Evolutionary Quantum Game Theory and Scientific Communication

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(Dated: 11th May 2010)

Quantum game theory is a mathematical and conceptual amplification of classical game theory. The space of all conceivable decision paths is extended from the purely rational, measurable space in the Hilbertspace of complex numbers. Trough the concept of a potential entanglement of the imaginary quantum strategy parts, it is possible to include corporate decision path, caused by cultural or moral standards. If this strategy entanglement is large enough, then, additional Nash-equilibria can occur, previously present dominant strategies could become nonexistent and new evolutionary stable strategies can appear. This article focuses on a quantum amplification of an evolutionary (2 player)-(2 strategy) - coordination game and shows, that the different publication norms of scientific authors additionally depend on their strategic entanglement strength γ. If the strength of entanglement exceeds a certain value, a phase transition within the whole population occurs, reaching the global optimum of the underlying coordination game.

I. INTRODUCTION

In 1928 the main inventor of game theory - Johann (John) von Neumann - published the first article on game theory [38]. The first book about game theory was published in 1944 by von Neumann and Morgenstern [37]. Evolutionary game theory [1, 12, 20, 29, 29, 32–35] was developed after J.M. Smith had found that the stationary solutions of the evolutionary differential equations are connected with game theory [31]. In the following years applications in respect to biological systems [4, 17, 22, 23, 30, 36] and socio-economic systems, e.g. "public good"-games [6], cultural or moral developments [10, 16], the evolution of languages [26], social learning [10], the evolution of social norms [2, 24], the financial crisis [15] and the evolution of social networks [9, 18, 35] came into the focus of research. In 1999 the first two articles on quantum games where published [8, 19]. In 2001 the first quantum game was realized on a quantum computer [7] (see also [28]). The extension to more than two players [3], the application to social networks [13, 27], social experiments [5, 14, 25] and first approaches towards an evolutionary quantum game theory [11, 15, 21][39] followed.

This article focuses on a simplified version of the open access game of scientific communication (for detail see [13]) and extends it to an evolutionary quantum coordination game. The payoff structure of the underlying game can be described by the payoff matrix illustrated in Table I.

A\B	o=s_1^B	ø=s_2^B
o=s_1^A	$(r+\delta, r+\delta)$	$(r-\alpha, r+\alpha)$
ø=s_2^A	$(r+\alpha, r-\alpha)$	(r, r)

Table I: Payoff of the underlying coordination game.

The players are the authors of scientific articles, the two strategies represent the authors' choice between publishing open access (o) or not (ø), parameter r describes the increase of reputation achieved by an author, if he/she publishes a new paper, δ is an additional benefit if both authors publish open access and the parameter α ($\alpha < \delta$) is responsible for the potential increase or decrease of reputation if one author publishes open access and the other not (for detail see [13]).

II. DEFINITIONS AND KEY ASPECTS OF CLASSICAL EVOLUTIONARY GAME THEORY

This section is dedicated to the introduction of the necessary definitions and fundamental basics of evolutionary game theory. In the following the presentation is constrained to the normal form of a symmetric (2 player)-(2 strategy) game Γ (for details see [12, 35]):

$$\begin{aligned} \Gamma &:= \left(\{A,B\}, \mathcal{S}\times\mathcal{S}, \hat{\$}_A, \hat{\$}_B \equiv \left(\hat{\$}^A\right)^T \right) \qquad (1)\\ \mathcal{S} &= \{s_1, s_2\} \quad \text{: Set of pure strategies}\\ \hat{\$}_A &= \begin{pmatrix} \$_{11} & \$_{12} \\ \$_{21} & \$_{22} \end{pmatrix} \quad \text{: Payoff matrix of Player A} \end{aligned}$$

The mixed strategy payoff function of player A has the following structure[40]

$$\tilde{\$}^A : \tilde{\mathcal{S}} \times \tilde{\mathcal{S}} \to \mathbb{R}\ , \quad \tilde{\$}^A(\tilde{s}_1^A, \tilde{s}_1^B) = \$_{11}\tilde{s}_1^A\tilde{s}_1^B + \qquad (2)$$
$$+\$_{12}\tilde{s}_1^A(1-\tilde{s}_1^B) + \$_{21}(1-\tilde{s}_1^A)\tilde{s}_1^B + \$_{22}(1-\tilde{s}_1^A)(1-\tilde{s}_1^B)$$

, where $\tilde{s}_1^A, \tilde{s}_1^B \in [0,1]$ and $\tilde{s}_2^A = 1-\tilde{s}_1^A$, $\tilde{s}_2^B = 1-\tilde{s}_1^B$. Inserting the payoff matrix of the coordination game of Table I into equation 2 yields to the following structure of the mixed strategy payoff function ($\tilde{s}^A := \tilde{s}_1^A$ and $\tilde{s}^B := \tilde{s}_1^B$):

$$\tilde{\$}^A(\tilde{s}^A, \tilde{s}^B) = \delta\, \tilde{s}^A\tilde{s}^B + \alpha\left(\tilde{s}^B - \tilde{s}^A\right) + r \qquad (3)$$

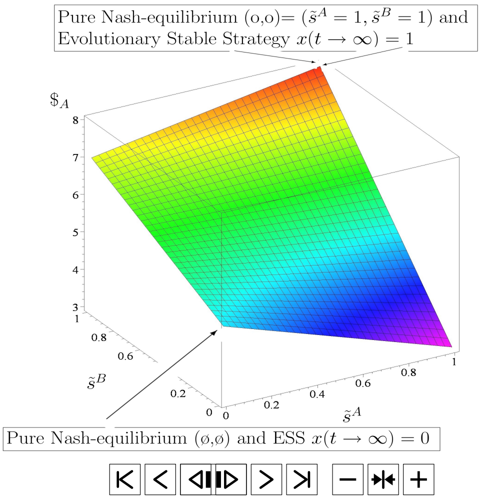

Figure 1: Payoff function in mixed strategies.

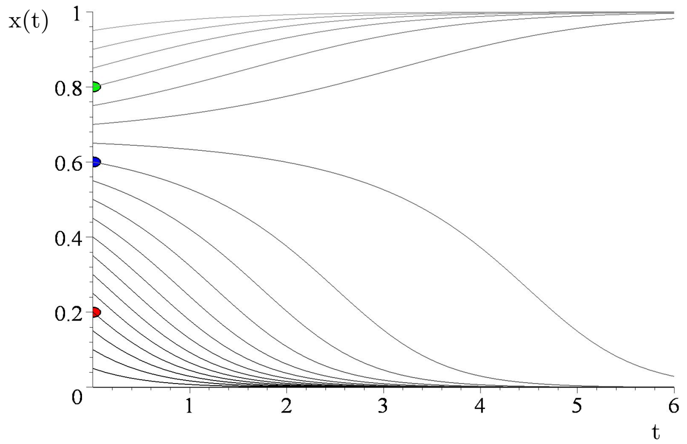

Figure 2: Fraction of players choosing strategy s_1 =o as a function of time ($x(t)$) for different starting values $x(t = 0)$. Results were calculated using the payoff matrix of Table I and the parameter set $r = 5, \alpha = 2$ and $\delta = 3$.

The coordination game of Table I has two symmetric, pure Nash-equilibria ((o,o)= $(s_1^A, s_1^B) = (\tilde{s}^A = 1, \tilde{s}^B = 1)$ and (ø,ø)= $(s_2^A, s_2^B) = (\tilde{s}^A = 0, \tilde{s}^B = 0)$) and one symmetric mixed strategy Nash-equilibrium $(s^{A*}, s^{B*}) = (\tilde{s}^A = \frac{\alpha}{\delta}, \tilde{s}^B = \frac{\alpha}{\delta})$. (s^{A*}, s^{B*}) can be calculated using the fact that the partial derivative of the mixed strategy payoff function of player A vanishes at the value of the mixed strategy Nash-equilibrium:

$$\left. \frac{\partial \tilde{\$}^A(\tilde{s}^A, \tilde{s}^B)}{\partial \tilde{s}^A} \right|_{\tilde{s}^B = s^{B*}} = \delta s^{B*} - \alpha = 0 \;\Rightarrow\; s^{B*} = \frac{\alpha}{\delta} \quad (4)$$

The three Nash-equilibria of the underlying coordination game (Table I, with $r = 5, \delta = 3$ and $\alpha = 2$) can be visualized (see Figure 1) by plotting the payoff of player A as a function of the mixed strategy of player A ($\tilde{s}^A$) and player B ($\tilde{s}^B$). The pure Nash-equilibrium (s_1^A, s_1^B) is actually present, because if one fixes the strategy of player B to $s_1^B = (\tilde{s}^B = 1)$ then the highest point on the payoff-surface for player A is realized, if he/she chooses s_1^A. The other pure Nash-equilibrium can be visualized the same way by fixing the strategy of player B to $s_2^B = (\tilde{s}^B = 0)$. The mathematical property of the mixed strategy Nash-equilibrium (equation 4) is visualized in Figure 1 by a transformation of the figures' viewpoint, as the three dimensional payoff surface shrinks to one point at s^{B*}, if one looks in direction of the $\tilde{s}^A$-axis.[41] To describe the time evolution of the repeated version of the game Γ, replicator dynamics were developed. Replicator dynamics, formulated within a system of differential equations, defines in which way the population vector $\vec{x} = (x_1, x_2)$ evolves in time. Each component $x_i = x_i(t)$ $(i = 1, 2)$ describes the time evolution of the fraction of different player types i in the whole population, where a type-i player is understood as an actor playing strategy s_i. The population vector has to fulfill the following conditions:

$$x_i(t) \geq 0 \quad \forall\, i = 1, 2\,,\; t \in \mathbb{R} \text{ and } \sum_{i=1}^{2} x_i(t) = 1 \qquad (5)$$

Because of condition 5, the population vector $\vec{x}(t) = (x_1(t), x_2(t))$ can be reduced to only one independent component ($x(t) := x_1(t)$, and $x_2(t) = 1 - x(t)$) and the replicator equation simplifies as follows:

$$\begin{aligned} \frac{dx}{dt} &= x\left[(\$_{11} - \$_{21})(x - x^2) + (\$_{12} - \$_{22})(1 - 2x + x^2)\right] \\ &= x\left[(\delta + \alpha)x - \delta x^2 - \alpha\right] := g(x) \qquad (6) \end{aligned}$$

Figure 2 visualizes the time evolution of the population fraction $x(t)$ for several different starting values ($x_o := x(t = 0)$). The two symmetric, pure Nash-equilibria are the two evolutionary stable strategies (ESSs). Which of these ESSs is developed, depends on the value of the initial condition x_o. If x_o is above the value of the mixed strategy Nash-equilibrium ($x_o > \frac{\alpha}{\delta}$), the population will evolve to a community choosing solely the strategy s_1 ($x = \tilde{s}^A = \tilde{s}^B = 1$), whereas if $x_o < \frac{\alpha}{\delta}$ the population will asymptotical reach $x = 0$, which means that every player will choose strategy s_2 =ø. In respect to the application under focus, the results of the classical evolutionary game indicate, that if a scientific community has a traditional publication norm (e.g. almost all of the scientists do not use open access repositories) it is not possible to overcome the dilemma of the game, and the population remains in the ESS with the lower payoff.

III. EVOLUTIONARY QUANTUM GAME THEORY

In quantum game theory, the measurable pure classical strategies (s_1 and s_2) correspond to the orthonormal unit basis vectors $|s_1\rangle$ and $|s_2\rangle$ of the two dimensional complex space $\mathbb{C}^2$, the so called Hilbert space $\mathcal{H}_i$ of player i $(i = A, B)$. A quantum strategy of a player i is represented as a general unit vector $|\psi\rangle_i$ in his/her strategic Hilbert space $\mathcal{H}_i$. The whole quantum strategy space $\mathcal{H}$ is constructed with the use of the direct tensor product of the individual Hilbert spaces: $\mathcal{H} := \mathcal{H}_A \otimes \mathcal{H}_B$. The main difference between classical and quantum game theory is that in Hilbert space $\mathcal{H}$ correlations between the players' individual quantum strategies are allowed, if the two quantum strategies $|\psi\rangle_A \in \mathcal{H}_A$ and $|\psi\rangle_B \in \mathcal{H}_B$ are entangled. The overall state of the system we are looking at is described as a 2-player quantum state $|\Psi\rangle \in \mathcal{H}$. The four basis vectors of the Hilbert space $\mathcal{H}$ are defined as the classical game outcomes ($|s_1 s_1\rangle := (1,0,0,0)$, $|s_1 s_2\rangle := (0,-1,0,0)$, $|s_2 s_1\rangle := (0,0,-1,0)$ and $|s_2 s_2\rangle := (0,0,0,1)$). The setup of the quantum game begins with the choice of the initial state $|\Psi_0\rangle$. We assume that both players are in the state $|s_1\rangle$. The initial state of the two players is given by $|\Psi_0\rangle = \widehat{\mathcal{J}}\,|s_1 s_1\rangle$, where the unitary operator $\widehat{\mathcal{J}}$ is responsible for the possible entanglement of the 2-player system (for details see [8, 13, 15]). The players' quantum decision (quantum strategy) is formulated with the use of a two parameter set of unitary 2×2 matrices:

$$\widehat{\mathcal{U}}(\theta,\varphi) := \begin{pmatrix} e^{i\,\varphi}\cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\,\varphi}\cos(\frac{\theta}{2}) \end{pmatrix} \quad (7)$$
$$\forall \quad \theta \in [0,\pi] \;\wedge\; \varphi \in [0,\tfrac{\pi}{2}] \qquad .$$

By arranging the parameters θ and φ, a player chooses his quantum strategy. The classical strategy s_1 is selected by appointing $\theta = 0$ and $\varphi = 0$ ($\widehat{s_1} := \hat{\mathcal{U}}(0,0)$), whereas the strategy s_2 is selected by choosing $\theta = \pi$ and $\varphi = 0$ ($\widehat{s_2} := \hat{\mathcal{U}}(\pi,0)$); in addition, the quantum strategy $\widehat{Q}$ is given by $\widehat{Q} := \hat{\mathcal{U}}(0,\pi/2)$. After the two players have chosen their individual quantum strategies ($\hat{\mathcal{U}}_A := \hat{\mathcal{U}}(\theta_A,\varphi_A)$ and $\hat{\mathcal{U}}_B := \hat{\mathcal{U}}(\theta_B,\varphi_B)$) the disentangling operator $\widehat{\mathcal{J}}^\dagger$ is acting to prepare the measurement of the players' state. The entangling and disentangling operator ($\widehat{\mathcal{J}}, \widehat{\mathcal{J}}^\dagger$; with $\hat{\mathcal{J}} \equiv \hat{\mathcal{J}}^\dagger$) depends on one additional single parameter $\gamma \in [0,\pi/2]$ which is a measure of the strength of the entanglement of the system. Finally, the state prior to detection can therefore be formulated as follows:

$$|\Psi_f\rangle = \hat{\mathcal{J}}^\dagger \left(\hat{\mathcal{U}}_A \otimes \hat{\mathcal{U}}_B\right) \hat{\mathcal{J}}\;|s_1 s_1\rangle \quad (8)$$

The expected payoff within a quantum version of a general 2-player game - which is an amplification of equation 2 - depends on the payoff matrix (see Table I) and on the joint probability to observe the four observable outcomes $P_{s_1 s_1}, P_{s_1 s_2}, P_{s_2 s_1}$ and $P_{s_2 s_2}$ of the game

$$\begin{aligned} \$_A &= \$_{11}\,P_{s_1 s_1} + \$_{12}\,P_{s_1 s_2} + \$_{21}\,P_{s_2 s_1} + \$_{22}\,P_{s_2 s_2} \\ \$_B &= \$_{11}\,P_{s_1 s_1} + \$_{21}\,P_{s_2 s_1} + \$_{12}\,P_{s_1 s_2} + \$_{22}\,P_{s_2 s_2} \\ \text{with:}\;\; P_{\sigma\sigma'} &= |\,\langle\sigma\sigma'|\Psi_f\rangle\,|^2 \;\;,\quad \sigma,\sigma' = \{s_1, s_2\} \quad . \quad (9) \end{aligned}$$

To visualize the payoffs in a three dimensional diagram it is necessary to reduce the set of parameters in the final state: $|\Psi_f\rangle = |\Psi_f(\theta_A,\varphi_A,\theta_B,\varphi_B)\rangle \to |\Psi_f(\tau_A,\tau_B)\rangle$. Within the following diagram, the same specific parameterization as Eisert et al. [8] was used, where the two strategy angles θ and φ depend only on a single parameter $\tau \in [-1,1]$.[42] Positive τ-values represent pure and mixed classical strategies, whereas negative τ-values correspond to quantum strategies, where $\theta = 0$ and $\varphi > 0$. The whole strategy space is separated into four regions, namely the absolute classical region (CC: $\tau_A, \tau_B \geq 0$), the absolute quantum region (QQ: $\tau_A, \tau_B < 0$) and the two partially classical-quantum regions (CQ: $\tau_A \geq 0 \wedge \tau_B < 0$ and QC: $\tau_A < 0 \wedge \tau_B \geq 0$). Fig. 3 depicts the expected payoff for scientist A ($\$_A$, intransparent surface) and scientist B ($\$_B$, wired surface) as a function of their strategies τ_A and τ_B in a separable quantum game ($\gamma = 0$). The outcome of this separable quantum game is similar to the classical solution outlined in section II. The animation in Figure 3 illustrates the change in the payoff surface, if one allows the strategic entanglement of the players to increase. For even tiny values of entanglement a new quantum Nash-equilibrium and additional ESS appears, for $\gamma > \frac{\pi}{4}$ the pure Nash-equilibrium (ø,ø) dissolves and the pure Nash-equilibrium (o,o) becomes the only observable ESS of the underlying game. A scientific community using a traditional publication norm can therefore overcome the dilemma of the game, if the strength of entanglement exceeds $\frac{\pi}{4}$. In such a case a spontaneous phase transition will occur reaching the global optimum of the underlying game.[43]

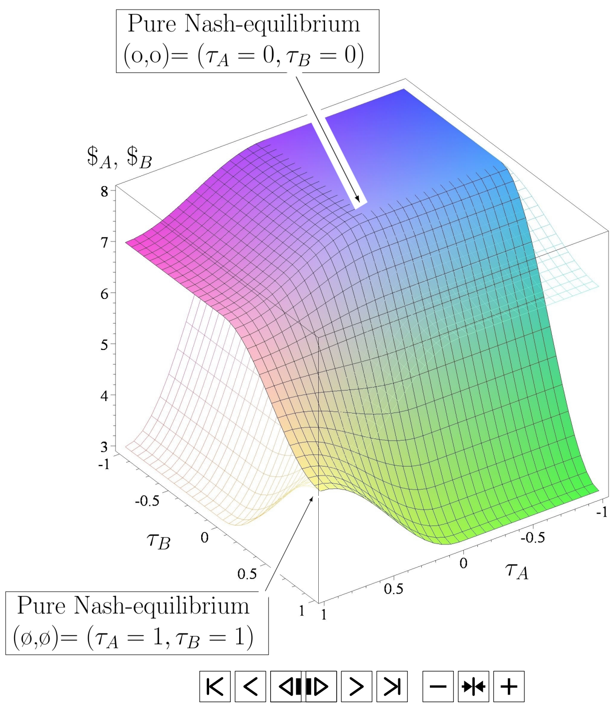

Figure 3: Payoff surface of player A (solid) and player B (wired) as a function of their strategies τ_A and τ_B.

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[39] ... - which is mathematically, most likely formulated with the use of the *von Neumann equation* -

[40] The mixed strategy payoff function of player B can be constructed simply by interchanging the indices ^{A} and ^{B}.

[41] The animations within this article are only viewable within its electronic version.

[42] The parameter τ corresponds to parameter t of [8].

[43] The electronic version of this article includes several dynamic animations. The LaTex-source files and the underlying Maple-worksheeds of all the calculations performed, are freely downloadable on the following internet page http://evolution.wiwi.uni-frankfurt.de/BWGT2010/.