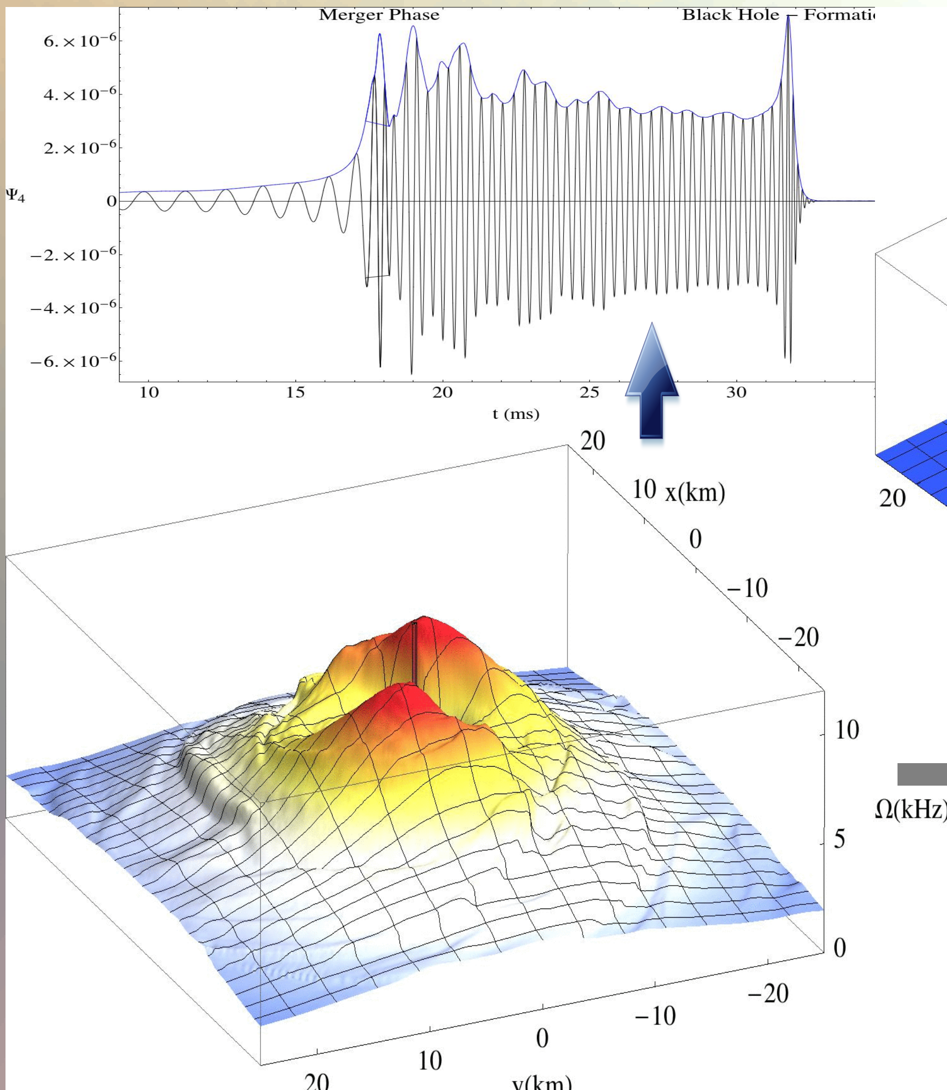
Merger phase of f1 and f3 frequencies

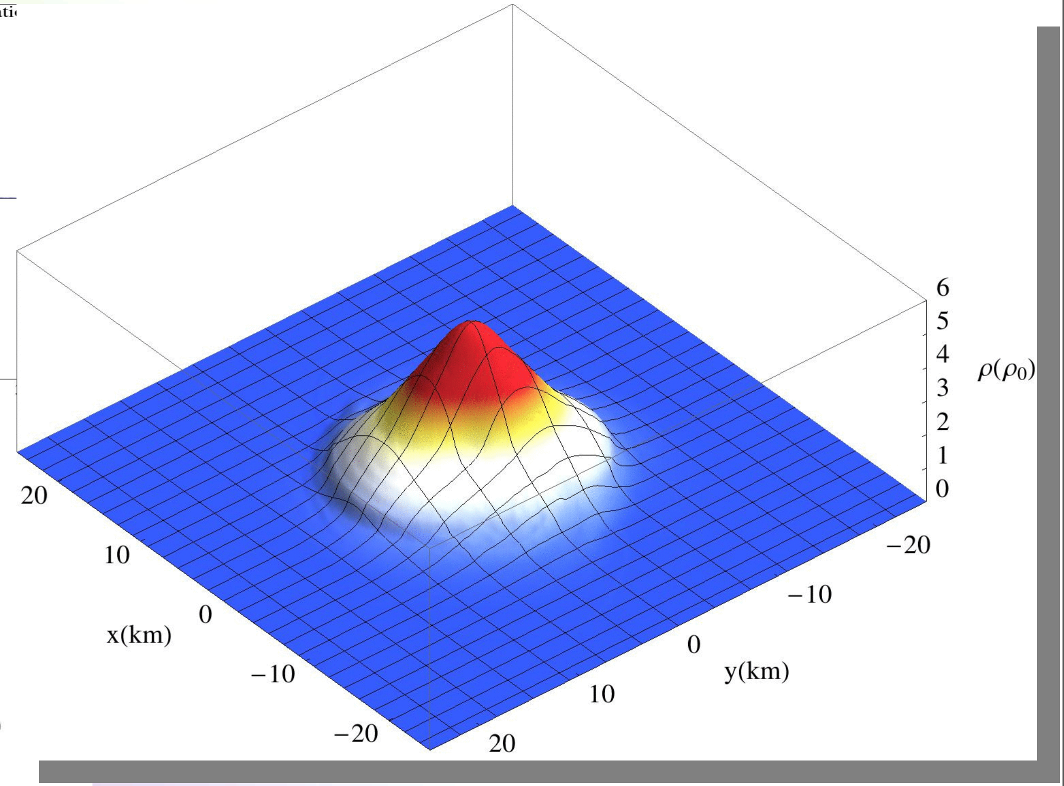
Talk by L.Rezzolla (GR 10.1)

<u>In Preparation</u>:
"Realistic Rotation Profile in Hyper Massive Neutron Stars", Kentaro Takami,Matthias Hanauske,Luciano Rezzolla,Filippo Galeazzi, Bruno Mundim, and Luke Bovard

Simulations done by Kentaro Takami

Mergers of two Hybrid Stars





Merger phase of the f2 frequency

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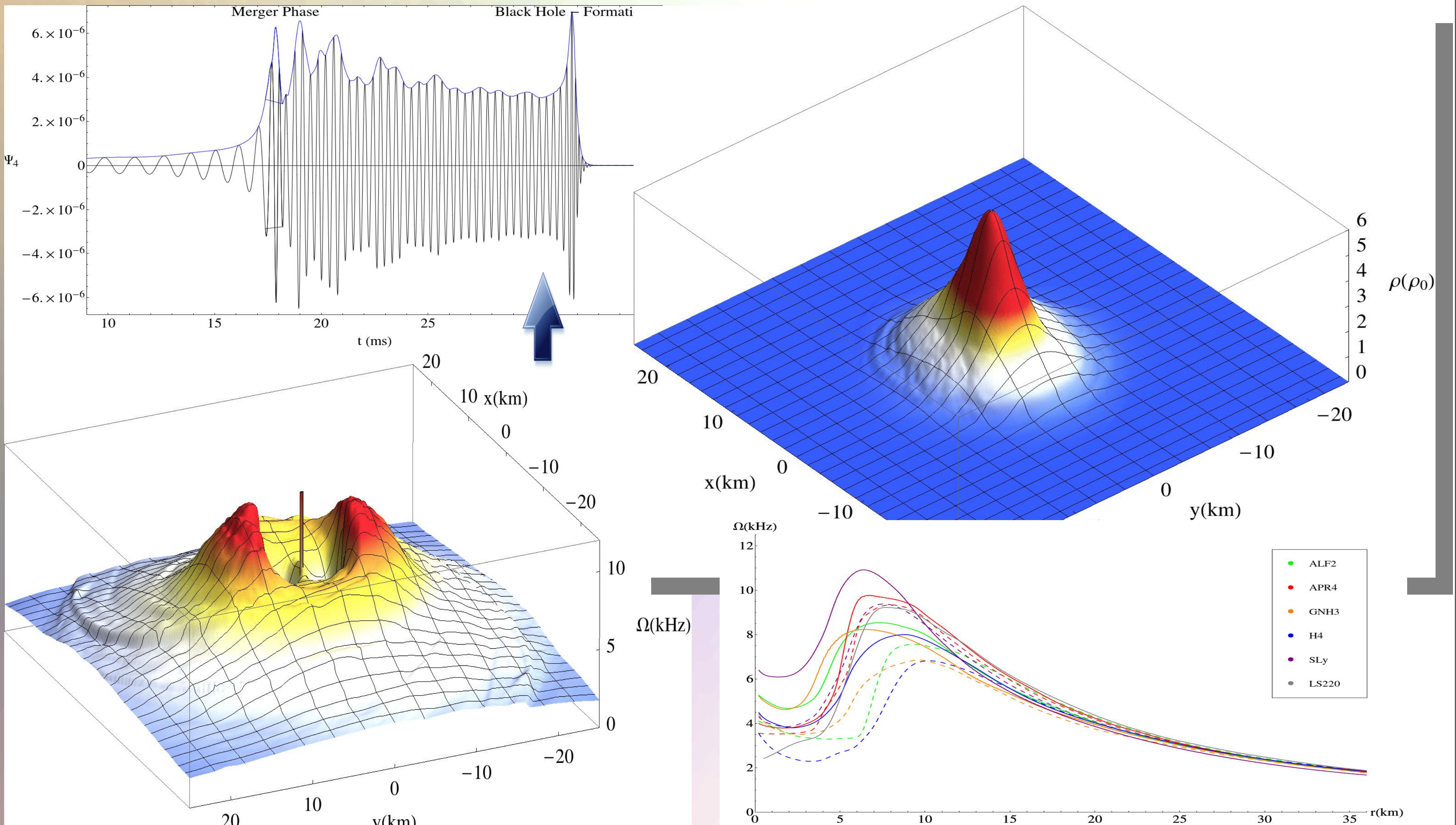

Simulations done by Kentaro Takami

Summary

- The transition from confined hadronic matter to deconfined quark matter is called the Hadron-Quark Phase Transition
- Depending on the value of the hadron-quark surface tension, the Maxwell- or Gibbs-phase transition construction should be used
- The star's density profile has a huge density jump (Riemann problem) at the phase transition boundary, if a Maxwell construction is used
- Realistic numerical simulations of collapsing neutron stars or neutron star mergers should use an EoS with a hadron quark phase transition
- Realistic rotation profile in hyper massive neutron stars indicate that the maximum of the rotation curve come along with the hadron quark phase transition boundary (mixed phase if Gibbs is used)
- The decisive amount of matter which is trapped behind the event horizon when a black hole is formed (in an merger or collapsing scenario of neutron stars) is deconfined quark matter

End

- ….Additional, backup-slides follow

Current Projects

- The twin star collapse
  (Collaborators: A.Zacchi, J.Schaffner-Bielich and L.Rezzolla)
- Oscillations of hybrid and quark stars within different models of the phase transition
  (Collaborators: A.Brillante, I.Mishustin, A.Sedrakian and L.Rezzolla)
- Hybrid and quark star merger calculations with temperature dependent equation of states
  (Collaborators: S. Schramm, B.Franzon, F.Galeazzi and L.Rezzolla)
- Differential rotation profiles of hypermassive hybrid stars and the effect Maxwell- vs. Gibbs-construction
  (Collaborators: K.Takami, F.Galeazzi, B.Mundim, L.Bovard and L.Rezzolla)
- Collapse scenario and black hole formation in the context of the hadron-quark phase transition

Summary

- The transition from confined hadronic matter to deconfined quark matter is called the Hadron-Quark Phase Transition
- Depending on the value of the hadron-quark surface tension, the Maxwell- or Gibbs-phase transition construction should be used
- The star's density profile has a huge density jump (Riemann problem) at the phase transition boundary, if a Maxwell construction is used
- In a narrow parameter window Twin Stars a possible: The collapse from a neutron to a hybrid quark star will emit a burst of gravitational waves, neutrinos and gamma rays
- Realistic numerical simulations of collapsing neutron stars or neutron star mergers should use an EoS with a hadron quark phase transition
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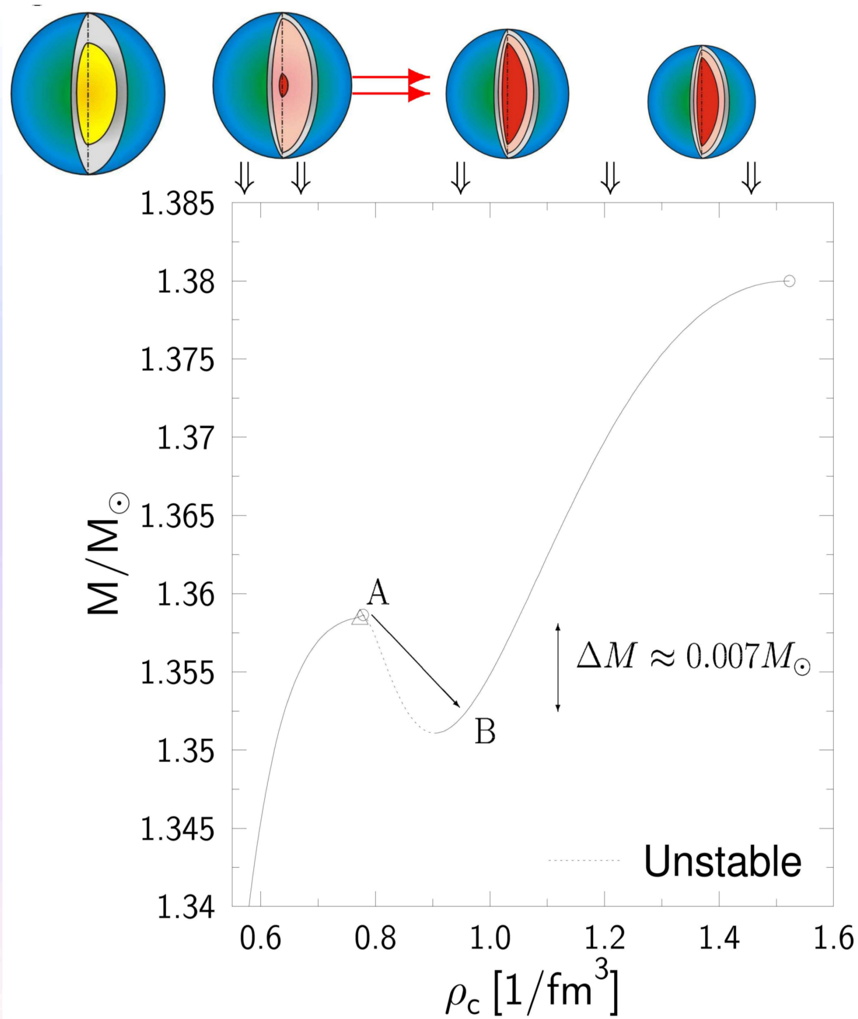
The Twin Star Collapse

Usually it is assumed that this loss of stability leads to the collapse into a black hole. However, realistic calculations open another possibility: the collapse into the twin star on the second sequence. A star from the first sequence which reaches the maximum mass (point A) will collapse to its twin star. The latter is the corresponding star on the second sequence, i.e. the one which has the same total baryon number (point B).



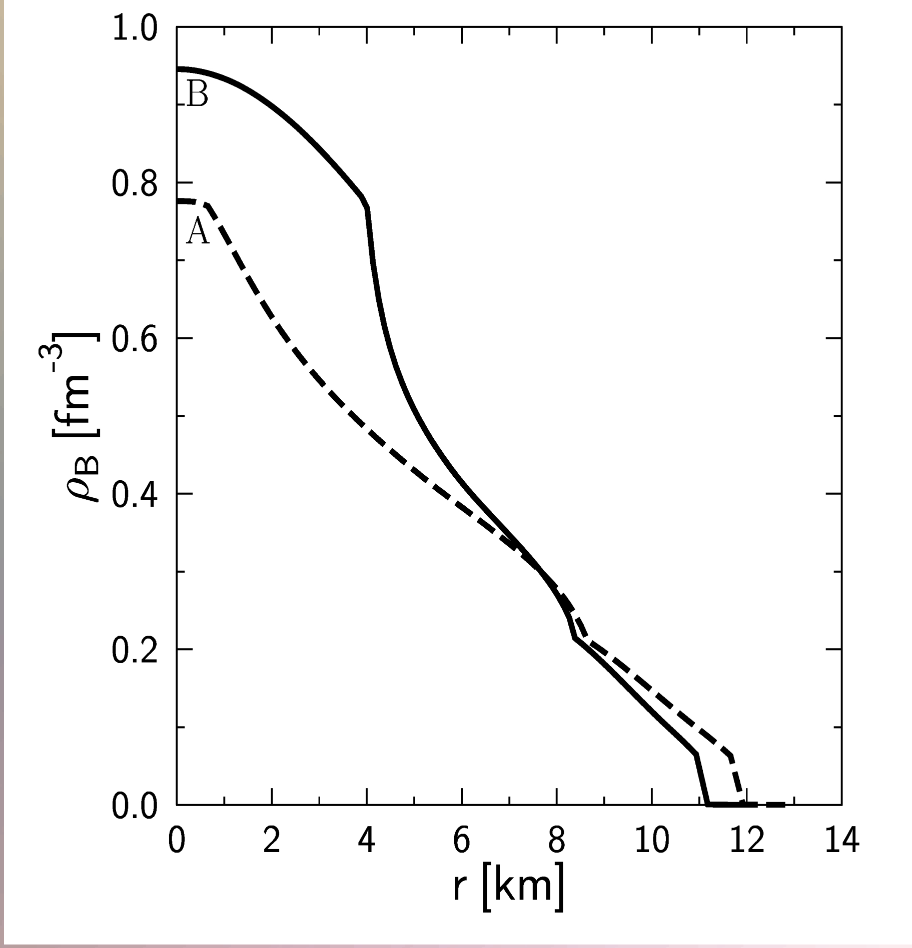
I.N. Mishustin, M. Hanauske, A. Bhattacharyya, L.M. Satarov, H. Stöcker, and W. Greiner, "Catastrophic rearrangement of a compact star due to quark core formation", Physics Letters B 552 (2003) p.1-8

J. Schaffner-Bielich, M. Hanauske, H. Stöcker, and W. Greiner, Phys. Rev. Lett. 89, 171101 (2002)

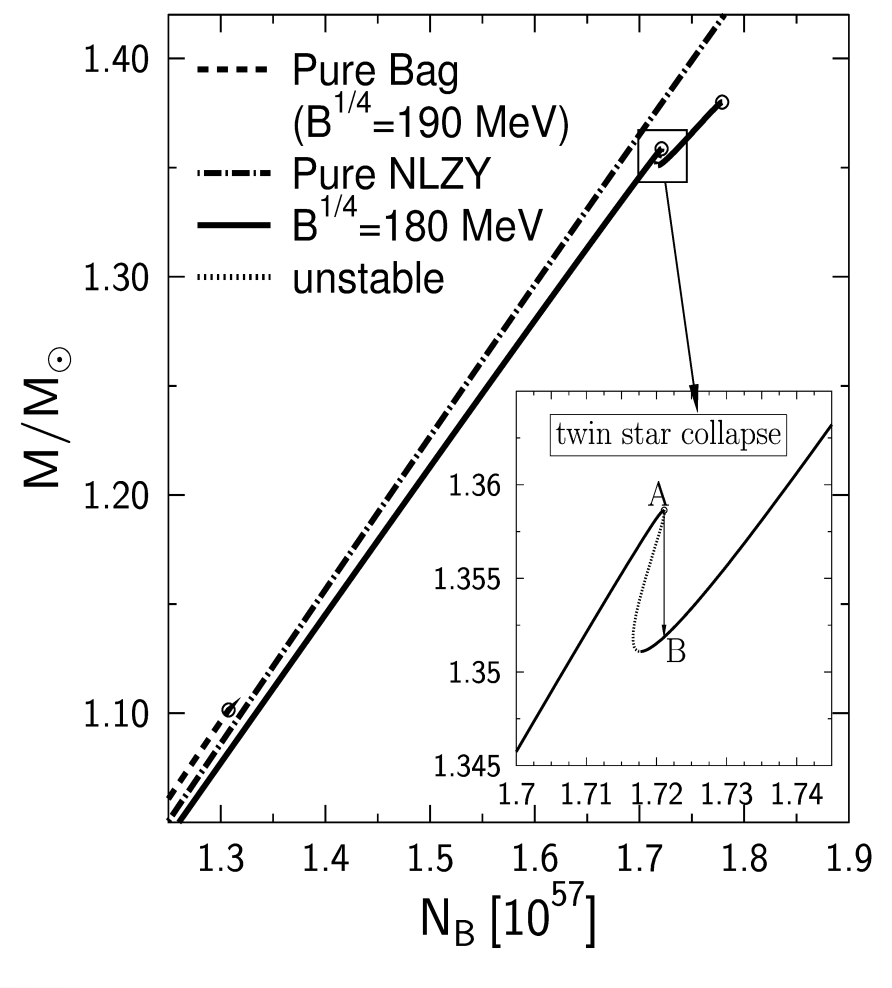
S. Banik, M. Hanauske, D. Bandyopadhyay, and W. Greiner, Phys. Rev. D 70, 123004 (2004)

The Twin Star Collapse

Density profiles of the two twins



Conservation of total baryonic mass



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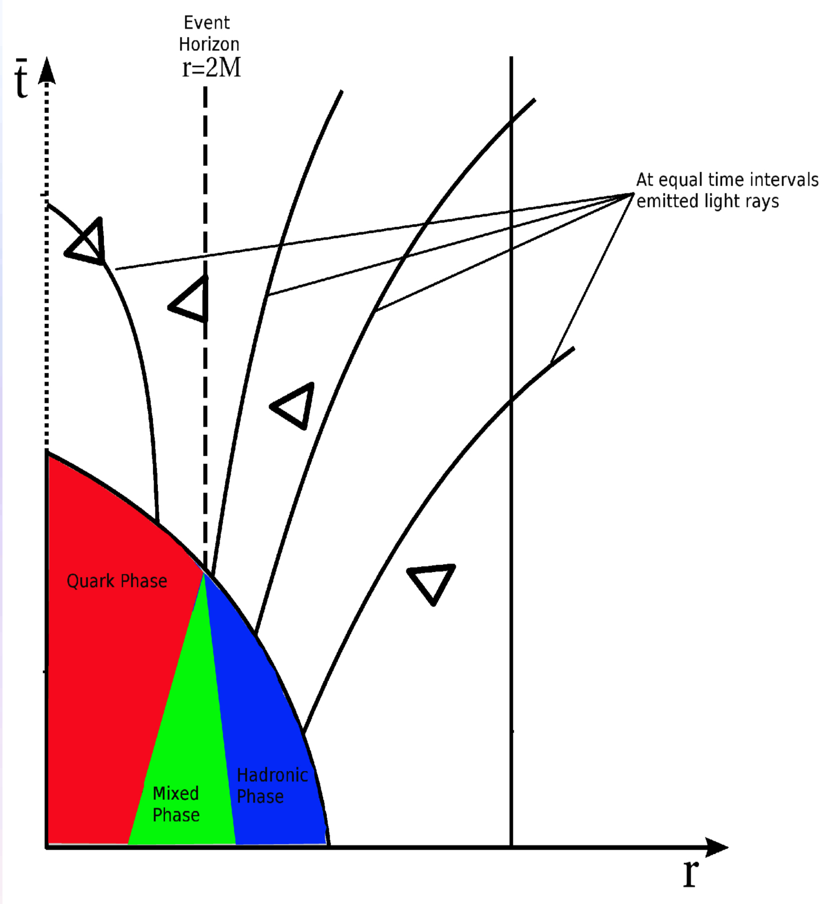
Collapse Scenario of a Hybrid Star

The gravitational collapse of a hybrid star to a black hole is visualized on the right side within a space-time diagram of the Schwarzschild metric in advanced Eddington-Finkelstein coordinates.

The formation of the apparent and event horizon of the black hole confines the quark star macroscopically. Finally the colour charge of the deconfined free quarks cannot be observed from outside.



M. Hanauske, Dissertation: "Properties of Compact Stars within QCD-motivated Models" ;
University Library Publication 2004 (urn:nbn:de:hebis:30-0000005872)

The QCD- Phase Diagram

The QCD phase diagram at temperature T and net baryon density is displayed on the right side.

Some regions can be accessed by heavy ion collisions at different energies.

Matter of the early universe and in the interior of compact stars are also indicated within the diagram.

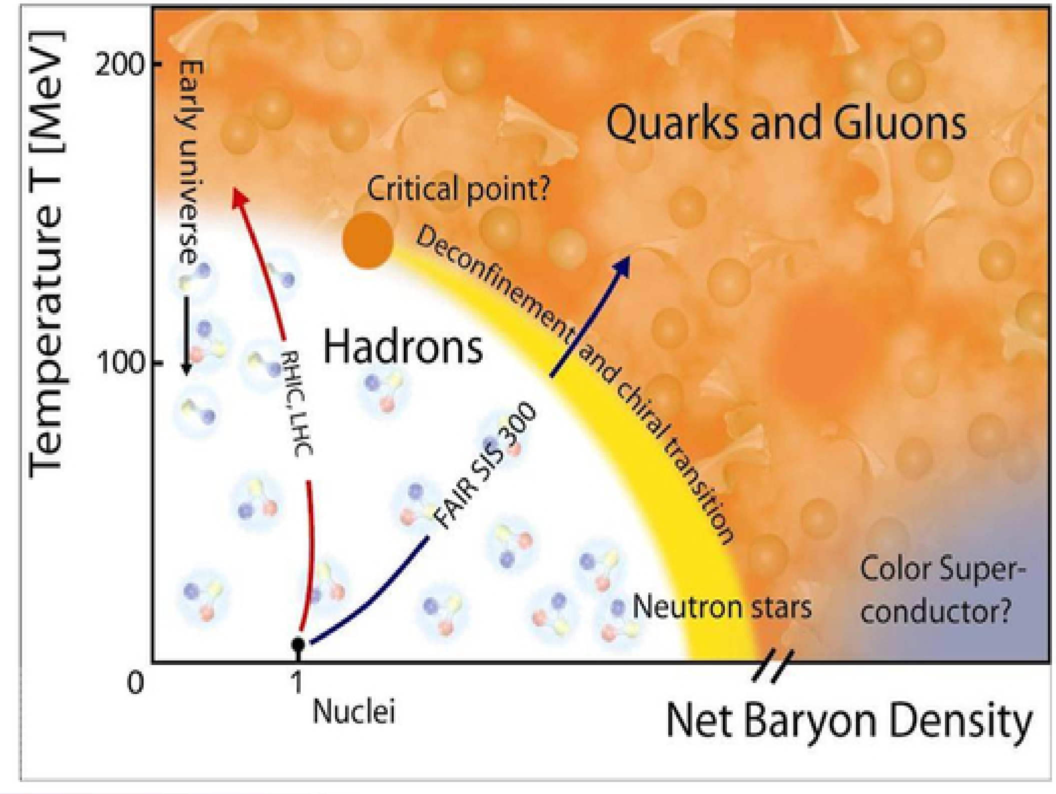


Image from http://webarchiv.fz-juelich.de/nic/Publikationen/Broschuere/Elementarteilchenphysik/hadron.jpg

The QCD- Phase Diagram II

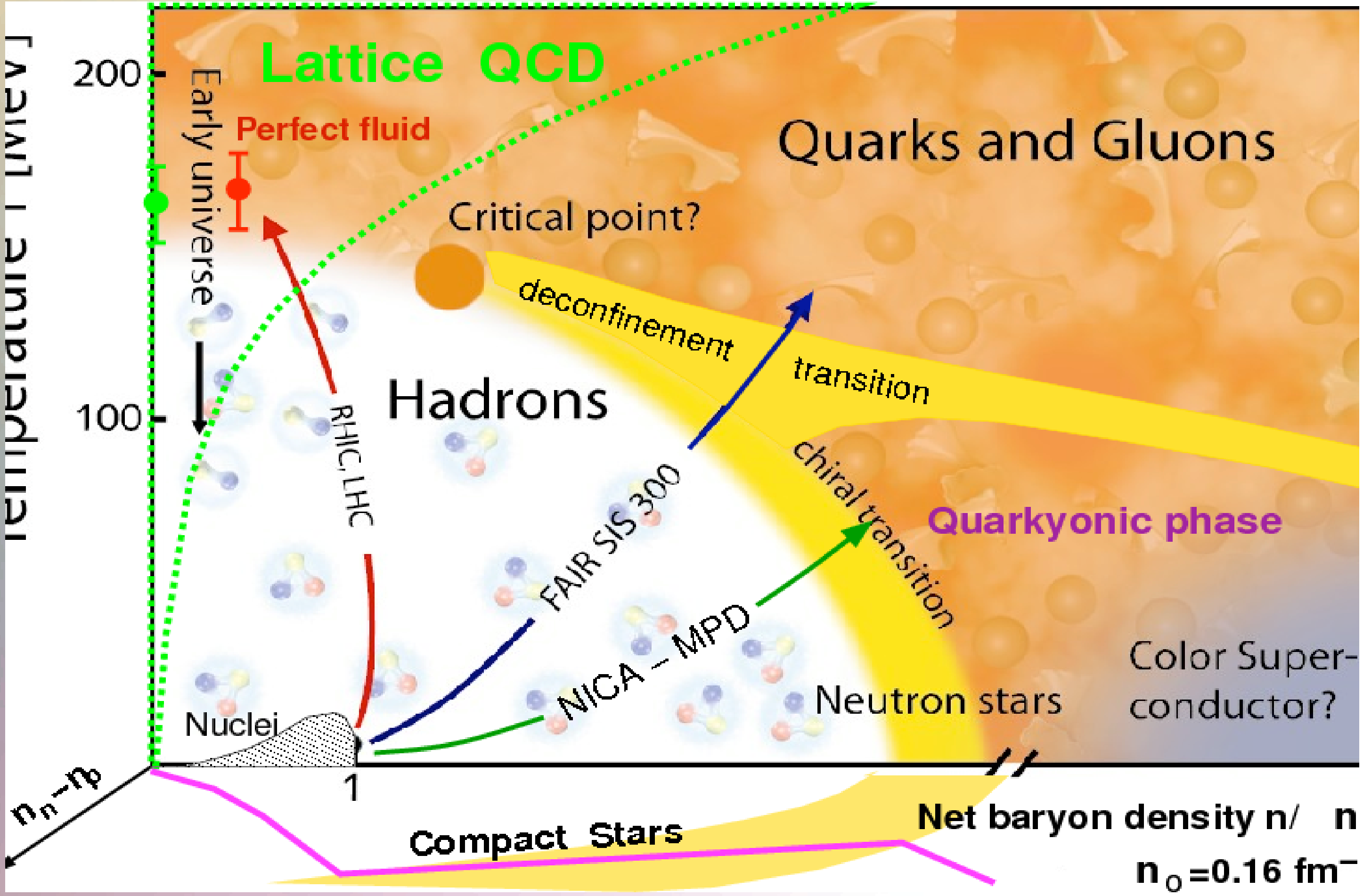
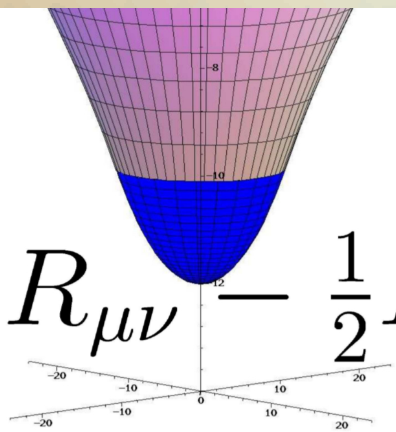


Image from http://inspirehep.net/record/823172/files/phd_qgp3D_quarkyonic2.png

Neutron Stars (NS)



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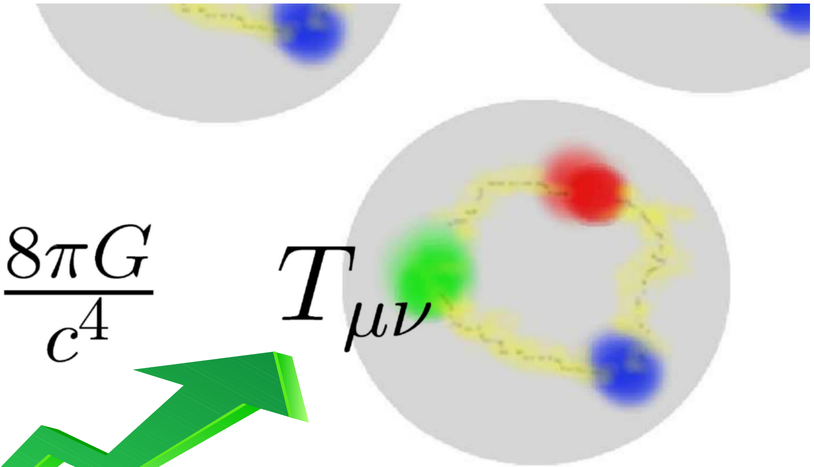


Relativistic Mean-Field Hadronic Models

$$\sum_B \quad (p, n, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0)$$

$$\begin{aligned}\mathcal{L} \;=\; & \sum_B \overline{\psi}_B \left(i\partial\!\!\!/ - m_B\right)\psi_B + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{a}{3}\sigma^3 - \frac{b}{4}\sigma^4 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} \\ & + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\vec{\rho}^{\,\mu\nu}\vec{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^{\,\mu}\vec{\rho}_\mu \; + \sum_B \overline{\psi}_B \left(g_{\sigma B}\sigma + g_{\omega B}\omega^\mu\gamma_\mu + g_\rho\vec{\rho}^{\,\mu}\gamma_\mu\vec{\tau}_B\right)\psi_B\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{YY} \;=\; & \frac{1}{2}\left(\partial^\mu\sigma^*\partial_\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) - \frac{1}{4}\phi^{\mu\nu}\phi_{\mu\nu} + \frac{1}{2}m_\phi^2\phi^\mu\phi_\mu \\ & + \sum_Y \overline{\psi}_Y \left(g_{\sigma^* Y}\sigma^* + g_{\phi Y}\phi^\mu\gamma_\mu\right)\psi_Y \quad ,\end{aligned}$$

$$\mathcal{L}_{\text{lep}} = \sum_{l=e,\mu} \overline{\psi}_l [i\gamma_\mu\partial^\mu - m_l]\psi_l$$

Composition of Neutron Star Matter

Neutron star matter conditions:

1) Charge Neutrality
2) β-equilibrium
3) Strangeness production

$$n \Leftrightarrow p + e + \tilde{\nu}_e$$

Λ-Hyperon	$n + n \leftrightarrow n + \Lambda + K^0$	$\mu_\Lambda = \mu_n$
Σ^--Hyperon	$n + n \leftrightarrow n + \Sigma^- + K^+$	$\mu_{\Sigma^-} = \mu_n + \mu_e$

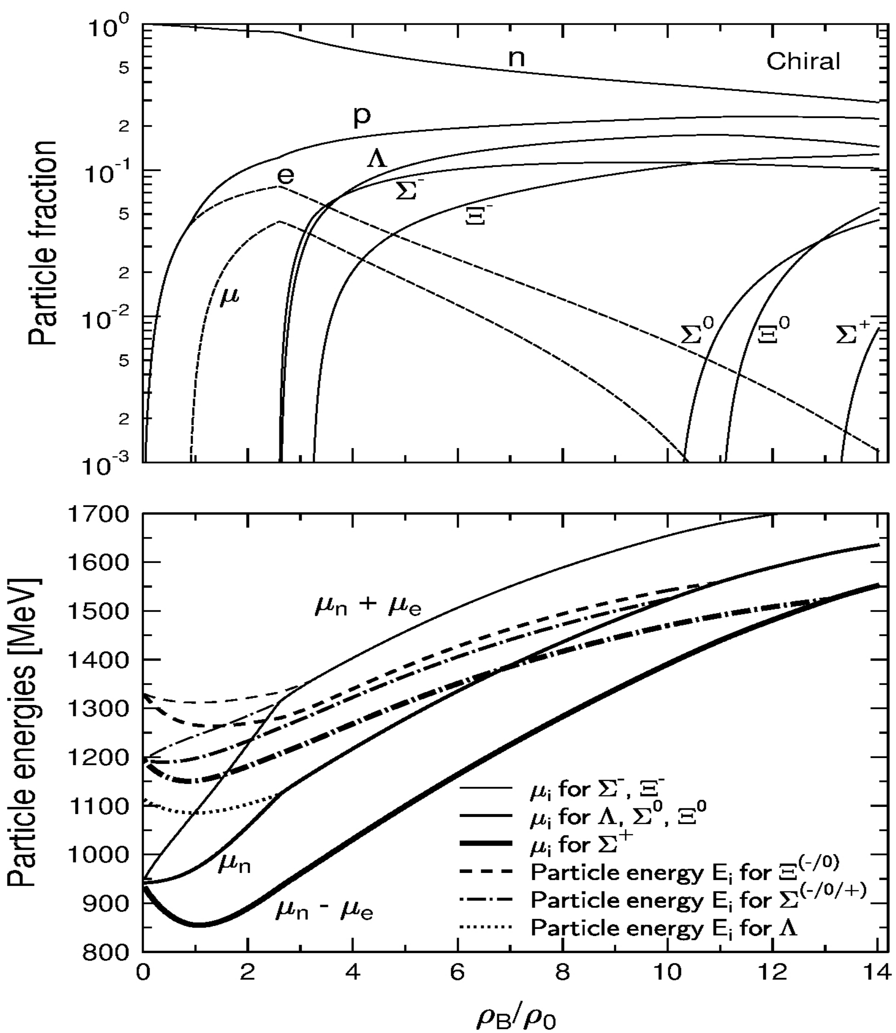
Chemical potentials and single particle energies of hyperons in dependence of the baryonic density.

$$E_i(k) = E_i^*(k) + g_{i\omega}\omega_0 + g_{i\phi}\phi_0 + g_{i\rho}I_{3i}\rho_0$$

$$E_i^*(k) = \sqrt{k_i^2 + m_i^{*2}}$$

$$\mu_i = b_i\mu_n - q_i\mu_e$$

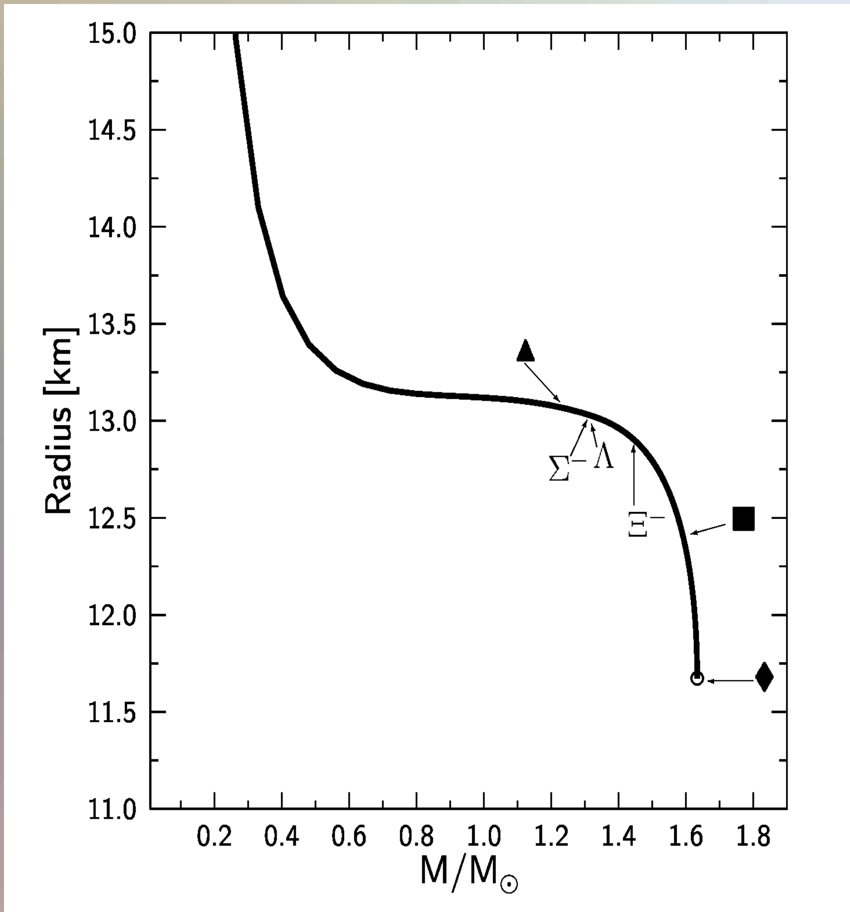
Particle composition vs baryonic density



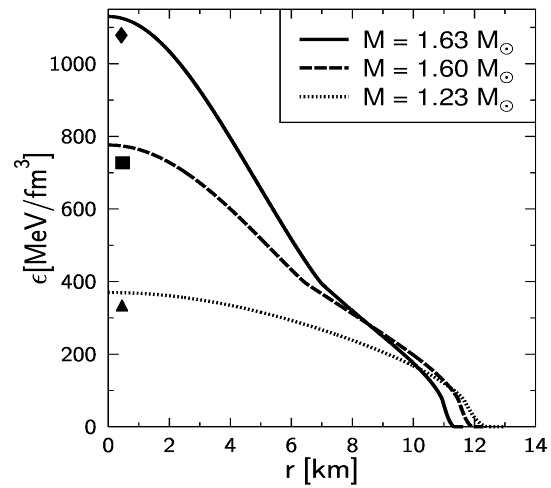
M. Hanauske, D. Zschiesche, S. Pal, S. Schramm, H. Stöcker, and W. Greiner, Astrophys. J. 537, 958 (2000)

Neutron Star Properties

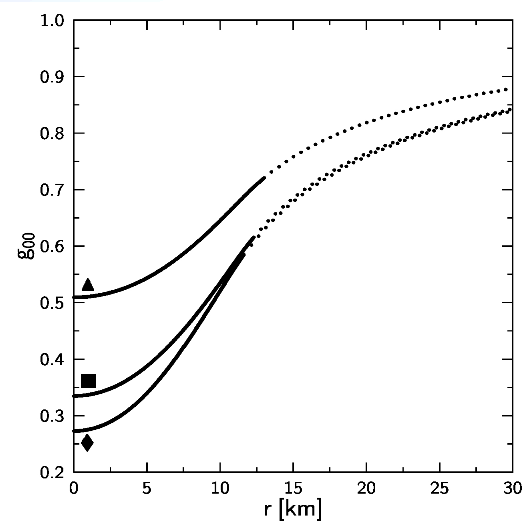
The neutron star radius as a function of its mass. A low, middle and high density star is displayed within the figure. Additionally the onset of hyperonic particles is visualized.



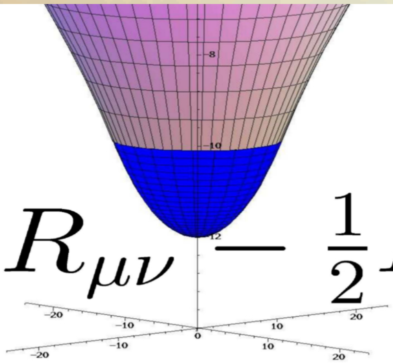
Energy density profiles of three neutron stars with different central densities and masses. The low density stars do not contain any hyperons, whereas the other two stars do have hyperons in their inner core.



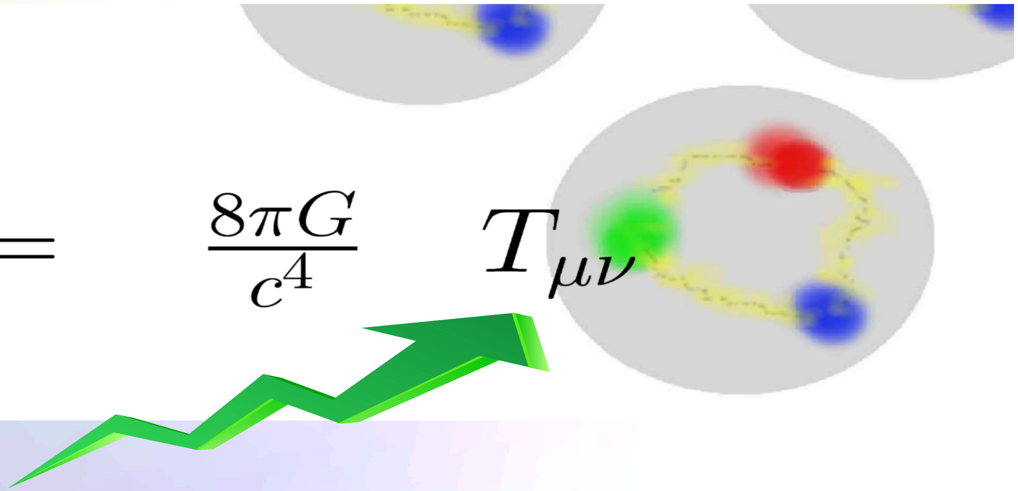
Time-time component of the metric tensor as a function of the radial coordinate. The solid line corresponds to the inner TOV-solution, whereas the doted curve depicts the outer Schwarzschild part.



Hybrid Stars



$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$



Hadronic Model + Quark Model (eg. NJL model or MIT-Bag model)

Lagrangian density of the NJL model

$$\mathcal{L} = \underbrace{\bar{\psi}\,(i\partial\!\!\!/ - \hat{m}_0)\,\psi}_{\text{Kinetische und Massenbeiträge}} \underbrace{+ G_S \sum_{j=0}^{8} \left[\left(\bar{\psi}\, \frac{\lambda_j}{2}\, \psi \right)^2 + \left(\bar{\psi}\, \frac{i\gamma_5 \lambda_j}{2}\, \psi \right)^2 \right]}_{\text{Skalare Wechselwirkung}}$$

$$\underbrace{- G_V \sum_{j=0}^{8} \left[\left(\bar{\psi}\, \gamma_\mu \frac{\lambda_j}{2}\, \psi \right)^2 + \left(\bar{\psi}\, \gamma_\mu \frac{\gamma_5 \lambda_j}{2}\, \psi \right)^2 \right]}_{\text{Vektorielle Wechselwirkung}} \qquad \psi \equiv \psi^f_{Aa}$$

$$\underbrace{- K \left[\det{}_f \left(\bar{\psi}\,(1-\gamma_5)\,\psi \right) + \det{}_f \left(\bar{\psi}\,(1+\gamma_5)\,\psi \right) \right]}_{\text{Flavour Mischterme}} + \underbrace{\mathcal{L}_L}_{\text{Leptonische Beiträge}}$$

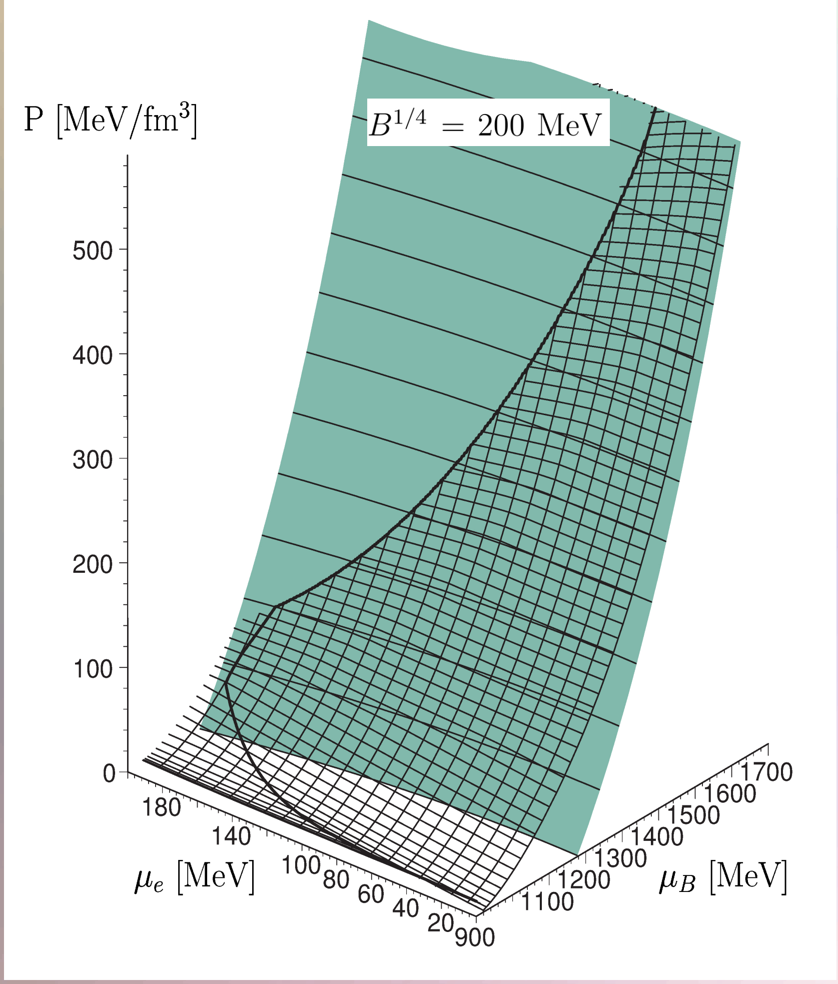
MIT-Bag model

$$\epsilon^Q = \sum_{f=u,d,s} \frac{\nu_f}{2\pi^2} \int_0^{k_F^f} k^2 \sqrt{m_f^2 + k^2}\, dk + B$$

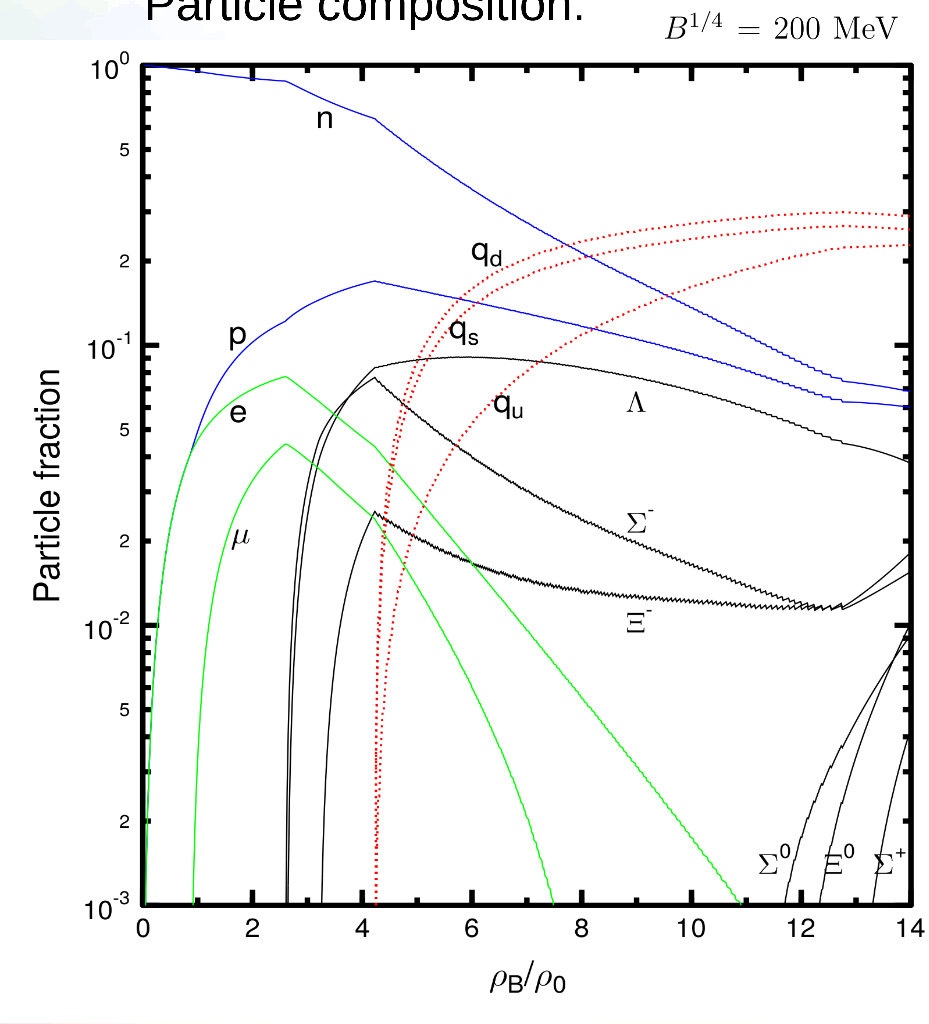
$$P^Q = \sum_{f=u,d,s} \frac{\nu_f}{6\pi^2} \int_0^{k_F^f} \frac{k^4}{\sqrt{m_f^2 + k^2}}\, dk - B\,,$$

The Gibbs Construction

Hadronic and quark surface:



Particle composition:

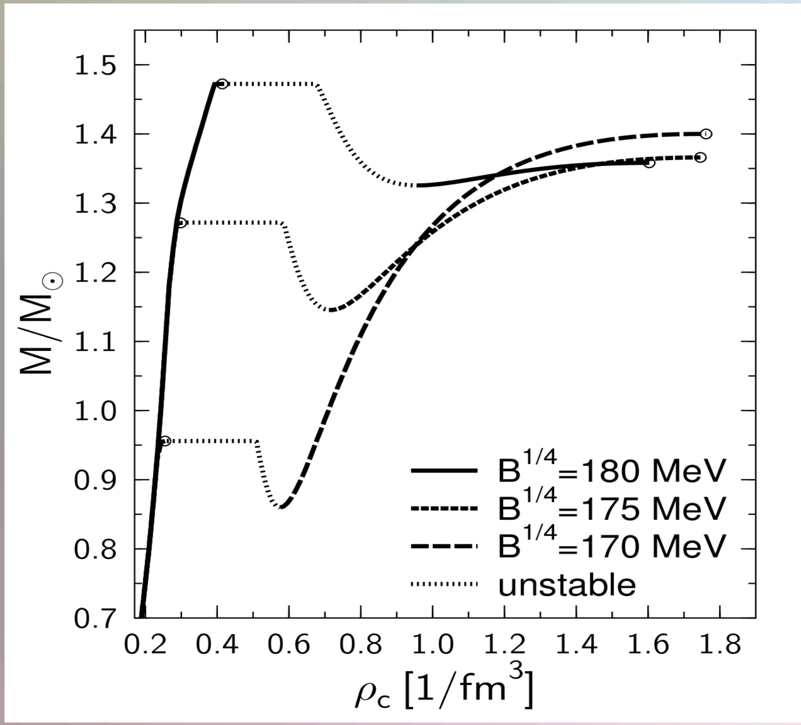


M. Hanauske, Dissertation, "Properties of Compact Stars within QCD-motivated Models"

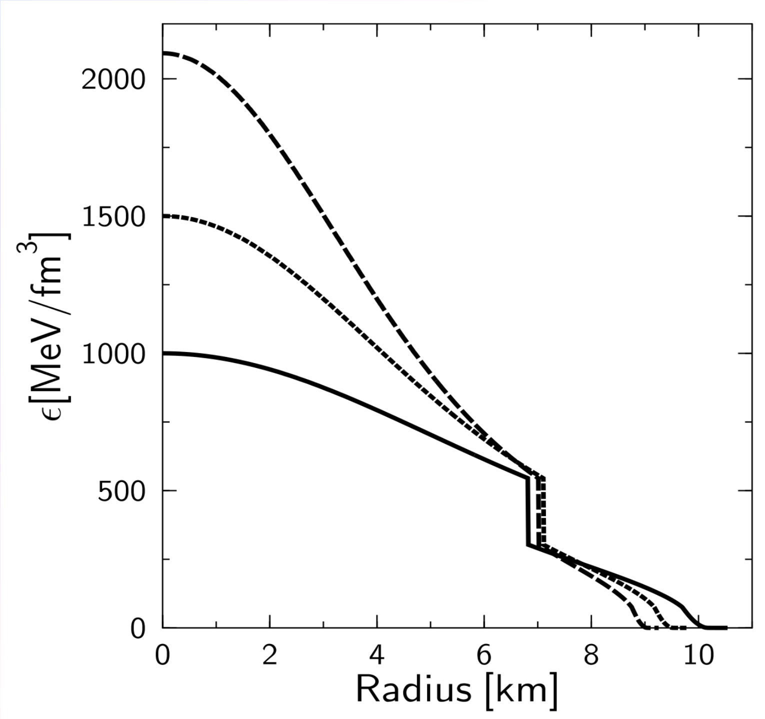
Hybrid Star Properties

In contrast to the Gibbs construction, the star's density profile within the Maxwell construction (see right figure) will have a huge density jump at the phase transition boundary. Twin star properties can be found more easily when using a Maxwell construction.

Mass-Density relation



Energy-density profiles



Matthias Hanauske: "How to detect the QGP with telescopes", GSI Annual Report 2003, p.96

The ADM equations

The ADM (Arnowitt, Deser, Misner) equations come from a reformulation of the Einstein equation using the (3+1) decomposition of spacetime.

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + \mathscr{L}_{\boldsymbol{\beta}} \gamma_{ij} = -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i$$

$$\partial_t K_{ij} = -D_i D_j \alpha + \beta^k \partial_k K_{ij} + K_{ik} \partial_j \beta^k + K_{kj} \partial_i \beta^k + \alpha \left({}^{(3)}R_{ij} + K K_{ij} - 2 K_{ik} K^k{}_j \right) + 4\pi\alpha \left[\gamma_{ij} (S - E) - 2 S_{ij} \right]$$

Time evolving part of ADM

$$D_j (K^{ij} - \gamma^{ij} K) = 8\pi S^i \qquad {}^{(3)}R + K^2 - K_{ij} K^{ij} = 16\pi E$$

Constraints on each hypersurface

Three dimensional covariant derivative

$$D_\nu := \gamma^\mu{}_\nu \nabla_\mu = (\delta^\mu_\nu + n_\nu n^\mu) \nabla_\mu$$

Three dimensional Riemann tensor

$${}^{(3)}R^\mu{}_{\nu\kappa\sigma} = \partial_\kappa {}^{(3)}\Gamma^\mu_{\nu\sigma} - \partial_\sigma {}^{(3)}\Gamma^\mu_{\nu\kappa} + {}^{(3)}\Gamma^\mu_{\lambda\kappa} {}^{(3)}\Gamma^\lambda_{\nu\sigma} - {}^{(3)}\Gamma^\mu_{\lambda\sigma} {}^{(3)}\Gamma^\lambda_{\nu\kappa}$$

$${}^{(3)}\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} \gamma^{\alpha\delta} \left(\partial_\beta \gamma_{\gamma\delta} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma} \right)$$

Spatial and normal projections of the energy-momentum tensor:

$$S_{\mu\nu} := \gamma^\alpha{}_\mu \gamma^\beta{}_\nu T_{\alpha\beta} \,,$$

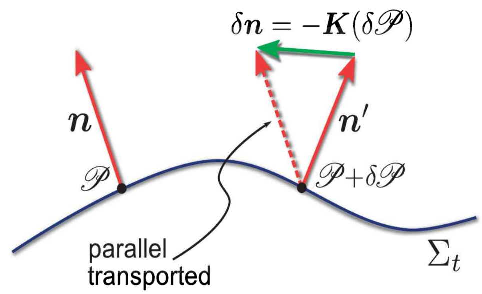
$$S_\mu := -\gamma^\alpha{}_\mu n^\beta T_{\alpha\beta} \,,$$

$$S := S^\mu{}_\mu \,,$$

$$E := n^\alpha n^\beta T_{\alpha\beta} \,,$$

Extrinsic Curvature:

$$K_{\mu\nu} := -\gamma^\lambda{}_\mu \nabla_\lambda n_\nu$$



All figures and equations from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

From ADM to BSSNOK

Unfortunately the ADM equations are only weakly hyperbolic (mixed derivatives in the three dimensional Ricci tensor) and therefore not "well posed". It can be shown that by using a conformal traceless transformation, the ADM equations can be written in a hyperbolic form. This reformulation of the ADM equations is known as the BSSNOK (Baumgarte, Shapiro, Shibata, Nakamuro, Oohara, Kojima) formulation of the Einstein equation. Most of the numerical codes use this (or the CCZ4) formulation.

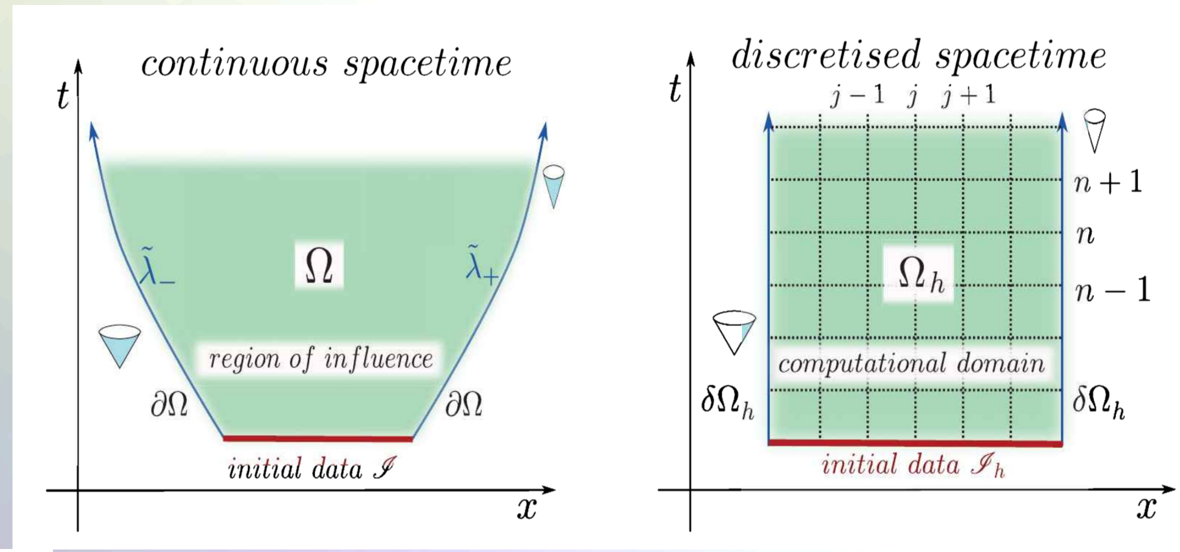
The 3+1 Valencia Formulation of the Relativistic Hydrodynamic Equations

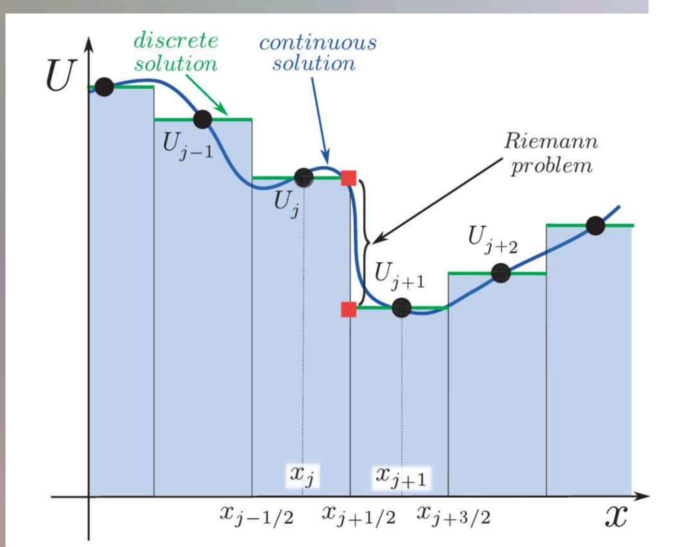
$$\nabla_\mu(\rho u^\mu) = 0\,,$$
$$\nabla_\nu T^{\mu\nu} = 0\,.$$

To guarantee that the numerical solution of the hydrodynamical equations (the conservation of rest mass and energy-momentum) converge to the right solution, they need to be reformulated into a conservative formulation. Most of the numerical “hydro codes” use here the 3+1 Valencia formulation.

Finite difference methods

Discretisation of a hyperbolic initial value boundary problem.





High resolution shock capturing methods (HRSC methods) are needed, when Riemann problems of discontinuous properties and shocks needs to be evolved accurately

All figures from: Luciano Rezzolla, Olindo Zanotti: Relativistic Hydrodynamics, Oxford Univ. Press, Oxford (2013)

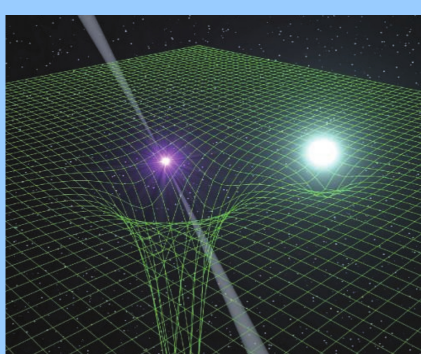
Observed Masses of Compact Star Binaries

PSR J1906+0746 Van Leeuwen et al, arXiv:1411.1518

144-ms pulsar, discovered in in 2004
Orbital period: 3.98 hours, Eccentricity: 0.085
Pulsar mass: 1.291(11), Companion mass 1.322(11)
Observed between 1998-2009,
then it disappeared due to spin precession

Double Pulsar PSR J0737-3039

Orbital period: 147 min, Eccentricity: 0.088
pulsar A: P=23 ms, M=1.3381(7)
pulsar B: P=2.7 s, M=1.2489(7)
Pulsar A is eclipsed once per orbit by B (for 30 s)
Kramer, Wex, Class. Quantum Grav. 2009



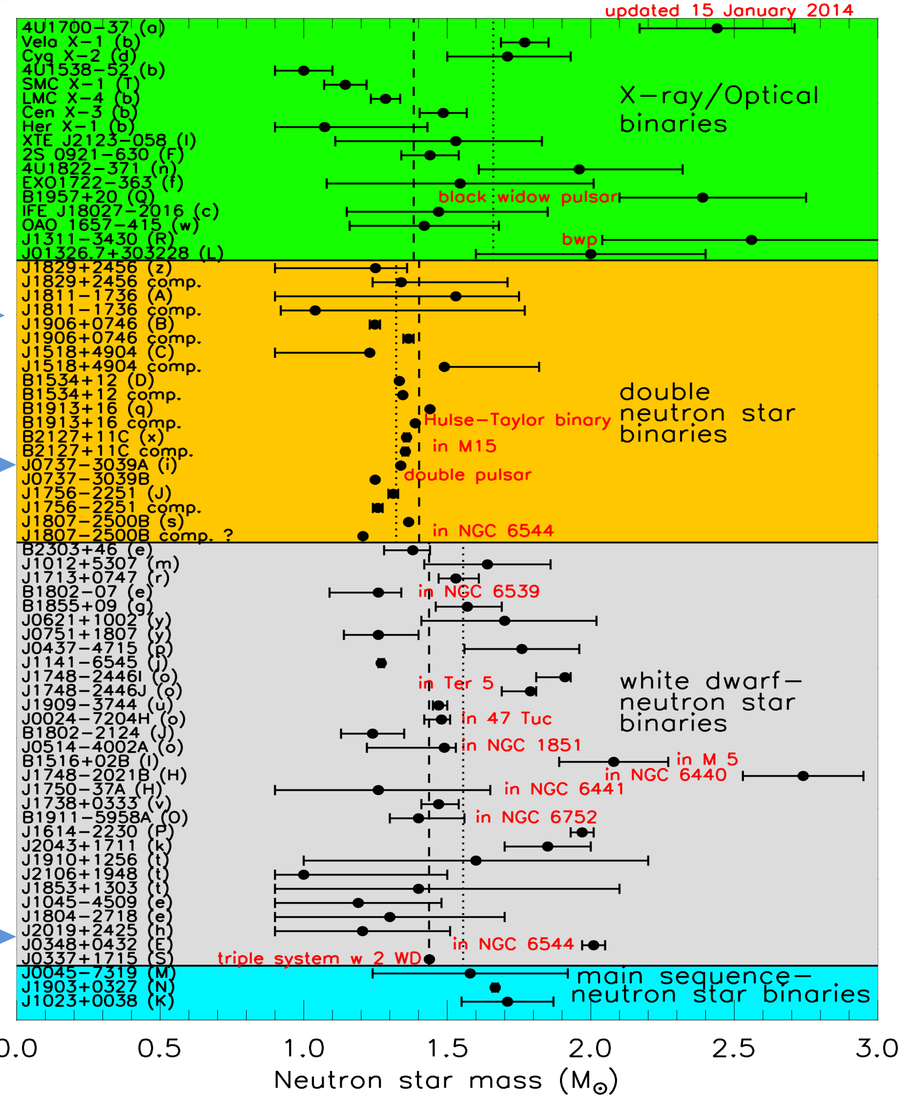
PSR J0348+0432

Orbital Period:
2.46 hours

Pulsar mass:
2.01+-0.04

white dwarf mass:
0.172+-0.003

Picture from J. Antoniadis et.al. Science 2013

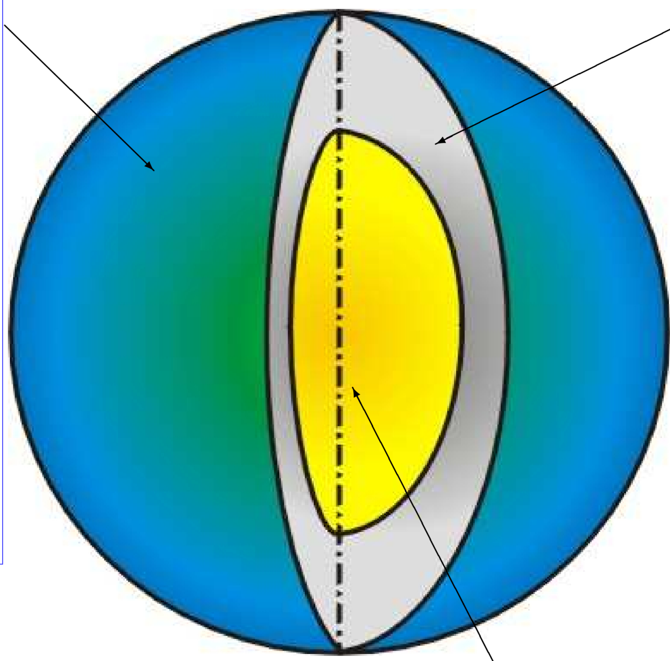


Summary: Neutron Stars

Schematic Structure of a Neutron Star

Outer Envelopes

The outer envelopes of a neutron star constist of a thin plasma atmosphere where the thermal radiation is formed and a outer and inner crust which consist of electrons and nuclei. The whole outer envelope is about one kilometer thick and it occupies the density range $\epsilon \leq 0.5\,\epsilon_0$.



Outer Core

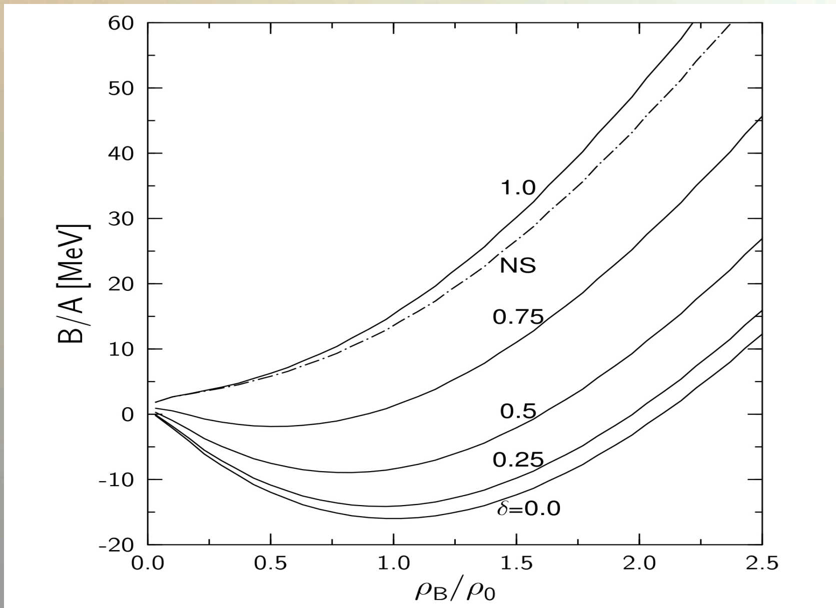
The outer core consists mainly of neutrons with several per cent admixture of protons, electrons and myons. It is several kilometers thick and occupies the density range $0.5\,\epsilon_0 < \epsilon \leq 2\,\epsilon_0$.

Inner Core

In the inner core of a neutron star hyperonic particles $(\Sigma^-, \Lambda, \Xi ...)$ are present. The inner core extends to the center of the star where its central density can be as high as $\epsilon \approx 15\,\epsilon_0$.

Nuclear matter density:
$\epsilon_0 = 2.8 \cdot 10^{14}$ g/cm^3

Neutron Star Matter



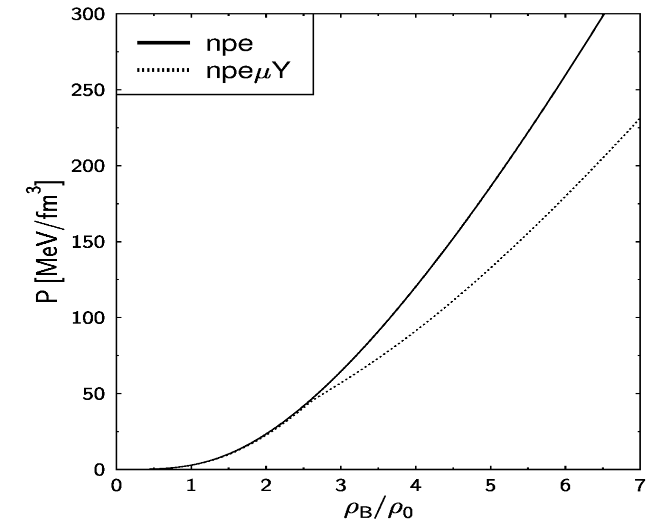
Binding energy per nucleon as a function of the baryonic density for different values of the neutron-proton asymmetry δ. 'NS' describes charge-neutral neutron star matter in β-equilibrium.

In contrast to normal nuclear matter, neutron star matter needs to fulfil three additional conditions:

1) Charge Neutrality
2) β-equilibrium $n \Leftrightarrow p + e + \tilde{\nu}_e$
3) Strangeness production

$$N + N \Rightarrow N + H + M \,,$$

1) The equation of state (pressure P of the hadronic matter vs. the baryonic density). The solid curve (npe) describes charge-neutral matter in β-equilibrium consisting of neutrons, protons and electrons, whereas the dotted curve (npeμY) includes muons and hyperons.



Hybrid Stars

To describe the properties of the Hadron-Quark phase transition happening in Hybrid stars an effective model for the quark phase is needed:

$$\lambda_j \equiv \lambda_{ja}{}^b$$

$$\mathcal{L} = \underbrace{\overline{\psi}\,(i\,\partial\!\!\!/ - \hat{m}_0)\,\psi}_{\text{Kinetische und Massenbeiträge}} + \underbrace{G_S \sum_{j=0}^{8} \left[\left(\overline{\psi}\, \frac{\lambda_j}{2}\, \psi \right)^2 + \left(\overline{\psi}\, \frac{i\gamma_5 \lambda_j}{2}\, \psi \right)^2 \right]}_{\text{Skalare Wechselwirkung}}$$

$$\underbrace{- G_V \sum_{j=0}^{8} \left[\left(\overline{\psi}\, \gamma_\mu \frac{\lambda_j}{2}\, \psi \right)^2 + \left(\overline{\psi}\, \gamma_\mu \frac{\gamma_5 \lambda_j}{2}\, \psi \right)^2 \right]}_{\text{Vektorielle Wechselwirkung}} \qquad \psi \equiv \psi^f_{Aa} \qquad \partial\!\!\!/ \equiv \partial\!\!\!/_A{}^B := \partial_\mu \gamma^\mu{}_A{}^B$$

$$\underbrace{- K \left[\det_f \left(\overline{\psi}\,(1-\gamma_5)\,\psi \right) + \det_f \left(\overline{\psi}\,(1+\gamma_5)\,\psi \right) \right]}_{\text{Flavour Mischterme}} + \underbrace{\mathcal{L}_L}_{\text{Leptonische Beiträge}}$$

Eg. the Nambu-Jona-Lasinio (NJL) Model or the MIT-Bag model

$$\epsilon^Q = \sum_{f=u,d,s} \frac{\nu_f}{2\pi^2} \int_0^{k_F^f} k^2 \sqrt{m_f^2 + k^2}\, dk \; + B$$

$$P^Q = \sum_{f=u,d,s} \frac{\nu_f}{6\pi^2} \int_0^{k_F^f} \frac{k^4}{\sqrt{m_f^2 + k^2}}\, dk \; - B\,,$$

A hybrid model of a compact star is realized by a construction of a phase transition between a hadronic model and a quark model. In contrast to the Maxwell construction of a phase transition, in a Gibbs construction a mixed phase is present in the stars interior, where both phases co-exist. In the mixed phase transition region each phase has a charge; only the overall electrical charge density vanishes. In the mixed phase, the pressure of the hadronic matter has to be equal to the pressure of the quark phase, whereas the particle and energy densities differ.

M. Hanauske, L.M. Satarov, I.N. Mishustin, H. Stöcker, and W. Greiner, Phys. Rev. D 64, 043005 (2001)

The Gibbs Construction

Since the charge neutrality condition is only globally realized, the pressure depends on two independently chemical potentials, the baryonic and charge chemical potential:

$$
\begin{aligned}
P^H(\mu_B, \mu_e) &= P^Q(\mu_B, \mu_e)\,, \\
\mu_B = \mu_B^H &= \mu_B^Q\,, \\
\mu_e = \mu_e^H &= \mu_e^Q
\end{aligned}
$$

Charge density neutrality condition:

$$\rho_e := (1-\chi)\,\rho_e^H(\mu_B, \mu_e) + \chi\,\rho_e^Q(\mu_B, \mu_e) = 0\,.$$

Overall baryonic density:

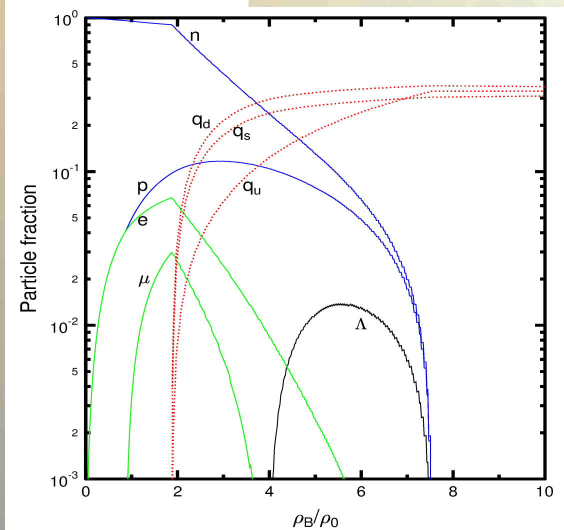
$$\rho_B = (1-\chi)\,\rho_B^H(\mu_B, \mu_e) + \chi\,\rho_B^Q(\mu_B, \mu_e)\,,$$

$$\mu_i = B_i\,\mu_B + Q_i\,\mu_e$$

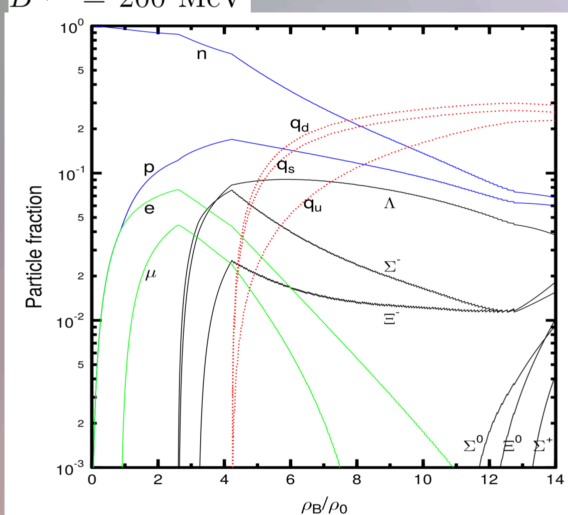
$$
\begin{aligned}
\mu_u &= \frac{1}{3}(\mu_B - 2\,\mu_e) \\
\mu_d = \mu_s &= \frac{1}{3}(\mu_B + \mu_e) \quad .
\end{aligned}
$$

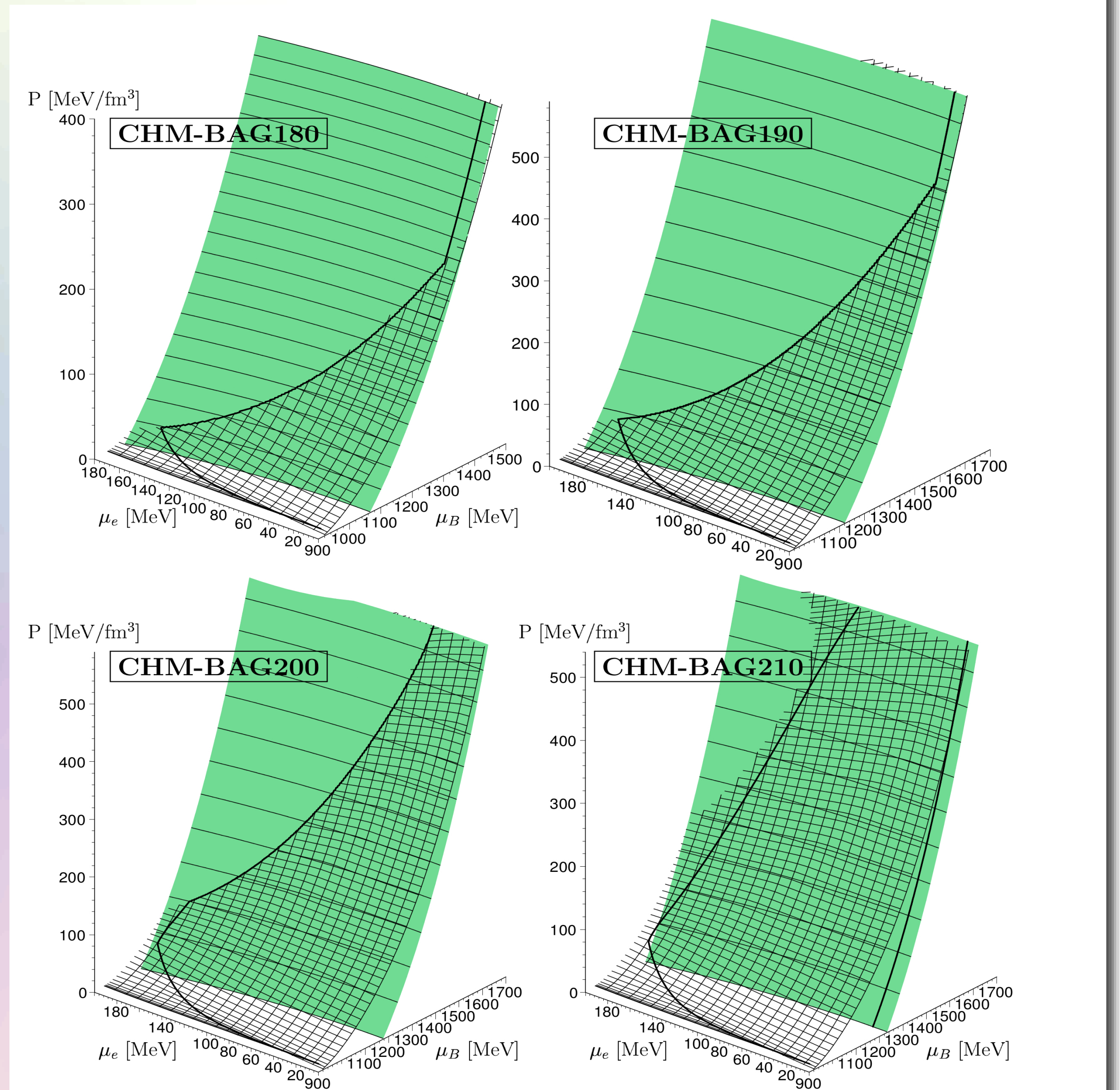
The Gibbs Construction

$B^{1/4} = 180$ MeV



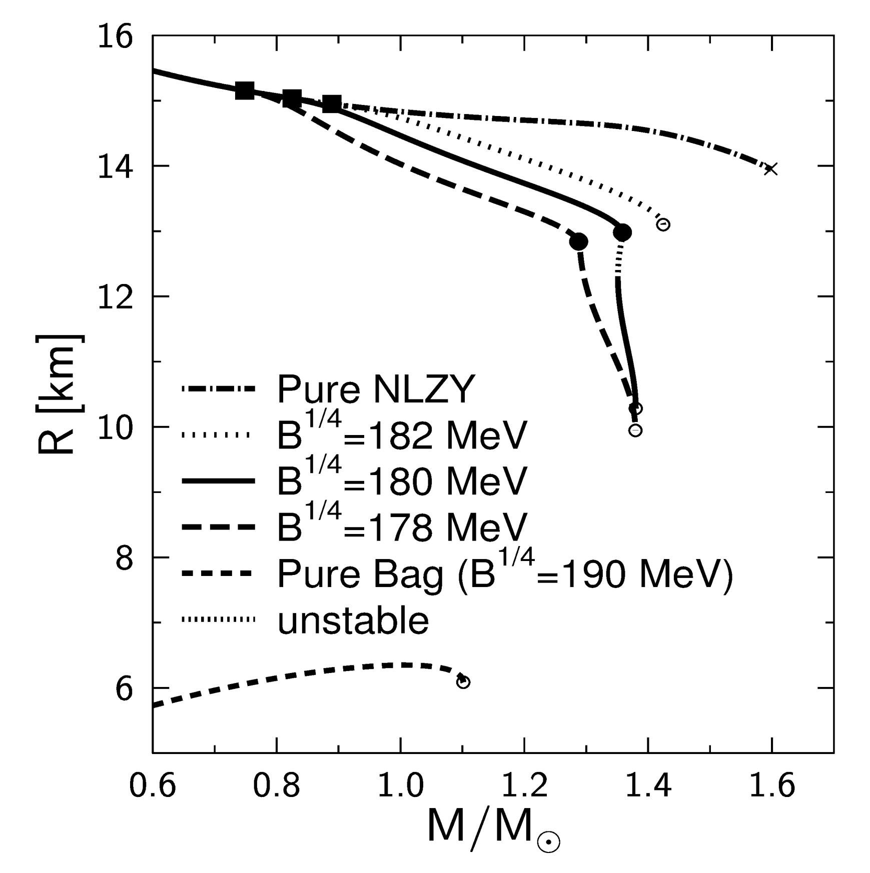
$B^{1/4} = 200$ MeV

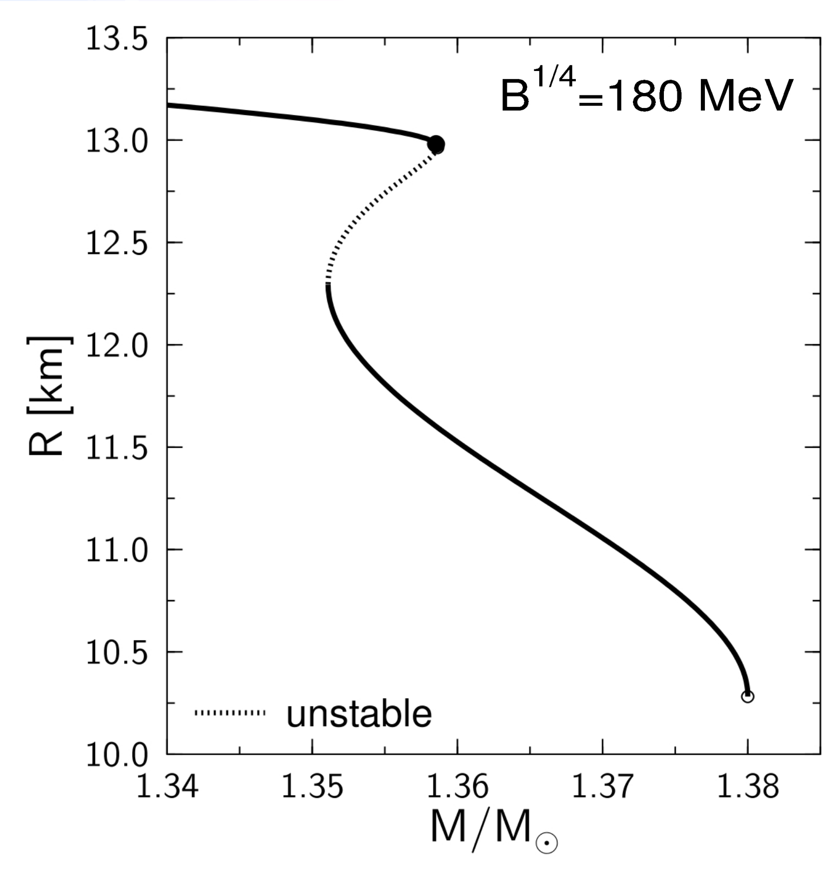




Hybrid Star Properties

Mass-Radius relations within a hybrid star model. The MIT-Bag model, for different values of the Bag parameter, was used for the quark phase, whereas for the hadronic phase the NLZY-model was used.



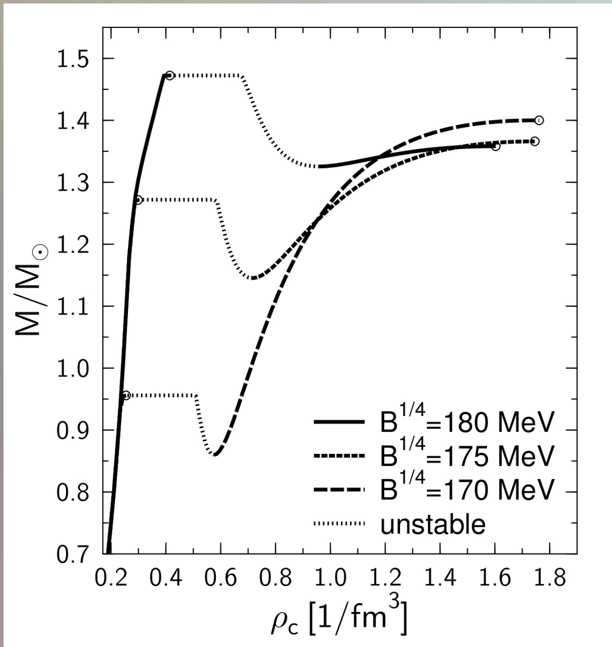


M. Hanauske, Dissertation, "Properties of Compact Stars within QCD-motivated Models"

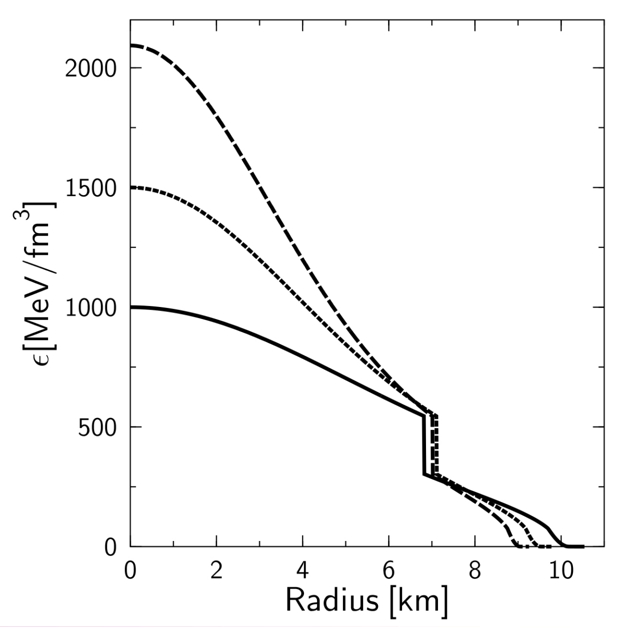
Hybrid Star Properties

In contrast to the Gibbs construction, the star's density profile within the Maxwell construction (see middle figure) will have a huge density jump at the phase transition boundary. Twin star properties can be found more easily when using a Maxwell construction (see left and right figure).

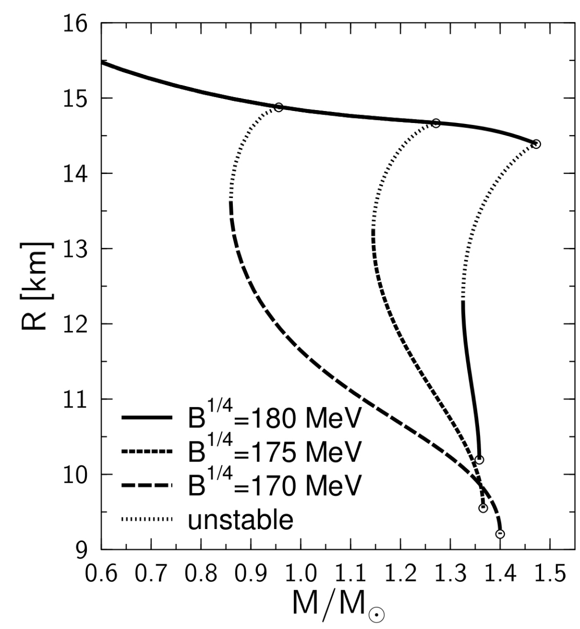
Mass-Density relation



Energy-density profiles



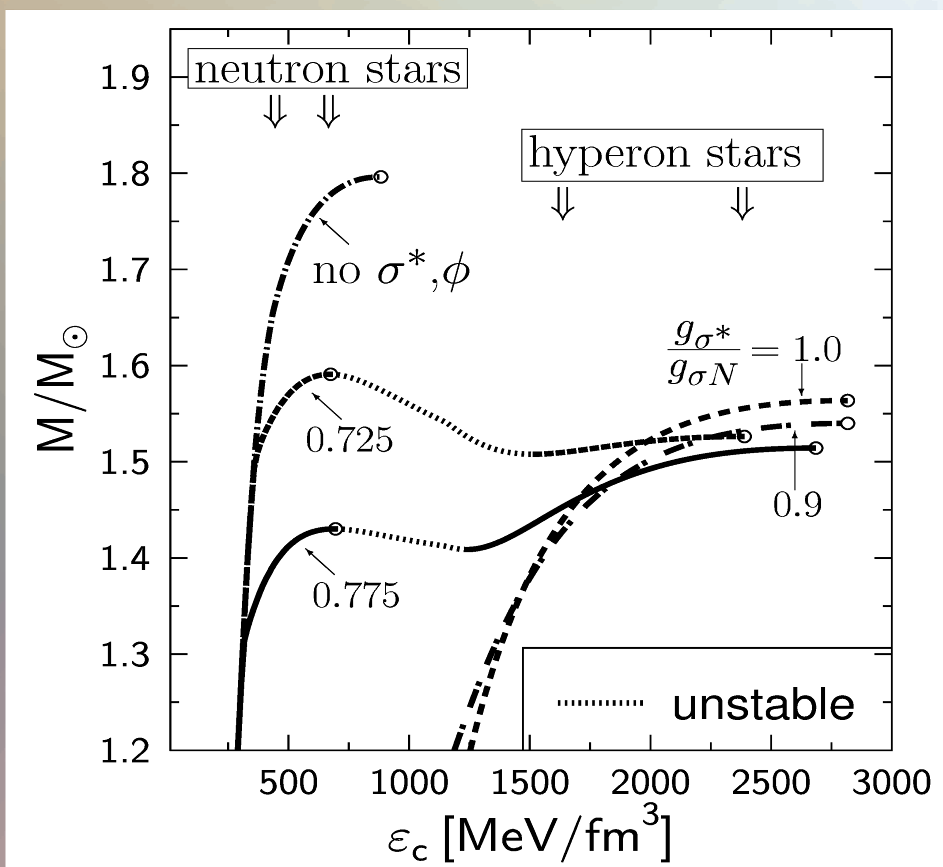
Radius-Mass relation

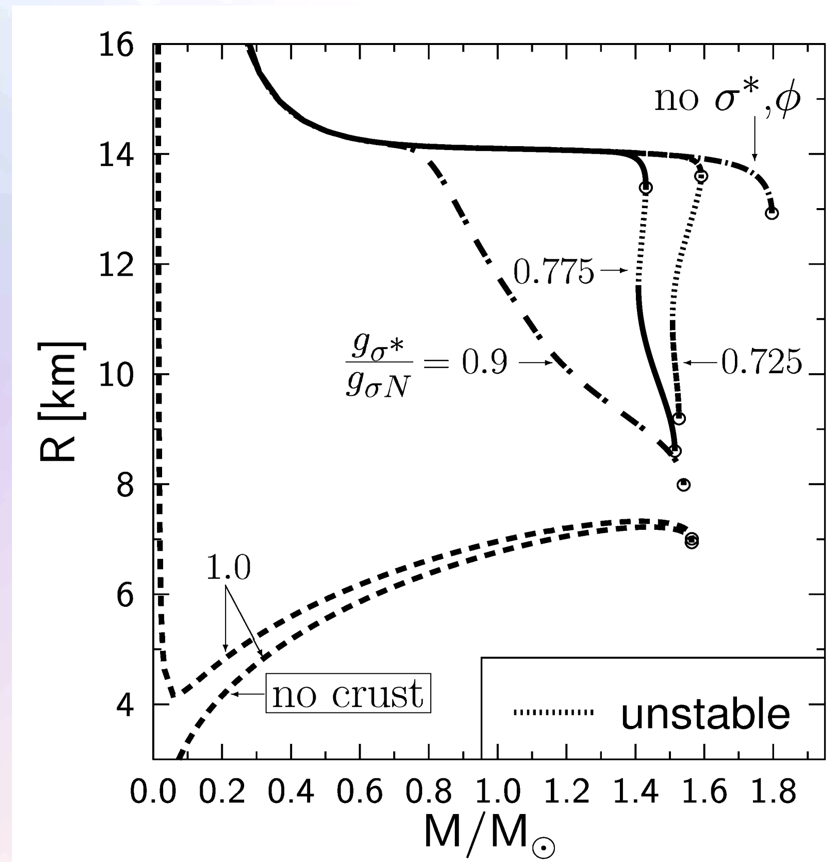


Matthias Hanauske: "How to detect the QGP with telescopes", GSI Annual Report 2003, p.96

Exotic Stars

But, unfortunately, twin stars can not be created solely by a Hadron-Quark phase transition. Extremely bound hyperon mater, or kaon condensation could also form a twin star behaviour.

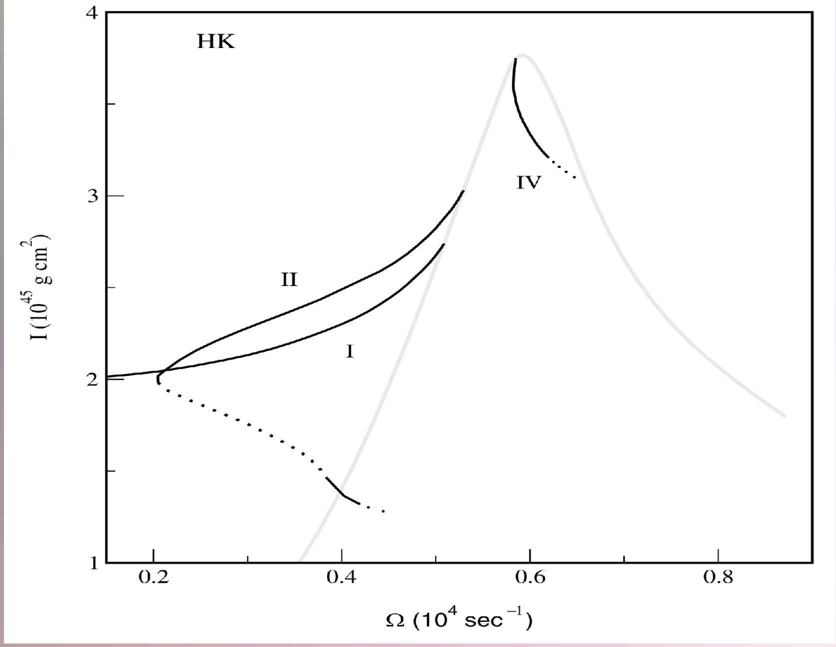


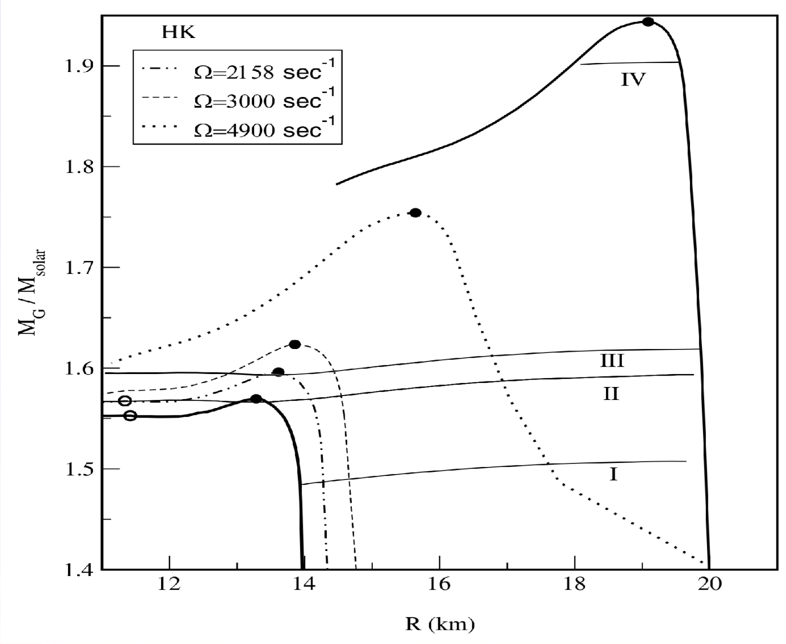


J. Schaffner-Bielich, M. Hanauske, H. Stöcker, and W. Greiner, Phys. Rev. Lett. 89, 171101 (2002)

The Spin Up Effect

A rotating neutron star slowly loses its energy and angular momentum through electromagnetic and gravitational radiation with time. However, it conserves the total baryon number during this evolution. The Figures below show results of uniformly rotating compact stars including a Bose-Einstein condensates of antikaons. The Figure on the right shows the behavior of angular velocity with angular momentum. The mass shedding limit sequence is shown by a light solid line. The stable parts of the normal and supramassive sequences are displayed by dark solid lines and the unstable parts by dotted lines. Curve II indicates a collapse of a neutron star to an exotic star belonging to the third family of compact stars.





S. Banik, M. Hanauske, D. Bandyopadhyay, and W. Greiner, Phys. Rev. D 70, 123004 (2004)

Gauge Conditions

On each spatial hypersurface, four additional degrees of freedom need to be specified: A *slicing condition* for the lapse function and a *spatial shift condition* for the shift vector need to be formulated to close the system. In an optimal gauge condition, singularities should be avoided and numerical calculations should be less time consuming.

Bona-Massó family of slicing conditions:

$$\partial_t \alpha - \beta^k \partial_k \alpha = -f(\alpha)\, \alpha^2 (K - K_0)$$

"1+log" slicing condition: $f = 2/\alpha$

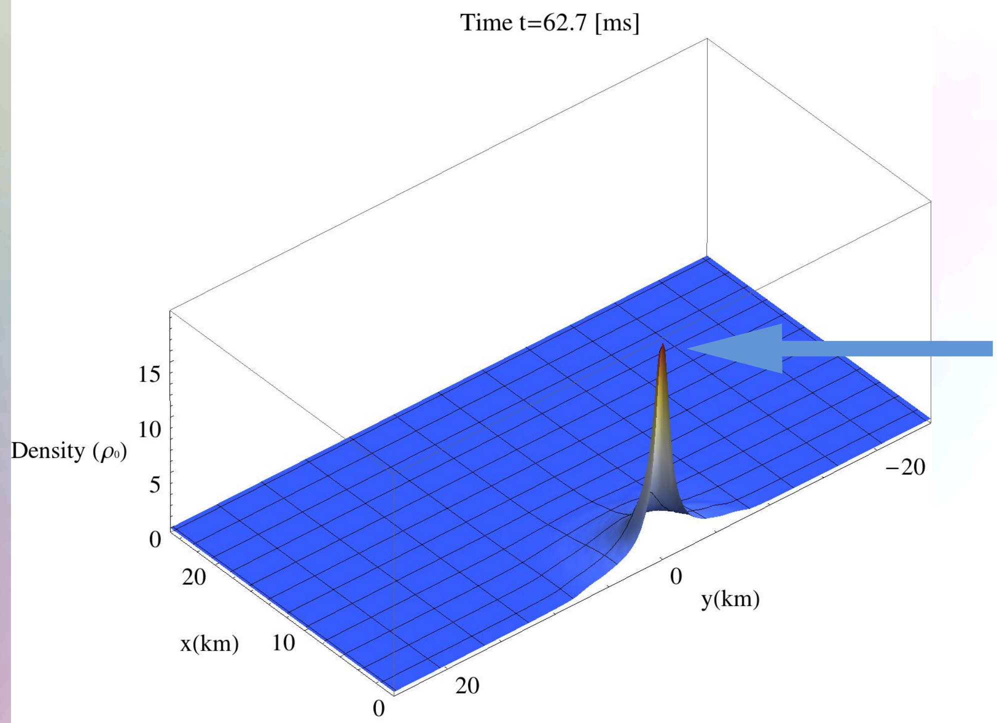
where $f(\alpha) > 0$ and $K_0 := K(t = 0)$

"Gamma-Driver" shift condition:

$$\partial_t \beta^i - \beta^j \partial_j \beta^i = \frac{3}{4} B^i,$$

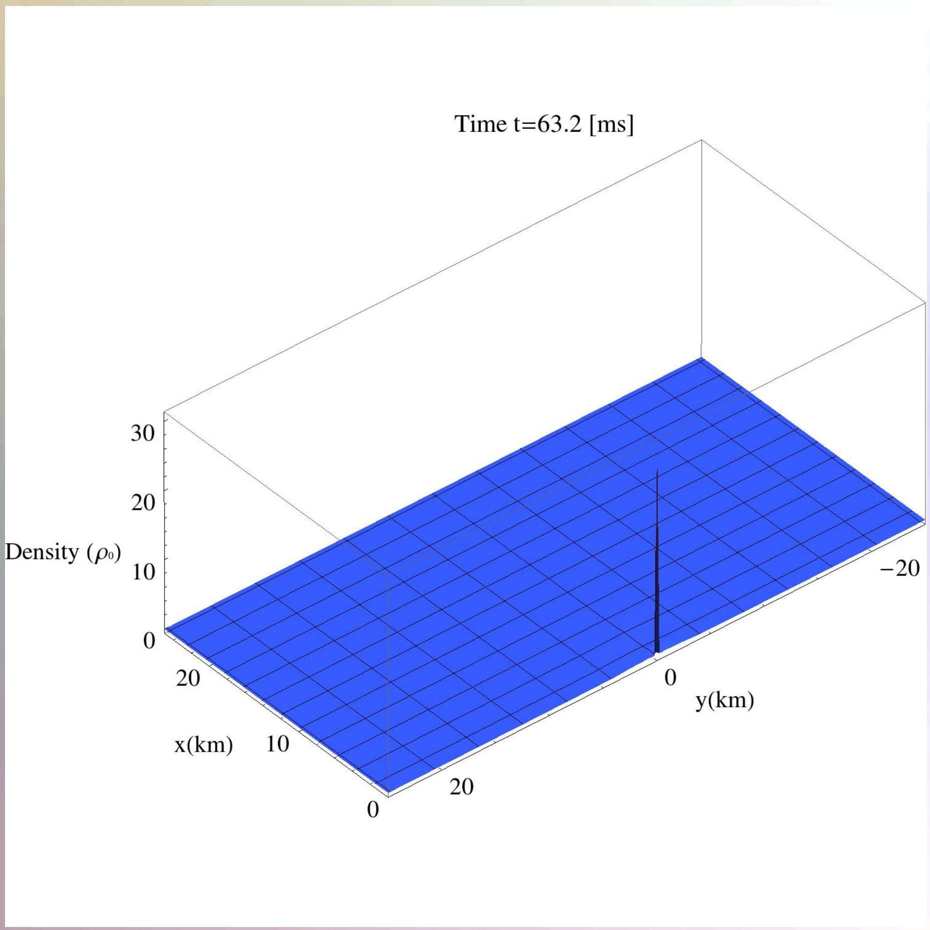
$$\partial_t B^i - \beta^j \partial_j B^i = \partial_t \tilde{\Gamma}^i - \beta^j \partial_j \tilde{\Gamma}^i - \eta B^i$$

Formation of the Black Hole



During the formation of the black hole, the star is mainly composed of quark matter

Formation of the Black Hole



The formation of the apparent and event horizon of the black hole confines the quark star macroscopically. Finally the colour charge of the deconfined, free quarks cannot be observed from outside.

Simulations done by Kentaro Takami